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"Interrelationships among Input Costs and Product Prices: Notes on the Empirical Use of a Price Input-Output Model".

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Advertência

Este "TDI" demonstra como a análise de insumo-produto pode ser utilizada para estudar a propagação de preços numa Economia.

Os resultados empíricos, entretanto, serão apresentados oportunamente em outro "TDI".

O texto reproduzido faz parte de um trabalho mais amplo e sofreu poucas modificações, o que explica a forma em que está organizado.

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INTERRELATIONSHIPS AMONG INPUT COSTS AND PRODUCT PRICES: NOTES ON THE EMPIRICAL USE OF A PRICE INPUT-OUTPUT MODEL

4.1 - Introduction

The purpose in Chapter IV is to set down, in a more formal and detailed manner, the cost-price relationships underlying the Brazilian price input-output model used in this study of the effects that the OPEC-induced extraordinary increase in oil prices has had on domestic prices in Brazil, a large oil-importing open developing economy. The chapter also shows how average product price changes, resulting from actual or simulated cost increases, can be evaluated through index numbers. The equations are introduced twice in order to clarify the exposition: first in a more expanded way, and then in compact form, using matrix algebra. The chapter is organized as follows. Section 2 introduces the standard input-output theory employed to investigate how raising wage income, non-wage income, taxes or imported product prices is translated into final product prices and, by extension, into the price level. In Section 3, the standard input-output theory is modified to answer questions concerning how raising the price of one or more strategic inputs is manifested in the price of final products and the price level. The possibility of more than one source and/or supply price of a given strategic input is taken into explicit account. The role of indexing and a way to compute, at least approximately, the first-round effect on product prices and on the price level of tying wages to the cost-of-living is discussed in Section 4. The main points and conclusions of the chapter are summarized at the end.

4.2 - The Effects of Wage Income, Non-Wage Income and Taxes on Product Prices

4.2.1 - Cost-Price Relationships

The following analysis is based on the cost-price relationships within a separate industry as conditioned by its technical structure.[1] Let P_1, P_2, ..., P_i, ..., P_m be the prices of the products in the m separate industries; a_{i1}, a_{i2}, ..., a_{ii}, ..., a_{im} the technical input coefficients showing the amounts of product of industry 1, 2, ..., i, ..., m, needed to produce commodity i; and R_i the total value added generated in industry i per unit of its product, i.e., the difference between the price of the commodity produced by a particular industry and that part of unit costs consisting of payments for materials and services. All the above elements can be combined into an equation showing the relationship between the price P_i of the product of any industry i and the prices of all other m-1 industries whose products are used as inputs in the production of the commodity produced by industry i.

(4.1) $$a_{i1}P_1 + a_{i2}P_2 + a_{i3}P_3 + \ldots + a_{im}P_m + R_i = P_i$$

$$(i = 1,2,3,\ldots,m)$$

Equation (4.1) can be set for each of the m industries in the economy.[2] Given the technical coefficients--the small a's--we have a system of m linear equations with 2m unknown variables: the m prices, P_1, P_2, ..., P_i, ..., P_m, and the m values added, R_1, R_2, ..., R_i, ..., R_m. By assigning some definite numerical magnitudes to any m of these 2m variables, we can solve equation system (4.1) for the remaining m variables. For example, if we knew the R's we could solve for prices, the P's. This solution could be written as a system of m equations, one for each of the unknown prices:

(4.2) $$P_i = A_{1i}R_1 + A_{2i}R_2 + A_{3i}R_3 + \ldots + A_{ii}R_i + \ldots + A_{mi}R_m$$

$$(i = 1,2,3,\ldots,m)$$

The A_{ki}'s show the relationships between the price, P_i, and the value added, R_k, derived by industry K per unit of its output.[3] They depend on all the small a's, which is to say that the A's are a function of the technical structure of all the m industries in the economy. The value added, R_i, can be easily broken into its components, here considered to be simply wages, W_i, and non-wage income or profits, including depreciation, π_i.[4]

$$(4.3) \qquad R_i = W_i + \pi_i \qquad (i = 1,2,3,\ldots,m)$$

which can be rewritten as:

$$(4.4) \qquad R_i = a_{in}w_n + \pi_i \quad \text{where} \quad W_i = a_{in}w_n$$

$$(i = 1,2,3,\ldots,m)$$

Here we have expressed the wage cost per unit of output as the wage rate, w_n, multiplied by the amount of direct labor used by industry i per unit of its output, a_{in}.[5] We have thus "opened" the input-output model with respect to households. Since equation system (4.2) gave us the solution of equation system (4.1) for prices, we can now substitute in for the R's:[6]

$$(4.5) \qquad P_i = (A_{1i}a_{1n} + A_{2i}a_{2n} + A_{3i}a_{3n} + \ldots + A_{mi}a_{mn})w_n + $$
$$+ A_{1i}\pi_1 + A_{2i}\pi_2 + A_{3i}\pi_3 + \ldots + A_{mi}\pi_m$$

$$(i = 1,2,3,\ldots,m)$$

Each equation in system (4.5) shows the price of each of the commodities produced as a function of the wage rate w_n paid in all industries and the profit rates π_1, π_2, ..., π_m earned by each, i.e. the total price effect of all possible combinations of separate and simultaneous changes in wage and profit rates can be easily determined. In order to derive the expression measuring, for example, the cost-push of a given increase in the wage rate, $\Delta\omega_n$, on the m endogenously determined prices, P_1, P_2, ..., P_m, we can re-write the above equation in terms of changes, assuming that profit rates per unit of production stay the same in all industries:

$$(4.6) \qquad \Delta P_i = (A_{1i}a_{1n} + A_{2i}a_{2n} + A_{3i}a_{3n} + \ldots + A_{mi}a_{mn})\Delta\omega$$

$$\text{assuming } \Delta\pi = 0$$

$$(i = 1,2,3,\ldots,m)$$

Similar formulation also permits estimates of the inflationary impact of a rise in the profit rate per unit of output of the various industries, given the constancy of the wage rate in all industries. Likewise, it allows for estimation of the price effects of combined variations of wage and profit rates.

