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DISCUSSION PAPER

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DISCUSSION PAPER

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CONTENTS

ABSTRACT

Real estate markets are subject to dynamic, ever-changing influences from location, amenities and neighborhoods; regulation, zoning and population changes; but also – macroeconomic variables, such as interest rates, inflation and economic cycles. The decision to buy or rent a durable, financially significant asset is a difficult one for both experts and amateurs alike. This paper provides a review of classic urban economic fundamentals and a more recent financial and macroeconomic analysis of the literature. There is consensus that a complex, long-term number of factors apply to the market and no single model can encompass all uncertainties revolving around the decision-making. Aiming at offering a practical tool, we propose a numerical simulation that takes the parameter space of renting versus purchasing homes to probabilistic estimate which choice would be more advantageous, more often. We analyze four concrete cases using Brazilian market sensible parameters. The simulation suggests that the amortization scheme known as SAC for the mortgage system is a better choice in nearly two-thirds of the cases. Renting is preferential on very low relative prices, very high real return or inflation. This pathway – from existing model and literature to a reasonable space of advantageous choice – leads to the suggestion that contracts in Brazil should acknowledge this level of uncertainty by including explicit mortgage renegotiations within shorter periods.

Keywords: real estate; housing market; mortgage finance; decision-making; uncertainties.

1 INTRODUCTION

Buying a house is typically the most relevant financial decision households make over a lifetime. However, the ability to plan, evaluate, act upon household tenure choices – simply, the decision to rent or buy, when and where – although financially and non-financially significant – is not an easy task.

On strict financial terms, future dynamics, such as economic cycles (Davis and Heathcote, 2005; Davis and Van Nieuwerburgh, 2015; Leamer, 2015) and the interactions with the rest of the economy (Saiz, 2018) make tenure choice hard even for experts, let alone the average family. Non-financially, a multitude of other factors – mainly coming from location and spatial influence – adds to the complexity of tenure choice.

Hence, an empirical question that is permanently uncertain is when is best to own a house or a business facility (buy one) or rent one. Higher homeownership levels have been related to more investment in social capital, lower crime rates, and higher real estate prices (Malmendier and Steiny, 2016). Renting on the other hand provides flexibility and mobility, does not mobilize capital and is less dependent on future uncertainties and market fluctuations. Tenure choice remains a relevant question even with the emergence of a new economic paradigm that seems to favor rent over ownership (MacAfee and Brynjolfsson, 2017).

Further, tenure choice is a complex one to make as housing prices display significant momentum (Case and Shiller, 1989; Dipasquale and Wheaton, 1994) mean reversion (Cutler, Poterba and Summers, 1991), and excess variance relative to fundamentals (Glaeser et al., 2014).

Obviously, there is no correct, definite answer at any given time because one cannot know ex-ante future behavior of interest rates, regulation, appreciation, inflation or neighborhoods dynamics, for example.

Historically, houses observe positive appreciation. Hence, owning is usually preferable because "the effect of leverage means that a mortgage borrower gets the benefit of the entire increase in the home's value despite only putting down a fractional equity stake" (Chan, Haughwout and Tracy, 2015, p. 40). Obviously, moments of depreciation result in comparatively larger wealth loss, which penalizes the mortgage borrower over the full value of the house as it happened in the housing crisis of 2008.

Despite the difficulties of forecasting and appraisal and the complexities of real estate, it is possible to work with the space of reasonable parameters and provide probabilistic results that may guide prospective households with their tenure choices.

We provide exactly that, a simulation decision-making tool to help households balance alternatives. As such, the paper has a twofold objective. On one side, it reviews both (a) the standard urban economic model of real estate[1] and (b) the mortgage and financial recent literature and, on the other side; it provides mechanism for the public to handle the complexities of the real estate market when making tenure choices. As such, we assume the perspective of the household proponent buyer to look forward and simulate probabilistic outcomes, weighting in likely variations of uncertain parameters. The motivation of the paper is to provide a simple decision-making tool. Moreover, it may offer insights to policymakers about which policy would be more efficient for social housing.

The main contribution of the paper is to provide an easy-to-use, open source, reproducible tool for the analysis of the Brazilian market, which can be easily adapted to other countries. As such, we try to illuminate the path across uncertainties using a numerical simulation to estimate most likely evolution paths for both renting and purchasing.

The simulation suggests that the SAC amortization system is preferable than the PRICE system; that renting becomes more advantageous only when inflation is too high, rent is cheap and real return decreases. In most cases, however, considering Brazilian parameter values, buying would be a better choice.

The literature review covers the basic of the real estate, reviewing the typical urban economics textbook model of DiPasquale and Wheaton (1994). Further, it emphasizes the multiple, dynamic influence of non-economic factors upon real estate. We then go on to detail the methods and assumptions made for the numerical simulation. We test the simulation in four cases: a comparison between amortization choice systems, a single-case parameter illustration and a systematic generalization of parameters, given a reasonable space of possibilities. Finally, we provide a test of a new monetary adjustment via inflation index for the case of Brazil. The paper ends with some suggestions and final considerations.

1. Similarly to the work done by Santos and Cruz (2000).

2 LITERATURE

The simple real estate model proposes that the demand for housing stock is based on the flow of services provided. The supply, in turn, for the most part, is the existing stock. The price of housing would be then determined from the interaction of demand and supply (Kau and Keenan, 1980). However, to realize these assumptions in practice, individual choices need a full spectrum of information that is unavailable.

Academic research suggests a rational decision path (Dipasquale and Wheaton, 1994; Xiao and Huang, 2010). The authors assume that (spatial) equilibrium represented by a mathematical convex optimal choice leads to a unique optimal solution. They assume further that prices contain all the necessary information about the market and that the market is efficient and always clears.

Other authors consider non-rational choices within the house market analysis (Case and Shiller, 1989; Glaeser et al., 2014). According to Case and Shiller (1989), predictable movements in real interest rates do not appear to be incorporated in house prices, thus suggesting that empirical housing markets are not efficient. They see no way of obtaining an accurate historical time-series on implicit rents of owner-occupied houses, as available property tax series appear to have major deficiencies. The authors suggest further that any effort to model tax effects runs into definitional problems because there are different income tax brackets. These kind of results support the idea that to estimate accuracy of real estate prices is complex. A balanced view may support a limited range within rational choices (Glaeser and Nathanson, 2017).

