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TREND, SEASONALITY AND SEASONAL ADJUSTMENT

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Abstract

The aim of this paper is to set out criteria for defining trend and seasonal components in a time series.

The criteria are set up primarily in terms of properties involving prediction.

Because a structural time series model is set up in terms of components of interest, the relevant information on these components is given directly.

It is shown that the Basic Stuctural Model has statistical properties, which are not dissimilar to the ARIMA model used by other authors, but the B.S.M. is only one model within a range of models all of which satisfy our proposed criteria.

This methodology is applied to two series: US Investment and Industrial Production in Brazil.

TREND, SEASONALITY AND SEASONAL ADJUSTMENT[1]

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INTRODUCTION

The aim of this paper is to set out criteria for definig trend and seasonal components in a time series. A structural time series model is formulated directly in terms of such components, and it will be shown that the proposed criteria are satisfied. The criteria are set up primarily in terms of properties involving

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prediction. Box, Pierce and Newbold (1987) also regard trend as being defined in terms of prediction but they are primarily concerned with ARMA models and the way in which such models can be broken down into separate components. They employ the notion of canonical decomposition, and this leads to one of our criteria, the continuity criterion, not being satisfied.

Because a structural time series model is set up in terms of components of interest, the relevant information on these components is given directly by the Kalman filter and the associated smoothing and prediction algorithms. This includes not only estimates of the components, but also their MSE's. For many practical purposes the appropriate model for monthly and quarterly data will be the basic structural model (BSM). This model was adopted in Harvey and Todd (1983), and is also used, with a minor variation, by Kitagawa and Gersch (1984) and Gersch and Kitagawa (1983). As Maravall (1985) has shown, its statistical properties are not dissimilar to those of the airline model, which is the ARIMA model used by Box, Pierce and Newbold (1987). However, the BSM is only one model within a range of models all of which satisfy our proposed criteria for modelling components. These criteria are also satisfied when structural models are set up in continuous time as in Harvey (1983) and Kitagawa (1984).

1. TREND

1.1 What is a Trend ?

The question of defining a trend is one which has exercised the minds of economists and statisticians for many year. Most people would claim to be able to recognize a trend when they saw one, but few would be able to go beyond the rather vague definition in the Concise Oxford Dictionary which is that a trend is a "general direction and tendency". Indeed there is a strong temptation to echo the sentiments expressed by Cairncross (1971, p. 139): " A trend is a trend is a trend ...".

Actually the Oxford Dictionary's definition is not a bad one, in that it defines the trend in terms of prediction. This is the view taken here. In much of the statistical literature, however, a trend is conceived of as that part of a series which changes relatively slowly over time. In other words, smoothness properties play a key role in the definition. There is no fundamental reason, though, why a trend should be smooth, except that it is somewhat easier on the eye. Suppose, for example, that a series, such as a stock market price, follows a random walk. The general direction of such a series at the current point in time is a horizontal line passing through the last observation. The last observation therefore is the trend. There is no difference between the trend and the observations and so the trend, like the series itself, is not smooth.

What is a trend? Viewed in terms of prediction, the *estimated trend is that part of the series which when extrapolated gives the clearest indication of the future long-term movements in the series*. Note that a trend does not necessarily have to show an upward or downward movement and in fact a series will always contain a trend unless the long-run forecasts are equal to zero. The definition makes no mention of smoothness and it is consistent with the idea of indicating a "general direction".

Having defined the trend in terms of its properties when extrapolated, we need a mechanism for making this extrapolation, just as we need a mechanism for making predictions of future values of the series itself. A mechanism for making predictions of the series is provided by a statistical model and such a model is fitted in the hope that it will provide the best possible predictions. A statistical model for the trend component can also be set up as part of the overall model for y_t. This sub-model should be such that the optimal estimator of the trend at the end of the series gives rise to a forecast function, satisfying the criterion of the previous paragraph. The optimal estimator of the trend component within the series is then defined automatically. The properties of the estimated trend within the series therefore emerge as a consequence of the required properties of the trend forecast fucntion and the characteristics of the data.

The trend components in structural time series models are always set up in such as way that their forecast functions indicate the long term movements in the series. Indeed, the motivation underlying the formulation of structural models is in terms of the predictions they yield. Thus, in the absence of seasonal and daily effects, the forecast function for the trend component tends towards the forecast function for the series as a whole, that is

$$\lim_{l \to \infty} (\tilde{y}_{T+l/T} - m_{T+l/T}) = 0 \qquad \text{(T1)}$$

Since the trend component in a structural model is usually set up so as to provide a local approximation to a continuous function of time, its forecast function can also be regarded as a continuous function of time and written as $m(T+l/T)$ for $l \geq 0$. This continuous forecast function is anchored at the estimated trend at the end of the series, that is

$$m(T+l/T) = m_T \text{ at } l=0. \qquad \text{(T2)}$$

we call this the <u>continuity criterion</u>.

Example 1.1 Consider the model

$$y_t = \mu_t + \nu_t + \varepsilon_t \qquad (1.1a)$$

$$\mu_t = \mu_{t-1} + \eta_t \qquad (1.1b)$$

$$\nu_t = \rho \; \nu_{t-1} + \varkappa_t \qquad |\rho|<1 \qquad (1.1c)$$

where, η_t, $\varkappa_t$ and ε_t are mutually independent white noise disturbances. On taking conditional expectations it can be seen that the forecast fucntion for future values of the series is

$$\tilde{y}_{T+l/T} = m_T + \rho^l \ \tilde{\nu}_T , \qquad l=0, 1, 2, \ldots \qquad (1.2a)$$

where $\tilde{\nu}_T$ is the optimal estimator of ν_T at time T. The forecast function for the trend is

$$m_{T+l/T} = m_T \qquad (1.2b)$$

and since $| \rho | < 1$ it is easily seen that it converges to the forecast function for the series as a whole as the lead time increases, i. e. (T1) is satisfied. However, because it is simpler than (1.2a), it gives a clearer indication of the long-run direction of the series. Indeed the continuous trend forecast function, $m(T+l/T)$, is simply a horizontal line at a height of m_T and so (T2) is satisfied in both the discrete and continuous form.

Example 1.2 The local linear trend model is

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t & \varepsilon_t &\sim NID(0, \sigma^2_\varepsilon) & (1.3) \\ \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t & \eta_t &\sim NID(0, \sigma^2_\eta) & (1.4a) \\ \beta_t &= \beta_{t-1} + \zeta_t & \zeta_t &\sim NID(0, \sigma^2_\zeta) & (1.4b) \end{aligned}$$

where μ_t is the level at time t, β_t is the slope and the three disturbances ε_t, η_t and ζ_t are mutually independent.

The continuous trend forecast function is a straight line, starting off from $\tilde{\mu}_T = m_T$ and having a slope of $\tilde{\beta}_T = b_T$, that is

$$m_{T+l/T} = m_T + b_T\, l \qquad l \geq 0 \qquad (1.5)$$

Example 1.3 In the damped trend model, equation (1.4b) is replaced by

$$\beta_t = \rho\ \beta_{t-1} + \zeta_t \qquad (1.6)$$

The trend forecast function is

$$m_{T+l/T} = m_T + \{ (1 - \rho^l)/(1 - \rho)\}\ b_T \qquad l=0, 1, 2,... \qquad (1.7)$$

The right hand side of (1.7) can also be regarded as a continuous forecast function, $m(T+l/T)$ for $l \geq 0$. All three properties (T1) amd (T2) are clearly satisfied.

Finally let us return to the question of smoothness. Other things being equal, it is nice to have a smooth trend if only because it is more appealing to policy makers. However, the view which has been put forward here is that once a suitable structural model has been selected and estimated, the properties of the estimated trend are fixed. Unless the variable is a flow and the model is formulated in continuous time or at a finer timing interval than the observation timing interval, a matter taken up in sub-section 1.3, there are no degrees of freedom left with which to impose smoothness. Smoothness can only be imposed on the trend by bringing in some restrictions. In the local linear trend model, the smoothness of the trend depends on the relative parameters $q_\eta = \sigma_\eta^2 / \sigma_\varepsilon^2$ and $q_\zeta = \sigma_\zeta^2 / \sigma_\varepsilon^2$, but q_η is constrained to be zero, as in Kitagawa (1981), Gersch and Kitagawa (1983) and Theil and Wage (1964), the trend component becomes

$$\Delta^2 \mu_t = \zeta_{t-1} \tag{1.8}$$

and the resulting estimated trend within the series is relatively smooth. The argument is given by Akaike (1980) in terms of *smoothness priors*. Since the model is still a structural one, the properties (T1) and (T2) are satisfied. The argument against setting σ_η^2 equal to zero is that such a model may give relatively poor predictions compared to a model where σ_η^2 is allowed to be positive. An obvious example would be if the data were generated by a random walk. It is interesting to note that the rationale for the smoothness induced by (1.8) was originally given by Whittaker (1923). However he was working with actuarial life tables where the case for imposing smoothness is much stronger than any conditions pertaining to predictions outside the range of the sample. The same is true when the corresponding continuous time version of the local linear trend model is used for fitting a spline function to cross-section data as in Wecker and Ansley (1983).