It must be stressed, however, that the expressions derived above have to be interpreted bearing in mind the assumptions, characteristics and limitations of input-output analysis.[7]

4.2.2 - Cost-Price Relationships Presented through Matrix Algebra

The ideas introduced above can also be expressed in a more compact form using matrix algebra, as follows.[8] If A is the direct unit input requirement matrix, then a_{iJ} is the technical input coefficient showing, as before, the amount of input i required to produce one unit of output J. Given our definition of value added, R, equation (4.1) can be re-written as:

$$\vec{P} = A'\vec{P} + \vec{R} \qquad (4.7)$$

where A' is matrix A transposed, $\vec{P}$ is the m-dimensional column vector of prices and $\vec{R}$ is the vector of value added per unit of output.[9] As explained before, given the technical coefficients and the m values added, the solution for the column vector of prices, $\vec{P}$, can be written as:

$$\vec{P} = \left[I - A' \right]^{-1} \vec{R} \qquad (4.8)$$

where I is the identity matrix with dimensions equal to those of matrix A.[10]

Except for the matrix algebra notation, (4.8) is identical to equation (4.2). The vector of value added per unit of output can now be broken into its component parts, once again considered to be simply wage and non-wage income or profits, as:

(4.9) $\vec{R} = \vec{W} + \vec{\pi}$

where $\vec{W}$ and $\vec{\pi}$ are m-dimensional column vectors of wage and non-wage income or profits, respectively. Equation (4.9) above can be further expanded to show explicitly the labor requirements per unit of output, a, and the wage rate, w.[11]

(4.10) $\vec{R} = \vec{a}w + \vec{\pi}$ where $\vec{W} = \vec{a}w$

Since (4.8) is the price solution of equation (4.7), we can now substitute in for the value added vector $\vec{R}$ to get:

(4.11) $$\vec{P} = \left[I - A' \right]^{-1} (\vec{W} + \vec{\pi})$$

where prices are a function of labor costs and profit, which, due to (4.10), can also be written as:

(4.12) $$\vec{P} = \left[I - A' \right]^{-1} (\vec{a}w + \vec{\pi})$$

Once again, the above expression can be formulated in terms of changes. For example, assuming that profit rates per unit of production stay the same in all industries, the expression measuring the cost-push of a given increase in wage rates is:

$$\Delta\vec{P} = \left[I - A' \right]^{-1} (\vec{a}\Delta\omega) \tag{4.13}$$

assuming $\Delta\pi = 0$

where prices are a function of labor productivity, here assumed to be constant, and of the wage rate.

Similar formulation, combined with the relevant data, can also be used to estimate the price effects of variations in profit rates, given constant wage rates, or of the effects on product prices of combined variations of wage and profit rates.

4.2.3 - Evaluation of Average Product Price Changes through Index Numbers

Such price changes can be better visualized if they are expressed in terms of an index number. The product price changes due, for example, to a general 10-percent wage rise can be expressed as an index number as follows. Let $\Delta P_1, \Delta P_2, \ldots, \Delta P_i, \ldots, \Delta P_m$ be the wage-induced set of product-price changes; $P_1^o, P_2^o, \ldots, P_i^o, \ldots, P_m^o$ prices in the base period; and $b_1, b_2, \ldots, b_i, \ldots, b_m$ the weights, for our purposes considered to be a function of the relative importance of each of the m industries in the economy.[12] The price index describing the corresponding wage-induced change in average wholesale prices is given by the following weighted average:

$$(4.14) \qquad \Delta I_\omega = \frac{\Delta P_1}{P_1^o} b_1 + \frac{\Delta P_2}{P_2^o} b_2 + \frac{\Delta P_3}{P_3^o} b_3 + \ldots + \frac{\Delta P_m}{P_m^o} b_m$$

$$\text{where} \quad \sum_{i=1}^{m} b_i = 1.00$$

Using equation (4.6) and factoring out $\Delta\omega_n$, (4.14) can be rewritten as:

$$(4.15) \qquad \Delta I_\omega = \Delta\omega_n \left[\left(\frac{A_{11}a_{1n} + A_{21}a_{2n} + A_{31}a_{3n} + \ldots + A_{m1}a_{mn}}{P_1^o}\right) b_1 + \right.$$

$$+ \left(\frac{A_{12}a_{1n} + A_{22}a_{2n} + A_{32}a_{3n} + \ldots + A_{m2}a_{mn}}{P_2^o}\right) b_2 +$$

$$+ \left(\frac{A_{13}a_{1n} + A_{23}a_{2n} + A_{33}a_{3n} + \ldots + A_{m3}a_{mn}}{P_3^o}\right) b_3 +$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$\left. + \left(\frac{A_{1m}a_{1n} + A_{2m}a_{2n} + A_{3m}a_{3n} + \ldots + A_{mm}a_{mn}}{P_m^o}\right) b_m \right]$$

$$\text{where} \quad \sum_{i=1}^{m} b_i = 1.00$$

Expressing each base year price, P_1^o, P_2^o, P_3^o, ..., P_m^o, as an index equal to 1.00, (4.15) becomes:

$$(4.16) \qquad \Delta I_\omega = \Delta\omega_n \left[(A_{11}a_{1n} + A_{21}a_{2n} + A_{31}a_{3n} + \ldots + A_{m1}a_{mn})b_1 + \right.$$
$$+ (A_{12}a_{1n} + A_{22}a_{2n} + A_{32}a_{3n} + \ldots + A_{m2}a_{mn})b_2 +$$
$$+ (A_{13}a_{1n} + A_{23}a_{2n} + A_{33}a_{3n} + \ldots + A_{m3}a_{mn})b_3 +$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$
$$\left. + (A_{1m}a_{1n} + A_{2m}a_{2n} + A_{3m}a_{3n} + \ldots + A_{mm}a_{mn})b_m \right]$$

$$\text{where} \quad \sum_{i=1}^{m} b_i = 1.00$$

The expression in brackets is the total weight of direct and indirect labor. If we designate it β_ω, (4.16), which represents the effect of an increase in the wage rate on the index, can be rewritten as simply:

$$(4.17) \qquad \Delta I_\omega = \beta_\omega \Delta\omega_n$$

This broad index includes goods and services produced by all the industries considered in the input-output tables used in the computations.[13]

It should be emphasized that an index like (4.17) above could be constructed for the average price changes resulting from a number of alternative assumptions about wages, such as different magnitudes of general or localized wage increases, on the assumption that profit rates remained unchanged. Analogously, an index could be constructed to measure the average impact on product prices of different assumptions about profits, assuming that wage rates remained unchanged. Finally, an index could be constructed to measure the average impact on prices of combined changes in wages and profits.