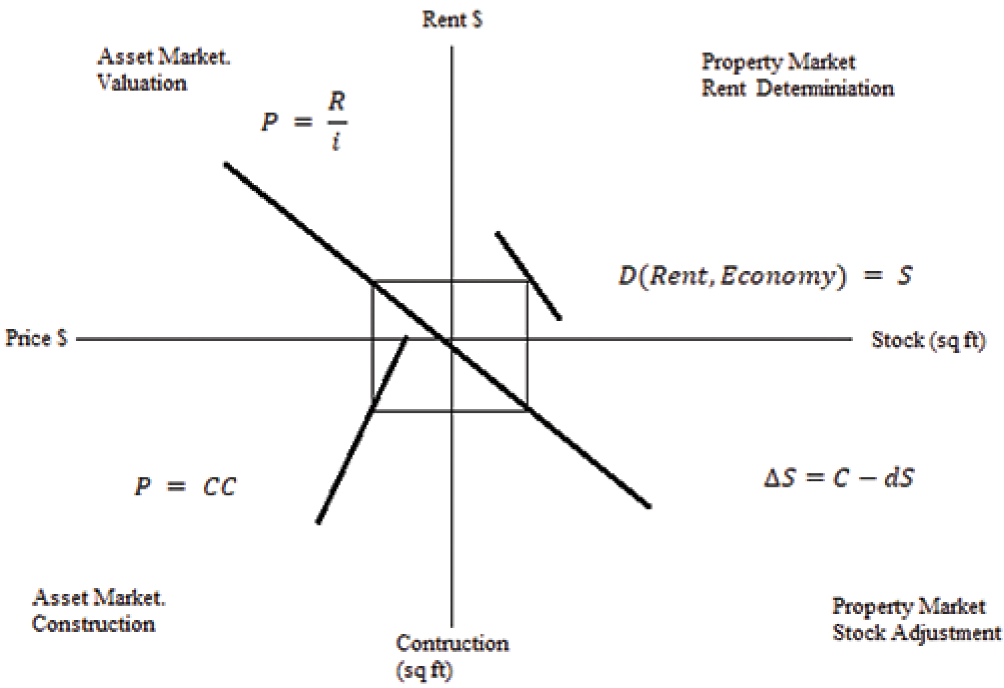
In this section, we review the typical urban economics model of DiPasquale and Wheaton – which is the representative of the rational choice – and then go on to a more contemporary view that includes the multitude of influences on real estate reasoning and ponders in empirically observed results.

2.1 DiPasquale and Wheaton model

The de facto model for real estate analysis is the one by DiPasquale and Wheaton (1992). The didactic, usefulness of the model comes from its capacity to put together prices and rents simultaneously with construction and retail markets.

DiPasquale and Wheaton's (1992) initial model show that in the short-run, in a market with inelastic supply curve, stock determines rent. In the end, stock of houses is a function of construction costs and depreciation rates. For a given price of rent and a given level of cost of capital, the ratio rent to prices is determined. Prices then influence replacement costs and the speed in which stock replaced and maintained.

FIGURE 1
Reproduction of DiPasquale and Wheaton (1992) explanatory diagram



Source: DiPasquale and Wheaton (1992, p. 188).

DiPasquale and Wheaton (1992) departs from a graphical analysis.

1) According to their proposal, rent is determined in the *Property Market* for space (Northeast quadrant). Given an inelastic supply of space (vertical supply), demand determines prices of rent.

   D(*Rent, Economy*) = S (1)

2) *Asset Market* valuation (Northwest quadrant) determines the second relationship. Prices of renting space (*R*) are equivalent to the interest rate (*i*) that compensates

the asset's prices (P) in the asset market. Thus, the proportion between rent prices and the asset price is the same that the asset would receive in the open market.

$$P = \frac{R}{i} \tag{2}$$

3) *Construction* of new assets happens in the Southwest quadrant. Here cost of construction (CC) depends on building levels. Prices then are equivalent to the construction costs of one space unit. Elasticity of the construction line relative to price depends on construction difficulties, such as availability of material and personnel.

$$P = CC \tag{3}$$

4) The Southeast quadrant is the *Stock Adjustment*. New stock, or the change in stock (of space) (ΔS) is equivalent to new construction (C) discounted by reductions of stock due to depreciation rate (d).

$$\Delta S = C - dS \tag{4}$$

The novelty of DiPasquale and Wheaton model was to put together, albeit abstractedly, the real estate market – the rents – and capital market – where property assets are negotiated, plus the construction market and its costs along with a long run mechanism to adjust the stock. In their own words:

> In summary, starting with a stock of space, the property market determines rents which then get translated into property prices by the asset market. These asset prices, in turn, generate new construction that, back in the property market, eventually yields a new level of stock (Dipasquale and Wheaton, 1992, p. 189).

That is the fusion of a spatial market, a capital market, a construction-labor market and the dynamics of slow adjustment.

2.1.1 The dynamic Wheaton model

In 1999, Wheaton proposed a dynamic version of his previous model. Wheaton explains that

> […] in its most simple form, where vacancy is ignored, it is assumed that the market clears in each period: rents adjust until demand (*ex post*) equals the current stock of space, In the long

run, the stock adjusts gradually because of lags in the delivery of new capital. *Capital investments decisions are based on a forecast of asset prices at the time of the new deliveries*. Thus rents and prices react quickly to change, while physical assets do not (Wheaton, 1999, p. 212, our emphasis).

Wheaton's dynamic model has the advantage of adding together both housing prices and rents, short and long runs, mediated by stocks and forecasting future asset prices, all within the same model. Although the analysis basis itself on specific functional forms and parameters, as made explicit by the author, its simplicity captures part of the intricacies of the real estate market.

The model contains five equations. The first assumes that office employment (E_t) and demand (D_t) to office space determine rents (R_t) with a constant elasticity ($-\beta$).

$$D_t = \alpha_1 E_t R_t^{-\beta_1} \tag{5}$$

There is no vacancy[2] and demand is equal to current stock (S_t). Thus, the relationship between space and utilization rates (St/Et) determines rents:

$$R_t = (\frac{S_t}{\alpha' E_t})^{\frac{-1}{\beta_1}} \tag{6}$$

Economic demand is unanticipated by agents and taken as constant (*E*), thus stock of space evolves following a difference equation.

$$\frac{S_t}{S_{t-1}} = 1 - \delta \frac{C_{t-n}}{S_{t-1}} \tag{7}$$

in which new space (*C*) is built and delivered after an *n*-long window. Delta is a constant rate of depreciation. Hence, the rate of construction $\frac{C_{t-n}}{S_{t-1}}$ is given by asset prices forecasted at the moment of delivery *t*.

$$\frac{C_{t-n}}{S_{t-1}} = \alpha_2 P_t^{\beta_2} \tag{8}$$

2. Vacancy could be introduced via a discount in current stock, making rents equal to: $R_t = (\frac{(1-V_t)S_t}{\alpha' E_t})^{\frac{-1}{\beta_1}}$.

