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"Indexation, Monetary Accommodation and Inflation in Brazil"

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INDEXATION, MONETARY ACCOMMODATION AND INFLATION IN BRAZIL

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This paper argues that between 1968 and 1979, due to indexation of exchange rates and wages, and because of monetary accommodation, the Brazilian inflation rate followed a random walk, while the real growth rate remained constant, except for uncorrelated shocks. 1/

The indexation rule for wages was introduced in 1965, but Brazilian economists and policy makers agree that up to 1968, wages were less than fully indexed and at least real minimum wages fell during the years between 1965 and 1967. 2/ After 1968, not only the wage indexation rule was brought more closely to observed past inflation rates, but a minidevaluations scheme was introduced, according to which, the exchange rate was devalued at short intervals, taking into account the difference between domestic and foreign inflation rates. Only at the end of 1979, the system suffered a discontinuity, when the exchange rate was devalued by 30 percent.

Evidence for accommodation policy can be found in Cardoso (1977) and Contador (1978). Using Sims' causality test,

1/ Taylor (1979) has shown such properties for models with full monetary accommodation and overlapping wage contracts under rational expectations.
they show that one cannot reject the hypothesis that money growth rates in Brazil respond to inflation rates.

Section 2 develops a model that permits to study the behavior of inflation and growth rates in an economy where wages are indexed to past inflation rates, purchasing power parity is guaranteed by a crawling peg, and where monetary authorities follow an accommodation rule. The paper concludes with the empirical evidence.

1. The model

The model consists of a standard Keynesian model, closed by a Phillips curve and a wage indexation rule. Variables are stated in growth rates. We assume output to be demand determined, and demand to depend on the real money stock and on the expected inflation rate. An increase in the real money stock reduces the nominal interest rate. Given the expected inflation rate, the expected real interest rate falls, stimulating demand. This is the Keynes–Hicks or liquidity effect. On the other hand, given the nominal interest rate, an increase in the expected inflation rate reduces the expected real interest rate and stimulates demand. This is known as the Mundell effect. Both effects are incorporated in the equation that describes output
as determined by demand: 3/

(1) \[ y_t = a(m_t - p_t) + c(p_t^* - p_{t-1}) \]

where: \( y_t \) = output growth rate in period \( t \); \( m_t \) = money growth rate; \( p_t \) = inflation rate and a star denotes its expected value.

3/ To obtain equation (1) consider the description of equilibrium in the money market and in the goods market given respectively by the LM and IS schedules: (I) \( i = L^{-1}(M/PY) \) and (II) \( Y = D(Y, i-p^*) \); where \( i \) denotes the nominal interest rate; \( Y \) = output level; \( M/P \) = the real money stock. Substituting (I) in (II) and log-differentiating, we obtain (1), where \( a \) and \( c \) depend on different elasticities.

In the open economy, demand for output equals domestic absorption plus the current account surplus, or net exports, NX:

(II) \[ y = A(y, i-p^*) + NX(y, E_{P}^f/P) \]

where \( E_{P}^f/P \) is the real exchange rate and we assume that a real depreciation increases net exports. In this case, equation (1) should be written as:

(1) \[ y_t = a(m_t - p_t) + b(e_t + p_t^f - p_t) + c(p_t^* - p_{t-1}) \]

where: \( e_t \) = the depreciation rate of the nominal exchange rate; \( p_t^f \) = foreign inflation rate; and \( a \), \( b \) and \( c \) depend on different elasticities, and on the share of domestic and foreign demand in total demand for the domestic output.

Assuming that the exchange rate is indexed to the difference between domestic and foreign inflation rates, we obtain the expression for equation (1), as it appears in the text.
The monetary authorities follow the rule:

\[ m_t = \bar{m} + g P_t^* \quad ; \quad 0 < g < 1 \]

where \( g \) is the accommodation parameter.