Once again, the results must be interpreted keeping in mind the assumptions, characteristics and limitations of input-output analysis.[14]

4.3 - The Effects of Energy Prices on Product Prices

4.3.1 - Cost-Price Relationships Restated

Often we may be interested not only in how a separate or combined rise in wage, profit or tax rates will affect the price of a certain commodity P_i (and, by extension, the price level, as was the case in Section 1), but also in how raising the price(s) of one or more strategic inputs will be reflected in the price of the final commodities P_i and, once again, the price level. Fortunately, with a few slight modifications, input-output theory is still appropriate. System (4.1) is solved for a different set of independent variables, omitting one of the equations. The price of the chosen strategic input, think of it as imported oil, P_2, is shifted from the set of dependent into that of the independent variables. This is done by rewriting equation (4.3) as follows:

$$(4.18) \qquad \bar{R}_i = (P_2)_i + W_i + \pi_i$$

$$(i = 1,3,4, \ldots, m)$$

which, to show explicitly the amount of oil and labor directly used by industry i per unit of output, can be rewritten as:

$$(4.19) \qquad \bar{R}_i = a_{i2}P_2 + a_{in}w_n + \pi_i \quad \text{where} \quad (P_{oil})_i = (P_2)_i = a_{i2}P_2$$

$$W_i = a_{in}\omega_n$$

$$(i = 1,3,4, \ldots, m)$$

Here the subscript 2 refers to the oil industry, i.e. P_2 is the price of oil and a_{i2} is the amount of oil required per unit of output of i. We have now "opened" the input-output model with respect to both households and the oil industry. Shifting oil prices from the set of dependent to that of the independent or exogenous variables implies not only a redefinition of the R's (as in [4.18] above), but also a redefinition of equation system (4.1) and its solution, originally given by system (4.2). These redefined systems can be rewritten as:

$$(4.20) \qquad a_{i1}P_1 + a_{i3}P_3 + \ldots + a_{im}P_m + \bar{R}_i = P_i$$

$$(i = 1,3,4, \ldots, m)$$

Equation system (4.20) is analogous to (4.1) and is rewritten here as a reminder that one particular link in the chain of cost-price relationships, that corresponding to the oil industry, has been omitted. Solving system (4.20) for prices, we arrive at:

$$(4.21) \qquad P_i = \bar{A}_{1i}\bar{R}_1 + \bar{A}_{3i}\bar{R}_3 + \ldots + \bar{A}_{ii}\bar{R}_i + \ldots + \bar{A}_{mi}\bar{R}_m$$

$$(i = 1,3,4, \ldots, m)$$

Once again, equation system (4.21) is analogous to (4.2), and the bars are a reminder that it gives the solution to the modified system (4.20), where prices for the oil industry are considered to be exogenously determined. We can now proceed as before, substituting in for the $\bar{R}$'s as defined by (4.19).[15]

$$P_i = (\bar{A}_{1i}a_{1n} + \bar{A}_{3i}a_{3n} + \ldots + \bar{A}_{mi}a_{mn})\, w_n + \tag{4.22}$$

$$+ (\bar{A}_{1i}a_{12} + \bar{A}_{3i}a_{32} + \ldots + \bar{A}_{mi}a_{m2})\, P_2 +$$

$$+ A_{1i}\pi_1 + A_{3i}\pi_3 + \ldots + A_{mi}\pi_m$$

$$(i = 1, 3, 4, \ldots, m)$$

Equation system (4.22), which is analogous to (4.5), shows the price of one commodity as an explicit function of not only the prevailing wage and profit rates in all industries, as is the case in (4.5), but also as a function of oil prices, here considered to be exogenously determined. The expressions in parentheses show how a particular price P_i varies for each monetary unit added to (or subtracted from) the wage rate, w_n, or the price of oil, P_2, respectively, assuming that profits earned per unit of output remain constant in all industries. The above equation can be rewritten in terms of changes in order to derive the expression measuring the cost-push of a given increase in oil prices, ΔP_2, on the m-1 endogenously determined prices, P_1, P_3, ..., P_m, assuming that wage and profit rates per unit of output stay constant in all industries:

$$\Delta P_i = (\bar{A}_{1i}a_{12} + \bar{A}_{3i}a_{32} + \ldots + \bar{A}_{mi}a_{m2})\Delta P_2 \tag{4.23}$$

assuming $\Delta\omega_n = \Delta\pi = 0$

$(i = 1,3,4, \ldots, m)$

Equation (4.23), combined with the relevant input-output and price data, measures the cost-push of a given variation in the price of oil, ΔP_2, on the endogenously determined P_i's.