Within this model, Wheaton considers forecast prices to be myopic "[which are] simply a constant capitalization (with a discount rate *r*) of known rents at the time the investment decision" (Wheaton, 1999, p. 215):

$$P_t = \frac{R_{t-n}}{r} \tag{9}$$

whereas a typical perfect foresight would preferably be the present value discounted by the value of future rents:

$$P_t = P_{t-1}(1 + r) - R_{t-1} \tag{10}$$

One way to investigate macroeconomic models such as the dynamic one proposed by Wheaton (1999) is via system dynamics. Medina de Lima (2018) conducts an exercise in which he applies an identification method to an adapted dynamic model of Wheaton in order to empirically validate the model. In doing so, he transforms Wheaton's equations into one equation in difference. After some algebraic steps, Medina de Lima represents the model as:

$$\frac{S_t}{S_{t-1}} = \tau + \omega \left(\frac{E_{t-n}}{S_{t-n}}\right)^{\xi} \tag{11}$$

where τ is the percentage of the stock that remains from one period to the next (discounted by depreciation) and ξ is the relation between elasticity of supply and elasticity of demand, $\boldsymbol{\omega}$ represents market factors that embed values of previous parameters (alpha, beta, and interest rate).

The dynamic model with myopic prices results in:

$$P_{t+n} = (1 - \delta + \alpha_2 P_t^{\beta_2})^{\frac{-1}{\beta_1}}.P_{t+n-1} \tag{12}$$

That is, prices at the time of new stock delivery will be a function of current prices and prices immediately before the delivery.

Medina de Lima (2018) finds adherence to Wheaton's (1999) work and provide results with simulated data that validate the method to correctly identify the dynamic generator system of the series. In all simulated cases, the learned parameters converged correctly to the target values. For Medina (2018), Wheaton's model works as a predictive tool for the real estate market during periods of stability. Zhang et al. (2018) have also tested the adequacy of the model to real data. Attempts at estimating it have also been made by Hendershott et al. (2010) and Steiner (2010).

In sum, the Wheaton's model (1999) suggests that rents depend on demand, which is a function of employment growth (and income), housing prices and prices of other goods, plus the macroeconomic prices, given by interest rate, credit availability, housing taxation, population size, migration and regulation. Supply is a function of depreciation and new construction. In a full information scenario in which all these influences are controlled, rate of construction determines prices. In practice, however, influences and information are not controlled and all of these factors interact interchangeably with asynchronous temporal lags. We look at those factors in the next section.

2.2 Real estate complexity

Housing is unique among major consumption goods. First, a residential property is a durable consumption good as well as an investment asset (Xiao and Huang, 2010). In the tradition research, housing prices reflect a spatial equilibrium, where prices are determined by local wages and amenities, so that local heterogeneity is natural (Glaeser and Nathanson, 2017). Further, real estate markets suffer influence from:

- microeconomic-generated effects, such as location and neighborhoods (Bourassa, Cantoni and Hoesli, 2007; Furtado, 2009; Galster, 2001; Rosenthal and Ross, 2015);
- accessibility and amenities (Cheshire and Sheppard, 1995, 2004);
- density, zoning and regulation (Glaeser, Gyourko and Saks, 2006);
- but also suffer simultaneously from macroeconomic dynamics, mainly economic cycles (Davis and Heathcote, 2005; Davis and Van Nieuwerburgh, 2015); and
- regional growth (Leamer, 2015; Saiz, 2018).

Further, real estate plays a relevant role in the economy as a whole contributing to GDP in construction, and renting services. That is despite the fact of constituting a large part of the stock of wealth.[3]

3. Eight trillion Reals in 2014 (R$ of 2010), according to Morandi (2016).

Hence, real estate markets are in a unique position that they pragmatically accommodate both spatial, structural, long-term dynamic local features with exogenous economic and policy drivers that oscillates within a shorter timeframe.

Moreover, real estate markets have some intrinsically complex features. Firstly, the market evolves in long temporal steps. Durability and immobility of real estate make market adjustments slow and illiquid, comparatively to other asset markets.

Secondly, transactions are costly, inertial and depend on asymmetric information. They include the difficulties of the searching process, the cost of moving and relocating, and the decisions on the timing, scope and design of the contract (Han and Strange, 2015). Further, these costly transactions happen among expert, retailers and brokers but also include decentralized amateurs with little to no experience.

Thirdly, real estate assets are heterogeneous, indivisible goods (Lambiri and Rovolis, 2014; Whitehead, 1999). Despite obvious distinction starting at the address – its location –, intrinsic quality or unique layout; estates influence and are influenced by agglomeration effects. Endogenously, the single fact of building a new flat of apartments, a new school or a shopping center changes the neighborhood and the valuation of location itself. Exogenously, a next neighbor decision to build a supermarket, a block of luxurious houses or a government decision to launch a new subway line or a penitentiary also affects land prices.

This spatial influence of proximity on prices – usually locked into perceptually known and distinguishable neighborhoods (Furtado, 2011; Galster, 2001) – create specific submarkets for which price dynamics may differ from close-by submarkets.

Another factor to impact real estate markets is the life cycle of individual households. Young families usually have less available resources and need to be more locationally flexible towards the labor market whereas more mature families, with children, tend to demand larger, more expensive housing.

A final relevant issue to consider when analyzing real estate markets is the fact that not only households look for a place to live. Investors also are searching for places to invest, thus making housing a wealth, speculative asset that interest both domestically and foreign interests.

All such complexities lead to a generous number of literature that covers specific areas of real estate analysis. A recent review, for instance, focus on the housing crisis period of 2000-2010 and tries to summarize *i*) what has been clearly understood; *ii*) what has been reasonably described; and *iii*) what is definitely missing within real estate knowledge (Chan, Haughwout and Tracy, 2015).

Others claim that the complexity is so large that it is difficult to even talk about a market that is comparable to regular goods markets: "this dispersed, idiosyncratic market means that *there is no such thing as the current price of housing*" (Glaeser and Nathanson, 2015, p. 709, our emphasis).

A good review on the interconnections among housing, finance and macroeconomics is the review by Davis and Nieuwerburgh (Davis and Van Nieuwerburgh, 2015). Their emphasis is on discussing models that have been empirically backed. They are also ready to state that economists *are not ready to account for all the volatilities of the 2000-2010 housing bubble*. Their vision in this case is shared by Glaeser and Nathanson (2015).

Nevertheless current macroeconomic models base their modeling on user costs that forecast shifts on demand. In such models, "prices equal the expected value of the exogenous flow of discounted future benefits from home-owning" (Glaeser and Nathanson, 2015, p. 703). However, the models cannot precise the magnitude of the shifts nor its location patterns. Indeed, the analysis often occurs at the aggregated metropolitan level.

Further, Chan et al. (2015) claim there is a clear favoring of mortgage lenders over those households opting for renting. Indeed, whereas homeowners do not pay taxes on their own (implicit) rent, they also benefit from tax discounts from interest and mortgage payments.

> Nonetheless, there is a need for further research intended to improve our understanding of how these various subsidies affect the spatial organization of activities across and within cities, and how they affect the kinds of housing occupied by both owners and renters (Chan, Haughwout and Tracy, 2015, p. 1004).