1.2 Canonical and Other Decompositions

The approach to trend estimation favoured by some statisticians is to first fit an unrestricted reduced form ARIMA model to the data and then to decompose this model into components. If the decomposition is to be into a trend and an irregular, one criterion which has been adopted is to maximize the variance of the irregular component, thereby making the trend as smooth as possible. Box, Hillmer and Tiao (1978) call this the <u>canonical decomposition</u> while Pierce (1979) refers to it as the <u>principle of minimum extraction</u>. Although Box, Pierce and Newbold (1987) show that the trend emerging from the canonical decomposition can be regarded as being defined in terms of prediction, its forecast function is not anchored at the trend estimate at the end of the series.

The way in which the canonical decomposition works can be illustrated by an ARIMA(0,1,1) process:

$$\Delta y_t = \xi_t + \theta \xi_{t-1}$$

Such a process can be decomposed as follows:

$$y_t = \mu^*_t + \varepsilon^*_t \qquad (1.9a)$$
$$\mu^*_t = \mu^*_{t-1} + \eta^*_t + \theta^*_t \eta^*_{t-1} \qquad (1.9b)$$

where $|\theta^*| \leq 1$ and ε^*_t and η^*_t are serially independent zero mean disturbances, with variances σ^2_ε and σ^2_η, which are distributed independently of each other.

The model can be reduced to stationarity by taking first differences, i. e.

$$\Delta y_t = \eta^*_t + \theta^* \eta^*_{t-1} + \varepsilon^*_t - \varepsilon^*_{t-1} \qquad (1.10)$$

The right hand side is equivalent to an MA(1) model since the autocorrelations are zero beyond lag one. The first order autocorrelation is

$$\rho(1) = \frac{\theta^* q - 1}{(1+\theta^{*2})q + 2} \qquad (1.11)$$

where $q = \sigma^2_\eta / \sigma^2_\varepsilon$. An immediate problem now arises with (1.10) since it contains three parameters, σ^2_η , σ^2_ε and θ^*, rather then two parameters associated with an MA(1) process. This is reflected in the two parameters, θ^* and q , in the expression for $\rho(1)$. There is an infinite number of combinations of values of θ^* and q giving the same value of $\rho(1)$. Hence, as it stands, model (1.10) is not identifiable. Restrictions must therefore be placed on it. The standard structural model has θ^* equal to zero, which reduces (1.9) to a random walk plus noise. In the canonical decomposition, the idea is to minimize q for a given value of $\rho(1)$ in (1.11), thereby determining the value of θ^*. It is not difficult to see that in the special case when the reduced form is a random walk, that is $\rho(1) = 0$, the canonical decomposition has $\theta^* = 1$ and q = 1. In fact it can be shown that the canonical decomposition has θ^*

= 1 for any value of $\rho(1)$; see Box, Hillmer and Tiao (1978).

If $\tilde{\eta}_T^*$ denotes the estimator of η_T^*, the forecast function for the trend, which in this case is just the level of the process, is

$$\tilde{\mu}_{T+l/T}^* = \tilde{\mu}_T^* + \tilde{\eta}_T^* \qquad l=1, 2, \ldots \qquad (1.12)$$

This is also the forecast function for the observations, so (T1) of the previous sub-section is satisfied. The trend forecast function can be regarded as a continuous horizontal line for $l \geq 1$. However, this line cannot be extended back to pass through the estimator of the current level, μ_T^*, unless η_T^* happens to be identically equal to zero. This compares with the model in Example 1.1 which reduces to the random walk plus noise when ρ is zero, and has as its continuous trend forecast function a horizontal straight line starting at $l = 0$. The implications of this failure to yield a continuous forecast function anchored at the estimated trend at the end of the series , Condition (T2), are brought home most clearly in the special case where the reduced form is a random walk. It can be shown that at the end of the series

$$\tilde{\mu}_T^* = \frac{9}{4} y_T + \frac{1}{4} y_T \qquad (1.13)$$

see Box, Hillmer and Tiao (1978, p.319). Of course the final forecast function is just

$$\tilde{y}_{T+l/T}^* = \tilde{\mu}_{T+l/T}^* = y_T \qquad l=1, 2, \ldots \qquad (1.14)$$

denoting the fact that the optimal forecast of future values of a random walk is the current value. It is difficult to see what $\tilde{\mu}_T^*$ represents since it cannot be regarded as an estimator of the current level of the process. Similarly the estimates of the trend within the sample, given by

$$\tilde{\mu}^*_{t/T} = \frac{1}{4} y_{t-1} + \frac{1}{2} y_t + \frac{1}{4} y_{t+1} \qquad 2 \leq t \leq T - 1$$

reflect an attempt to smooth a process which is inherently unsmoothable.

The paper by Beveridge and Nelson (1981) proposes a definition of a trend which is based primarily on prediction. The underlying philosophy is therefore very close to that expounded in the previous sub-section. As with the structural approach, the Beveridge-Nelson methodology indicates that the trend in a random walk process coincides with the observations themselves. Indeed they cite this as a check on the reasonableness of their results.

Beveridge and Nelson focus attention on fitted ARIMA(p,1,q) models which contain a constant term.For such models the final forecast function is linear, and the Beveridge-Nelson definition of the trend at the end of the series is the value obtained when the final forecast function is extrapolated back to the end of the series. The decomposition of the series is then expressed in terms of the ARIMA disturbance term. In the ARIMA(0,1,1) case the decomposition is:

$$y_t = \mu^*_t + \varepsilon^*_t \qquad (1.15)$$

where

$$\mu_t^* = (1 + \theta)\ \xi_t \ /\ \Delta \qquad \text{and} \qquad \varepsilon_t^* = -\ \theta\ \xi_t$$

The model is therefore of the form of a random walk plus noise in which the components are correlated and we may write

$$y_t = \frac{\eta_t^*}{\Delta} + \varepsilon_t^* \tag{1.16}$$

where $\mathrm{Var}(\eta_t^*) = (1 + \theta)^2\ \sigma^2$ and $\mathrm{Var}(\varepsilon_t^*) = \theta^2\ \sigma^2$, with σ^2 being the variance of ξ_t. As can be seen, μ_t^* does indeed become equal to y_t when θ is equal to zero.

In the corresponding structural random walk plus noise model, the filtered estimate of the trend component is equal to the long-run prediction of the observations made at that point in time. This is equal, by definition, to the trend component in the Beveridge-Nelson decomposition. Thus in terms of (1.1), μ_t^* is estimated with zero MSE and there is no need to even contemplate smoothing; see, for example, Watson (1986, p.65). However, in the structural model, a better estimator of the underlying trend, μ_t, can be obtained by smoothing and, except when θ is equal to zero or minus one, it will generally be different to the filtered estimator. Hence it will also be different to the trend, μ_t^* , given by the Beveridge-Nelson decomposition. The fact that observations beyond time t are ignored in constructing the Beveridge-Nelson trend within the series must surely lessen its appeal.

The Beveridge-Nelson definition of trend will not always correspond to the filtered estimate of trend in analogous structural model. Thus in the damped trend model, the trend forecast function, (1.7), only goes towards its final form, which is a horizontal straight line, as l goes to infinity. Hence

$$\mu_t^* = m_t + b_t / (1-\rho)$$

In the special case when $\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$ in the damped trend model, the trend is given by the observations themselves. The Beveridge-Nelson decomposition, on the other hand, yields the trend

$$\mu_t^* = y_t + \rho \, \Delta y_t / (1-\rho) = \xi_{t-1} / \{(1-\rho L)\Delta\}$$

This trend is a random walk in which the variance of the disturbance term is $\sigma_\xi^2 / (1-\rho)^2$. This suggests that, for $\rho > 0$, it will be more erratic than the actual observations which are driven by a disturbance term with variance σ_ξ^2.

1.3 Smoothness and the Model Timing Interval

As was indicated in the introductory paragraphs, the criteria for trend- and seasonal components are satisfied by models formulated in continuous time. They are also satisfied in discrete models which are formulated at a finer timing interval than the observation timing interval. One interesting aspect of such models is that for flow data they can lead to a somewhat smoother trend than would be obtained if the model were formulated at the same timing interval as the observations.

Consider a random walk plus noise structural model in which observations are made every δ time periods. This may be written as

$$y_t^{\dagger} = \mu_t + \varepsilon_t \qquad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2) \qquad (1.17a)$$

$$\mu_t = \mu_{t-1} + \eta_t \qquad \eta_t \sim NID(0, \sigma_\eta^2) \qquad (1.17b)$$

with the observations defined by

$$y_\tau = \sum_{r=0}^{\delta-1} y^{\dagger}_{\delta\tau-r} \qquad \tau = 1, \ldots, T \qquad (1.17c)$$

The reduced form of the above model is the ARIMA(0,1,1) process

$$\Delta y_\tau = \xi_\tau + \theta\, \xi_{\tau-1} \qquad (1.18)$$

with $-1 \le \theta \le 0.27$; see Tiao (1972). When $\delta = 1$, θ cannot be positive, and so formulating the model at a finer timing interval effectively extends it, even though the parameter space of the original structural parameters is unaltered.