4.3.2 - Cost-Price Relationships Restated and Presented through Matrix Algebra

As in Section 1, the modified equations introduced above can also be expressed in compact form using matrix algebra. Since the oil price, P_2, is now exogenously determined, the dimension of matrix A is reduced by a factor dependent on the number of input prices assumed to be exogenously determined (one in the present case). Call this new matrix $\bar{A}$. As explained before, the price of our exogenously determined input, oil, is here redefined as part of value added:

$$(4.24) \qquad \vec{\bar{R}} = \vec{P}_2 \div \vec{W} + \vec{\pi}$$

which, to show explicitly the amount of oil and labor directly used by industry i per unit of output, can be rewritten as:

$$(4.25) \qquad \vec{\bar{R}} = \vec{a}P_2 + \vec{a}w + \vec{\pi}$$

where $\vec{\bar{R}}$ is the vector of value added per unit of output, the $\vec{a}$'s being the oil and labor requirements per unit of output, P_2 the price of oil and w the wage rate. We saw that redefining $\vec{\bar{R}}$, as in (4.24), implies redefining equation (4.20) and its solution (4.21), which, using matrix algebra, become:

$$(4.26) \qquad \vec{\bar{P}} = \bar{A}' \vec{\bar{P}} + \vec{\bar{R}}$$

where the bars over the variables are a reminder that we now have an equation system in which the price of oil is exogenously determined, i.e., $\vec{\bar{P}}$ is the m-1 dimensional column vector of endogenously determined prices, $\bar{A}$ is the new smaller input-output matrix (m-1 x m-1) and $\vec{\bar{R}}$ is the redefined value added vector per unit of output. Given the technical coefficients and the m-1 values added, the solution for the column vector of endogenously determined prices, $\vec{\bar{P}}$, can be written as:

$$(4.27) \qquad \vec{P} = \left[I - \bar{A}' \right]^{-1} \vec{\bar{R}}$$

Once again, the bars are a reminder that (4.27) gives the solution to the modified equation system (4.26), where oil prices are exogenously determined. Substituting in for the value added vector $\vec{\bar{R}}$, as defined by (4.24), we have:

$$(4.28) \qquad \vec{\bar{P}} = \left[I - \bar{A}' \right]^{-1} (\vec{P}_2 + \vec{W} + \vec{\pi})$$

where prices are a function of oil and labor costs and profits. Due to (4.25), this can also be written as:

$$\vec{P} = \left[I - \bar{A}' \right]^{-1} (\vec{a}P_2 + \vec{a}w + \vec{\pi}) \tag{4.29}$$

where the m-1 dimensional column vector of endogenously determined prices, $\vec{P}$, is shown as an explicit function of not only wage and profit rates but also of oil prices. The theory is simply that price changes in the endogenous sectors reflect cost changes due to exogenous price movements. In order to derive the expression measuring the cost-push of a given increase in the price of oil, ΔP_2, on the m-1 dimensional vector of endogenously determined prices, $\vec{P}$, we can rewrite the above equation in terms of changes:

$$\Delta\vec{P} = \left[I - \bar{A}' \right]^{-1} (\vec{a}\Delta P_2) \tag{4.30}$$

assuming $\Delta w = \Delta\pi = 0$

Recalling that the input-output coefficients are assumed to be constant and that there is no change in per unit wage or profit rates, the above equation, combined with the relevant data, measures the cost-push of a given variation in the price of oil, P_2.

4.3.3 - Evaluation of Average Product Price Changes through Index Numbers

Equation (4.23), as explained before, measures the cost-push of a given increase in the price of oil on the price of i. In order to better evaluate the relative importance of such a set of product price changes, the ΔP_i's, they can be expressed in terms of an index. To construct this index, which measures the average impact on product prices of changes in costs resulting from extraordinary price increases of an important input--like the recent OPEC-induced oil price increase--we have to slightly modify index (4.14).

Let $\Delta P_1, \Delta P_3, \ldots, \Delta P_i, \ldots, \Delta P_m$ be the oil-induced set of product price changes; $P_1^o, P_2^o, P_3^o, \ldots, P_i^o, \ldots, P_m^o$ prices in the base period; and $b_1, b_3, \ldots, b_i, \ldots, b_m$ the weights (as in Section 4.2.3), considered to be a function of the relative importance of each of the m-1 industries in the economy.[16] The price index describing the corresponding oil-induced change in average wholesale prices is given by the following weighted average:

$$(4.31) \qquad \Delta I_2 = \frac{\Delta P_1}{P_1^o} b_1 + \frac{\Delta P_3}{P_3^o} b_3 + \ldots + \frac{\Delta P_m}{P_m^o} b_m + \frac{\Delta P_2}{P_2^o} b_2$$

$$\text{where} \quad \text{for } i \neq 2 \ \sum_{i=1}^{m} b_i = 1.00$$

$$b_2 = 0$$

Recall that the subscript 2 refers to the oil industry. Using equation (4.23) and factoring out ΔP_2, (4.31) can be rewritten as:

$$(4.32)\qquad \Delta I_2 = \Delta P_2 \left[\frac{(\bar{A}_{11}a_{12} + \bar{A}_{31}a_{32} + \ldots + \bar{A}_{m1}a_{m2})b_1}{P_1^o} + \right.$$

$$+ \frac{(\bar{A}_{13}a_{12} + \bar{A}_{33}a_{32} + \ldots + \bar{A}_{m3}a_{m2})b_3}{P_3^o} +$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$+ \frac{(\bar{A}_{1m}a_{12} + \bar{A}_{3m}a_{32} + \ldots + \bar{A}_{mm}a_{m2})b_m}{P_m^o} +$$

$$\left. + \frac{(1)\, b_2}{P_2^o} \right]$$

where $\quad$ for $i \neq 2 \quad \sum_{i=1}^{m} b_i = 1.00$

$$b_2 = 0$$

If $P_1^o = P_3^o = P_m^o = P_2^o = 1.00$, i.e. if each base year price is expressed as an index equal to 1.00, (4.32) becomes:

$$(4.33)\qquad \Delta I_2 = \Delta P_2 \left[(\bar{A}_{11}a_{12} + \bar{A}_{31}a_{32} + \ldots + \bar{A}_{m1}a_{m2})b_1 + \right.$$

$$+ (\bar{A}_{13}a_{12} + \bar{A}_{33}a_{32} + \ldots + \bar{A}_{m3}a_{m2})b_3 +$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$+ \quad (\bar{A}_{1m}a_{12} + \bar{A}_{3m}a_{32} + \ldots + \bar{A}_{mm}a_{m2})b_m +$$

$$\left. + \quad b_2 \right]$$

$$\text{where} \quad \text{for} \quad i \neq 2 \quad \sum_{i=1}^{m} b_i = 1.00$$

$$b_2 = 0$$

The expression in brackets is the total weight of oil, direct and indirect. Designating it β_2, (4.33), which represents the effect of an increase of the price of oil on the index, can be simplified as

(4.34) $\Delta I_2 = B_2 \Delta P_2$

This broad index includes goods and services produced by all industries considered in the input-output tables used in the computations.[17]

Clearly, an index such as (4.34) can be constructed for average price changes resulting from a number of alternative assumptions not only about oil prices, as suggested above, but also about the price of any other energy source of interest, assuming that all other value added components remain unchanged.

