MEASUREMENTS AND CONCEPTS OF THE INEQUALITY

OF INCOMES:

AN ALTERNATIVE BEING STUDIED IN BRAZIL

By

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MEASUREMENTS AND CONCEPTS OF THE INEQUALITY OF INCOMES: AN ALTERNATIVE BEING STUDIED IN BRAZIL

JOSÉ CARLOS PEREIRA PELIANO

Thesis under the direction of Professor Samuel Morley

The main purpose of this work should be understood as a preliminary attempt to show that, in general, measures of inequality of income imply their own concepts of the process of income distribution between individuals or groups. These underlying concepts should be brought out through a non-conventional analysis of the currently used measures and their mathematical-statistical properties. In our view, this should become the centre of attention of researchers who are interested in understanding the causes and effects of inequality of income in a given economic system.

One should emphasize that the conceptual link between the measures of inequality and their subsequent socio-economic interpretations must exist in order to obtain better theoretical and empirical use of research on income distribution. Consequently a knowledge of the characteristics and properties of the measures of inequality of income makes it possible not only to choose those which are most appropriate to the theoretical models, but also to supply valuable additional elements for a better utilization in empirical analysis. This implies that these measures, so frequently used, are not limited only to furnishing quantitative estimates of inequality of income; in addition to this, being developed as they are on their own concepts of the process of income distribution, they allow the empirical verification and proof of these concepts by means of

appropriate manipulation of their operational characteristics and properties.

In this context the characteristics and properties of three well known measures of inequality of income are presented and commented on in the first part of this work; these are Pareto's index, Gini's coefficient of concentration and the variance of logarithms. Their respective concepts of the process of income distribution are brougth out and evaluated in accordance with our interpretations and observations of other works. In the second part, taking as a basis a summary analysis of classical economic theory, an alternative measure on inequality of income is proposed. Its concept, mathematical formulation, operative statistical form as well as its fundamental characteristics and properties are developed. In the third part, the proposed measure is applied to estimate the inequality of income in Brazil in two specific cases: (a) Individual income distribution 1960/1970 and (b) Income distribution in the formal urban sector 1970/1973. Finally, the fourth part is directed to presenting the most important conclusions and suggestions for future works in this field.

Approved _____ Date_____
Adviser

... a man... can stare stupidly at phenomena; but in the absence of imagination they will not connect themselves in any rational way. (Italics mine.)

C. S. Pierce

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Obviously, I accept full blame for any remaining errors or omissions.

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INTRODUCTION

Considerable research has been carried out recently concerning income distribution both in theoretical approaches and empirical exercises. This interest is not merely academic but also political, to the extent that the results of these studies, inasmuch as they contribute to a better understanding of the phenomenon and its implications for the economic relationships in a given system of production, can lead to policy options for government programmes.

In the theoretical approaches, one notes as a general rule the "resurrection" of the classicists[1] and the improvement of the neo-classicists[2] in an attempt to grasp more clearly the causes and effects of the functioning of the economic system in the determination of income distribution.

In the empirical exercises we discover:

    (1) Analysis of the distribution of income of

        countries, regions, etc., through the application

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[1]See, for example, Maurice Dobb, "The Straffa System and Critique of the Neo-Classical Theory of Distribution", A Critique of Economic Theory, ed. by E.K. Hunt and Fesse G. Schwartz (Harmondsworth, Middlesex: Penguin Books, Inc., 1972), and Teoria del Valor y de la Distribución desde Adam Smith (Argentina: Siglo XXI Editores S.A., 1975); Joan Robinson, "Capital Theory Up to Date" and "Prelude to a Critique of Economic Theory", A Critique of Economic Theory, op. cit.

[2]Chiefly, Gary Becker, Human Capital (N.Y.: National Bureau of Economic Research, 1964) and Theodore W. Schultz, O Capital Humano (Rio de Janeiro: Zahar Editores, 1973).

of the models and measures available[3].

(2) Works directed only at the perfection of the measurement of the inequality of income distribution; these usually start with the manipulation of mathematical/statistical relationships which provide operational instruments for estimating the inequality of incomes, with special references to particular features of the phenomenon of income distribution[4].

(3) A further group of researchers try to analyse the "performance" of the measures of inequality of incomes. They are placed alongside each other to check the extent to which they estimate the "depth" and precision of income inequalities.

One begins to discover, on the basis of these works[5], that suitable measurements can be developed for the kinds of hypotheses which should be tested in studies of income distri

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[3]See, for example, Albert Fishlow, "Brazilian Size Distribution of Income", American Economic Review (May, 1972), 391-402; Carlos Geraldo Langoni, Distribuição da Renda e Desenvolvimento Econômico do Brasil (Rio de Janeiro: Editora Expressão e Cultura S.A., 1973); Rodolfo Hoffman and João Carlos Duarte, "A Distribuição de Renda no Brasil", Revista de Administração de Empresas (São Paulo), No. 2 (June, 1972), 46-66.

[4]See, for example, Anthony B. Atkinson, "On the Measurement of Inequality", Journal of Economic Theory, II (1970), 244-263; O. Elteto and E. Frigyes, "New Income Inequality Measures as Efficient Tools for Causal Analysis and Planning", Econometrica, XXXVI (April, 1968), 383-396.

[5]See, for example, Ramonaval Augusto Costa, "Medidas de Desigualdade de Renda", Boletim Geográfico (Rio de Janeiro), No. 238 (Jan./Feb., 1974), 45-72; João Carlos Duarte, "Aspectos da Distribuição da Renda no Brasil em 1970" (unpublished M.A. Thesis, University of Piracicaba, São Paulo, 1972); Rodolfo Hoffman, "Contribuição à Análise da Distribuição da Renda e da Posse de Terra no Brasil" (unpublished Livre Docência Dissertation, University of Piracicaba, São Paulo. 1971).

bution. Indirectly, one can also perceive that each measure of inequality of income brings with it, explicitly or implicitly, a concept of the operation of the process of income distribution even though this group of researchers does not explicitly mention this conceptual relationship. The link between the concept and the operation of the process becomes credible as a way of making more valid these measures and their respective analytical properties.

In view of these considerations, this thesis has three objectives. Firstly, to list some of the measures traditionally employed in estimating the inequality of income and attempt to evaluate their concepts of income distribution and their measurement properties. Secondly, to supply an alternative interpretation of the process through a further concept of the mechanism underlying the determination of the distribution of income generated by the productive process between groups of individuals. Thirdly, to operationalize an interpretation of the process through an alternative measure of inequality of income, applying it to a few income distributions available for Brazil.

Combining the first and second objectives, the intention is to provide systematic view of the analysis of income inequality based on a specific conceptual reference. With this, one may hope to warn the user that the consistent use of a measure of income inequality implies a certain conceptualization of the process of income distribution. It is hoped to have provided some elements which may remind researchers that, ultimately, underlying a certain measure of income distribution there is an understanding of how inequality of income comes about. This has tended to be forgotten in many of the studies in this field. The importance of the last objective, the operational aspect, is secondary to the extent to which one intends to understand the process as such, while making it clear that the improve

ment of studies on income distribution, especially the
estimates of inequality, requires a systematic effort at
understanding the features which involve the economic system.

CHAPTER I

CONCEPTS OF THE DISTRIBUTION OF INCOME

It is intended in this section to present in summary form, on the one hand how the phenomenon of the distribution of income is conceived by some of the measures of inequality of income and, on the other to develop some comments about the features of these measures.

Pareto's Index

Vilfredo Pareto[6] introduced a measure of inequality of income which tried to show, in a simplified way, how the distribution of incomes manifests itself among individuals. His concept of the phenomenon can be said to be "empirical" insofar as it originated in the observations of the profiles of income distribution in various countries of his time.

The mathematical representation of his function of income distribution would be[7]

$$N = AY^{-b}$$

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[6]Regarding Pareto's index see: Ramonaval Augusto Costa, "Medidas de Desigualdade de Renda", op.cit.; Rodolfo Hoffman, "Contribuição à Análise da Distribuição da Renda e da Posse de Terra no Brasil", op.cit.; Jorge Kingston, "A Desigualdade na Distribuição das Rendas", Revista Brasileira de Economia (Rio de Janeiro), No. 1 (March, 1952), 9-13.

[7]This is known as the first of Pareto's laws, that is the simplified form of his function of income distribution. Concerning Pareto's second law see: Rodolfo Hoffman, "Contribuição à Análise da Distribuição da Renda e da Posse de Terra no Brasil, op.cit.; Ramonaval Augusto Costa, "Medidas de Desigualdade de Renda", op.cit.

where N represents the number of people with income Y or
more; A and b are constants. The constant $\underline{b}$ is called
Pareto's coefficient (or index).

It will be noted from the interpretation of the
Pareto function (in hyperbolic form) that the number of
people with an income Y_i or more decreases as income
increases, in other words the number of people with an income
$\overline{Y}_i$ or more is inversely proportional to the levels of income
$\overline{Y}_i$. This inverse relationship is shown logically by the
coefficient b, the differential marginal coefficient of the
Pareto function. The graphical presentation of this statement
can be seen in figure 1.

Some points may be underlined among the observations
made about Pareto's index:

(1) The implicit concept of inequality of income
among individuals (or groups of individuals) can
be understood as the visual presentation of a
social pyramid where the individuals at the base
receive average incomes less than the individuals
at the top. This inverse relationship between the
number of individuals and the average levels of
income would be, according to Pareto, the most
common aspect of the distributions of income among
the units observed (countries, regions, etc.)
independently of the socio-economic conditions
which generated these distributions. Consequently,
his index represents the degree of inequality of
income existing in a particular distribution,
given the productive relationships preceding the
distribution of the proceeds among the indi-
viduals.