The temporal aggregation in model (1.17) can be handled within a state space framework by using the methods developed in Harvey and Stock (1988). The first step is to define the cumulator variable

$$y^f_{\delta(\tau-1)+r} = \sum_{s=1}^{r} y^f_{\delta(\tau-1)+s} \qquad \tau=1,\ldots,T \qquad r=1,\ldots,\delta \quad (1.18)$$

Substituting from (1.17) gives

$$\begin{aligned} y^f_t &= \psi_t\, y^f_{t-1} + \mu_t + \varepsilon_t \\ &= \psi_t\, y^f_{t-1} + \mu_{t-1} + \eta_t + \varepsilon_t \end{aligned}$$

with the indicator variable, ψ_t, defined as

$$\psi_t = \begin{cases} 0 & t = \delta(\tau-1) + 1 \qquad \tau=1,\ldots T \\ 1 & \text{otherwise} \end{cases} \qquad (1.20)$$

The cumulator variable y^f_t, is used to augment the state vector to give the transition equation

$$\begin{bmatrix} \mu_t \\ y^f_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \psi_t \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ y^f_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \eta_t \\ \varepsilon_t \end{bmatrix} \qquad (1.21a)$$

while the measurement equation is

$$\bar{y}_\tau = (\,0 \quad 1\,) \begin{bmatrix} \mu_t \\ y^f_t \end{bmatrix} \qquad t=\delta\tau \qquad \tau = 1,\ldots T \qquad (1.21b)$$

No starting values are posed by the introduction of y^f_t into the state vector as $y^f_c = 0$ and the model can be handled by the Kalman filter and smoothing algorithms.

Figure 1 shows ten observations y_τ , $\tau = 1,\dots,10$ generated by a random walk. The broken line is the trend obtained from fitting (1.17) with $\delta = 4$ and setting the ratio $\sigma^2_\eta / \sigma^2_\varepsilon$ to 0.4. It is this ratio of variances which gives a random walk at the observation timing interval. The forecast function is a horizontal straight line passing through the last observation. Thus all the criteria of section (1.1) are satisfied, but the trend is smoother than the series itself, which, of course, is the same as the trend in the random walk model.

2. SEASONALITY

2.1 Definition of Seasonality

As with a trend, a seasonal component is defined in terms of predictions it yields. *The estimated seasonal is that part of the series which, when extrapolated, repeats itself over any one year time period and averages out to zero over such a time period.* More formally, if $\tilde{c}_{T+l/T}$ denotes the estimated seasonal component at time $T+l$, the seasonal forecast function satisfies the condition:

$$\tilde{c}_{T+l/T} = \sum_{j=1}^{s-1} \tilde{c}_{T+l-j/T} \qquad l = 1, 2, \dots \qquad (2.1)$$

Hence the forecast function depends solely on the current estimates of the seasonal effects over the last s-1 periods, that is $\tilde{c}_{T-j/T}$, $j = 0,1,\ldots,(s-2)$. This is somewhat akin to Property (T2) which was defined for a trend.

The seasonal forecast function contains no information on the general direction of the series, either in the long run or the short run. It is therefore often sensible to focus attention on the seasonally adjusted forecast function for the series which is

$$\tilde{y}^{a}_{T+l/T} = \tilde{y}_{T+l/T} - \tilde{c}_{T+l/T} \qquad l = 1,2,\ \ldots \qquad (2.2)$$

Replacing $\tilde{y}_{T+l/T}$ by $\tilde{y}^{a}_{T+l/T}$ in Property (T1) of sub-section 1.1 allows the definition of a trend to be extended to a seasonal series.

The seasonal component in a structural model is usually modelled in terms of stochastic dummy variables or stochastic trigonometric components. The dummy variable model is

$$\sum_{j=0}^{s-1} \gamma_{t-j} = \omega_t \qquad \text{or} \qquad \gamma_t = \sum_{j=1}^{s-1} \gamma_{t-j} + \omega_t \qquad (2.3)$$

where $\omega_t \sim NID(0,\sigma^2_\omega)$. The bigger the value of the variance σ^2_ω relative to σ^2_ε , the more are past observations discounted in constructing a seasonal pattern for the forecast function. The forecasts satisfy the recursion in (2.1) where the starting values are given by the estimated seasonal effects at time T. Thus the seasonal pattern projected into the future is fixed, and its components sum to zero over any period of s consecutive months. The same is true of the trigonometric seasonal model which is formulated as

$$\gamma_t = \sum_{j=1}^{[s/2]} \gamma_{t,j} \qquad (2.4a)$$

with

$$\gamma_{j,t} = \gamma_{j,t-1} \cos \lambda_j + \gamma^*_{j,t-1} \sin \lambda_j + \omega_{jt} \qquad j=1,\ldots,[s/2]$$

$$\gamma^*_{j,t} = -\gamma_{j,t-1} \sin \lambda_j + \gamma^*_{j,t-1} \cos \lambda_j + \omega^*_{jt} \qquad (2.4b)$$

were ω_{jt} and ω^*_{jt} are zero mean white noise processes which are uncorrelated with each other with a common variance σ^2_ω for $j = 1, \ldots, [s/2]$, and $[s/2]$ is $s/2$ when s is even and $(s-1)/2$ when s is odd. Whem s is even the equation for $\gamma^*_{\frac{s}{2},t}$ is redundant The forecast function is

$$\tilde{\gamma}_{T+l/T} = \sum_{j=1}^{[s/2]} (\tilde{\gamma}_{j,T} \cos\lambda_j l + \tilde{\gamma}^*_{j,T} \sin\lambda_j l) \qquad l=0,1,2,\ldots \qquad (2.5)$$

and from standard trigonometric identities (2.1) holds.

More generally, if $S(L)\gamma_t \sim MA(s-2)$, the forecast function satisfies (2.1). The seasonal component in the canonical decomposition of Hillmer and Tiao (1982) is such that $S(L)\gamma_t \sim MA(s-1)$ and in this case (2.1) only holds for $l=2$ onwards. Similarly the seasonal components

$$S(L)\gamma_t = (1 + \theta_s L^s)\, \omega_t \qquad (2.6)$$

and

$$S(L)\gamma_t = (1 + \theta_s L^s + \theta_{2s} L^{2s}) \omega_t \tag{2.7}$$

where ω_t is white noise, satisfy (2.1) from $t=3$ onwards and $t=s+3$ onwards respectively. The rationale behind (2.6) and (2.7) is discussed in Cleveland and Tiao (1976) and Burridge and Wallis (1984).

The model

$$\gamma_t = \gamma_{t-s} + \omega_t \tag{2.8}$$

where ω_t is white noise is sometimes used for modelling seasonality. It is not suitable for this purpose. The forecast function repeats itself every year, but the sum of the terms over a year will not, in general, be zero. Thus the predictions of the seasonal component are confounded with the predictions of the trend. For similar reasons, it is not possible to separate out the trend and seasonal components in the sample; see Pierce (1979).

Introducing a parameter φ into (2.8) yields the stationary model

$$\gamma_t = \varphi \gamma_{t-s} + \omega_t \qquad |\varphi| < 1 \tag{2.9}$$

This model has a forecast function which damps down to zero as $l \rightarrow \infty$. There may be occasions where a component of this kind is useful for picking up certain seasonal effects, but these effects are transient and do not constitute a seasonal component as such. As Bell and Hillmer argue (1984, p.304), it would be inappropriate to attempt to remove them as part of a process of seasonal adjustment.

We now consider briefly how the notion of seasonality may be generalized. The crucial feature of a seasonal component is that it should be based on the seasonal summation operator, S(L). Thus the notion of a seasonal component might be generalized by defining it such that $S^2(L)\gamma_t$ follows a MA Process. In the dummy variable seasonal model, the seasonal effects could be allowed to increase over time by adding a slope term to (2.3). If this slope term is denoted by $\beta_t^{(s)}$, the model becomes

$$\sum_{j=0}^{s-1} \gamma_{t-j} = \beta_{t-1}^{(s)} + \omega_t \tag{2.10a}$$

$$\sum_{j=0}^{s-1} \beta_{t-j}^{(s)} = \zeta_t^{(s)} \tag{2.10b}$$

where $\zeta_t^{(s)}$ is a white noise disturbance term. Using the seasonal summation operator the model can be rewritten as

$$S^2(L)\gamma_t = \zeta_{t-1}^{(s)} + S(L)\omega_t \tag{2.11}$$

which shows that $S^2(L)\gamma_t \sim MA(s-1)$. Trigonometric seasonality can also be extended by the introduction of an additional component in a 2 x 1 vector, $\beta_{j,t}^{(s)}$, at each seasonal frequency. Thus if (2.4) is written as

$$\underset{\sim}{\gamma}_{j,t} = T_j \underset{\sim}{\gamma}_{j,t-1} + \underset{\sim}{\omega}_{j,t} \qquad j=1,\ldots,[s/2] \tag{2.12}$$

it becomes

$$\underset{\sim}{\gamma}_{j,t} = T_j \; \underset{\sim}{\gamma}_{j,t-1} + \underset{\sim}{\beta}^{(s)}_{j,t} + \underset{\sim}{\omega}_{j,t} \qquad (2.13a)$$

$$\underset{\sim}{\beta}^{(s)}_{j,t} = T_j \; \underset{\sim}{\beta}^{(s)}_{j,t-1} + \underset{\sim}{\zeta}^{(s)}_{j,t} \qquad (2.13b)$$

where $\underset{\sim}{\zeta}^{(s)}_{j,t}$ contains two uncorrelated white noise disturbances with common variance. Whether models of this kind are a useful extension of the notion of seasonlaity remains to be seen.