The results, once again, must be interpreted holding in mind the assumptions, characteristics and limitations of input-output analysis.[18]

4.3.4 - Consideration of More Than One Source and/ or Supply Price of Energy Commodities

4.3.4.1 - Cost-price relationships

The theory presented so far can be used in an empirical study to estimate, for example, the cost-push of a given increase in the price of oil on other prices, assuming there is only one oil price, which is reasonable if all oil, whether produced domestically or imported, is sold at the same international price. A single supply source is not, however, what is observed in most cases. In Brazil, the subject of this study, oil is produced domestically (20%) and imported (80%),[19] so there are at least two supply prices, namely the average supply price of domestically produced oil and the average supply price of imported oil.

The possibility of more than one source of oil, which may signify different supply prices, is important enough to be explicitly considered in the input-output model used in this study. The modifications required, to allow for this diversity of sources and prices are introduced here.[20] Recall that the equations in Section 3 resulted from the assumption that oil prices, P_2, are no longer endogenously determined as in Section 1. Since it is now assumed that the prices of domestically produced and imported oil are not necessarily the same, the new supply price, P_2^*, is a weighted average of the two prices, with the weights being determined by the proportion of total consumption produced domestically:

$$(4.35) \quad P_2^* = D_2 P_{d2} + (1-D_2) P_{M2} \qquad \text{where} \quad D_2 + (1-D_2) = 1.00$$

Here, P_2^*, P_{d2} and P_{M2} are the new average supply price of oil, the average price of domestically produced oil and the average price of imported oil, respectively. D_2 is the weight of domestically produced oil in relation to imported oil. This new average supply price, P_2^*, can now substitute the previous price, P_2, as follows:

$$(4.36) \qquad a_{i1}P_1 + a_{i3}P_3 + \ldots + a_{im}P_m + \bar{R}_i^* = P_i$$

$$(i = 1,3,4,\ldots, m)$$

$$(4.37) \qquad P_i = \bar{A}_{1i}\bar{R}_1^* + \bar{A}_{3i}\bar{R}_3^* + \ldots + \bar{A}_{ii}\bar{R}_i^* + \ldots + \bar{A}_{mi}\bar{R}_m^*$$

$$(i = 1,3,4,\ldots, m)$$

$$(4.38) \qquad \bar{R}_i^* = (P_2)_i^* + W_i + \pi_i$$

$$(i = 1,3,4,\ldots, m)$$

$$(4.39) \qquad \bar{R}_i^* = a_{i2}P_2^* + a_{in}W_n + \pi_i$$

$$(P_{oil})_i^* = (P_2)_i^* = a_{i2}P_2^*$$

where

$$W_i = a_{in} w_n$$

$$(i = 1,3,4,\ \ldots,\ m)$$

(4.40) $$P_i = (\bar{A}_{1i} a_{1n} + \bar{A}_{3i} a_{3n} + \ldots + \bar{A}_{mi} a_{mn})w_n +$$

$$+ (\bar{A}_{1i} a_{i2} + \bar{A}_{3i} a_{32} + \ldots + \bar{A}_{mi} a_{m2})P_2^* +$$

$$+ (\bar{A}_{1i} \pi_1 + \bar{A}_{3i} \pi_i + \ldots + \bar{A}_{mi} \pi_m$$

$$(i = 1,3,4,\ \ldots,\ m)$$

(4.41) $$\Delta P_i = (\bar{A}_{1i} a_{12} + \bar{A}_{3i} a_{32} + \ldots + \bar{A}_{mi} a_{m2})\Delta P_2^*$$

assuming $\Delta w_n = \Delta \pi = 0$

$$(i = 1,3,4,\ \ldots,\ m)$$

The starred values and the resulting new set of equations listed here are in every respect analogous to their pairs introduced earlier, the only difference being that we think of the new supply price, P_2^*, as a weighted average in order to allow for a multitude of sources that offer oil at different prices.

4.3.4.2 - Cost-price relaticnships presented through matrix algebra

For completeness, this new set of equations, allowing for cases when there is more than one supply price of oil, is also written using matrix algebra:

(4.42) $$\vec{P} = \bar{A}'\vec{P} + \vec{\bar{R}}^{*}$$

(4.43) $$\vec{P} = [I - \bar{A}']^{-1} \vec{\bar{R}}^{*}$$

(4.44) $$\vec{\bar{R}}^{*} = \vec{P}_2^{*} + \vec{W} + \vec{\pi}$$

(4.45) $$\vec{\bar{R}}^{*} = \vec{a}P_2^{*} + \vec{a}w + \vec{\pi}$$

(4.46) $$\vec{P} = [\; I - \bar{A}' \;]^{-1} \left(\vec{P}_2^{*} + \vec{W} + \vec{\pi}\right)$$

(4.47) $$\vec{P} = [\; I - \bar{A}' \;]^{-1} \left(\vec{a}P_2^{*} + \vec{a}w + \vec{\pi}\right)$$

(4.48) $$\Delta\vec{P} = [\; I - \bar{A}' \;]^{-1} \left(\vec{a}\Delta P_2^{*}\right)$$

assuming $\Delta w = \Delta\pi = 0$

4.3.4.3 - Average product price changes

To compute the average oil-induced price change, taking into account more than one supply source and/or price of oil, the relevant equations are:

(4.49) $$\Delta I_2 = \frac{\Delta P_1}{P_1^o} b_1 + \frac{\Delta P_3}{P_3^o} b_3 + \ldots + \frac{\Delta P_m}{P_m^o} b_m + \frac{\Delta P_2^{*}}{P_2^o} b_2$$

where for $i \neq 2 \quad \sum_{i=1}^{m} b_i = 1.00$

$$b_2 = 0$$

(4.50) $$\Delta I_2 = \Delta P_2^* \left[\frac{\left(\bar{A}_{11}a_{12} + \bar{A}_{31}a_{32} + \ldots + \bar{A}_{m1}a_{m2}\right)}{P_1^o} b_1 + \right.$$

$$+ \frac{\left(\bar{A}_{13}a_{12} + \bar{A}_{33}a_{32} + \ldots + \bar{A}_{m3}a_{m2}\right)}{P_3^o} b_3 +$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$+ \frac{\left[\bar{A}_{1m}a_{12} + \bar{A}_{3m}a_{32} + \ldots + \bar{A}_{mm}a_{m2}\right]}{P_m^o} b_m +$$

$$\left. + \frac{(1)}{P_2^o} b_2 \right]$$

where for $i \neq 2 \quad \sum_{i=1}^{m} b_i = 1.00$

$$b_2 = 0$$

Expressing $P_1^o = P_3^o = P_m^o = P_2^o = 1.00$

(4.51) $$I_2 = P_2^* \left[\left(\bar{A}_{11}a_{12} + \bar{A}_{31}a_{32} + \ldots + \bar{A}_{m1}a_{m2}\right) b_1 + \right.$$

$$+ (\bar{A}_{13}a_{12} + \bar{A}_{33}a_{32} + \ldots + \bar{A}_{m3}a_{m2}) b_3 +$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots +$$

$$+ A_{1m}a_{12} + A_{3m}a_{32} + \ldots + A_{mm}a_{m2}) \, b_m \; +$$

$$+ b_2 \Big]$$

$$\text{where for } i^{*} = 2 \quad \sum_{i=1}^{m} b_i = 1.00$$

$$\dot{b}_2 = 0$$

(4.52) $\Delta I_2 = B_2 \Delta P_2^{*}$

4.4 - <u>The Implications of Indexing</u>

4.4.1 - Monetary Correction

"Monetary correction" is the name given to a policy of widely applied indexing employed in Brazil in recent years.[21] It is a second-best solution to the problem of inflation, a way of adapting to price rises that cannot be completely eliminated.

Although there are several problems and misconceptions concerning the true effects of this policy of widespread indexing, for the purposes of this study, it is enough to understand the underlying principles, which are: (1) nominal values such as bond principals, savings accounts, mortgage payments, rents, the exchange rate, and fixed physical assets are adjusted to corresponding recent inflation rates; (2) wage increases are based on the average real wages that prevailed over the previous 24 months, plus prospective increases in productivity and prices.[22]

In practice, the non-wage adjustments listed under (1) have characterized an attempt to restore relative prices as a guide to the allocation of resources. The wage adjustments mentioned under (2) have served to curb inflation, since the formulas have been aimed at restoring the average real wages recorded over the previous 2 years rather than the peak real wage at the time of the last nominal increase, and have been based on projected rather than observed inflation (and productivity)--which basically means that Brazilian wage policy has been discretionary.

The purpose here is not to discuss the details of indexing or of how it has been employed in Brazil. It is sufficient to state without proof that, while indexing might have been expected to improve income distribution, it seems to have had rather regressive results, since it has been applied against wages. As pointed out previously, the prime objective was to curb inflation.[23]

4.4.2 - Wages Tied to Cost of Living

What would be the additional inflationary effect of revising wages in the face of modifications in the Brazilian labor market such as (1) productivity gains, (2) changes in demand conditions in the relevant product markets or (3) changes in the domestic cost of living? An attempt is made to use the price input-output model to give at least an approximate quantitative answer to the third point (no explicit effort is made to answer the first two points). As in the rest of this study, productivity is assumed to remain unchanged and the possible effects of changing demand conditions in the relevant product markets are ignored.

In order to allow for wages (w) to be maintained in real terms rather than kept constant, they must now be considered as a variable in the analysis. Theoretically, an additional relationship is introduced into the model, i.e. wages are fully indexed to the cost of living. The cost of living, in turn, is defined as a weighted average price index with the weights representing the proportion of total consumption devoted to the purchase of each of the goods and services, domestic and imported, included in the consumer basket. In practice however, the cost-of-living index was represented by a proxy, the WAI.[24] The WAI was computed as a broad weighted average price index including the goods and services produced by all industries considered in the input-output computations whose prices were assumed to be endogenously determined. The weights were a function of the relative importance of each of these industries in the economy.[25] The approximate additional quantitative impact of a one-round wage indexation to the cost of living on product prices and the price level was computed as follows. In a first run of the model, wages were held constant, and the WAI values obtained were assumed to be representative of the cost-of-living changes resulting from each of the simulations. In a second run, wages were allowed to rise according to the WAI changes obtained from the first run. The WAI^* changes obtained from the second run are in every respect analogous to those obtained from the first, except that in the latter case wages are assumed to follow the cost of living. Thus, the price input-output model is now solved not only for the vector of endogenously determined product prices and for a general price index (WAI) that summarizes the average product-price changes resulting from each of the simulations, but also for the wage level (W).

The first observation that can be made about the results obtained from the second run of the model, where wages were assumed to be flexible rather than fixed, is that prices rise more in all sectors of the economy.[26] The second observation is that, if wages are flexible, the pattern of relative vulnerability of the various sectors, in response to each one of the simulations, is not necessarily the same as that observed when the exercise was done assuming fixed wages. The third observation is that the differential between the WAI and WAI^* computed for each of the simulations can be considered a rough estimate of the cost of a flexible wage level, or, to be more specific, the cost of allowing for a one-round wage indexation to the cost of living. The results, however, must be interpreted and qualified due to the limitations of the technique employed, assumptions made, and practical computational problems.[27] Moreover, since the wage variable in this analysis includes salaries and employers' social security contributions, the actual impact of wage revisions alone on the cost of living would be less than indicated by the figures generated within our model.