(2) It can be said that his index reflects a
"partial" concept of the phenomenon of distri-
bution. "Partial" to the extent that it reflects

FIGURE 1

PARETO'S FUNCTION

$$(N = AY^{-b})$$
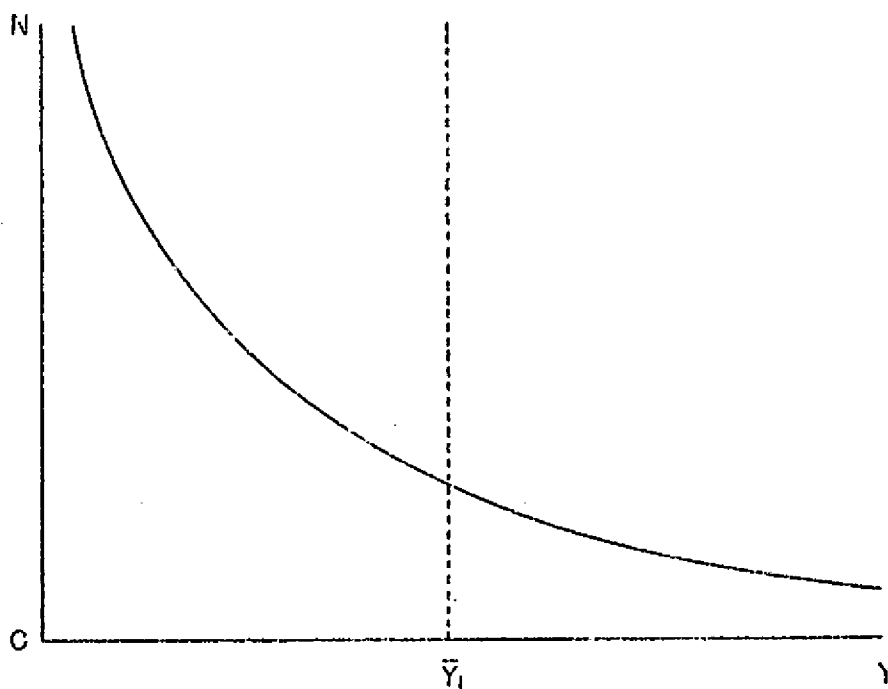
a simple functional relationship between
individuals and the respective average levels of
income without any attempt to perceive the
mechanism of distribution of income as such.
(3)   Some other obvious characteristics[8] of
Pareto's index separated into two groups are
summarized briefly below:.

(a)   Advantages:

.i)   Its calculation is relatively simple.

ii)   It is generally used for estimating
the average incomes of the highest and
lowest groups (when these are represented
with the respective lower and upper limits
left open).

iii)   Its use is immediate irrespective of
the presentation of the distribution.

(b)   Disadvantages:

i)   It is not very sensitive to small
differences between the distributions of
income.

ii)   Because of the fact that the ad-
justment of the curve to the observed
data is only significant above a certain
level of income $\overline{Y}_i$ (see figure 1), it
often happens that only the upper part of
the distribution is well interpreted by
the Pareto function; this does not happen
with the lower part of the distribution
where the inequality of income (as a

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[8]For better information on these characteristics,
see: Ramonaval Augusto Costa, ibid.

general rule) is less, so that Pareto's
reasoning is less applicable.

iii) A poor adjustment of the function
to the observed data produces an
inadequate interpretation of the concen-
tration of income distribution through
his index b.

Gini's Coefficient of Concentration

Corrado Gini made a dual contribution to the study of
the concentration of incomes. He developed two measures, one
of them little known to researchers and described as Gini's
index, and the other generally used, the Gini's coefficient of
concentration. We are dealing here with the latter, not only
because of its greater use by researchers, but also because
of its practical significance when associated with the
Lorenz curve.

Given the Lorenz curve, Gini's coefficient of concen-
tration (G) [9] can be visualised (see figure 2). One observes
that the area OW (neutral area) refers to the situation of
perfect equality of income (where all the individuals have
the same share of total income); that the area OZW refers to
the situation of extreme inequality of income (where only one
individual receives the whole income of the distribution);
and finally that the area OXW refers to the situation of
intermediate inequality of income (where the shares of some
individuals or groups in the total income exceed the shares
of others in a differential manner).

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[9]The Lorenz curve is obtained by placing on the hori-
zontal axis the cumulative percentages of population and on
the vertical axis the corresponding cumulative percentages of
total income. In this way, one can visualise the partici-
pation in total income of each respective group of indi-
viduals being considered.

FIGURE 2

LORENZ CURVE
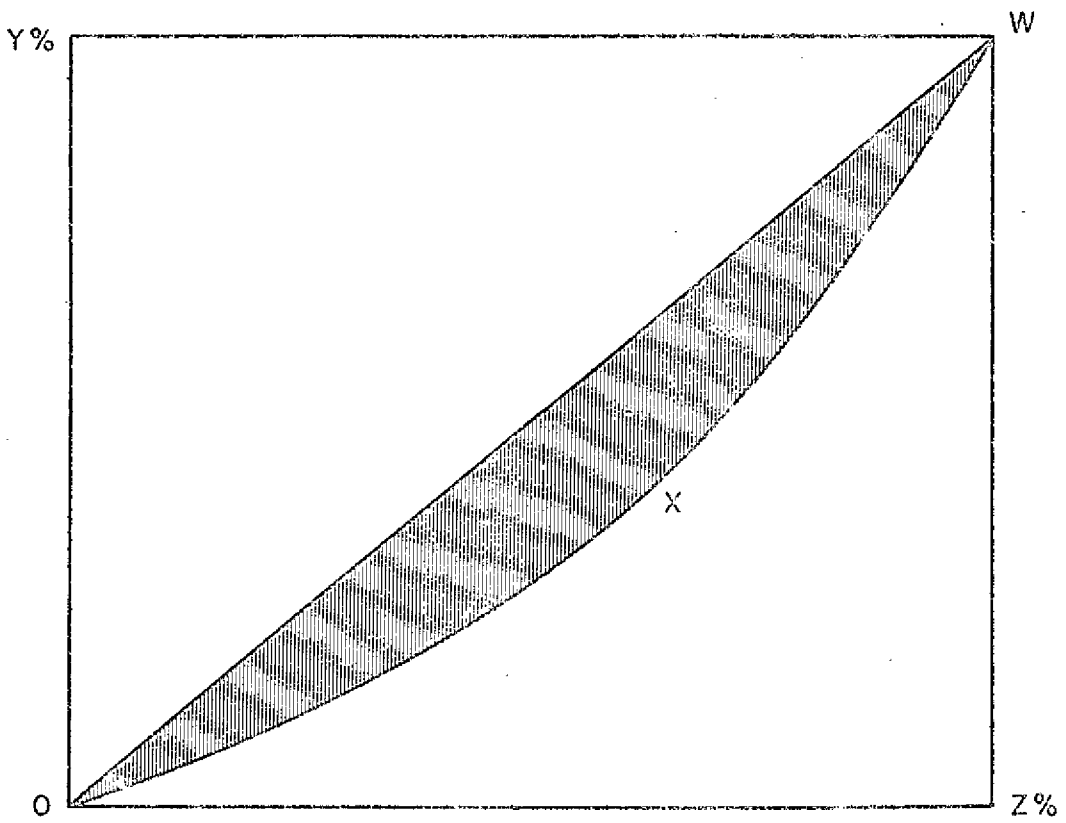
To obtain G through the Lorenz curve, it is sufficient to relate the areas OXW and OZW as follows:

$$G = \frac{OXW}{OZW}$$

whose limits of variance are $\underline{0}$ (area OW) and $\underline{1}$ (area OZW).

Given these initial observations, we may consider below the main aspects related to Gini's coefficient of concentration (G):

(1) The implicit concept of the inequality of income in G can be interpreted simply by the different income shares of individuals generated by the productive process. This relative differentiation therefore will reflect the degree of the inequality of income existing in the distribution.

(2) Compared with a simple functional relationship as in the case of Pareto's index, G shows in a clearer way the phenomenon of income distribution. Specifically a better or worse apparent concentration of income in a given distribution will depend directly on the share of total income of the individuals which produced it; the more unequal the shares, the worse is the degree of concentration of income in G and vice-versa.

(3) Summarized below are some characteristics of Gini's coefficient of concentration:[10]

(a) Advantages:

i) It is sensitive to small variations between distributions of income.

ii) It is a simple and clear concept (particularly when linked with the Lorenz curve).

iii) It is easily and quickly calculated.

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[10]See Ramonaval Augusto Costa, "Medidas de Desigualda de de Renda", op.cit.

iv) It reflects the inequality of income in
the whole of the distribution.

v) It can be disaggregated into components
of concentration "within" and "between"
distributions.

(b) Disadvantages:

i) Its underestimation of the area of
concentration of income[11] by the methods of
calculation traditionally used.

ii) The interpretation of its value becomes
ambiguous in the case where two Lorenz
curves intersect with the same area of
concentration.

Variance of Logarithms

This deals basically with a measure of dispersion
relative to the average income of the distribution, with the
specification that one takes instead of absolute values the
logarithms of the incomes (individuals or groups of indi-
viduals). The use of logarithms arises from the effort to
minimize the effect of the absolute differences of income.

The expression for the variance of logarithms is the
following:

$$\sigma^2 \log = \frac{1}{X} \cdot \sum_{i=1}^{X} (\text{Log } \overline{Y}_i / \overline{\overline{Y}})^2$$

where X = the number of individuals in the distribution

$\overline{Y}_i$ = individual income

$\overline{\overline{Y}}$ = geometric average of the individual incomes;

where the data provide a distribution by classes of income
the expression is corrected:

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[11]For more details see Rodolfo Hoffman, "Contribuição
à Análise da Distribuição da Renda e da Posse de Terra no Bra
sil", op.cit.

$$\sigma^2 \log = \frac{1}{X} \cdot \sum_{I=1}^{K} X_I \ (\text{Log} \ \overline{Y}_I / \overline{\overline{Y}})^2$$

where X = the number of individuals in the distribution;

X_I = the number of individuals in class I;

$\overline{Y}_I$ = the average income of individuals of class I

$\overline{\overline{Y}}$ = geometric mean of average incomes of K income classes;

K = the number of income classes.

With reference to the concept of income distribution implicit in $\sigma^2 \log$, it is worth pointing out that this represents a rather static view of the phenomenon insofar as it takes the difference of the squares of the deviations of average incomes in relation to the geometrical average of the distribution. Although this concept is correct and workable it brings nothing new to the understanding of the phenomenon if we wish to treat it in a wider perspective, which at least clarifies further the complex mechanisms underlying the process of income distribution. At first sight, one has the impression that the $\sigma^2 \log$ may be only a mathematical device, although suitably developed which makes more efficient the use of σ^2, which in Statistics has basic applications.

With respect to the operational characteristics of this measure one may add:

(a) Advantages:

i) easy to interpret since it consists of a measure of dispersion;

ii) disaggregated into components of the variation of income "within" and "between" classes or distributions, in accordance with the property of σ^2.

iii) it has the result of minimizing the effect

of absolute differences of income through the

use of logarithms of the incomes;

iv) easy to calculate because it uses the

logarithms of the incomes when available.

v) it is a measure sensitive to small

variations between income distributions.