Substituting (2) in (1) and assuming perfect foresight, we obtain:

\[ y_t = \bar{a} m + \left[ c - a(l - g) \right] P_t - c P_{t-1} \]

Equation (1), or aggregate demand, is represented in figure 1 as the upward sloping schedule AA. We assume both the accommodation parameter and the Mundell effect to be large enough to guarantee that \( c > a(l - g) \). Consider a point along AA, at which output equals demand. Suppose that the inflation rate increases. On one hand, the increase in the inflation rate reduces the real interest rate; on the other, it decreases the real money stock increasing the nominal interest rate and thus the real interest rate. Since the monetary policy in part accommodates the price acceleration, the effect on the nominal interest rate is reduced. As the Mundell effect predominates over the liquidity effect, the rise in the inflation rate induces an increase in the demand for output. Equilibrium would require a higher output growth rate. Also observe that AA was drawn for a given past inflation rate. As the inflation rate evolves, AA will be shifting. Dynamics are discussed below.

We now turn to the supply side. Assuming that prices
are cost determined, the inflation rate equals the wage inflation rate, \( w \):  \(^4\)

\[(3) \quad p_t = w_t \]

The wage inflation is assumed to depend on two factors: on the state of the labor market and on the indexation rule, that establishes that wage rates must rise in proportion with the increase in prices one period before:

\[(4) \quad w_t = -hu_t + p_{t-1} \quad ; \quad h > 0 \]

The unemployment rate increases whenever output growth falls below trend growth, \( \bar{y} \):

\(^4\) In the open economy, total costs must also include the costs with imported intermediates. The inflation rate would thus be written as: \( \hat{3} \) \( p_t = n w_t + v p_t^o \); where \( n \) and \( v \) represent the shares of wages and imported intermediates in total costs.

The inflation rate of domestic prices of imported intermediates is: \( p_t^o = e_t + s_t + p_t^{of} \); where: \( s_t \) is the rate of change of subsidies to imported intermediates and \( p_t^{of} \) is the dollar inflation rate of prices of imported intermediates. Assuming that increases in the prices of imported intermediates are washed out by subsidies, \( s_t = -(e_t + p_t^{of}) \), we obtain equation (3), where the constant \( n \) was made equal to one. Evidence shows that domestic prices of imported intermediates in Brazil where in part subsidized by the government and did not follow the international prices.
\[ u_t - u_{t-1} = -k(y_t - \bar{y}) \quad ; \quad k > 0 \]

Using (3), (4) and (5), we can write:

\[ (3') \quad p_t = hk(y_t - \bar{y}) + 2p_{t-1} - p_{t-2} \]

Equation (3'), or the short run supply schedule, is represented by SS in figure 1. Also observe that SS was drawn for given past inflation rates. As the inflation rate evolves, SS will be shifting.

In the steady state, the output growth rate equals trend growth, \( \bar{y} \), and the inflation rate is constant. To the right of \( \bar{y} \), in figure 1, prices are accelerating and inversely to its left. On the other hand, output growth is constant only if inflation equals its trend level \( \bar{p} \). Above \( \bar{p} \), output is accelerating and inversely below it.

Point E represents a saddle point equilibrium. The perfect foresight path FF is: \(^5/\)

\[ (6) \quad p_t = \lambda(y_t - \bar{y}) + \bar{p} \]

where: \( \bar{p} \equiv (a\bar{m} - \bar{y})/\left[a(1-g)\right] \)

\[ \lambda \equiv \left[q/(\Theta q - c)\right] \quad ; \quad \lambda < 0 \quad ; \quad |\lambda| > 1 \]

\[ \Theta \equiv c - a(1-g) \quad ; \quad \Theta > 0 \]

\[ q \equiv \left[(2-hc)/(2\Theta k - 1)\right] + \left\{ \left[(2-hc)/(\Theta k - 1)\right]^2 + \left[1/(\Theta k - 1)\right]\right\}^{(1/2)} \quad ; \quad 0 < q < 1 \]

\(^5/\) The perfect foresight path is derived in Appendix 1.
Observe that the more accommodating the monetary policy, the higher trend inflation, \( \bar{p} \). Also observe that \( g=1 \) implies \( \theta=c, \ q=1, \ \lim_{q \to 1} \lambda \to \infty \) and \( \lim_{q \to 1} \bar{p} \to \infty \)

2.1. Disinflation

Next we turn to some exercises on comparative dynamics.