There is an example of the use of a seasonal component of the form (2.10) in Kitagawa and Gersch (1984, p.386). In fact, they impose the constraint that σ^2_ω is zero, so that (2.10a) simply has a white noise disturbance term on the right hand side.

2.2 Model Based Seasonal Adjustment

Seasonal adjustment entails the removal of a seasonal component from a series. In a structural model, the various components are defined explicitly. Given a linear model with normally distributed disturbances, the mechanics of seasonal adjustment are straighforward. Once the parameters have been estimated, the seasonal component can be obtained by a smoothing algorithm and subtracted from the original series to yield the seasonally adjusted series

$$y^a_t = y_t - \tilde{c}_{t/T} \qquad t=1,...T \qquad (2.14)$$

The estimated RMSE of $\tilde{c}_{t/T}$, and hence y^a_t, are also given by the smoother. As Hausman and Watson (1985) note, as early as 1962, the

President's Committee to Appraise Employment and Unemployment Statistics in the U.S. recommended "that estimates of the standard errors of seasonally adjusted data be prepared and published as soon as the technical problems have been solved".

The seasonally adjusted figures at the end of the series will be subject to revisions as new observations become available. The nature and extend of these revisions may be investigated by setting up a fixed point smoother. It should be noted that such revisions are inevitable, but that they are minimized for a correctly specified model. The question of revising arising from the fact that the latest observations are often preliminary is a separate one. Suffice to say that such revisions can be handle by the Kalman filter using a suitable extension of the model.

Seasonal adjustment can be carried out for any structural model containig a seasonal component. The usual choice will be the basic structural model (BSM)

$$y_t = \mu_t + \gamma_t + \varepsilon_t \tag{2.15}$$

where μ_t is the stochastic trend, (1.4), and γ_t is dummy variable or trigonometric seasonal component defined earlier. The model may possibly contain a cycle or, as Hausman and Watson (1985), an irregular component which reflects the sampling scheme. Because y_t^a is constructed by fitting a model to the series, the procedure is known as model based seasonal adjustment.

The notion of model based seasonal adjustment could be extended further by introducing explanatory variables into the model. The seasonal component then becomes

$$\gamma_t^{\dagger} = \gamma_t + \gamma^{*} x_t \qquad (2.16)$$

where γ_t is the standard dummy variable or trigonometric seasonality model, x_t is an observable explanatory variable and γ^{*} is a unknown coefficient. The most likely candidates for x_t are weather variables such as temperature or rainfall. If the x_t's are measured in terms of deviations from the average for each particular season, the $\gamma_t^{\dagger}$'s will still satisfy the condition (2.1) as the value of x_t entering into the forecast function will all be zero.

Calendar effect can also be handle by introducing explanatory variables into the model.For instance, if the x_t's are measured in terms of deviations from the average for each year, as for working days, the $\gamma_t^{\dagger}$'s will still satisfy the condition (2.1) for every year as the value of x_t entering into the forecast function will sum zero in erevy year.

A final aspect of model based seasonal adjustment is that the state space form enables data irregularities, such as missing observations, to be handle. It also provides a framework for dealing with outliers.

Model based seasonal adustment can also be carried out by fitting a seasonal ARIMA model to the series and then decomposing it into trend, seasonal and irregular components. This is the approach adopted by Burman (1980) and Hillmer and Tiao (1982). Hillmer and Tiao (1982) examine three models which give rise to what they call "an acceptable decomposition". The most important of these is the airline model.

$$y_t = (1 + \theta L)(1 + \Theta L^s)\xi_t \tag{2.17}$$

where the decomposition is such that

$$\begin{aligned} \Delta^2 \mu_t &\sim MA(2) \\ S(L)\gamma_t &\sim MA(\underline{s}-1) \end{aligned} \tag{2.18}$$

while the irregular component is white noise. Without further restrictions the components would not be identifiable. This problem is solved by means of the canonical decomposition whereby the variance of the irregular component is maximized. The mechanics of the canonical decomposition were described for a simple case in sub-section 1.2; for a seasonal model the technical details are considerably more complex.

The decomposition obtained from the airline model is very similar to the UCARIMA form of the BSM. The difference is that in the BSM, the orders of the MA components in $\Delta^2\mu_t$ and $S(L)\gamma_t$ are one less than in (2.18), that is they are one and s-2 respectively. Maravall (1985) shows that the BSM and airline models can be applied to much the same kind of series. If both models are fitted to such a series it follows from the discussion in sub-section 1.2 that the trend extracted by the canonical decomposition will be smoother than the trend estimated for the BSM. The same is true for the seasonal component. A corollary to this is that the seasonally adjusted series will be correspondingly less smooth when the seasonal component is estimated by the canonical decomposition.

The objections to the canonical decomposition as a method of trend extraction were set out in sub-section 1.2. These objections apply with equal force in the context of seasonal adjustment. It is also important to point out that very few seasonal ARIMA models are capable of producing an acceptable decomposition, and that even in the case of the airline model, it is necessary to impose the restriction that the seasonal MA parameter Θ is less than or equal to zero. By contrast, a structural model can always be decomposed into components since it is explicitly set up in that way in the first place.

2.3 X-11 and Model Based Seasonal Adjustment

The U.S. Bureau of the Census X-11 procedure has been widely used for the seasonal adjustment of official time series. It is based on a series of filters. These filters are two-sided, but it is clearly not possible to apply a two-sided filter at the end of the series. Instead a one-sided filter must be applied. The result is that the latest adjusted figures must be revised as new observations become available and it becomes possible to apply a two-sided filter. Dagum (1975) suggested that a more satisfactory aprroach would be to fit an ARIMA model to the series, forecast future values of the series and then seasonally adjust the whole series, actual and predicted, by X-11. In this way the latest observations are actually adjusted by a two-sided filter. Revisions are still necessary as actual observations come in to replace the predicted values, but they should be smaller than before. This modified version of X-11 is known as X-11-ARIMA.

The basic drawback to X-11 is its inflexibility. Although there is some scope for adjustment, the same procedure is essentially applied irrespective of the properties of the series. This can lead to the adjusted series having very undesirable properties, something which has now been well documented; see Pierce (1978) and Hausman and Watson (1985).

A model based seasonal adjustment procedure is tailor made to a particular series. As noted in the previous section, the model specification may not be the same for all series. Futhermore, for a particular specification, such as the BSM, there is the additional flexibility deriving from the estimation of the model hyperparameters. Once these parameters have been estimated the Kalman filter and smoother produce the optimal estimates of the trend and seasonal components. There is no need to consider some modification based on forecasting future observations as in X-11-ARIMA. Revisions will still be needed as new observations become available, but, as pointed out in the previous sub-section, if the model is correctly specified, the size of these revisions will be minimized.

The theoretical argument in favour of model based seasonal adjustment is very strong. However, since X-11 has been found to perform reasonably well for many series, it is important to determine whether the BSM is applicable to such series. This question is addressed in Maravall (1985): If y_t denotes the unobserved components model for a monthly series for which X-11 yields the optimal estimates of the components, the a.c.f. of $\Delta\Delta_{12}y_t$ may be derived. All the constraints on the a.c.f. of the dummy variable BSM are satisfied by this a.c.f. In particular, setting

$$q_\eta = 0.133 \qquad q_\zeta = 0.167 \quad \text{and} \qquad q_\omega = 0.067 \tag{2.19}$$

where $q_\omega = \sigma^2_\omega / \sigma^2_\varepsilon$ makes the two a.c.f.'s practically identical. (There is a misprint in Maravall's paper in equation (3.1) where $0.050\gamma_0$ should read $0.150\gamma_0$.) Thus the BSM can certainly be fitted to series for which X-11 is appropriate. On the other hand, an examination of the estimated parameters in many BSM's shows them to be some way removed from the values in (2.19). For real series, σ^2_ζ is usually dominated by σ^2_η while σ^2_η is often of a similar size, or larger, than σ^2_ε . The seasonal variance σ^2_ω , is often close to zero; see Pierce (1978). The trend and seasonal components extracted from a BSM with hyperparameters set as in (2.19) will therefore typically be smoother than if the parameters had been estimated.