(b) Disadvantages:

i) the non-availability of the logarithms of

the incomes makes immediate calculation more

difficult;

ii) the fact that it is related to the geometric

averages of the individual incomes, and not to

the arithmetical average of the distribution

accentuates dispersal effect of the measure;

iii) the non-availability of the individual

income data makes the measure less efficient for

estimating the degree of inequality of the

income distribution.

Further Observations

The summary consideration of Pareto's index, Gini's
coefficient of concentration and the variance of logarithms
allows one however to highlight some aspects of the evaluation
of the inequality of income distributions. These are necessary
for the understanding of proper use of the available measures
of inequality.

First, that all the measures of inequality of income
have an implicit concept of the process of income distri-
bution.[12]

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[12]See Anthony B. Atkinson, "On the Measurement of
Inequality", op.cit.; O. Elteto and E. Frigyes, "New Income
Inequality Measures as Efficient Tools for Causal Analysis
and Planning", op.cit.

Secondly, that some measures reflect some aspects of concentration of income in comparison with others. In the cases analysed, Pareto's index tends to be a more efficient measure of inequality for the upper range of the distribution (beginning with the level of average income $\overline{Y}_i$); whilst Gini's coefficient of concentration and the Variance of Logarithms give a good representation of the inequality of income of the whole distribution.

These two observations indicate that, depending on the characteristics to be brought out in the evaluation of the inequality of an income distribution, there will be appropriate measures to be used.

Finally, it is interesting to point out that, in operational terms, the proper use of a measure of inequality of income requires that it should be at least:

(a) easy and precise to interpret;
(b) quickly calculated;
(c) independent of the form of the distribution;
(d) sensitive to differences between distributions;
(e) sensitive to ambiguity between distributions.[13]

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[13]Ambiguity, for example, as in the case of the Lorenz curve when it may happen that the comparison of two of them intersecting provides the same value for the Gini's coefficient of concentration.

CHAPTER II

AN ALTERNATIVE CONCEPT

In this section a further way of visualizing the
phenomenon of income distribution is developed in order to
propose later a measure of inequality of income. Following
the lines of the previous section the concept of income
distribution is approached first, followed by the mathe-
matical formulation of this concept, and finally the charac-
teristics of the proposed measure of inequality of income.
The Means of Production and Income Distribution

The classic theories (including Marx) about the value
and distribution of goods and services[14] bring out the
importance of the ownership of means of production in the
determination of the distribution of the incomes, although
with varying approaches.[15] In reality, they are practically
the only theories (with of course their followers) which try
to go beyond the process of distribution of goods and
services. They do not take this (the Neo-Classic and
Keynesian positions) as being established in a competitive
market of factors of production where the prices of goods
and services are determined by the marginal productivities of

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[14]See, especially, Maurice Dobb, Teoria del Valor y de
la Distribución desde Adam Smith, op.cit.

[15]These variations are clear from Adam Smith to Marx.
The latter, in turn, maybe has succeeded best in integrating
the theories of production and distribution.

those factors. In the end, they start from socio-economic reality, historically determined; through an analysis of the social structure to distinguish the difference in the owner-ship of the means of production (social relationships of production) as determinants of the distribution of goods and services produced for the market.

It is evident that political and historical factors exist which are hard to measure - which interacting with the classical economic variables - determine the distribution of individual incomes.[16] However, one may suppose a close relation ship between the ownership of the means of production and the structure of political power historically determined. It is reasonable to suppose that both vary in the same direction. The exceptions may become apparent insofar as political power, other things being equal, manifests itself with support of economic power.[17]

It is clear that the inter-relation between the owner ship of the means of production and the structure of income or rather the historical-structural determination of the latter by the former has an outstanding place in economic writing, particularly in developing countries. The external relationships of these with developed countries clearly reflect an unequal social division of labour in which the latter, with the greater accumulation of capital and technology, determine the distribution of factors in the former and, consequently bring about concentrated income

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[16]See Irma Adelman and Cinthia Taft Morris,"¿Quien se beneficia con el desarrollo economico?", Distribución del Ingreso, ed. by Alejandro Foxley (Mexico: Fondo de Cultura Economica, 1974).

[17]This position is reinforced by Jorge Graciarena, "Estrutura de Poder y Distribución del Ingreso en America La-tina", ibid.

structures.[18] In short, it may be supposed that the domination
or existing influence of the "strong" over the "weak",
economically and politically speaking, has been maintained
historically, just as much between countries as between their
social classes. This inequality of economic opportunities,
determined by the marked stratification of the social
structure, is reflected objectively in the different
appropriations of goods and services produced for the market.
It is towards this aspect that the objective of this section
is directed which is that of highlighting productive relation
ships which, within the economic system, determine the
relationships of distribution of the factor rewards.
The Question of Interdependence

The ownership of the means of production in a given
economic structure determines the distribution of goods and
services (real incomes) or of the factor rewards (money
incomes). So the allocations of income between individuals
(or groups) is a consequence of the entire flow of goods and
services produced in a given period. In this way, within the
absolute limits of this flow of production, the ownership of
the means of production acts and confers on the individuals
(or groups) differential access to the rewards. In other
words, the rewards of individuals (or groups) are not
determined separately in the process of production and
therefore differentiated in the process of distribution of
goods and services; in reality, they are determined jointly
in the productive process in accordance with the possessions
of means of production by the individuals (or groups), given
the flow of goods and services produced within the period.

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[18]See, especially, Celso Furtado, Análise do Modelo
Brasileiro (Rio de Janeiro: Civilização Brasileira S.A.,
1973).

In the flow of total income generated over a given [19]
period, there is an interdependence of incomes of individuals
(or of groups). This reflects the fact that mainly for
reasons of their respective economic bargaining power (or
through the influence of differential ownership of the means
of production), the higher allocation of income to some
(individuals or groups) necessarily implies a smaller
allocation to others.

The Interpretation of the Process of Income Distribution

The previous sections commented on the inter-relation
ship between the ownership of the means of production and the
allocation of income as well as on the question of inter-
dependence of incomes. It becomes now possible to interpret
in summary form how the distribution of rewards between
individuals (or groups) takes place.

In the first place, the proposition concerning the
inter-relationship between the ownership of the means of
production, (Z), and the allocation of income, (Y), suggests
a direct proportional relationship in the sense that a larger
(Z) means a larger (Y), and vice-versa. Without doubt this
situation can be generalised although the given historical
setting of social-economic structures show specific variations
around this proportional relationship. However, it seems
impossible to deny that this will always be positive, mainly
because of the fact that individuals (or groups) tend to
maximize their relative benefits (or rewards) in their
economic activities.

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[19]This aspect is not new, because it is found
explicitly in most of the measures of inequality of income.
The objective here was to highlight this aspect because of
its decisive importance for understanding the process of
income distribution - a fact which is generally neglected
in other studies.

In the second place, the question of interdependence
in the allocations of income between individuals (or groups),
(X), tends to show the dynamics of the social relationships
of production in the sense that, by the destination of the
flow of goods and services produced in a given period,
individuals will compete among themselves (or groups) in such
a way as to define their respective shares (or allocations).
In this case, the ownership of the means of production plays
a decisive part since it already reserves for its owners a
large part of the rewards obtained in the market.
Consequently, the interdependence of incomes will specify
the differences in shares of income between individuals (or
groups), that is some will necessarily appropriate more than
others within the total flow of income generated in the
period.

Thirdly, it is necessary to attempt an integrated
view of the process of income distribution. The conjunction
of the processes of production and distribution[20] of goods
and services, which we are trying here to make operational,
gives the analysis of income distribution a more dynamic
perspective with the specific structural implications. In
this sense, it starts from the proposition that differential
ownership of the means (or factors) of the productive process
determines the differential allocation of the rewards of
those factors. This difference, in its turn, establishes in
the market the distribution of the flow of goods and services
produced among the individuals (or groups) through the
mechanism of interdependence of incomes, which specify the
absolute levels of income differentially allocated.

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[20]The distribution of goods and services in itself
only signifies the distribution of real incomes which will be
converted into rewards (monetary) by entry to the market.

Thus, the differential allocation of incomes is determined a priori through the differential ownership of the means of production and realised a posteriori through the mechanism of interdependence of incomes.

An Operational Form of the Interpretation of the Process of Income Distribution

Taking the conceptual sequence of the previous sections we present the mathematical formulation of the interpretation of the process of income distribution.

To give effect to that interpretation the following form is chosen:

(a) Let $\bar{Z}_I$ and $\bar{Y}_I$ respectively be the average stock of the means of production and the average income of the group of individuals X_I.

Then, the expression

$$\sum_I X_I \bar{Z}_I \bar{Y}_I \quad , \quad (1)$$

indicates the sum of all groups of individuals in the distribution with the respective allocations of income and means of production.

(b) If all the individuals in the distribution were allocated the same level of income $(\bar{Y})$, we could write

$$\sum_I X_I \bar{Z}_I \bar{Y} \quad , \quad (2),$$

(c) Verification of the absolute inequality of income between all the individuals in the distribution is possible by utilizing the expressions (1) and (2) as follows:

$$\sum_I X_I \bar{Z}_I \bar{Y}_I - \sum_I X_I \bar{Z}_I \bar{Y} = \sum_I X_I \bar{Z}_I (\bar{Y}_I - \bar{Y}), \quad (3),$$

which final form signifies the weighted sum of all the absolute differences of income of the groups of individuals between the observed distribution and the

ideal distribution,

(d)   The relativity[21] of the measure is obtained by the conjunction of the expressions (1) and (3) or

$$\sum_I X_I \bar{Z}_I (\bar{Y}_I - \bar{Y}) / \sum_I X_I \bar{Z}_I \bar{Y}_I^{22} \quad , \quad (4).$$

Practical Application of the Operation

It is difficult to find operational support for the proposition that the ownership of the means of production determines income distribution since the usual systems of information used do not identify separately the first variable.   On the one hand this is because it has become customary to simply reproduce traditional systems of economic information based on Keynesian theory[23] and on the other hand because the very concept of means (or factors) of production makes the question open to different interpretations which seriously impedes its statistical application.

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[21]Obtaining positive values for this relative measure depends on the proposition that the ownership of the means of production (stock) is directly proportional to the absolute levels of income of the group of individuals holding them.

[22]In reality, the expression (4) can be understood as the weighting of the relative differences of income by the share of the groups of individuals I = 1, . . . K, in the total ownership (stock) (Z) of the means of production, i.e.

$$\sum_I X_I \bar{Z}_I (\bar{Y}_I - \bar{Y}) / \sum_I X_I \bar{Z}_I \bar{Y}_I = \frac{\sum_I X_I \bar{Z}_I}{\sum_I X_I \bar{Z}} \cdot (\bar{Y}_I - \bar{Y}) / \frac{\sum_I X_I \bar{Z}_I}{\sum_I X_I \bar{Z}} \cdot \bar{Y}_I .$$

[23]In this case, Simon Kuznets is one of the exponents of that part which relates to statistical manipulation of macro-economic aggregates.