Another typical idiosyncrasy of real estate markets is vacancy (and unoccupied retail places). Wheaton (1999) claims that vacancy has been found to correlate with future rents. That is, higher levels of vacancy leads to lower future rents and a reduction of vacant estates would increase future rents. Once more, however, there is no clear understanding of the frequency of effect transmission, from vacancy into rents.

This section shows that even though a general model of real estate is available, the complexity of estates per se – especially its financial aspects – constitute a relevant difficulty when making decisions under uncertainties of such magnitude. This paper circumvents such complexity by providing a numerical simulation that estimates possible evolution paths for real estate mortgage and renting conditions, considering this overview.

3 METHODS

We present a numerical simulation as a means to provide systematically probabilistic boundaries over the space of probabilities. Given a reasonable parameter space and a distribution, one can calculate choices expected values.

The numerical simulation compares the expenditures of a mortgage scheme with all its details to rental in an equivalent estate. At the end of the period, gains (or losses) for either choice are brought back to present value. Specifically, the simulation weights in a number of different parameters to combine them in a systematic way to get a single, comparable number as a result. After having a single simulated output as a result of all parameters, it is easy to vary the parameters and observe optimal decision-making alternatives.

The open source, reproducible code is available at <https://bit.ly/2tER0ZP>. The code follows the methods and equations described and the commands to run each part of the cases (with some parameter adjustments) is in each Case description. However, a specific section (3.5) explains how the code is organized.

3.1 Choice of amortization system

The first relevant parameter refers to the amortization system. Currently, the user can choose between 'SAC', the most common one, or an alternative named 'PRICE'.

SAC is a Portuguese acronym that stands for *system of constant amortization*. That consists in having two distinct components. Amortization itself – which is the result of the division between the total loan amount and the number of the months – and interest. Interest is a result of current debt times the monthly rate.

$$p_i = \frac{D}{n} + B * (1 + r) \tag{13}$$

The PRICE system in turn is the payment against loan principal plus interest, with future value at the end of the period set to zero.

$$Fv + Pv * (1 + r)^n + \frac{Pmt * (1+r)}{r * [(1+r)^r - 1]} = 0 \tag{14}$$

where *Fv* is the future value of the payment, *Pv* is the present value of the payment, *Pmt* is the payment and *r* is the real interest rate of the payment.

In practice, amortization is fixed and interests paid decrease rapidly in the SAC system whereas monthly payment is reasonably stable in the PRICE system with low amortization and high interests at first, followed by lower interest and higher amortization at the end of the period.

3.2 Insurance

In Brazil, mortgage financing includes two compulsory types of insurance. One – called DFI – which refers in Portuguese to Physical Damage to the Estate and is calculated as a fixed fee[4] applied to the total value of the estate.

The second portion of insurance – MIP – is a death and permanent disability insurance. It depends on the current outstanding debt and the age rounded to years of each of the borrowers at a given month. The age at the signature of the contract, determines the entry line in table A.1 (appendix A). Current age applies to the column, so the older the borrower is, he or she moves rightwards on the row of entrance.

$$MIP_i = D_i * fee_{signature,current} \tag{15}$$

4. For the simulation of this paper, this value is set at 7.8e-05, following official CAIXA data – the public bank that has roughly 70% of the mortgage market (Fioravante and Furtado, 2018).

3.3 Rent

Rent is calculated straightforwardly from initial monthly payment and an annual rate of increase. The raise in rent follows the same parameter considered for house appreciation.

3.4 Simulation schedule

In practice, the simulation follows the sequence:

- calculates each month debt monetary correction, amortization and interest, given system choice;
- calculates the outstanding debt for each month;
- calculates insurance, based on debt and total value of estate, then total monthly payment;
- calculates equivalent rental expenditure;
- calculates house appreciation value and rental savings return, given parameters;
- calculates equity for the purchasing choice, discounting selling brokerage and capital gains tax; and
- calculates present value of the difference between purchase and rental options.

3.5 Code organization

The key class of the model is Mortgage (mortgage.py file). The instantiation of a Mortgage object with given parameters of interest, number of months, amount to lend, debt monetary adjustment rate and choice of scheme (either 'SAC' or 'PRICE') produces a table with monthly payment schedule.

Next, another file (comparisons.py) is responsible for setting up a Contract (another class) that contains the parameters of the Mortgage (with an import from mortgage) and of the borrowers. Then a class of Comparison sets up a Contract, a Mortgage, imports insurance details from insurance.py and instantiates within itself a Rental class.

The two files above are enough to compare tenure choices. Further, a generalization.py file contains a number of functions that enables the automatic change of parameters. A default value for parameters can be set at conf directory at params.py.

However, changing those parameters on the fly requires some consistency checks and adjustments. For example, it is in the generalization file that the user may alter initial, final and number of times he or she wants to automatically change parameters.

Hyperspace.py is the file that calls generalization, conf and plotting to organize the automatic change of parameters and produce the plot. Finally, randomizing.py articulates the multiple random generation of parameters, calls comparisons and plots the results.

The user interested in applying the tool for their own intent should follow the steps of each case description in the next section and may adjust values at the files: params.py (any of the desired parameters), generalization.py (function prepare – for the values within linspace), or rnd.py file for the interval of the randomizing process.

3.6 Specificities of the design for the case of Brazil

Some common practices used in the simulation might vary for different countries.

In Brazil, for instance, the burden of both property taxes and condominium fees, if any, fall for the current living tenant, whether an owner or not. Thus, those would make no difference whether you buy or rent a place and did not go into our calculations.

A second relevant difference for the case of Brazil is that *there is no income tax deduction for mortgage interest paid*. There is a fixed 15% tax rate on top of both capital gains from a house appreciation (when selling) or annually when investing in the treasure bond market.

However, the borrower is allowed to update the house capital by all payments made towards mortgage. That includes interests, amortization and insurance fees. The update does not include inflation or a new estate appraisal. Thus, at the end of period, if down payment plus all payments made exceed selling value, then the tax applies on the capital gain. That is how the current simulation implements it. However, by the current tax legislation, if the owner does not own another estate and buys a new residence within a period of six months, this tax is waived.

Finally, years of mortgage payment plus age of borrower at the moment of the contract signature cannot exceed 80 years. Hence, if the user tries a simulation with contract date for 2019, then borrowers cannot have been born before 1969, if mortgage plan is for 30 years.

4 ILLUSTRATION FOR THE CASE OF BRAZIL

Given the simulation in place, a number of outputs and analysis are possible.

First, we evaluate a single case with a set of default parameters, checking the amortization system. Then, we generalize some parameters, departing from default values to see their general influence in overall gains towards renting or buying. The third exercise involves generating 10,000 different simulations considering a wide specter of possible parameters evaluating the pseudo-probability of making it more advantageous to buy or to rent. Finally, we check a different implementation of monetary adjustment for the outstanding debt.

4.1 Case 1 – SAC system versus PRICE system

Running python comparisons.py at a Terminal – after cloning the model from the GitHub repository using the link provided at the Methods section – will produce two outputs. We present them for the default case – based on parameters from table 1 – and for the case in which MORTGAGE_CHOICE is set to 'PRICE'.