To study a disinflation process, we look at an economy in steady state, where both growth and inflation rates equal their trend values, as represented by point E in figure 2. Trend inflation is very high and therefore the monetary authorities reduce money growth by decreasing \( \bar{m} \). Since wages are indexed, current inflation equals last period inflation and the reduction in money growth is translated into a reduction in the growth of real cash balances, increasing the nominal and the real interest rate. The impact of the new monetary policy is a reduction in output growth rate below trend while inflation is left unchanged. Next period we face an increase in the rate of unemployment and wage inflation falls below the inflation rate of the previous period, inducing a fall in the current inflation rate. This
would permit real liquidity to recover, interest rates to fall
and income growth to improve again, moving the economy to point
E" in figure 2.

Observe that the vertical shift of the perfect
foresight path in figure 2 equals \( \tilde{\phi} = (1/(1-g))\tilde{\phi}_m \), and the speed
of adjustment from the old to the new equilibrium is given by
\((1-q)/q\). The speed of adjustment is faster, the less accommodating
the monetary policy. If the parameter of accommodation is close to
one, the recession that will bring about disinflation, will have to last for
many periods. Assume that the monetary authorities decide to
fasten the disinflation process by changing the accommodation
rule, and decreasing \( g \). What will happen is that, although the
adjustment period might last for a shorter time, the recession
during the initial periods will be much more severe than under
the first program. This is illustrated in figure 3, where the
decrease in both \( \tilde{\phi}_m \) and \( g \) shifts and rotates the perfect foresight
path to the left.

2.2 An adverse productivity shock

Figure 4 illustrates the effects of an adverse
productivity shock that reduces trend growth. For an unchanged
monetary policy, the new trend inflation will also be higher.

Assume money growth was reduced in proportion with
the fall in money demand induced by the fall in output. If this
was the case, the economy would move to \( B \), where we observe the
same inflation rate as before, and output growth has fallen to
its new lower trend level.

Assume money growth is left unchanged. In this case, the fall in output growth in the current period would be less
than the fall we will observe in the long run. The decrease in
output growth rates reduces the growth of money demand in relation
to money supply inducing a fall in interest rates. Output growth
rates consistent with the level of demand can only be obtained
through overemployment, given the fall in productivity.
Overemployment induces nominal wage rates and prices to accelerate,
reducing liquidity, increasing interest rates and reducing the
level of activity till the economy sets down at the new lower
trend growth and higher inflation rates, illustrated by point E''
in figure 4.

2. The Empirical Evidence

Now that we understand how the model works, we look
back to the expressions of aggregate demand and supply. The
complete solution of the system formed by the two difference
equations (1') and (3') is:

\( p_t = (p_{t_o} - \bar{p})q_t + \bar{p} \)  \hspace{1cm} (7)

\( y_t = (1/\lambda)(p_{t_o} - \bar{p})q_t + \bar{y} \)  \hspace{1cm} (8)
By subtracting $p_{t-1}$ from equation (7) and admitting the possibility of unanticipated shocks we can write:

\[(9) \quad p_t = (1-q)\bar{p} + q p_{t-1} + \varepsilon_t\]

If there is full accommodation ($q = g = 1$) the inflation rate is a random walk:

\[(10) \quad p_t = p_{t-1} + \varepsilon_t\]

On the other hand, full accommodation also implies that $\lim_{q \to 1} \lambda = \infty$, and thus, from (8), admitting the possibility of random shocks, we obtain:

\[(11) \quad y_t = \bar{y} + \eta_t\]

We now want to examine the empirical evidence for the period 1968-1979, in Brazil, when both wages and the exchange rate were nearly fully indexed to the inflation rate.

Table 1 reports the results of the regressions for the inflation rate.
### TABLE 1

The inflation rate: 1968/79

\[(10): \quad p_t = \alpha_0 + \alpha_1 p_{t-1}\]