A second issue considered by Maravall (1985) is the nature of the X-11 and BSM decompositions for series in which X-11 is appropriate. Following Cleveland (1972), the X-11 decomposition explicitly corresponds to the following unobserved component model:

$$y_t = \frac{1 + 0.26L + 0.30L^2 - 0.32L^3}{\Delta^2}\,\zeta^*_{1t} + \frac{1 + 0.26L^{12}}{S(L)}\,\zeta^*_{2t} + \zeta^*_{3t} \qquad (2.20)$$

where ζ^*_{1t}, ζ^*_{2t} and ζ^*_{3t} are independent white noise disturbances with

$$\mathrm{Var}(\zeta^*_{1t})/\mathrm{Var}(\zeta^*_{3t}) = 0.099 \text{ and } \mathrm{Var}(\zeta^*_{2t})/\mathrm{Var}(\zeta^*_{3t}) = 0.030 \qquad (2.21)$$

It follows that $\mathrm{Var}(\Delta\Delta_{12}y_t) = 6.067\ \mathrm{Var}(\zeta^*_{3t})$. In the case of the BSM with relative hyperparameters given by (2.19), $\mathrm{Var}(\Delta\Delta_{12}y_t) = 6.667\ \sigma^2_\varepsilon$. The net result is that the variance of the X-11 irregular component is approximately 10% greater than that of the BSM. The implicit model underlying X-11 is therefore quite similar to the UCARIMA form of the BSM. The only difference is the additional MA terms in (2.20) which result in some additional smoothing for the trend and seasonal components and a corresponding increase in the variance of the irregular.

For most series, therefore, X-11 will tend to produce a smoother trend component than the BSM. This arises both because of the implied parameter values in the underlying reduced form and because of the decomposition given by X-11 for such a reduced form. Nevertheless, there are occasions when the BSM will give a smoother trend and it follows from the discussion at the end of sub-section 1.1, that one such case is when σ_{η}^{2} is found to be small relative to σ_{ζ}^{2} .

2.4 Why Seasonally Adjust Anyway?

Although the pattern of seasonal effects may change slowly over time, seasonality is basically a repetitive feature of s time series. This emerges explicitly in the seasonal forecast function and there is an obvious rationale for looking at a forecast function for the series with the seasonal effects removed as in (2.2). By a similar token, it could be argued that it is useful to look at a series in order to gain some idea of its history and that if this is to be done it is helpful to abstract from its seasonal movements; hence the need for seasonal adjustment.

Why would one want to look at a seasonally adjusted series apart from historical interest? One reason is the desire to extrapolate. However, if a series has been seasonally adjusted by fitting a model to it, this becomes pointless since optimal predictions can be made directly from the model. Indeed, in a structural model all the information needed to make predictions is explicitly available in the final estimated state vector. Thus the ability to carry out the model based seasonal adjustment effectively removes one of the main reasons for wishing to seasonally adjust in the first place!

There is perhaps some case to be made for working with a seasonally adjusted series at the model formulation stage. The use of a relatively robust procedure, such as X-11, may be helpful in enabling one to identify non-seasonal movements. For example, the seasonally adjusted series may give a clearer indication as to whether a trend is subject to a saturation level, in which case a damped trend might be fitted in preference to the usual local linear trend. It may also make it easier to detect breaks and structural changes.

The production of seasonally adjusted series is standard practice for government agencies. While adjusted series may be marginally easier for the causal user to interpret, it has to be recognized that the original series has been distorted in what is often a rather arbitrary way. There are usually better and more direct ways of providing the information which the user actually requires. Certainly the idea that a seasonally adjusted series sould be published *instead of* the original series is open to very serious criticism.

3. Applications

The applications below illustrate some of the points made above. The calculations were all carried out on an IBM AT using the program STAMP (Structural Time series Analyser Modeller and Predictor), developed by S. Peters.

The estimates given are maximum likelihood estimates computed in the frequency domain by the method of scoring, for the first two examples and for the last example they are computed in the time domain.

3.1 Cycle in US Investment

The additive cycle takes the form

$$y_t = \mu_t + \psi_t + \varepsilon_t \tag{3.1}$$

where μ_t is the stochastic trend model of (1.4) and ψ_t is a stationary stochastic cycle defined by

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos \lambda_c & \sin \lambda_c \\ -\sin \lambda_c & \cos \lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varkappa_t \\ \varkappa_t^* \end{bmatrix} \tag{3.2}$$

where $\varkappa_t$ and $\varkappa_t^*$ are mutually uncorrelated normally distributed white noise processes with mean zero and common variance $\sigma_\varkappa^2$, λ_c is the cyclical frequency and ρ is a damping factor such that $0 \leq \rho \leq 1$; see Harvey (1985).

Fitting (3.1) to data on the logarithms of US investment (in seasonally adjsuted form!) over the period 1951 Q1 to 1985 Q4 gave the following results:

$$\tilde{\sigma}_\eta^2 = 0 \quad \tilde{\sigma}_\zeta^2 = \underset{(0.07 \times 10^{-4})}{.11 \times 10^{-4}} \qquad \tilde{\sigma}_\varkappa^2 = \underset{(2.9 \times 10^{-4})}{21.8 \times 10^{-4}} \quad \tilde{\sigma}_\varepsilon^2 = 0$$

$\tilde{\rho} = 0.90$ $\qquad\qquad$ $\tilde{\lambda}_c = 0.299$
(0.03) $\qquad\qquad\qquad$ (0.045)

The model satisfies the standard diagnostics, with the Box-Ljung Q-statistics based on the first 12 residuals autocorrelations being Q(12) = 12.38. The period of the stochastic cycle is $2\pi/ .299 = 20.9$, which is just over five years.

The main features of the estimated model are that the irregular component disappears, and the estimate of the level variance σ^2_η is zero. This second feature of the model is responsible for the trend being relatively smooth as shown in Figure 2. Note that the extrapolation of the trend follows on continuously from the estimated trend within the sample. Figure 3 shows the estimated cycle within the series, while Figure 4 shows the forecast function for the series as a whole. This consists of the trend forecast function of Figure 2 plus the transitory forecast function for the cyclical component. If $\tilde{\psi}_T$ and $\tilde{\psi}^*_T$ denote the expected values of ψ_T and ψ^*_T at time T, the l-step ahead predictions of the cyclical component is

$$\tilde{\psi}_{T+l/T} = \rho^l (\tilde{\psi}_T \cos \lambda_c l + \tilde{\psi}^*_T \sin \lambda_c l) \qquad l=1,2,... \qquad (3.3)$$

and the forecast fucntion is a damped sine, or cosine, wave.

3.2 Industrial Production in Brazil

Data on industrial production in Brazil is published by the IBGE (Brazilian Institute of Geography and Statistics). The series considered here is the Total Industrial Production, which is the aggregation of Consumer Goods, Capital Goods and Intermediate Goods series. In order to model this series, the SM is supplemented by a dummy variable, x_t, recording the discrepancy between the working days in each particular month and the average of working days in a year; see equation (2.18).

$$\tilde{\sigma}_{\varkappa}^{2} = \underset{(2.763)}{4.267} \qquad \tilde{\rho} = \underset{(0.08)}{0.86} \qquad \tilde{\lambda}_c = \underset{(0.143)}{0.243}$$

$$\tilde{\sigma}_{\eta}^{2} = 0.0 \qquad \tilde{\sigma}_{\zeta}^{2} = \underset{(0.025)}{0.03} \qquad \tilde{\sigma}_{\omega}^{2} = \underset{(0.251 \times 10^{-2})}{0.334 \times 10^{-2}}$$

$$\tilde{\sigma}_{\varepsilon}^{2} = \underset{(1.56)}{2.220}$$

$$\text{Working Days(88M4)} = \underset{(.301)}{1.784}$$

The model satisfies the standard diagnostics, with Box-Ljung Q-statistics based on the first 24 residuals autocorrelations being Q(24) = 15.54. The period of the stochastic cycle is 2π / .243 = 25.8, which is just over two years.

Figures 5, 6, 7, 8, and 9 show the trend, cyclical, seasonal and irregular components together with the part of the series which is explained by the exogenous variable representing the working days effect. Adding the seasonal effects picked up by the working days variable to the seasonal component in Figure 7 would give the composite seasonal effect $\gamma_t^{\dagger}$. It is interesting to see the way in which the seasonal component cahnges in response to the working days effect of March 1988. The effectiveness of the model is brought home in Figure 10 which shows the extrapolations made over the 1988 year. If the working days effects were not present in the model, the effect of March 1988 would distorce the extrapolations.