Firstly, the printed output is: R$ -64,440.11 and -23,213.6. That indicates that in the SAC amortization system the present value of buying surpasses that of renting by almost 65 thousand reals, whereas in the PRICE system the gain is much smaller.

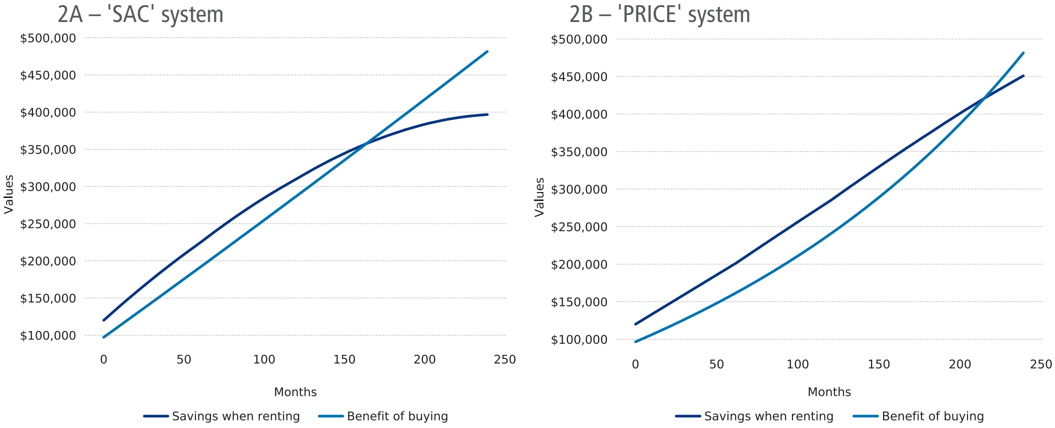
Additionally, a table with the schedule payment is generated.[5] The table contains monthly information on: the monetary correction applied to the outstanding debt (if any), the amount amortized, interest paid, outstanding debt, DFI (estate damage insurance), MIP (full estate insurance), monthly payment due, equivalent rent, savings when renting, home value, equity of home purchaser and savings when purchasing.

5. The tables are available at: <https://bit.ly/2sW2k3u>.

The analysis of the generated tables for both the default 'sac' system and the 'price' system indicates that purchasing becomes better at the 165th month using 'SAC' and at the 216th month using 'PRICE'.

Further, typing python plotting.py generates plots with the behavior of savings and purchasing gains (figure 2).

FIGURE 2
Plots of default simulation for 'SAC' system and 'PRICE' system



Authors' elaboration.

TABLE 1
Default parameters' values for a comparison run

Original Parameters	Value	Units	Observations
Purchase_price	400000	BRL R$	
Downpayment	120000	BRL R$	
Loan_amount	Calculated	BRL R$	Derived From Purchase Price Minus Downpayment
House_appreciation	0.05	Annual Percentage	
Financing_rate	0.07	Annual Percentage	
Mortgage_choice	'Sac'		Sac: Constant Amortization System, Price: Payment Against Loan Principal Plus Interest
Selling_cost	0.06	Applied Once	Broker's Cost When Selling
Contract_date	2019, 9, 17	Date (Year, Month, Day)	Date Of Mortgage Contract Signature
Birth1	1969, 9, 23	Date	First Borrower's Date Of Birth
Birth2	1970, 9, 1	Date	Second Borrower's Date Of Birth
Perc_borrower1	1	Percentage	Percentage Share Of First Borrower Income, Relative To Both Borrowers
Perc_borrower2	Calculated	Percentage	Residual Percentage In Relation To First Borrower
Inflation	0.0375	Annual Percentage	
Treasure_return	0.0537	Annual Percentage	Free Of Risk Interest Paid By Governmental Bonds
Rent_percentage	0.0029	Monthly Percentage	Proportion Of Rent Price Of Rental Compared To Value Of That Same House

(Continues)

(Continued)

Original Parameters	Value	Units	Observations
Rent_raising_period	12	Months	Observed Period To Adjust Rent Payment
Amortization_months	240	Months	Duration Of Mortgage Contract
Tax	0.15	Percentage	Taxes Applied To Both Treasure Bonds And Capital Gain Of House Appreciation
Custody_fee	0.0003	Percentage	Custody Fee Charged On Treasure Bonds Annually
Real_return	Calculated	Annual Percentage	Treasure Bond Return Minus Inflation Minus Taxes
Real_house_appreciation	Calculated	Annual Percentage	House Appreciation Minus Inflation Minus Taxes

Authors' elaboration.

4.2 Case 2 – Parameters comparison

A second analysis we provide is the variation of parameters by a given successive multiplier.[6] The default alteration varies a given parameter from a quarter of its value (multiplied by .25) to its double. Taking inflation, for example, that would mean that we vary the parameter from 0.9% to 7.5% yearly. This values can be changed in file python generalization.py within function prepare, altering the parameters of linspace (initial multiplier, last multiplier, number of intervals). You may generate the plots typing python hyperspace.py.

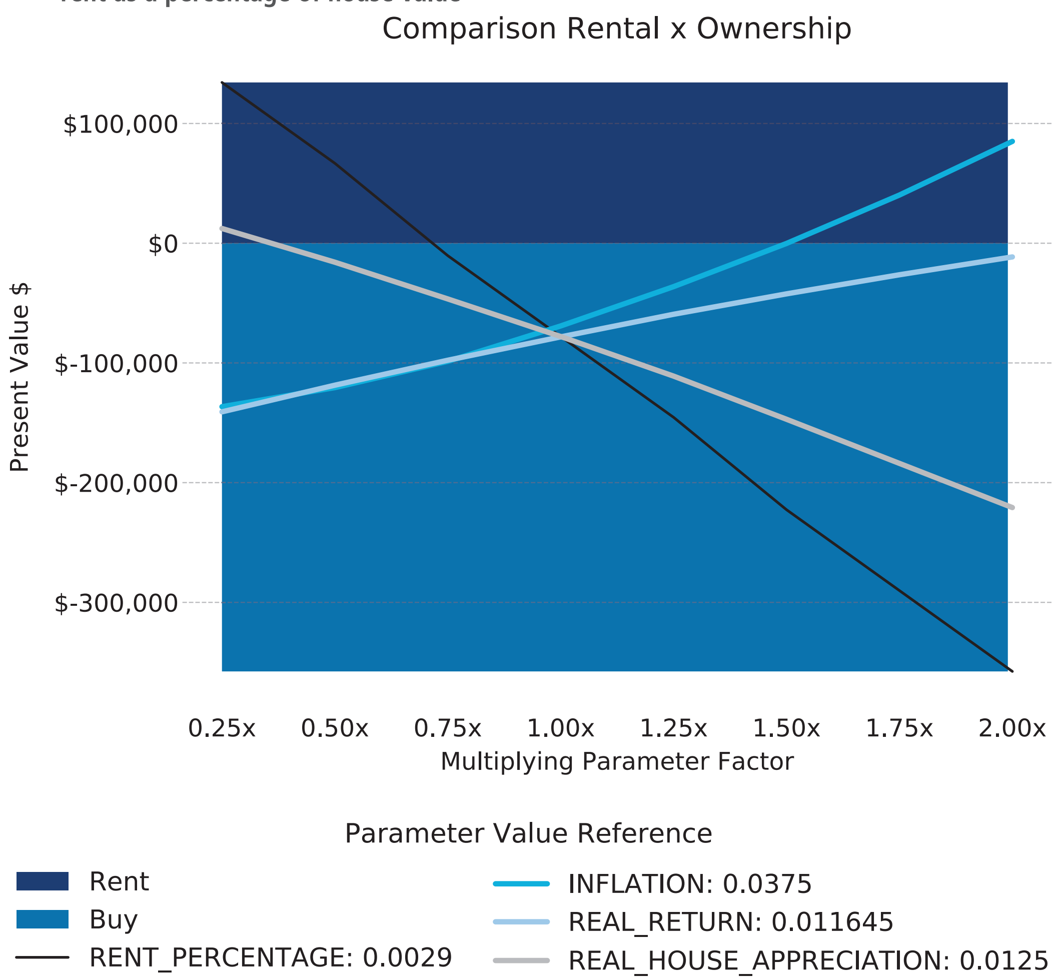
As some of the parameters depend on one another, some consistency checks are applied. For example, if one is testing 'inflation' or 'tax' variations, real return (from treasure bonds) and real house appreciation also have to change accordingly.