In this way, except for specific studies previously
defined methodologically, the alternative proposed here for
measuring the inequality of income is at a disadvantage.
The practical application of this alternative is not totally
unserviceable because it is possible, with a reasonable
degree of confidence, to adapt it suitably to the established
conceptual aims.

Thus, starting from the proposition that the owner-
ship of the means of production determines income distri-
bution, and that, consequently, there exists a direct
relationship between the two variables (i.e. both vary in
the same direction) it becomes possible by a workable method
to substitute the first by the second. In other words, one
may use the level of income ($\bar{Y}_I$) of the group of individuals
(I) as a "proxy" variable for the average stock of the means
of production ($\bar{Z}_I$) of this group of individuals.[24] It is
evident that the results will not be the same if the initial

---

[24]A more specific interpretation of this proposal
could arise from the relations $\bar{Y}_I = r_I \bar{Z}_I$, where r_I is the
average rate of return on the stock of the means of
production of the group of individuals I.
Given that,

$$\bar{Y}_I = \sum_j r_{Ij} \bar{Z}_{Ij} / \sum_j r_j \bar{Z}_j ,$$

$$I = 1, \ldots K \text{ e } j = 1, \ldots J$$

and,

$$rj = \sum_{Ij} r_{Ij} X_{Ij} / \sum_j X_j \text{ for each } j,$$

then,

$$\bar{Y}_{Ij} = r_j \bar{Z}_{Ij} ,$$

where rj = the average rate of return on the stock of
means of production of the distribution j.

Furthermore, one may adapt the neo-classical variation
of the theory of human capital, following the justification
above, taking $\bar{Z}_I$ as the average stock of human capital of the
group of individuals I and, consequently, altering the
significance of r_I and r_j .

expression (4) is used; however it is probable that in periodical comparisons the variations of the two measures (the original and the adapted) follow the same directions.

Thus the indicated operational adaptation reduces to:

$$\sum_I X_I \bar{Y}_I (\bar{Y}_I - \bar{Y}) / \sum_I X_I \bar{Y}_I^2 = (\sum_I X_I \bar{Y}_I^2 - \sum_I X_I \bar{Y}_I \bar{Y}) / \sum_I X_I \bar{Y}_I^2 , \quad (5),$$

As $\sum_I X_I \bar{Y}_I = X \bar{Y}$ and substituting in (5) we see

$$(\sum_I X_I \bar{Y}_I^2 - X \bar{Y}^2)/\sum_I X_I \bar{Y}_I^2 = \sum_I X_I (\bar{Y}_I^2 - \bar{Y}^2)/\sum_I X_I \bar{Y}_I^2 , \quad (6).$$

Given that,

$$\sum_I X_I (\bar{Y}_I^2 - \bar{Y}^2)/X \bar{Y}^2 = C V^2 , \quad (7),$$

the following relationship is obtained,

$$(\sum_I X_I \bar{Y}_I^2 - \sum_I X_I \bar{Y}^2)/X \bar{Y}^2 = \sum_I X_I \bar{Y}_I^2 /X\bar{Y}^2 - 1 = C V^2$$

and finally,

$$\sum_I X_I \bar{Y}_I^2 /X\bar{Y}^2 = C V^2 + 1, \quad (8)$$

Substituting the expressions (7) and (8) in (6), we obtain:

$$\sum_I X_I (\bar{Y}_I^2 - \bar{Y}^2)/\sum_I X_I \bar{Y}_I^2 = \frac{\sum_I X_I (\bar{Y}_I^2 - \bar{Y}^2)}{X\bar{Y}^2} \div \frac{\sum_I X_I \bar{Y}_I^2}{X\bar{Y}^2} = C V^2/C V^2 + 1, (9)$$

where $C V^2$ = the square of the coefficient of variation.[25]

---

[25]Note that the expression $C V^2$ used in the text refers, as was shown, to grouped income data. Also note that $C V^2 = (\sigma/\bar{Y})^2$, where σ = the standard deviation of the distribution.

In this way the expression (9) is the final form of the proposed measure of inequality of incomes.[26]

The Range of Variation of the Measure

From (9) the limits of variation of the measure are:

(a) Maximum Equality of Income

In this case, where all the individuals in the distribution, whatever their number may be, receive equal incomes, or $\bar{Y}_I = \bar{Y}$, so that $\sigma^2 = 0$, the measure consequently is equal to 0.

(b) Maximum Inequality of Income

In this case, where only 1 (one) individual receives all the income in the distribution, when $X \to \infty$, $\bar{Y} \to 0$, and consequently the measure tends to the limit 1.

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[26] In a recent study not known to us till the final stage of this thesis, Jorge de Souza, "Duality and Concentration", Brasília, 1974 (Mimeographed), arrived at an alternative measurement of the inequality of income of a distribution, whose final expression is the same as the one suggested here. Though the results of both works may be the same in their final expression, the two attempts are not exclusive since the concepts and methodologies used by the two authors are themselves different. While Souza starts with the property of the "duality of function", econometric approach, our work takes an economic approach. Far from constituting similar works, the two attempts are on the one hand specific, to the extent to which the fields of analysis are different, and on the other hand complementary to the extent to which the statistical/mathematical framework is necessary in giving operational form to the measurable economic interpretations. Finally, it is relevant to point out, throughout our work, that our concern was directed more to the concept of the phenomenon of income distribution, evidently capable of being made operational than to elaborating a measure of inequality as such.

In brief, $0 \leq \dfrac{CV^2}{CV^2 + 1} < 1.$

Graphical Presentation

The expression (6) permits the following graphical presentation:

from $\dfrac{C V^2}{C V^2 + 1} = \dfrac{\sum\limits_{I=1}^{K} X_I \, \overline{Y}_I^2 - \overline{X} \, \overline{Y}^2}{\sum\limits_{I=1}^{K} X_I \, \overline{Y}_I^2} = \dfrac{\sum\limits_{I=1}^{K} X_I \, (\overline{Y}_I^2 - \overline{Y}^2)}{\sum\limits_{I=1}^{K} X_I \, \overline{Y}_I^2}$

we have (see figure 3).

Thus, $\dfrac{\sum\limits_{I=1}^{K} X_I \, (\overline{Y}_I^2 - \overline{Y}^2)}{\sum\limits_{I=1}^{K} X_I \, \overline{Y}_I^2} = \dfrac{(AREA\ C + AREA\ B) - (AREA\ C + AREA\ A)}{AREA\ C + AREA\ B} =$

$= \dfrac{AREA\ B + AREA\ A}{AREA\ C + AREA\ B},$ where $AREA\ A = |-AREA\ A|$

Disaggregation of the Measure

An important characteristic of the proposed measure of inequality of income is the fact of its being disaggregated into two components, one which measures the inequality of income observed "within" the classes or distributions and the other which measures the inequality of income "between" the classes or distributions. In the case of measuring the inequality of income "within" and "between" the classes of a distribution, it is indispensable to have data on individual incomes, which will furnish the necessary information for the disaggregation.

FIGURE 3

GRAPHICAL PRESENTATION OF $CV^2/CV^2 + 1$
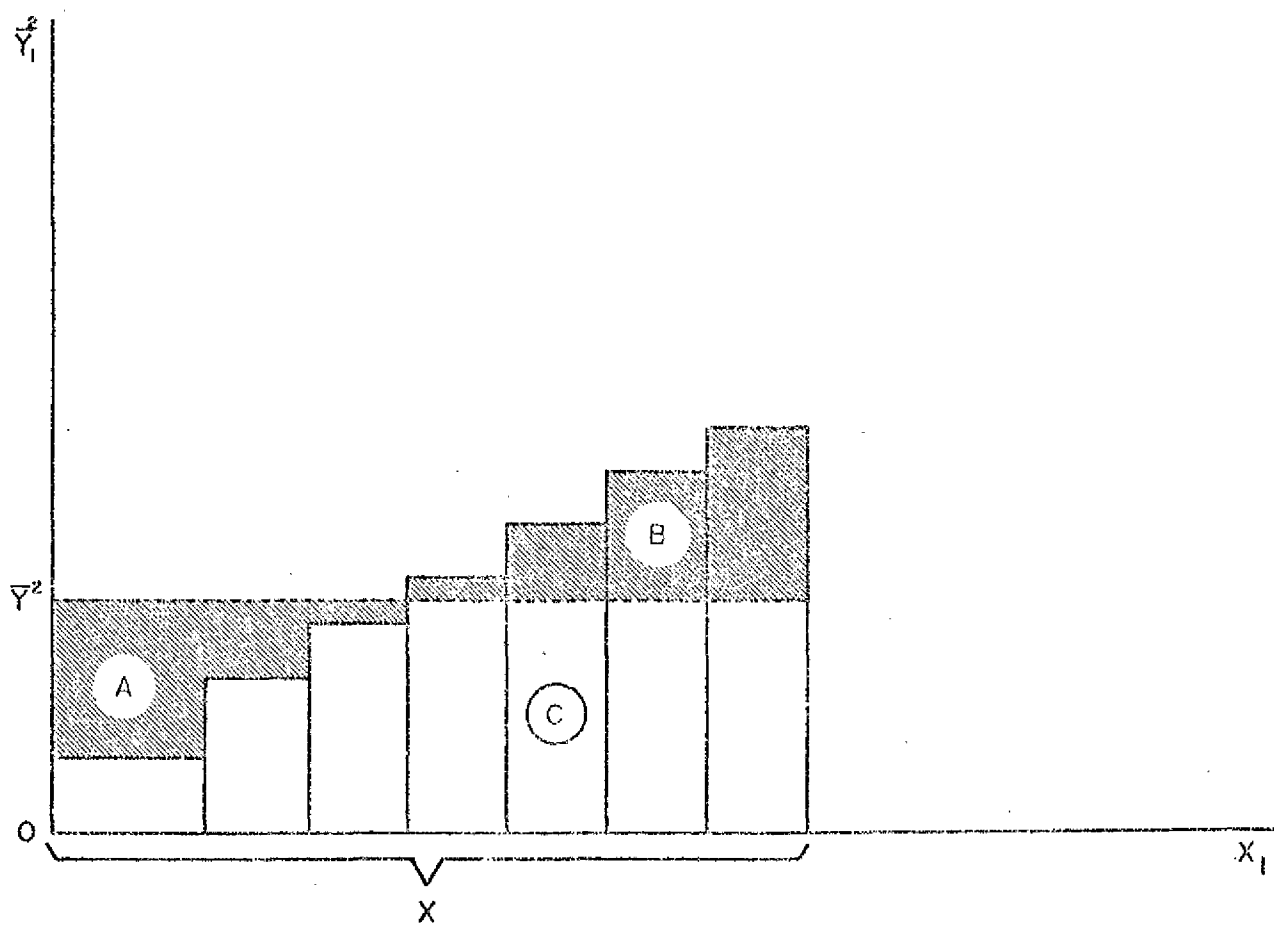
The idea underlying the disaggregation is that the degree of inequality of income of a given distribution is formed by the component of inequality between the incomes of the individuals included "within" each class I and by the component of inequality "between" incomes of individuals in each class I (in this case, the incomes of individuals of each class I are represented by the average income $(\overline{Y}_I)$ of the respective class). In the case where the distribution is formed by J distributions, the measure of inequality of income of this distribution will be also disaggregated into the two previous components. One which would measure the inequality of income between the incomes of the population classified "within" each distribution j and the other which would measure the inequality "between" the incomes of the populations of the j distributions (in this case, the incomes of the populations of each distribution j would be represented by the average income $(\overline{Y}_j)$ of the respective distribution). So suming up the proposed measure of inequality of income can be explained just as much by the components of inequality between the individual incomes "within" and "between" each class I - when individual income data are available - , as by the components of inequality between the individual incomes "within" and "between" each distribution j - when the distribution of total income is formed by J distributions.