4.5 - Summary and Conclusions

This chapter has presented in detail the cost-price relationships (within separate industries as conditioned by their respective technical structures) underlying the empirical computations to be introduced in the next.[28] This was done by setting up an equation system where the prices of each of the commodities produced were designed as a function of the wages paid in all industries, the profits earned by each, the prices of domestic and imported strategic inputs used in production (energy, agricultural products) and indirect taxes paid. An expression was derived to show how a particular price varies for each monetary unit added to (or subtracted from) any of these "exogenous" price components. The theory is simply that price changes in the endogenous sectors reflect cost changes due to exogenous price movements. It was also shown how average sectoral price changes, resulting from simulated or actual cost increases, can be evaluated through index numbers.

Furthermore, an attempt was made to determine what the additional inflationary effect of revising wages in the face of changes in the cost of living would be. In this case, prices in all sectors of the economy could be expected to rise more, and the pattern of relative vulnerability of the various sectors would not necessarily be the same as what would be observed in the absence of wage indexing.

NOTES

1. The computational procedure suggested here is a somewhat modified version of the methods described by W. Leontief and [1950] and [1947] .

2. This equation is in fact:

$$a_{i1}P_1 + P_{i2}P_2 + a_{i3}P_3 + \ldots + a_{ii}P_i + \ldots + a_{im}P_m + R_i = P_i$$

$$(i = 1, 2, 3, \ldots, m)$$

But since, for computational reasons, products consumed by the same industry in which they have been produced are excluded, i.e.. $a_{ii} = 0$, the diagonal elements in the equation system disappear. The complete equation system introduced in 4.1 is:

$$\begin{array}{lllllllll}
 & a_{12}P_2 & +a_{13}P_3 & + \ldots & +a_{1i}P_i & + \ldots & +a_{1m}P_m & +R_1 & = P_1 \\
a_{21}P_1 + & & a_{23}P_3 & + \ldots & +a_{2i}P_i & + \ldots & +a_{2m}P_m & +R_2 & = P_2 \\
a_{31}P_1 & +a_{32}P_2 + & & \ldots & +a_{3i}P_i & + \ldots & +a_{3m}P_m & +R_3 & = P_3 \\
\vdots & & & & \vdots & & & & \vdots \\
a_{i1}P_1 & +a_{i2}P_2 & +a_{i3}P_3 + & & & \ldots & +a_{im}P_m & +R_i & = P_i \\
\vdots & & & & \vdots & & & & \vdots \\
a_{m1}P_1 & +a_{m2}P_2 & +a_{m3}P_3 & + \ldots & +a_{mi}P_i & + \ldots & & +R_m & = P_m
\end{array}$$

3. The complete equation system introduced in 4.2 is:

$$
\begin{aligned}
P_1 &= A_{11}R_1 + A_{21}R_2 + A_{31}R_3 + \dots + A_{i1}R_i + \dots + A_{m1}R_m \\
P_2 &= A_{12}R_1 + A_{22}R_2 + A_{32}R_3 + \dots + A_{i2}R_i + \dots + A_{m2}R_m \\
P_3 &= A_{13}R_1 + A_{23}R_2 + A_{33}R_3 + \dots + A_{i3}R_i + \dots + A_{m3}R_m \\
&\vdots \\
P_i &= A_{1i}R_1 + A_{2i}R_2 + A_{3i}R_3 + \dots + A_{ii}R_i + \dots + A_{mi}R_m \\
&\vdots \\
P_m &= A_{1m}R_1 + A_{2m}R_2 + A_{3m}R_3 + \dots + A_{im}R_i + \dots + A_{mm}R_m
\end{aligned}
$$

The A_{ki}'s are computed as follows: let A be the matrix of technical input coefficients. Using the complete equation system 4.1 given in footnote 2, compute (I - A'), where I is the identity matrix and A' is the transposed matrix A, i.e.:

$$
\begin{matrix}
(1-a_{11}) & -a_{21} & -a_{31} & \dots & -a_{i1} & \dots & -a_{m1} \\
-a_{12} & (1-a_{22}) & -a_{32} & \dots & -a_{i2} & \dots & -a_{m2} \\
-a_{13} & -a_{23} & (1-a_{33}) & \dots & -a_{i3} & \dots & -a_{m3} \\
\vdots & & & & \vdots & & \vdots \\
-a_{1i} & -a_{2i} & -a_{3i} & \dots & (1-a_{ii}) & \dots & -a_{mi} \\
\vdots & & & & \vdots & & \vdots \\
-a_{1m} & -a_{2m} & -a_{3m} & & -a_{im} & \dots & (1-a_{mm})
\end{matrix}
$$

Recalling footnote 2, where we made $a_{ii} = 0$, this matrix becomes:

$$\begin{matrix}
1 & -a_{21} & -a_{31} & \cdots & -a_{i1} & \cdots & -a_{m1} \\
-a_{12} & 1 & -a_{32} & \cdots & -a_{i2} & \cdots & -a_{m2} \\
-a_{13} & -a_{23} & 1 & \cdots & -a_{i3} & \cdots & -a_{m3} \\
\vdots & & & & \vdots & & \vdots \\
-a_{1i} & -a_{2i} & -a_{3i} & \cdots & 1 & \cdots & -a_{mi} \\
\vdots & & & & \vdots & & \vdots \\
-a_{1m} & -a_{2m} & -a_{3m} & \cdots & -a_{im} & \cdots & 1
\end{matrix}$$

Using the matrix above, let D_{mm} be the minor of element $-a_{mm}$ and D_{mm-ki} be the minor of elements $-a_{mm}$ and $-a_{ki}$, then

$$A_{ki} = \frac{D_{mm-ki}}{D_{mm}} .$$

4. This presentation of the theory decomposes the value added, R_i, in two parts: wage income, W_i, and non-wage income or profits, π_i. This is a simplification. In the empirical computation, taxes and imported products are also defined as part of R.