Figure 3 shows what happens for four different parameters: inflation, real house appreciation, real return and rent as a percentage of house value. Inclination of the curves and values above or below zero indicate their dynamics. In this case, rent is preferable only in the cases in which inflation is much above the default value of 3.75% or when rent is relatively cheap, much below the default annual percentage of 0.29% of the estate value.

The figure further suggests that the higher the real return less and less advantageous purchasing becomes, according to intuition. In the exact opposite direction, the less the house appreciates; more unlikely it is to be a better option in comparison to renting.

6. Running the second case requires setting the DATA parameter to None.

FIGURE 3
Example of parameters variation using: inflation, house appreciation, real return and rent as a percentage of house value

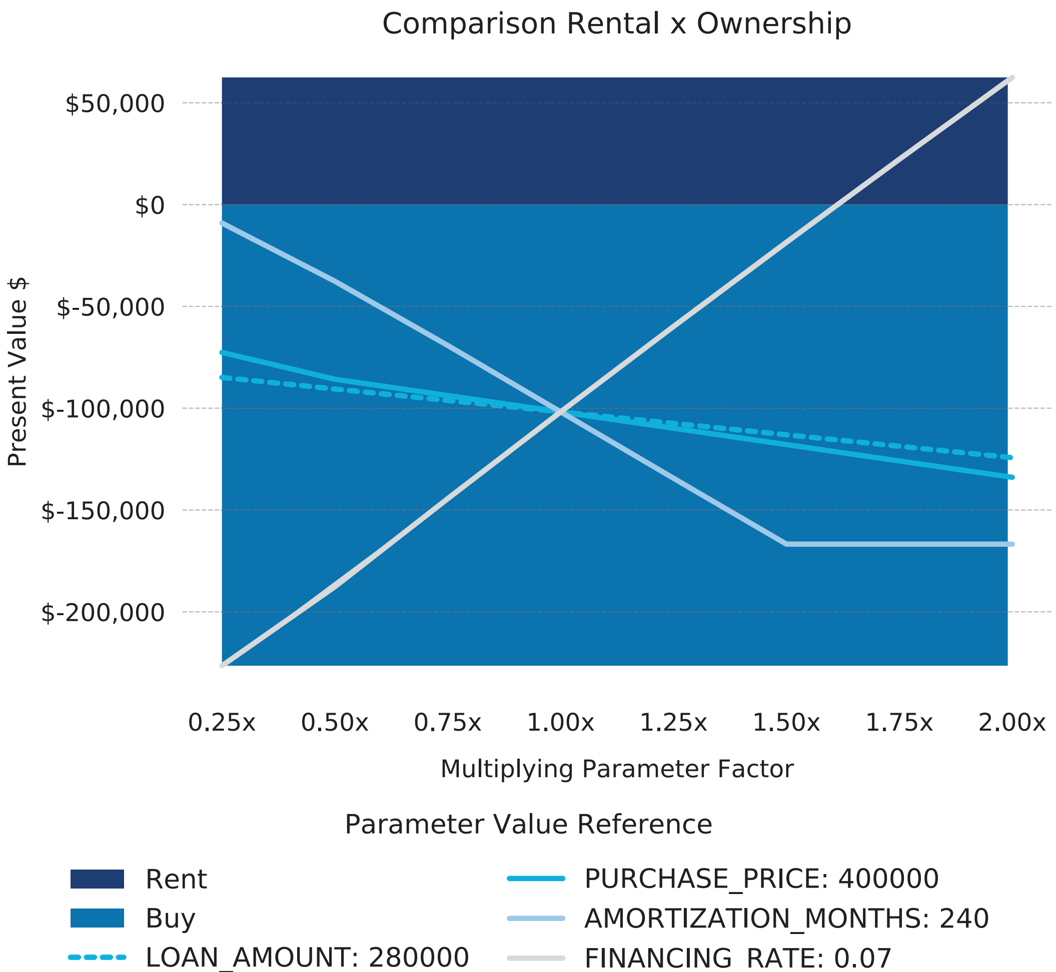


Authors' elaboration.
Obs.: 1. Original parameter values are those in the legend.
2. Image displayed in low resolution due to the technical characteristics of the original files provided by the authors for publication (editorial note).

It is also possible to change the parameters been tested. The reader interested in doing so should uncomment the parameters at the list at the bottom of file hyperspace.py and include the respective variable to the line that calls function generalization.prepare.

Figure 4 shows that the results for the variation of the purchasing price and loan amount do not result in large variance in results with larger values favoring purchasing, due to appreciation of larger values, *ceteris paribus* the value of the other parameters as default.