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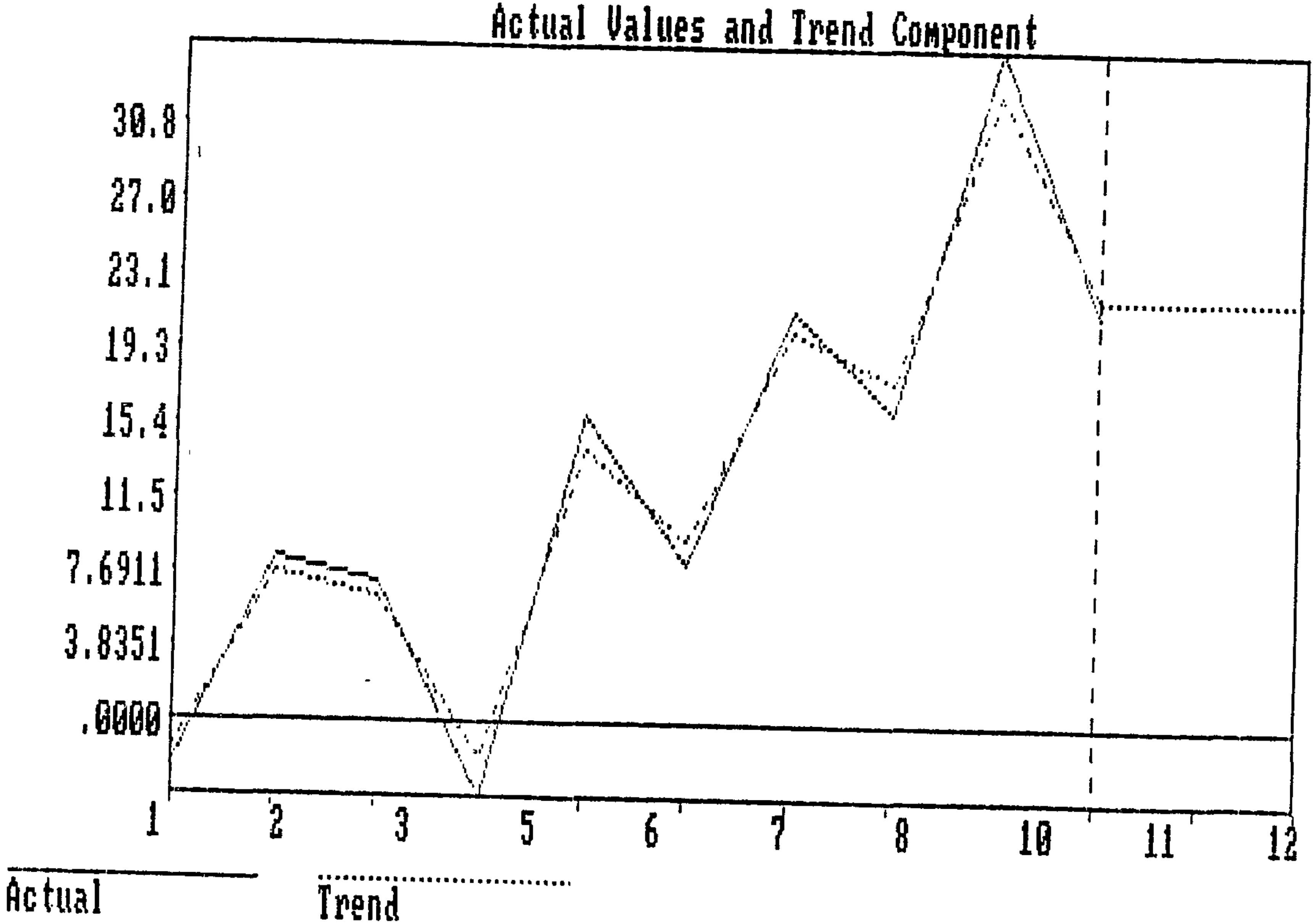