This characteristic of the proposed measure of inequality of income is of great value in the sense that it gives greater explanatory power to the measure and provides additional elements to the analysis of the inequality of income.

A practical example of the disaggregation can be imagined when one wishes to evaluate the degree of inequality of income of a given economy, and one has available, at the same time, information about the income distributions of economic sectors. In this way, the measure of inequality of

income calculated for this economy will be formed by the
component of inequality of income "between" the sectors and
by the component of inequality of income "within" the sectors.
The same example applies to the case where one also has
information about the income distribution of the social
classes in this economy. In this case, the same value of
the measure of inequality of income of the economy will be
formed by the component of inequality of income "within" each
social class and by the component of inequality of income
"between" the social classes.

This being so, given an aggregate income distribution,
formed by $\underline{K}$ classes or $\underline{J}$ distributions, one can disaggregate
the proposed measure of inequality of income as follows:

(1) By Income Classes

Taking the expression of the measure in (9), and
transforming it for individual income data, we have

$$\frac{C V_T^2}{C V_T^2 + 1} = \frac{\sigma^2 / \bar{Y}^2}{\sigma^2 / \bar{Y}^2 + 1} = \frac{\sum_{i=1}^{X} (\bar{Y}_i^2 - \bar{Y}^2) / X\bar{Y}^2}{\sum_{i=1}^{X} (\bar{Y}_i^2 - \bar{Y}^2) / X\bar{Y}^2 + 1}$$

taking solely the numerator we have

$$\frac{\sum_{i=1}^{X} (\bar{Y}_i^2 - \bar{Y}^2)}{X\bar{Y}^2} = \frac{\sum_{I=1}^{K} X_I (\bar{Y}_I^2 - \bar{Y}^2)}{X\bar{Y}^2} + \frac{\sum_{I=1}^{K} X_I \bar{Y}_I^2}{X\bar{Y}^2} \cdot \frac{\sum_{i=1}^{I} (\bar{Y}_{iI}^2 - \bar{Y}_I^2)}{X_I \bar{Y}_I^2}$$ [27]

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[27]
A proof of this disaggregation is given in the
appendix.

substituting the disaggregated numerator in the measure, we have

$$\frac{C V_T^2}{C V_T^2 + 1} = \frac{\dfrac{\sum\limits_{I=1}^{K} X_I (\bar{Y}_I^2 - \bar{Y}^2)}{X\bar{Y}^2} + \dfrac{\sum\limits_{I=1}^{K} X_I \bar{Y}_I^2}{X\bar{Y}^2} \cdot \dfrac{\sum\limits_{i=1}^{I} (\bar{Y}_{iI}^2 - \bar{Y}_I^2)}{X_I \bar{Y}_I^2}}{\dfrac{\sum\limits_{i=1}^{X} (\bar{Y}_i - \bar{Y}^2)}{X\bar{Y}^2} + 1},$$

or

$$\frac{C V_T^2}{C V_T^2 + 1} = \frac{C V_B^2 + \dfrac{\sum\limits_{I=1}^{K} X_I \bar{Y}_I^2}{X\bar{Y}^2} \cdot C V_W^2}{C V_T^2 + 1};$$

where,

$$C V_B^2 = \frac{\sum\limits_{I=1}^{K} X_I (\bar{Y}_I^2 - \bar{Y}^2)}{X\bar{Y}^2} = \text{component of income inequality "between" the classes of the distribution;}$$

X_I = total of individuals in income class I;

X = total of individuals in the distribution;

$\bar{Y}_I$ = average income of individuals in class $\underline{I}$;

$\bar{Y}$ = average income of the distribution;

K = total of income classes.

$$C V_W^2 = \frac{\sum\limits_{i=1}^{I} (\bar{Y}_{iI}^2 - \bar{Y}_I^2)}{X_I \bar{Y}_I^2} = \text{component of income inequality "within" the classes of the distribution;}$$

X_I = total of individuals in the income class I,

X = total of individuals in the distribution;

$\bar{Y}_{iI}$ = individuals income of class I;

$\bar{Y}_I$ = average income of the individuals in class I;

K = number of income classes.

$$CV_T^2 = \frac{\sum\limits_{i=1}^{X} (\bar{Y}_i - \bar{Y})^2}{X\bar{Y}^2} = \text{component of income inequality of the distribution,}$$

$\bar{Y}_i$ = individual income;

$\bar{Y}$ = average income of the distribution;

X = total number of individuals in the distribution.

(2) By Distributions

In this case, the disaggregation of the measure is made by starting from a distribution built up by the combination of J distributions. Thus, taking only the numerator from the expression (9), we have

[28]

$$\frac{\sum\limits_{I=1}^{K} X_I (\bar{Y}_I - \bar{Y})^2}{X\bar{Y}^2} = \frac{\sum\limits_{j=1}^{J} X_j (\bar{Y}_j - \bar{Y})^2}{X\bar{Y}^2} + \frac{\sum\limits_{j=1}^{J} X_j \bar{Y}_j^2}{X\bar{Y}^2} \cdot \frac{\sum\limits_{I=1}^{K_j} X_{Ij} (\bar{Y}_{Ij} - \bar{Y}_j)^2}{X_j \bar{Y}_j^2}$$

substituting the disaggregated numerator in the measure we have

$$\frac{CV_T^2}{CV_T^2 + 1} = \frac{\dfrac{\sum\limits_{j=1}^{J} X_j (\bar{Y}_j - \bar{Y})^2}{X\bar{Y}^2} + \dfrac{\sum\limits_{j=1}^{J} X_j \bar{Y}_j^2}{X\bar{Y}^2} \cdot \dfrac{\sum\limits_{I=1}^{K_j} X_{Ij} (\bar{Y}_{Ij} - \bar{Y}_j)^2}{X_j \bar{Y}_j^2}}{\dfrac{\sum\limits_{I=1}^{K} X_I (\bar{Y}_I - \bar{Y})^2}{X\bar{Y}^2} + 1}$$

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[28] The proof of this disaggregation follows the logic which was used in the appendix.

or

$$\frac{C V_T^2}{C V_T^2 + 1} = \frac{C V_B^2 + \dfrac{\sum\limits_{j=1}^{j} X_j \bar{Y}_j^2}{X\bar{Y}^2} (C V_W^2)}{C V_T^2 + 1}$$

where,

$$C V_B^2 = \frac{\sum\limits_{j=1}^{J} X_j (\bar{Y}_j^2 - \bar{Y}^2)}{X\bar{Y}^2} = \text{component of income inequality}$$
"between" the $\underline{J}$ distributions;

X_j = total number of individuals in the distribution $\underline{j}$;

X = number of individuals of the $\underline{J}$ distributions;

$\bar{Y}_j$ = average income of distribution $\underline{j}$;

$\bar{Y}$ = average income of the $\underline{J}$ distributions;

J = number of distributions which together form the total distribution;

$$C V_W^2 = \frac{\sum\limits_{I=1}^{K_j} X_{Ij} (\bar{Y}_{Ij}^2 - \bar{Y}_j^2)}{X_j \bar{Y}_j^2} = \text{component of income inequality}$$
"within" the $\underline{J}$ distributions;

X_j = total number of individuals of the distribution j;

X = total number of individuals of the $\underline{J}$ distributions;

$\bar{Y}_j$ = average income of the distribution $\underline{j}$;

$\bar{Y}$ = average income of the $\underline{J}$ distributions;

$\bar{Y}_{Ij}$ = average income of class I of distribution $\underline{j}$;

K_j = total of income classes of the distribution $\underline{j}$;

$$C V_T^2 = \frac{\sum\limits_{I=1}^{K} X_I (\bar{Y}_I^2 - \bar{Y}^2)}{X\bar{Y}^2} = \text{component of income inequality of}$$
the aggregated distribution.

X_I = number of individuals in class $\underline{I}$ of the aggregated distribution;

X = number of individuals in the aggregated distribution;

$\overline{Y}_I$ = average income of class $\underline{I}$;

$\overline{Y}$ = average income of the aggregated distribution;

K = total number of income classes.

Properties of the Measure

We try next to comment on the properties of the proposed measure of inequality of income, given the observations made in the previous sections, as a final procedure to appreciate and evaluate its ability to estimate the concentration of income in a given distribution. These properties are arranged in two groups, one which brings together its characteristics[29] considered advantageous, and the other characteristics considered disadvantageous, or at least as limiting its analytical power.

(a) Advantages

i) The measure has a structural concept of the process of income distribution to the extent to which it adopts the approach of classical economic theory.

This concept allows a wider interpretation of the working of the processes of generation and income distribution in a given economic system.

ii) The measure is decomposed in components of inequality of income "within" classes or distributions and "between" classes or distributions.

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[29]These characteristics are grouped on the basis of the analysis of similar measures as well as a critical appreciation of other authors concerning income distribution.

This property confers on the measure greater power to analyse and interpret the process of income distribution.

iii)   The measure is independent of the form taken by the distribution.