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_0)</th>
<th>(\alpha_1)</th>
<th>(R^2)</th>
<th>D.W.</th>
<th>S.E.R.</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Price Index</td>
<td>0.013</td>
<td>1.029</td>
<td>0.63</td>
<td>2.08</td>
<td>0.078</td>
<td>12</td>
</tr>
<tr>
<td>(annual data)</td>
<td>(0.071)</td>
<td>(0.250)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(4.120)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Price Index</td>
<td>0.009</td>
<td>0.914</td>
<td>0.60</td>
<td>1.87</td>
<td>0.022</td>
<td>48</td>
</tr>
<tr>
<td>(Quarterly data)</td>
<td>(0.008)</td>
<td>(0.110)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.122)</td>
<td>(8.298)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Prices</td>
<td>0.030</td>
<td>0.958</td>
<td>0.59</td>
<td>1.84</td>
<td>0.082</td>
<td>12</td>
</tr>
<tr>
<td>(annual data)</td>
<td>(0.073)</td>
<td>(0.251)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.412)</td>
<td>(3.818)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Prices</td>
<td>0.009</td>
<td>0.903</td>
<td>0.66</td>
<td>1.96</td>
<td>0.019</td>
<td>48</td>
</tr>
<tr>
<td>(Quarterly data)</td>
<td>(0.007)</td>
<td>(0.097)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.344)</td>
<td>(9.341)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The equations were estimated by ordinary least squares. Standard-errors and t-statistics appear in parentheses. The first two equations use the inflation rate of the general price index, column 2, in *Conjuntura Econômica* (C.E.). The last two equations use the inflation rate of consumer prices, column 6 in C.E. (or line 64 in IMF, *International Financial Statistics*, (IFS). The equation for annual data was also run using the inflation rate of wholesale prices, column 3 in C.E. (or line 63 in IFS). Results thus obtained do not differ from those in table 1.
According to results in table 1, one cannot reject the hypothesis that the intercept of equation (10) is zero and that the coefficient of the lagged inflation rate is one. The Durbin-Watson statistic, in all equations in table 1, is close to 2, and thus one cannot reject the hypothesis that the errors are (first order) serially uncorrelated. Reported in table 2 are the results of testing the joint restriction that the constant term is zero and that the coefficient of the lagged inflation rate is one. The relevant statistic for testing the null hypothesis is an F-statistic obtained from the regression $P_t - P_{t-1} = \gamma_0 + \gamma_1 P_{t-1}$. The results show that the F-statistics fall well below the critical values corresponding to the 95 and 99 percent confidence level. In all cases the null hypothesis cannot be rejected and it is concluded therefore that the inflation rate followed a random walk without trend in the period 1968-79.

We turn now to the evidence on the real growth rate, which is shown in Table 3. Using annual data, one cannot reject the hypothesis that yearly output growth rates remained constant at the trend value, except for white noise. This is not true though for quarterly output growth rates. Using quarterly data, the estimate for the coefficient of the lagged quarterly output growth rate is different from zero and smaller than one in absolute value. It is also negative, indicating that convergence is oscillatory, and its size implies that adjustment to a random shock takes on average three quarters and a half.
Table 2

\[ P_t - P_{t-1} = \gamma_0 + \gamma_1 P_{t-1} \]

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Null hypothesis</th>
<th>F-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_t - P_{t-1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. General price index</td>
<td>( \gamma_0 = \gamma_1 = 0 )</td>
<td>.014</td>
</tr>
<tr>
<td>annual data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. General price index</td>
<td>( \gamma_0 = \gamma_1 = 0 )</td>
<td>.608</td>
</tr>
<tr>
<td>quarterly data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Consumer prices</td>
<td>( \gamma_0 = \gamma_1 = 0 )</td>
<td>.028</td>
</tr>
<tr>
<td>annual data</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Consumer prices</td>
<td>( \gamma_0 = \gamma_1 = 0 )</td>
<td>.999</td>
</tr>
<tr>
<td>quarterly data</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The F-statistic is distributed as F (1,10) in equations 1. and 3.; and as F (1, 46) in equations 2. and 4. Critical values for F (1, 10) are 4.96 (95 percent) and 10.0 (99 percent). Critical values for F (1,46) are 4.1 (95 percent) and 7.3 (99 percent).
Table 3

Real GDP growth rates: 1968/79

(11): \( y_t = \alpha_0 + \alpha_1 y_{t-1} \)

<table>
<thead>
<tr>
<th></th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>( R^2 )</th>
<th>D.W.</th>
<th>SER</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual</td>
<td>0.061</td>
<td>0.341</td>
<td></td>
<td></td>
<td></td>
<td>0.13</td>
<td>1.65</td>
<td>.03</td>
<td>12</td>
</tr>
<tr>
<td>data</td>
<td>(0.027)</td>
<td>(0.280)</td>
<td></td>
<td></td>
<td></td>
<td>(2.280)</td>
<td>(1.217)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quarterly</td>
<td>0.044</td>
<td>-0.365</td>
<td>0.064</td>
<td>0.007</td>
<td>0.008</td>
<td>0.68</td>
<td>1.90</td>
<td>.02</td>
<td>48</td>
</tr>
<tr>
<td>data</td>
<td>(0.008)</td>
<td>(0.141)</td>
<td>(0.010)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(5.246)</td>
<td>(2.581)</td>
<td>(6.293)</td>
<td>(0.509)</td>
<td>(0.747)</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Note: D1, D2 and D3 are seasonnal dummies.