FIGURE 1 - SMOOTHED ESTIMATES FROM A RANDOM WALK

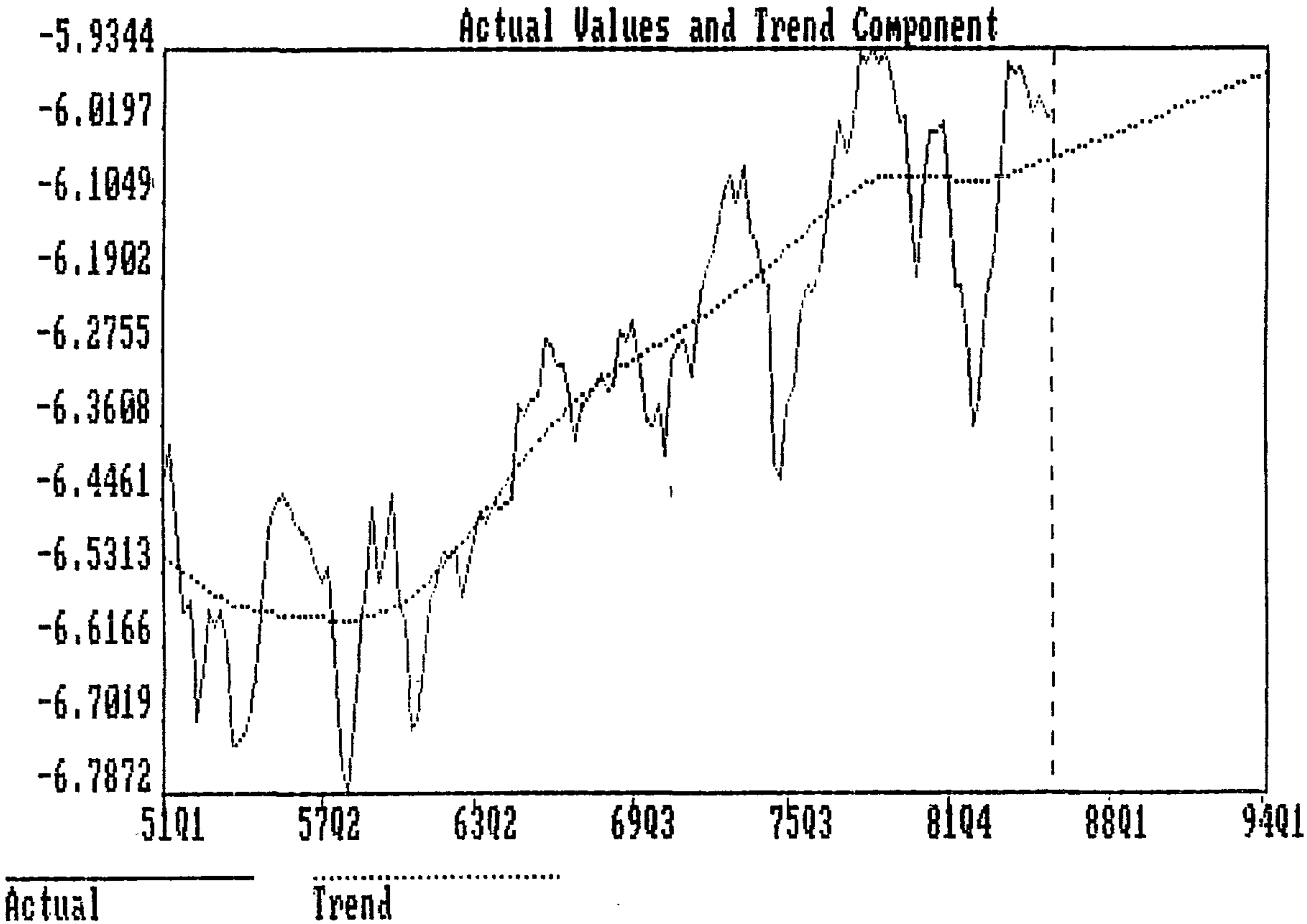

FIGURE 2 - TREND IN US INVESTMENT

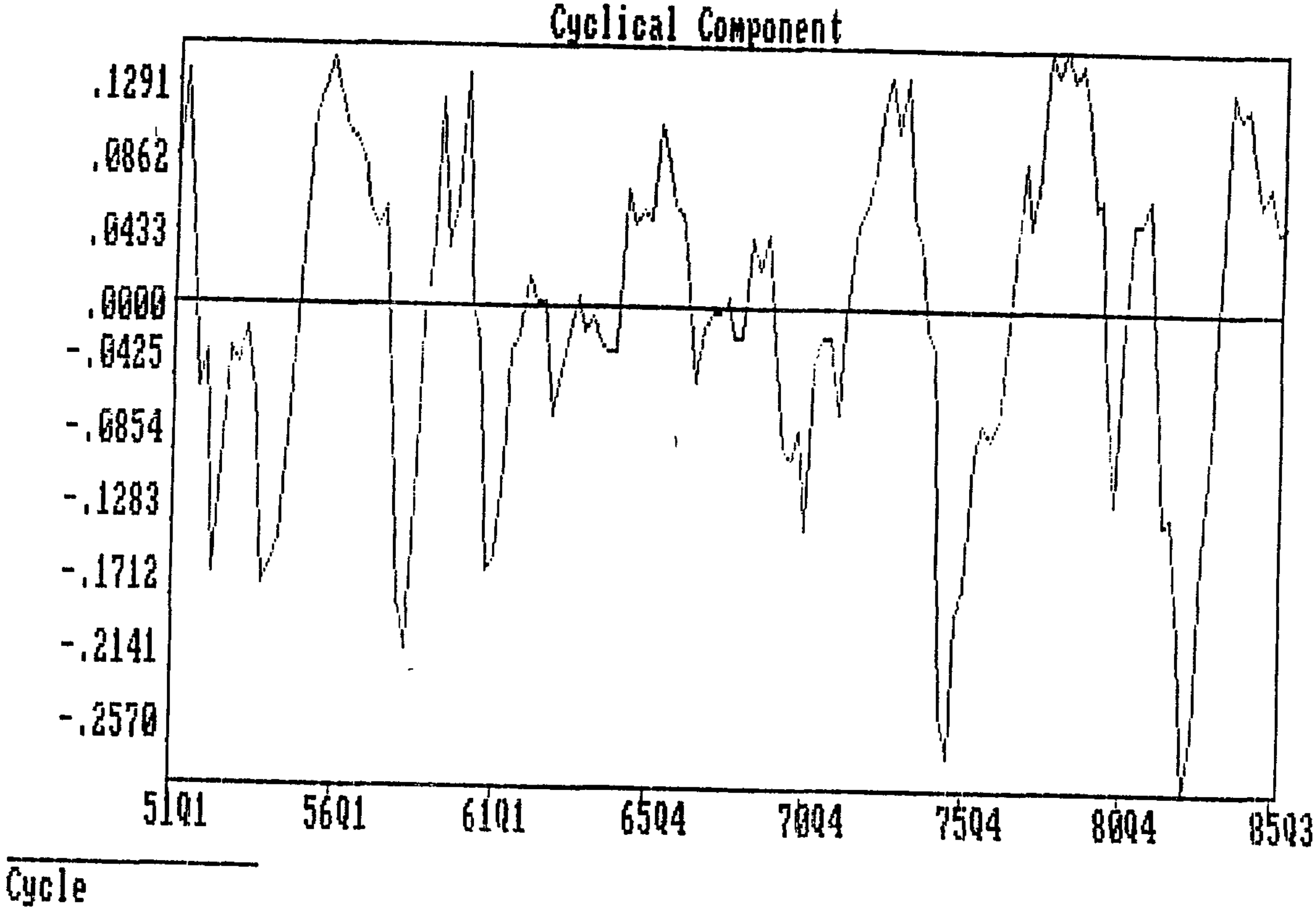

FIGURE 3 - ESTIMATED CYCLE IN US INVESTMENT

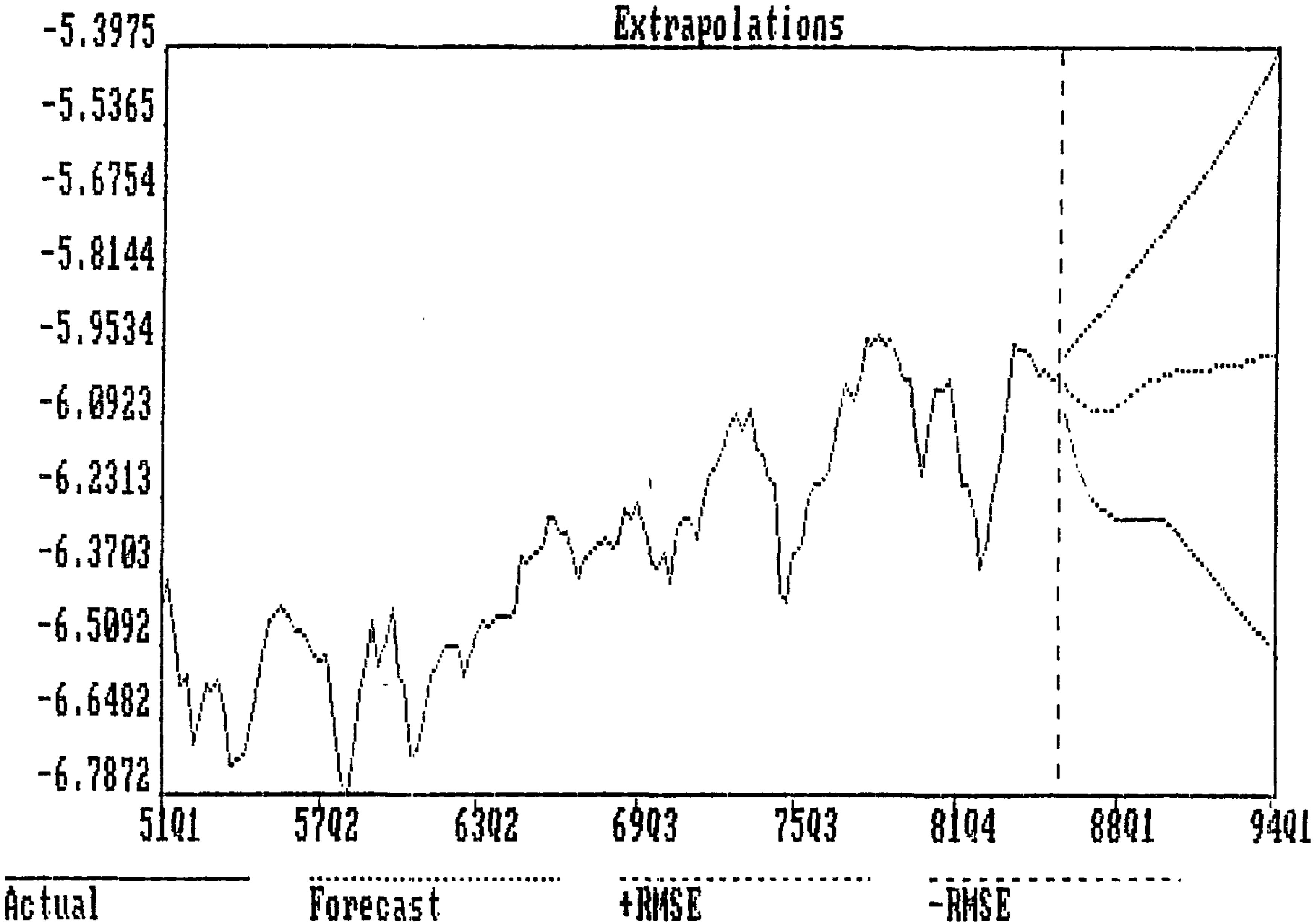

FIGURE 4 - PREDICTIONS FOR US INVESTMENT