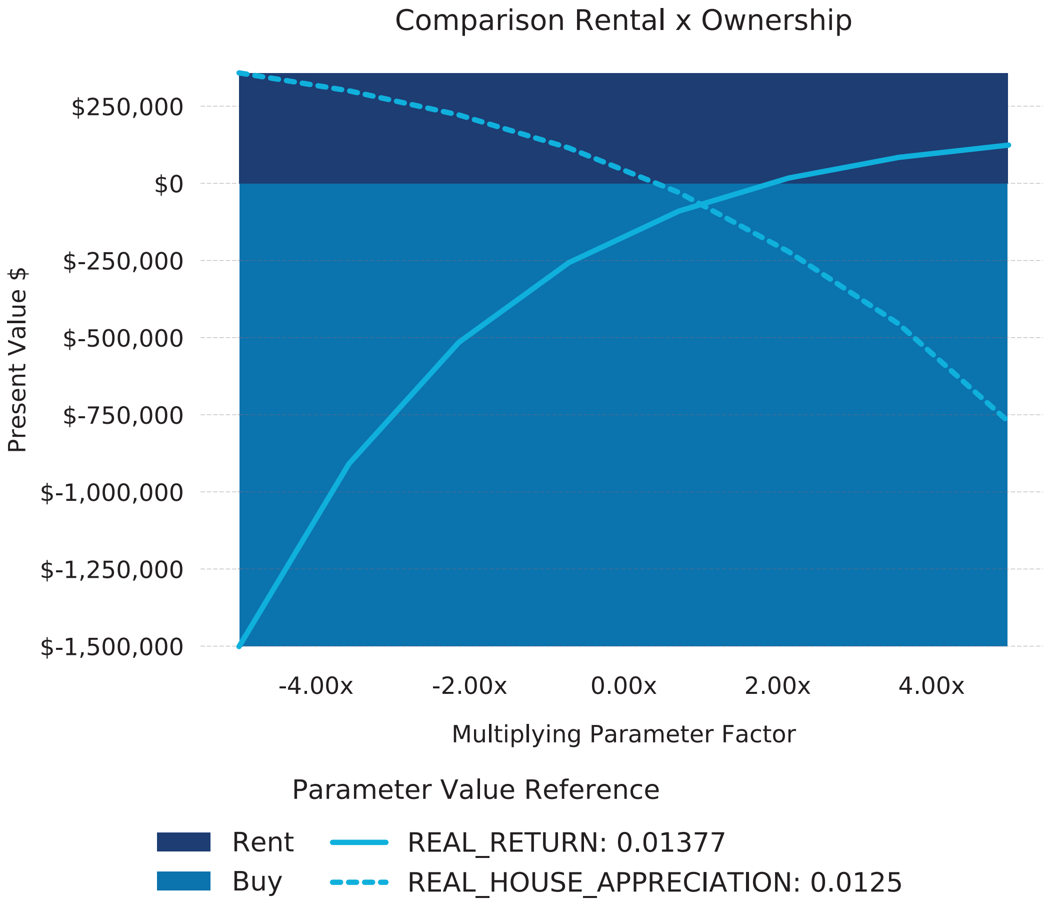
FIGURE 4
Example of parameters variation using: loan amount, purchase price, financing rate and amortization months



Authors' elaboration.
Obs.: 1. Original parameter values are those in the legend.
2. Amortization months are truncated at values lower than 60, or higher than 360.

Finally, a last example in which the variation of the parameters is pushed to five times positive and five times negative over real return and house appreciation (figure 5). It clearly shows that a house depreciation can lead to losses and better choice of renting. Appreciation of real returns, in turn, also leads to favor renting, although with less relevance.

FIGURE 5
Example of parameters variation using real return and real house appreciation for larger numbers of positive and negative variation



Authors' elaboration.

4.3 Case 3 – Full generalization of parameters

Although the cases reported above provide a general description of the problem, it still does not provide much in terms of quantitative help on the uncertainty scenarios.

Our third case presents the possibility of varying all parameters of the model simultaneously, generating the consolidated results of all the runs.

Still, results depend on the *ad hoc* assumptions made about the interval of each parameter and the sampling distribution chosen. We provide what we consider a reasonable set of choices below (table 2). The reader should try out different configurations. Individual plots are available at the GitHub repository.

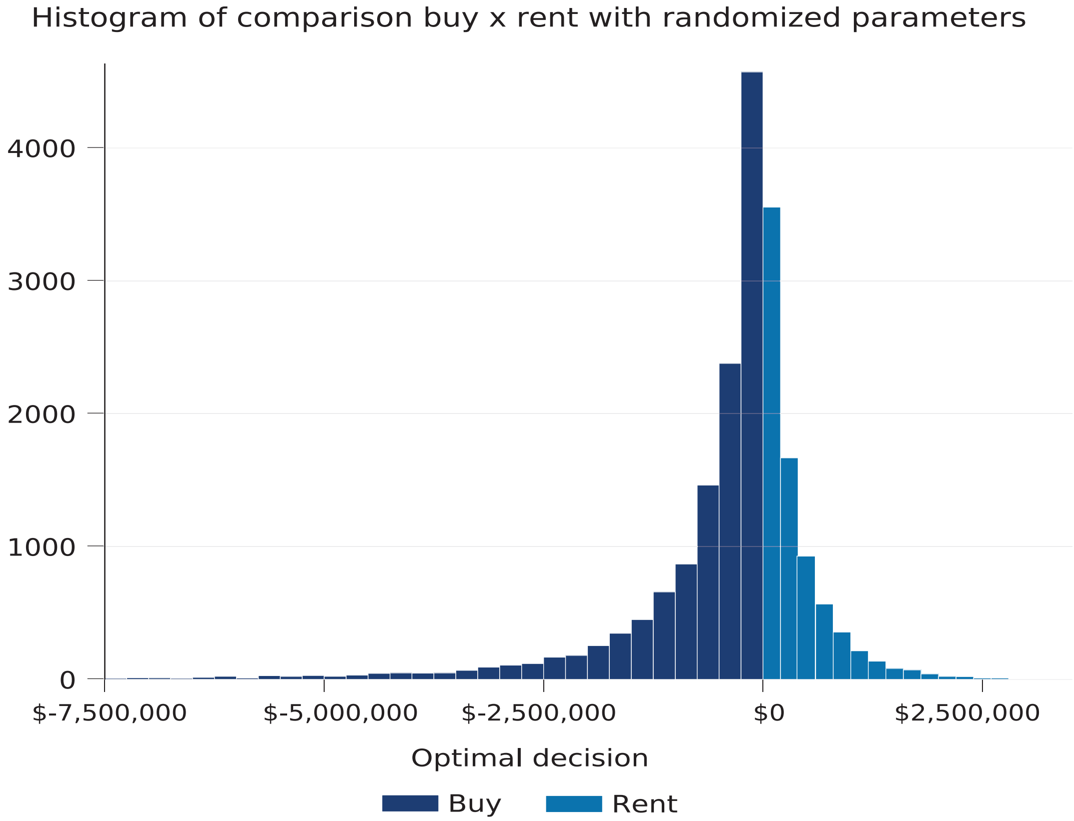
The reader might run this case by typing python randomization.py at the Terminal. The simulation will plot and save the parameters and then produce the histogram of all cases, as of figure 6. On average, the simulation with 20,000 cases results and the chosen configuration of parameters distributions suggests that renting is a better option in 38.52% of the cases.