Equations were estimated by ordinary least squares. Standard-error and t-statistics appear in parentheses. Annual data for GDP growth rates was obtained in C.E., various issues. The quarterly data comes from Cardoso (1981), Table A.
Conclusions:

Using a Keynesian macro model, closed by monetary accommodation and indexation rules, this paper has argued that Brazilian inflation rate followed a random walk between 1968 and 1979, while annual growth rates were kept roughly constant. The model presented in section 2 helps to understand why observed inflation and growth rates in Brazil are negatively correlated (for an unchanged policy, inflation and growth rates lie on a downward sloping perfect foresight path).

The empirical evidence in section 3 permits to understand why economists have found it difficult to detect the effect of cyclical components in price equations for Brazil when using yearly data.
Appendix 1

This appendix solves the system formed by equations (1') and (3'):

(1') \[ y_t = \bar{a}m + \theta p_t - c p_{t-1} \]

where \( \theta \equiv c-a \ (1-g) > 0 \)

(3') \[ p_t = h k (y_t - \bar{y}) + 2 p_{t-1} - p_{t-2} \]

We define: \( x_t \equiv p_{t-1} \), implying \( p_{t-2} = x_{t-1} \). Next we re-write the system above to which, we add the equation:

\[ x_t = p_{t-1}, \text{ in matrix form:} \]

\[
\begin{bmatrix}
1 & -\theta & c \\
-hk & 1 & -2 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_t \\
p_t \\
x_t
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
p_{t-1} \\
x_{t-1}
\end{bmatrix}
= 
\begin{bmatrix}
\bar{a} \bar{m} \\
-h k \bar{y}
\end{bmatrix}
\]

Looking for the particular integrals, we find:

\[
\begin{bmatrix}
\bar{y} \\
\bar{p} \\
\bar{x}
\end{bmatrix}
= \frac{1}{\bar{\Delta}}
\begin{bmatrix}
0 & \theta-c & \theta-c \\
hk & 1 & 1-hkc \\
hk & 1 & 1+hk\theta
\end{bmatrix}
\begin{bmatrix}
\bar{a} \bar{m} \\
-hk\bar{y}
\end{bmatrix}
= 
\begin{bmatrix}
(\bar{a}m-\bar{y})/a(1-g) \\
(\bar{a}m-\bar{y})/a(1-g)
\end{bmatrix}
\]

where: \( \bar{\Delta} \equiv h k a \ (1-g) \)

\( \theta-c \equiv a \ (1-g) \)
To find the complementary functions we calculate the characteristics roots:

\[ \Delta = q\left( q^2 + \frac{(hk - 2)}{(1 - hk\theta)}q + \frac{1}{(1 - hk\theta)} \right) = 0 \]

The conditions for a non-oscillatory convergent path to equilibrium are:

\[ \theta hk > 1 \]
\[ hk < 2 \]

And the stable root is:

\[ q = -\frac{(2 - hk)}{2(\theta hk - 1)} + \left[ \frac{(2 - hk)^2}{4(\theta hk - 1)^2} + \frac{1}{(\theta hk - 1)} \right]^{1/2} \]

It can be shown that if there is full accommodation, \( g=1 \), which implies \( q=1 \).

Choosing the initial conditions as to rule out the
non-stable root, the solution for the homogeneous part of the system is:

\[ p_t = A_0 q^t \]

\[ y_t = (1/\lambda)A_0 q^t \]

where \( \lambda \equiv (\theta q - c)/q \)

Adding the particular integrals we come to:

(i) \[ p_t = (p_{t_0} - \bar{p}) q^t + \bar{p} \]

(ii) \[ y_t = (1/\lambda)(p_{t_0} - \bar{p}) q^t + \bar{y} \]

From equations (i) and (ii) we obtain the perfect foresight path:

\[ p_t = \lambda (y_t - \bar{y}) + \bar{p} \]
References