5. This formulation permits alternative assumptions about labor productivity. In the computations, labor productivity is assumed to be constant, although increasing labor productivity may be introduced without much additional effort by specifying a rule by which the a_{in}'s decrease through time.

6. (4.1) $a_{i1}P_1 + a_{i2}P_2 + a_{i3}P_3 + \ldots + a_{im}P_m + R_i = P_i$ $(i = 1, 2, 3, \ldots, m)$

(4.2) $P_i = A_{1i}R_1 + A_{2i}R_2 + A_{3i}R_3 + \ldots + A_{ii}R_i + \ldots + A_{mi}R_m$ $(i = 1, 2, 3, \ldots, m)$

(4.4) $R_i = a_{in}w_n + \pi_i$ $(i = 1, 2, 3, \ldots, m)$

Substituting 4.4 in 4.2

$$P_i = A_{1i}(a_{1n}w_n + \pi_1) + A_{2i}(a_{2n}w_n + \pi_2) + A_{3i}(a_{3n}w_n + \pi_3) + \ldots + A_{ii}(a_{in}w_n + \pi_i) + \ldots + A_{mi}(a_{mn}w_n + \pi_m) \quad (i = 1, 2, 3, \ldots, m)$$

$$P_i = A_{1i}a_{1n}w_n + A_{1i}\pi_i + A_{2i}a_{2n}w_n + A_{2i}\pi_2 + A_{3i}a_{3n}w_n + A_{3i}\pi_3 + \ldots + A_{ii}a_{in}w_n + A_{ii}\pi_i + \ldots + A_{mi}a_{mn}w_n + A_{mi}\pi_m \quad (i = 1, 2, 3, \ldots, m)$$

$$P_i = (A_{1i}a_{1n} + A_{2i}a_{2n} + A_{3i}a_{3n} + \ldots + A_{ii}a_{in} + \ldots + A_{mi}a_{mn})w_n + A_{1i}\pi_1 + A_{2i}\pi_2 + A_{3i}\pi_3 + \ldots + A_{ii}\pi_i + \ldots + A_{mi}\pi_m \quad (i = 1, 2, 3, \ldots, m)$$

Which can be written simply as:

(4.5) $$P_i = (A_{1i}a_{1n} + A_{2i}a_{2n} + A_{3i}a_{3n} + A_{mi}a_{mn})w_n + A_{1i}\pi_1 + A_{2i}\pi_2 + A_{3i}\pi_3 + \ldots + A_{mi}\pi_m \quad (i = 1, 2, 3, \ldots, m)$$

7. See W. Leontief [1952] and R. Dorfman [1954].

8. See W. D. Nordhaus and J. B. Shoven [1974].

9. The complete equation system introduced in 4.7 is:

$$
\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{bmatrix}
\equiv
\begin{bmatrix}
 & a_{21} & a_{31} \cdots a_{i1} & \cdots a_{m1} \\
a_{12} & & a_{32} \cdots a_{i2} & \cdots a_{m2} \\
a_{13} & a_{23} & \cdots a_{i3} & \cdots a_{m3} \\
\vdots & & \vdots & \vdots \\
a_{1i} & a_{2i} & a_{3i} \cdots & \cdots a_{mi} \\
\vdots & & \vdots & \vdots \\
a_{1m} & a_{2m} & a_{3m} \cdots a_{im} \cdots &
\end{bmatrix}
\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{bmatrix}
+
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_i \\ \vdots \\ R_m \end{bmatrix}
$$

$$\vec{P} \equiv A' \vec{P} + \vec{R}$$

10. The complete equation system introduced in 4.8 is:

$$
\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_i \\ \vdots \\ P_m \end{bmatrix}
=
\begin{bmatrix}
1 & -a_{21} & -a_{31} \cdots -a_{i1} & \cdots -a_{m1} \\
-a_{12} & 1 & -a_{32} \cdots -a_{i2} & \cdots -a_{m2} \\
-a_{13} & -a_{23} & 1 \quad \cdots -a_{i3} & \cdots -a_{m3} \\
\vdots & & \vdots & \vdots \\
-a_{1i} & -a_{2i} & -a_{3i} \cdots \quad 1 & \cdots -a_{mi} \\
\vdots & & \vdots & \vdots \\
-a_{1m} & -a_{2m} & -a_{3m} \cdots -a_{im} \cdots & 1
\end{bmatrix}^{-1}
\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ \vdots \\ R_i \\ \vdots \\ R_m \end{bmatrix}
$$

$$\vec{P} = [I - A']^{-1} \vec{R}$$

11. See footnote 5.

12. In practice, the official Wholesale Price Index weights for Brazil ("IPA, OG.") will be used in the empirical computations.

13. In practice, the input-output tables will be aggregated to match as close as possible the components of the Wholesale Price Index for Brazil ("IPA, OG.").

14. See footnote 7.

15. The procedure here is analogous to that in footnote 6.

16. See footnote 12.

17. See footnote 13.

18. See footnote 7.

19. In fact, the proportions of domesticaly produced oil in recent years are 36,3% (1970), 34,2% (1973), 21,4% (1976), 14,8% (1979), 16,9% (1980) and 18,6% (1981).

20. See T. Watanabe [1974].

21. See A. Fishlow [1971] and C. R. Contador [1981].

22 What is described here is the situation prevailling in the late 1960's and does not necessarily reflect what has happened in more recent years.

23. See A. Fishlow [1971].

24. WAI: weighted average price index. To the extent that it will be possible to aggregate the FIBGE input-output sectors to match the FGV sectors, the WAI will be equal to the "IPA , OG".

25. See footnote 12.

26 The empirical results suggested here will be presented and discussed in detail in another "TDI".

27. See footnote 7.

28. See footnote 26.