This property is due to the fact that it is not necessary to know the form of the distribution in order to apply the measure.

iv)   The measure does not show ambiguities of interpretation of degrees of inequality of income when two or more distributions are compared.

This property is acquired through the characteristics of CV^2, according to which the variance and the average will only be equal in the case of having two or more similar income distributions, that is to say identical or proportional.

v)   The measure is sensitive to small differences between income distributions.

This property arises from the characteristics of the squares of the weighted differences of income.

vi)   The simplicity of its meaning: the ratio between the sum of the squares of the weighted differences of income (of individuals or groups of individuals and the average income of the distribution) and the sum of the squares of the weighted incomes (of individuals or groups of individuals).

vii)   The measure shows well the inequality of income observed in all the distribution.

viii)   The method of calculation of the measure is simple and quick.

(b)   Disadvantages

i)   The measure only achieves greater precision in estimating the degree of inequality of income of a distribution when individuals' income data are available.

This is explained by the fact that the measure takes the simplified form of the ratio between two factors in which CV^2 occurs. This, being a measure of dispersion, inevitably requires all the data about the distribution in order to make it a precise measuring instrument.

ii)   In the disaggregation of the measure between components of inequality of income "between" and "within", the sum of the weights is at the minimum equal to 1.

This fact means that this sum attains the value $1 + CV^2_E$ [30], which implies that the component of inequality of income "within", including its weights, depends on the value taken by the component of inequality of income "between".

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[30] Henry Theil, Economics and Information Theory (Amsterdam: North-Holland, 1967), p. 125.

CHAPTER III

APPLICATION OF THE MEASURE

In this section we make use of the proposed measure
to estimate the relative inequality of income in two cases:
(1) employing the overall measure and (2) employing its
property of disaggregation. As the objective is only to
test the power of analysis and interpretation of the measure
in two specific cases, we shall not try to enter into the
details about the implications of the results obtained.

(1) We shall now use the proposed measure to
estimate the degree of concentration existing in the
income distributions of Brazil from 1960 to 1970:

(a) The income distribution for 1960 was
obtained in: Brazilian Institute of Geography
and Statistics, Demographic Census - Preliminary
Results - Special Series, 1960 (Rio de Janeiro:
IBGE Printing Office, 1965), p.9. The mid-
points of the income classes with the exception
of the last were used. For calculating the
average income of the latter a Pareto curve was
fitted to the last two ranges of income, giving
the coefficient b = 1,8764.

(b) The income distribution for 1970 was
obtained in: Brazilian Institute of Geography
and Statistics, Advanced Tabulations of the
Demographic Census, 1970 (Rio de Janeiro: IBGE
Printing Office, 1973), p.6. As in the previous
distribution, the mid-points of the ranges of
income below the last were used. For calculating

the average income of this a Pareto curve was
fitted to the last two ranges of income, giving
the coefficient b = 1,6613.

The two income distributions with the
respective values of the measure are shown below
in tables 1 and 2.

As will be seen, the indicator of concen-
tration of income in Brazil in the decade of the
60s through the estimate given by the proposed
measure, agrees with the results of other works.
Thus for the years 1960 and 1970 were found
respectively: Fishlow (0.59 and 0.63), Langoni
(0.50 and 0.56), Duarte and Hoffman (0.49 and
0.57) and Kingston and Kingston (0.42 and 0.53)-
values corresponding to the Gini's coefficient
of concentration.

Of course the different values obtained reflect
not only the different estimates of average
incomes of the classes, but also adaptations in
the content of the data used (inclusion or not
of the individuals who do not receive incomes).
However, it is not the objective of this work to
make qualitative comparisons of the different
estimates of income concentration. It is only
intended to show the appropriateness of the
proposed measure, as opposed to instruments
traditionally used.

(2) Finally, we used the proposed measure to estimate
the degree of concentration existing in the distri-
butions by salary groups of the formal urban sector
for regions of Brazil in the period 1970/73.

The distributions by salary groups were obtained
from the 2/3 Law, National Manpower Department,
Ministry of Labour. This information refers to the

TABLE I

DISTRIBUTION OF INDIVIDUAL INCOMES[a]

IN BRAZIL - 1960

(In Current Cruzeiros)

Including Individuals Without Incomes			Excluding Individuals Without Incomes		
Ranges of Income	Average Income $(\bar{Y}_I)$	EAP[b]	Ranges of Income	Average Income $(\bar{Y}_I)$	EAP[b]
0	0	3 322	2,1	1,05	4 788
2,1	1,05	4 788	2,1 - 3,3	2,70	3 242
2,1 - 3,3	2,70	3 242	3,3 - 4,5	3,90	2 464
3,3 - 4,5	3,90	2 464	4,5 - 6,0	5,25	2 887
4,5 - 6,0	5,25	2 887	6,0 - 10,0	8,00	3 180
6,0 - 10,0	8,00	3 180	10,0 - 20,0	15,00	1 733
10,0 - 20,0	15,00	1 733	20,0 - +	42,82	649
20,0 - +	42,82	649			
$CV^2/CV^2 + 1$	0,677		$CV^2/CV^2 + 1$	0,620	

[a]Source: Brazilian Institute of Geography and Statistics.
[b]Economically active population in thousands.

TABLE 2

DISTRIBUTION OF INDIVIDUAL INCOMES[a]

IN BRAZIL - 1970

(In Current Cruzeiros)

Including Individuals Without Incomes			Excluding Individuals Without Incomes		
Ranges of Income	Average Incomes $(\overline{Y}_I)$	EAP[b]	Ranges of Income	Average Incomes $(\overline{Y}_I)$	EAP[b]
0	0	2 755	100	50	9 374
100	50	9 374	101 - 150	125	3 770
101 - 150	125	3 770	151 - 200	175	4 604
151 - 200	175	4 604	201 - 250	225	1 321
201 - 250	225	1 321	251 - 500	375	4 307
251 - 500	375	4 307	501 - 1 000	750	1 738
501 - 1 000	750	1 738	1 001 - 2 000	1 500	660
1 001 - 2 000	1 500	660	2 001 - +	5 027	306
2 001 - +	5 027	306			
$CV^2/CV^2 + 1$	0,824		$CV^2/CV^2 + 1$	0,806	

[a]Source: Brazilian Institute of Geography and Statistics.
[b]Economically active population in thousands.

wages which enterprises declared as paid to their employees and is usually supplied in April each year, by means of standard questionnaires which by law the enterprises must send to the Regional Labour Delegations.

These data refer to the "formal" sector of the urban economy, i.e. they cover industry, commerce and services (excluding unpaid, personal and own-account services). In terms of representativeness the data covered about 45% of the individuals declared to be employed in the population census of 1970.

Moreover, the 2/3 Law does not furnish the remuneration of employers and other administrative and technical professions (consultants, advisers, administrators, high-level and similar technicians), liberal professions and other workers on their own account, nor individuals connected with "informal" activities (those exercised outside the legal framework which regulates the urban economy).

We present in Table 3 and the corresponding figure the values shown by the measure with the respective components of inequality of total income, within and between distributions. The distributions of wages by groups and regions are shown in Tables 4 and 7.

The interpretation of the data contained in Table 3 and the corresponding figure suggests the following observations:

(a) The tendency to concentration of income occurring in the period 1970/73 was increasing slightly - 0.602 in 1970 to 0.606 in 1973 -, although with a very low growth - just under 1%. Although this result for Brazil may on the one hand show a very slow growth of concentration of wages, it also shows a certain rigidty in the wage structure in

Year	1970	
Component	Value	%
Total	0,602	100,0
Between	0,003	1,4
Within	0.594	98,6
Regions: North	0,005	0,9
Northeast	0,043	7,2
Southeast	0,494	81,9
South	0,044	7,3
Center West	0,008	1,3

[a]Calculated from tables 4 to 7.

3

OF $CV^2/CV^2 + 1$[a]

	1971		1972		1973	
	Value	%	Value	%	Value	%
	0,601	100,0	0,605	100,0	0,606	100,0
	0,008	1,3	0,010	1,6	0,009	1,5
	0,593	98,7	0,595	98,4	0,597	98,5
	0,009	1,5	0,007	1,1	0,006	1,0
	0,041	6,8	0,035	5,8	0,038	6,3
	0,485	81,1	0,497	82,2	0,493	81,7
	0,052	8,7	0,046	7,7	0,046	7,6
	0,006	1,0	0,010	1,6	0,014	2,3

FIGURE 4
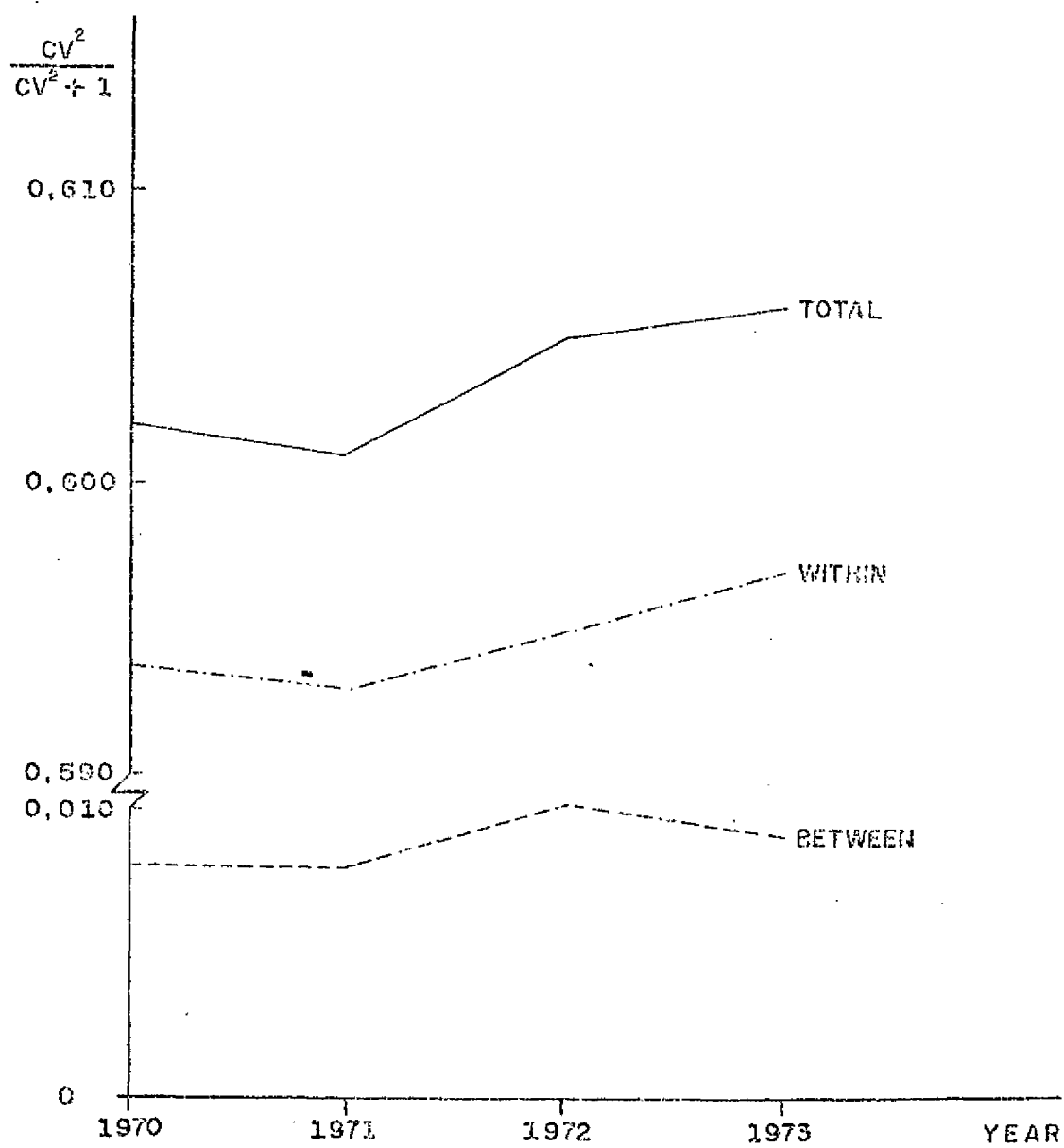
DISAGGREGATION OF $CV^2/CV^2 + 1$