TABLE 2
Interval of parameters used in the generalization case

Original Parameters	Minimum/Parameters	Maximum/Parameters	Distribution
Purchase_price	R$100,000.00	R$1,800,000.00	Uniform
Downpayment	10%	80%	Uniform -- Percentage Of Purchase Price
Loan_amount	R$90,000.00	R$1,440,000.00	Calculated
House_appreciation	Mean=0.02	Standard-Deviation=0.05	Normal -- Percentage, Annually
Financing_rate	Mean=0.08	Standard-Deviation=0.05	Normal -- Percentage, Annually
Mortgage_choice			Constant= 'Sac'
Selling_cost			Constant=0.06%, Applied Once
Contract_date			Constant=17/09/2019
Birth	22/07/1945	29/08/1993	Uniform
Perc_borrower	0.00	1.00	Continuous Uniform
Inflation	Mean=0.04	Standard-Deviation=0.025	Normal -- Percentage, Annually
Treasure_return	Min=0, Max=.06	Mean=0.02, Std=0.5	Truncated Normal, Percentage, Annually
Rent_percentage	Min=0, Max=.01	Mean=0.0029, Std=0.5	Truncated Normal, Percentage, Monthly
Amortization_months	60	360	Normal, Conditional On Birth Date, Months
Tax			Constant=0.15 -- Percentage, Annually

Authors' elaboration.
Obs.: Real return and real house appreciation are calculated from the other parameters and have no minimum and maximum set a priori.

FIGURE 6
Results of generalization of parameters and decision to buy or rent with indication of monetary gains for either decision



Authors' elaboration.
Obs.: Simulation for 10,000 cases. For better visualization, the x axis was truncated for outliers' values.

4.4 Case 4 – SAC system plus inflation: new alternative in Brazilian real estate market

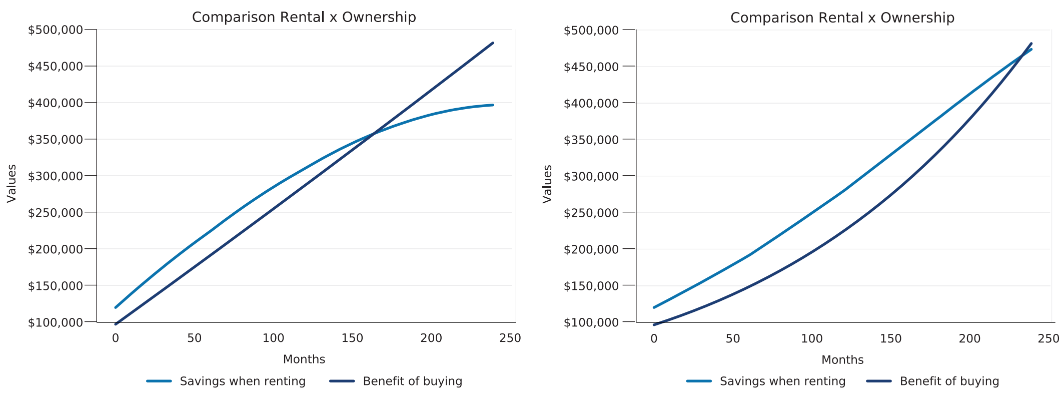
Resolution 4,739, of August 19, 2019 of the Central Bank of Brazil revokes item III of the caption of article 13 of Resolution 4,676, of July 31, 2018. Hence, the imposition of using the basic remuneration of savings deposits is waived. In practice, when SELIC is below 10%, the basic savings remuneration, that is, the Referential Rate (TR), is zero. That is, as the explanatory memorandum of Resolution 4,739 comments, the lender needs to estimate future inflation and incorporate such amounts into the interest on the loan. With the revocation, the financial authority allows – within the scope of SFH system – the free choice of monetary indexation, still limited to the maximum effective cost of 12% annually.

The objective of the authority is to allow the basic interest rate of the loan to fall while transferring to the borrower any risks of inflationary fluctuation over the period of the loan.

Using the same procedure as Case 1, we tested the SAC setting with default parameters and additionally with financing interest reduced to 3.25% – so that interest plus inflation is nominally equivalent to 7% of interest on original financing (figure 7).[7]

Using inflation as the monetary adjustment of the loan, the decision between purchasing and rent becomes nearly indifferent at present value.

FIGURE 7
Plots of default simulation for 'sac' system and 'sac' + inflation system
7A – 'Sac' system
7B – 'Sac' + inflation system



Authors' elaboration.

5 FINAL REMARKS

We made a review of basic economic real estate modeling and the complexity of factors that influence the real estate market and hence makes tenure choice decision a hard one to make. Despite the level of uncertainties and unknowns about the future, a simulation of possible parameter space allows for a probabilistic view of outputs in future scenarios.

Results suggest that buying is a better option for the case of Brazil in more than 60% of the cases simulated with random parameters within the proposed intervals. When buying is a better option, the gains are also of a higher magnitude when compared to renting.

7. Change the REFERENCIAL_RATE as factor of adjustment in parameters.

Further, the simulation suggest that buying using the SAC system without inflation as monetary adjustment as the best option for the choice of parameters made.

More than the results of the simulation run, our intention is to make available the calculator so that the interested reader may choose the window of parameter and simulate each specific case. Bearing the fact that multiple factors influence real estate analysis and that it is not possible to know how the future will come to realize.

Finally, the discussion promoted by the paper suggests that uncertainties related to the real estate market and their effects on tenure choice are of high magnitude.

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APPENDIX A

TABLE A.1
Fees for MIP insurance calculation

Current age (in years)	25	30	35	40	45	50	55	60	65	70	75	80
At the date of signature												
25	0.0115	0.0119	0.0169	0.0211	0.0254	0.0336	0.0525	0.1015	0.1807	0.2129	0.2824	0.325
30		0.0121	0.0174	0.0222	0.0286	0.0359	0.0563	0.1093	0.1807	0.2307	0.2824	0.352
35			0.0178	0.0229	0.0302	0.0404	0.06	0.01171	0.1946	0.2307	0.306	0.325
40				0.0234	0.0312	0.0426	0.0675	0.1249	0.2085	0.2484	0.306	0.325
45					0.0318	0.0439	0.0713	0.1405	0.2224	0.2662	0.3295	0.325
50						0.0448	0.0675	0.1327	0.2085	0.2484	0.3295	0.3295
55							0.0751	0.1327	0.2308	0.2662	0.306	0.3295
60								0.1561	0.2641	0.3372	0.4236	0.3766
65									0.2781	0.3372	0.4472	0.4236
70										0.3549	0.4472	0.4236
75											0.4707	0.4472
80												0.4707

Obs.: Age at the moment of mortgage signature determine the entry row. Then fee evolves to the next value when the age in years at the top of the columns are reached. A borrower that signs a contract at 49 will start on row 6 and column 6 (0.448). As soon as she or he turns 50, the fee will move forward on the same row (0.0675). Such table prevents borrowers that are 80 or over at the end of the mortgage period.

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