TABLE 4
DISTRIBUTION OF WAGES[a] IN THE FORMAL URBAN SECTOR[b]
BY REGIONS — 1970

Ranges of Wages[c]	Regions										Brazil	
	North		Northeast		Southeast		South		Center West			
	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I
−140	121,4	24 280	113,4	230 287	107,0	193 033	116,8	91 990	119,2	30 662	112,4	570 252
140−159	148,0	11 425	148,6	54 856	152,8	542 593	147,1	206 883	149,3	26 290	151,0	842 047
160−179	165,3	6 123	167,7	32 233	168,3	372 868	168,7	89 538	168,8	12 594	168,3	513 356
180−199	189,9	3 611	188,4	26 210	188,4	272 728	187,9	79 955	187,9	8 428	188,3	390 932
200−299	244,0	12 272	240,6	90 282	240,6	880 451	239,3	189 675	237,5	33 143	240,3	1 205 823
300−399	341,8	6 030	399,4	45 097	343,8	567 047	338,9	85 626	339,8	15 894	342,8	719 694
400−499	442,4	3 582	442,6	25 772	445,5	331 817	441,3	44 473	427,0	9 429	444,4	415 073
500−599	545,1	2 646	541,8	16 13.	542,9	219 295	540,0	27 943	537,5	4 593	542,5	270 608
600−699	648,3	1 614	639,9	11 072	642,2	143 706	643,2	20 118	649,4	4 500	642,4	181 010
700−799	739,9	877	746,6	8 451	742,3	98 242	743,0	11 235	747,4	1 853	742,7	120 658
800−899	844,8	1 270	843,3	7 295	842,4	71 984	839,2	8 256	842,7	1 190	842,2	89 995
900−999	950,9	530	946,9	4 521	945,3	47 372	946,8	6 031	940,8	795	945,6	59 249
1000−1199	1 078,1	1 076	1 088,3	10 068	1 085,0	76 902	1 086,7	9 363	1 078,7	1 840	1 085,3	99 249
1200−1599	1 392,3	1 206	1 374,8	10 361	1 371,4	81 376	1 364,4	9 875	1 378,5	2 292	1 371,4	105 110
1600−1999	1 779,8	653	1 771,9	4 591	1 766,3	42 579	1 758,4	4 695	1 751,1	1 049	1 766,0	53 567
2000−2399	2 195,3	229	2 162,5	2 328	2 169,0	25 383	2 158,0	2 512	2 071,2	492	2 166,3	30 944
2400 and +	3 460,5	469	3 314,8	4 079	3 807,1	53 761	3 500,4	3 416	3 165,2	616	3 749,1	62 341
$\overline{Y}$, ΣX_I	305,6	77 893	304,1	583 634	402,8	4 021 137	295,7	891 584	302,4	155 660	384,6	5 729 908
CV^2/CV^2+1	0,611		0,631		0,597		0,546		0,540		0,602	

[a] Source: Ministry of Labor − National Department of Labor − 2/3 Law.
[b] Industry, commerce and services (excluding unpaid, personal and own − account services).
[c] Wages declared to be paid by enterprises to their employees (Cr$ 1,00).

Ranges of Wages[c]	North		Northeast		South
	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$
−140	128,3	5 363	129,1	134 437	108,1
140−159	146,7	20 859	148,2	91 027	149,4
160−179	167,3	9 654	167,7	46 681	175,3
180−199	189,4	11 305	188,9	38 244	189,0
200−299	240,9	17 716	241,1	101 952	238,1
300−399	343,3	11 657	340,7	61 852	342,7
400−499	443,7	7 171	442,5	33 877	443,8
500−599	545,4	4 049	539,6	20 103	542,9
600−699	646,6	2 361	642,9	15 351	641,4
700−799	748,3	2 085	744,8	12 156	743,0
800−899	848,7	1 489	843,4	9 529	842,3
900−999	943,6	1 304	944,3	7 988	945,3
1000−1199	1 072,3	2 441	1 085,0	12 764	1 082,0
1200−1599	1 371,1	2 013	1 377,4	11 722	1 369,3
1600−1999	1 756,7	1 037	1 778,2	6 317	1 777,3
2000−2399	2 167,2	823	2 163,3	4 255	2 170,9
2400 and +	3 971,8	1 209	3 394,0	6 556	3 936,9
$\overline{Y}$, ΣX_I	395,3	102 536	365,3	614 811	500,4
$CV^2/CV^2 + 1$	0,630		0,613		0,

TABLE 5

a IN THE FORMAL URBAN SECTOR[b]

REGIONS - 1971

| Regions | | | | | Brazil | |
| east | South | | Center West | | | |
X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I
116 560	111,8	36 115	111,3	2 542	118,5	295 017
91 362	153,9	51 954	148,5	17 362	149,6	272 564
182 879	170,2	141 022	176,1	17 777	172,4	398 013
510 164	186,7	134 170	189,1	9 572	188,6	703 455
1 139 676	238,1	283 721	241,6	33 068	238,4	1 576 133
600 596	339,8	123 707	337,1	17 191	342,0	815 003
426 931	439,9	65 777	428,8	12 310	442,9	546 066
248 194	541,0	37 356	541,4	2 681	542,5	312 383
186 780	640,9	23 274	654,3	4 538	641,8	232 304
134 467	739,7	16 597	742,4	782	742,9	166 087
100 975	834,5	13 799	852,7	533	841,7	126 325
70 671	943,3	9 483	944,2	1 091	945,0	90 537
95 443	1 078,1	16 338	1 066,5	945	1 081,5	127 931
107 679	1 363,3	16 864	1 383,9	1 314	1 369,4	139 592
58 071	1 764,4	8 895	1 797,2	934	1 775,8	75 254
36 080	2 167,2	4 833	2 173,0	420	2 169,8	46 411
85 853	3 556,5	7 509	3 243,0	1 251	3 866,9	102 378
4 192 381	364,2	991 414	344,5	124 311	459,2	6 025 453
597	0,558		0,563		0,601	

of Labor - 2/3 Law.

id, personal and own - account services).

ir employees (Cr$ 1,00).

Ranges of Wages[c]	North		North		Northeast		Northeast		South
	$\overline{Y}_I$		X_I		$\overline{Y}_I$		X_I		$\overline{Y}_I$
−140	112,5			646	108,0	11	157		117,7
140−159	145,5			430	151,0	47	091		148,6
160−179	172,3		5	052	166,7	136	392		168,8
180−199	187,8		24	077	187,8	63	411		187,2
200−299	237,2		33	084	239,5	139	493		244,9
300−399	343,7		16	142	339,9	75	149		341,1
400−499	446,6		10	518	439,7	47	956		444,6
500−599	541,3		5	570	540,1	29	222		543,5
600−699	642,7		4	217	640,3	19	485		642,0
700−799	739,9		2	481	743,1	12	953		743,9
800−899	843,9		1	934	845,3	10	717		843,9
900−999	947,3		1	880	944,7	9	694		944,9
1000−1199	1 087,9		2	051	1 090,3	14	324	1	086,4
1200−1599	1 351,1		3	265	1 361,6	16	450	1	368,7
1600−1999	1 771,8		1	387	1 764,1	8	755	1	771,2
2000−2399	2 187,3		1	061	2 166,5	5	345	2	172,7
2400 and +	3 503,9		2	778	3 684,8	12	007	4	223,7
$\overline{Y}$, ΣX_I	483,8		117	073	437,2	659	601		646,8
$CV^2/CV^2 + 1$		0,598				0,625			0

TABLE 6

[a] IN THE FORMAL URBAN SECTOR[b]
REGIONS - 1972

| Regions | | | | | | Brazil | |
| east | South | | Center West | | | | |
X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	
90 101	111,6	25 234	113,5	2 623	115,6	129 761	
29 323	151,3	15 222	148,7	1 301	150,2	93 367	
59 188	169,0	13 451	172,5	22 525	168,0	236 608	
61 615	187,8	52 886	186,8	15 588	187,6	217 577	
1 488 859	236,7	491 510	235,7	65 546	242,4	2 218 492	
763 089	339,3	183 075	339,6	32 702	340,7	1 070 157	
512 943	440,6	99 044	445,3	24 502	443,7	694 963	
367 840	539,4	58 895	536,9	9 262	542,6	470 789	
256 579	640,4	39 423	645,3	8 739	641,8	328 443	
203 060	739,2	27 672	741,2	4 693	743,3	250 859	
146 435	839,5	19 345	841,4	3 193	843,5	181 624	
118 970	941,8	15 673	937,8	1 829	944,5	148 046	
167 590	1 085,5	21 677	1 079,3	3 276	1 086,5	208 918	
173 850	1 360,2	23 281	1 371,4	4 774	1 367,1	221 620	
97 422	1 768,1	13 466	1 755,5	2 235	1 770,1	123 765	
62 248	2 151,9	7 437	2 131,2	1 823	2 169,5	77 914	
169 349	3 750,1	14 853	3 460,9	3 846	4 132,8	202 833	
4 768 461	444,3	1122144	453,1	208 457	585,0	6 875 736	
,598	0,558		0,573		0,605		

f Labor - 2/3 Law.
d, personal and own - account services).
r employees (Cr$ 1,00).

Ranges of Wages[c]	Regions					
	North		Northeast		Southeast	
	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I
-140	94,7	1 087	99,9	10 397	69,6	66 097
140-159	148,8	265	149,1	4 043	148,0	58 021
160-179	168,0	324	169,1	4 354	169,1	30 906
180-199	189,0	442	192,0	125 189	187,8	20 948
200-299	239,0	50 462	237,6	213 733	271,9	1 026 945
300-399	345,0	17 181	341,9	94 237	345,2	1 024 164
400-499	447,7	11 991	441,0	61 513	445,0	616 406
500-599	545,8	8 994	542,1	39 547	545,7	465 072
600-699	647,8	5 048	643,1	27 100	644,2	362 290
700-799	746,0	3 576	742,7	18 509	744,0	278 103
800-899	847,7	2 866	849,8	14 874	845,1	217 499
900-999	940,9	1 597	945,8	10 287	945,3	161 653
1000-1199	1 097,6	2 790	1 082,6	18 158	1 090,3	243 349
1200-1599	1 373,7	4 196	1 371,1	22 601	1 373,0	269 992
1600-1999	1 758,6	2 320	1 765,5	11 798	1 773,4	138 991
2000-2399	2 183,8	1 633	2 178,1	6 610	2 176,8	89 798
2400 and +	3 879,3	4 393	4 149,5	23 161	4 625,2	282 618
$\overline{Y}$, ΣX_I	603,6	119 165	557,3	706 111	811,8	5 352 852
$CV^2/CV^2+ 1$	0,605		0,648		0,598	

FORMAL URBAN SECTOR[b]
- 1973

South		Center West		Brazil	
$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I	$\overline{Y}_I$	X_I
121,8	17 993	97,4	1 494	83,2	97 068
149,2	9 794	151,4	1 285	148,3	73 408
168,4	8 675	168,4	1 041	168,9	45 300
187,9	15 109	188,6	829	191,1	162 517
259,0	387 901	246,1	76 100	262,8	1 755 141
339,7	273 038	340,4	38 007	343,8	1 446 627
444,4	136 743	442,2	28 439	444,6	855 092
542,7	87 848	539,9	19 795	544,9	621 256
638,7	57 471	637,0	10 658	643,3	462 567
743,1	39 337	741,1	7 222	743,9	346 747
846,2	30 250	844,3	6 601	845,5	272 090
945,7	21 411	943,6	3 808	945,3	198 756
1 081,8	30 903	1 083,7	6 778	1 088,9	301 978
1 375,2	35 701	1 374,3	6 935	1 373,1	339 475
1 770,9	17 013	1 772,8	4 418	1 772,4	174 540
2 162,6	10 872	2 167,4	4 872	2 175,2	113 785
3 973,6	29 033	3 635,5	11 635	4 497,7	350 840
556,6	1209 092	672,0	229 967	740,2	7 617 187
0,566		0,582		0,606	

/3 Law.
and own - account services).
(Cr$ 1,00).

relation to the various ranges of·salary considered.

The interpretation of these results would suggest immediately the importance of the wages policy adopted. The latter as well as fixing annual increments - to be observed by the public and private sectors - for the workers receiving the minimum salary, also influences directly and proportionally the salary levels of the remaining workers. In this case the results would be simply reflecting the maintenance, more or less constant, of the indices of wages concentration in the period 1970/73.

(b) In terms of the disaggregation of the indices of concentration in the period 1970/73, one observes that the component of wages concentration "between" was not significant - approximately 1.5%. It follows that it was the component of wages concentration "within" regions which really explained the values of the indices encountered.

The southeast region makes the greatest contribution to the concentration of wages observed in the period - about 80% of the values of the indices. This weight is due more to the greater share of its employees in the total for the regions, rather than the value in itself of its index of concentration of wages obtained.

In relative terms, it is worth mentioning that the northeastern, southern and central-west regions were those which showed a growing tendency to concentration of wages in the period. However, wages concentration "within" the southeastern reg on was highest because of the

greater relative weight of its total of
employees.

(c) Finally, it is appropriate to mention the
quality of the information used. It is obvious
that one cannot take the results obtained as
precise indicators of wages concentration
occurring in the period 1970/73, because the
degree of precision and correctness of the
information given by enterprises is not known;
furthermore these data do not necessarily refer
to the same establishments – which makes the
representativeness of the same sample question-
able, and finally the fact that the coverage of
the information in terms of occupational
categories is not known for certain. The most
that can be said is that the information refers
to those occupational categories which, through
individuals, have a professional registration
through the issue of work books (C.T.P.S.).
These observations certainly restrict any more
detailed analysis.

CHAPTER IV

SUMMARY, CONCLUSIONS AND SUGGESTION

The present thesis was divided into three main parts.
In the first, three well-known measures of inequality of
income were presented and commented on - Pareto's index,
Gini's coefficient of concentration and the variance of
logarithms. The analysis suggested that among these measures,
the last two were the most efficient, both in terms of
interpreting the variation of income among individuals (or
groups) in the population and in relation to their mathe-
matical/statistical properties.

In the second part, an alternative for understanding
the distribution of income was proposed, based on a summary
interpretation of the ideas contained in the classic economic
theory. From this interpretation, it became possible to
elaborate a measure of inequality of income, and also to
examine its characteristics, properties and limitations.

Finally, in the third part, the proposed measure was
tested in two different cases. On the one hand, it was
applied to the distributions of income in Brazil from 1960
to 1970, and on the other to the wage distributions of the
employees in the formal urban sector in the period 1970-1973.
In this case the implications of the results obtained were
commented upon briefly. The most important conclusions were:

(1) The existing measures of inequality of income
have, as a general rule, implicit concepts about the
process of income distribution. These concepts can
be clearly observed on the basis of the analysis of
their operational characteristics and specific

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properties. It follows that the proper use of a
measure of inequality of income requires the previous
verification of its peculiarities in order to ensure
compatibility between the hypothesis and the repre-
sentative empirical results. The ensuing interpre-
tations must necessarily be influenced by the choice
of the measures.

(2) The proposed alternative for measuring inequality
of income originated in a summary interpretation of
the relevant ideas contained in the Classic Economic
theory. It was later observed that the Human Capital
approach of Neo-Classical theory could also be
adopted in applying the measure (see footrote 24).
The original interpretation may have been jeopardised
in the sense of not being the only one to be put into
operational form in the way assumed by the measure.
But we demonstrated the possibility of adapting to
the measure, other theories dealing with the process
of income distribution on the basis of comparable
analytical structure.

(3) A principal conclusion of this thesis is that
one should try to understand a problem before one
starts measuring it. Before chosing or applying a
mathematical-statistical model or formula the
researcher has to go back to his theory of development,
conceptualize the problem, and only then choose or
develop a measure which can help him to quantify the
relationships which he considers important. In this
case, the understanding of the Classics concerning
the process of income distribution furnished the
development of a measure of inequality with
operational characteristics and properties which place
it, along with others traditionally known, in

equivalent conditions of application. This is
particularly the case of the practical application of
the measure suggested in the text, although by this
transformation its structural concept of the process
of income distribution is simplified rendering it
conventional.

Finally, as a suggestion, it is worthwhile high-
lighting the importance of future studies in this field,
especially in the economic circumstances of developing
countries, when one observes the insufficiency of traditional
models of growth with respect to distribution and rewards
generated in the productive process. Its better understand-
ing through the improvement of analytical techniques and
methods is fundamental, not only to detect the principal
bottlenecks between production and distribution of goods and
services, but also for suitable planning of the economic
measures attempting to correct the inequality of income
arising from the differential ownership of the means of
production.

The proof of the disaggregation of CV^2 in relation to information by income classes (see text) is the following:

$$CV_T^2 = \frac{\sum_{i=1}^{X} (\bar{Y}_i^2 - \bar{Y}^2)}{X\bar{Y}^2} = \frac{\sum_{i=1}^{X} (\bar{Y}_i - \bar{Y})^2}{X\bar{Y}^2} = \frac{\sum_{i=1}^{X} \left[(\bar{Y}_i - \bar{Y}_I) + (\bar{Y}_I - \bar{Y})\right]^2}{X\bar{Y}^2} =$$

$$= \frac{\sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I)^2 + \sum_{i=1}^{X} (\bar{Y}_I - \bar{Y})^2 - 2 \sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I)(\bar{Y}_I - \bar{Y}).}{X\bar{Y}^2}$$

For any class of income $\underline{I}$, the term $(\bar{Y}_I - \bar{Y})$ is the same for all the values of $\underline{i}$, hence the last term of the above expression reduces to:

$$2 \sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I) \cdot (\bar{Y}_I - \bar{Y}) = 2 \sum_{I=1}^{K} (\bar{Y}_I - \bar{Y}) \cdot \sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I)$$

and for any class of income I,

$$\sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I) = \sum_{i=1}^{X} \bar{Y}_i - \sum_{j=1}^{X} \bar{Y}_I = \sum_{i=1}^{X} \bar{Y}_i - \sum_{I=1}^{K} X_I \bar{Y}_I = 0$$

consequently,

$$2 \sum_{i=1}^{X} (\bar{Y}_i - \bar{Y}_I) \cdot (\bar{Y}_I - \bar{Y}) = 0$$

Finally,

$$CV_T^2 = \frac{\sum\limits_{i=1}^{X} (\overline{Y}_i - \overline{Y}_I)^2 + (\overline{Y}_I - \overline{Y})^2}{XY^2}$$

but, as

$$\sum\limits_{i=1}^{X} i = \sum\limits_{I=1}^{K} X_I = X$$

we have

$$CV_T^2 = \frac{\sum\limits_{I=1}^{K} X_I \overline{Y}_I^2}{XY^2} \cdot \frac{\sum\limits_{i=1}^{X} (\overline{Y}_i - \overline{Y}_I)^2}{X_I \overline{Y}_I^2} + \frac{\sum\limits_{I=1}^{K} (\overline{Y}_I - \overline{Y})^2}{X \overline{Y}^2} \quad ,$$

or simply,

$$CV_T^2 = \frac{\sum\limits_{I=1}^{K} X_I \overline{Y}_I^2}{X\overline{Y}^2} \cdot (CV_W^2) + CV_B^2 \; .$$

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