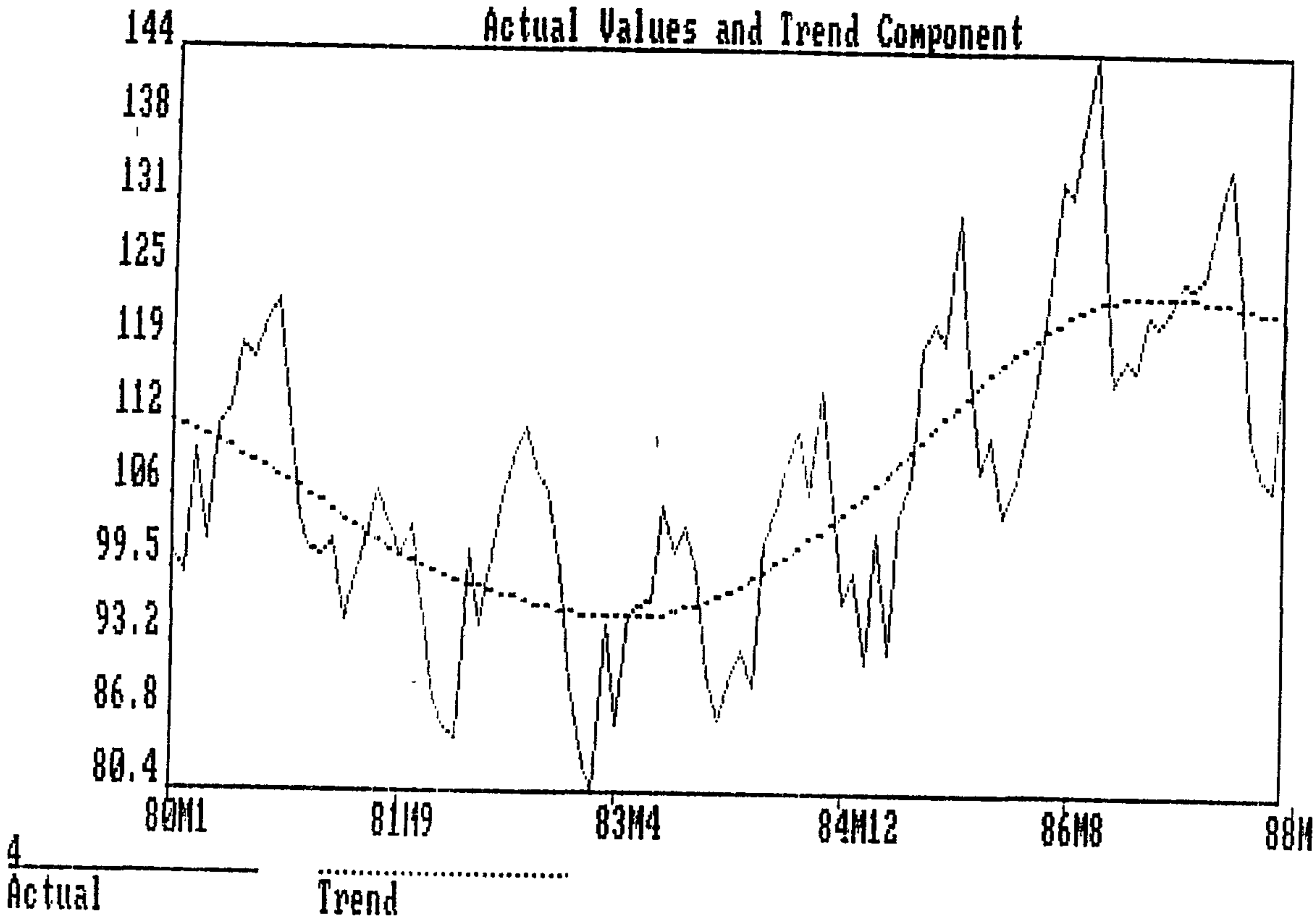

FIGURE 5 - TREND FOR BRAZILIAN INDUSTRIAL PRODUCTION

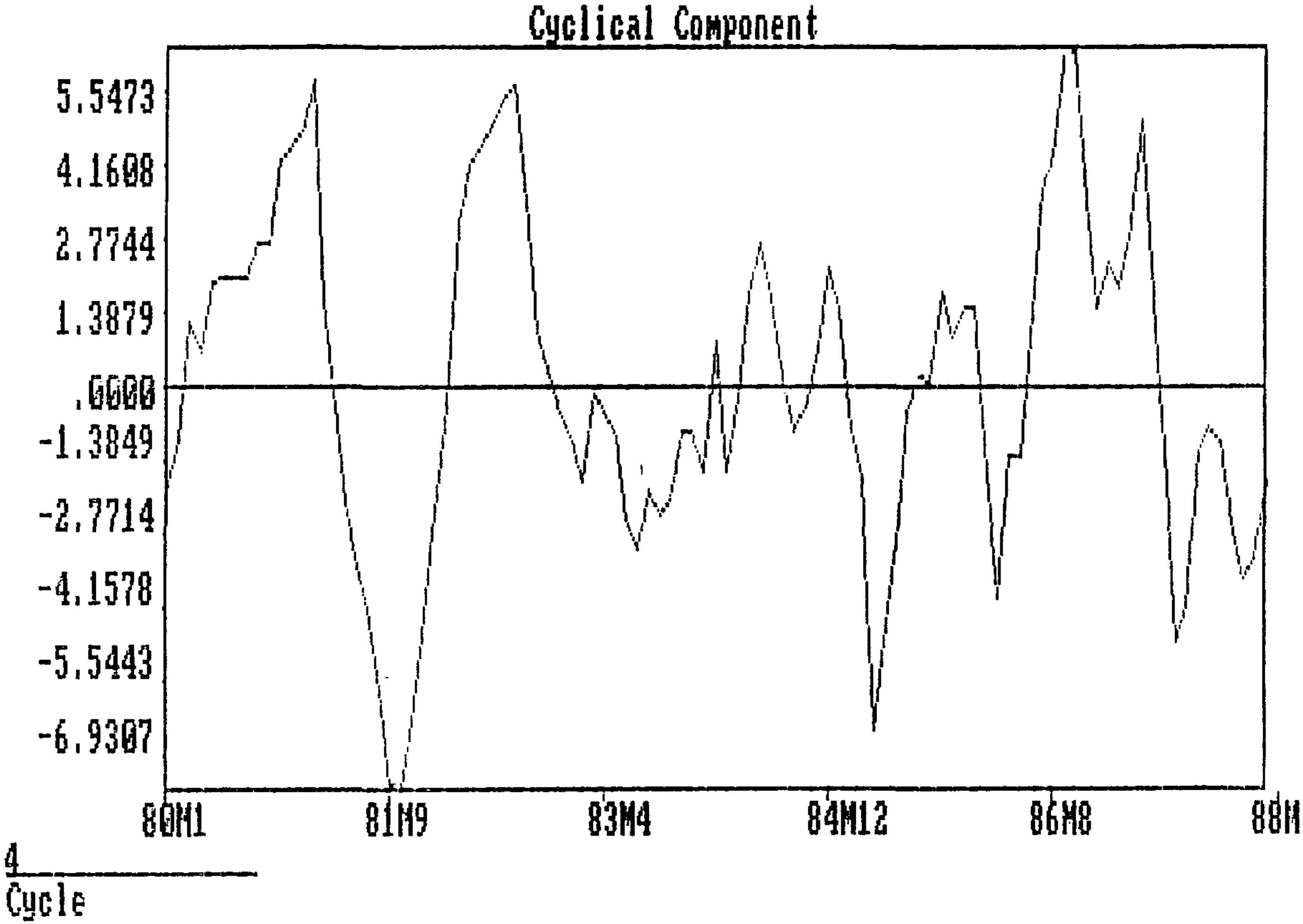

FIGURE 6 - ESTIMATED CYCLE FOR BRAZILIAN INDUSTRIAL PRODUCTION

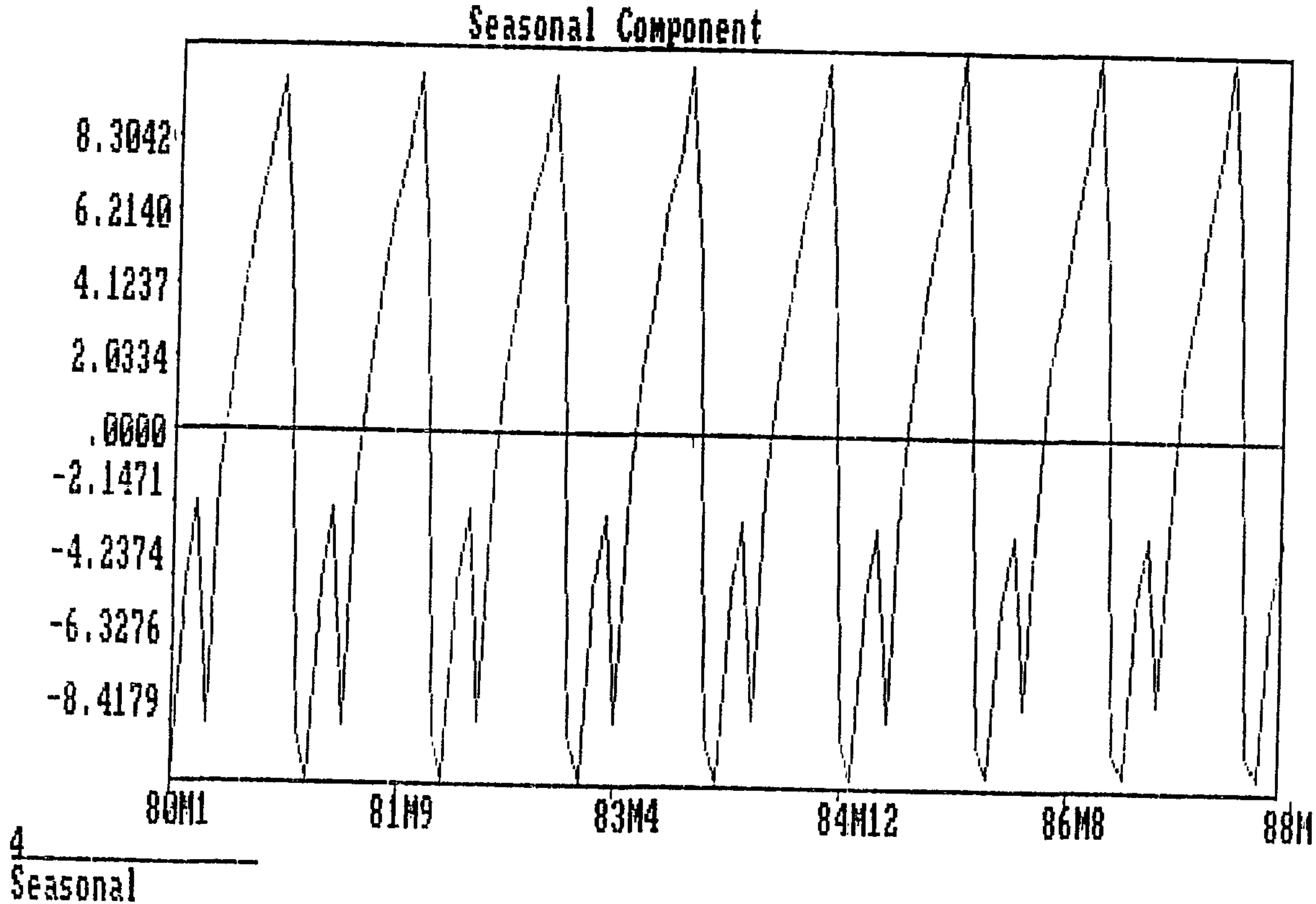

FIGURE 7 - SEASONAL FOR BRAZILIAN INDUSTRIAL PRODUCTION

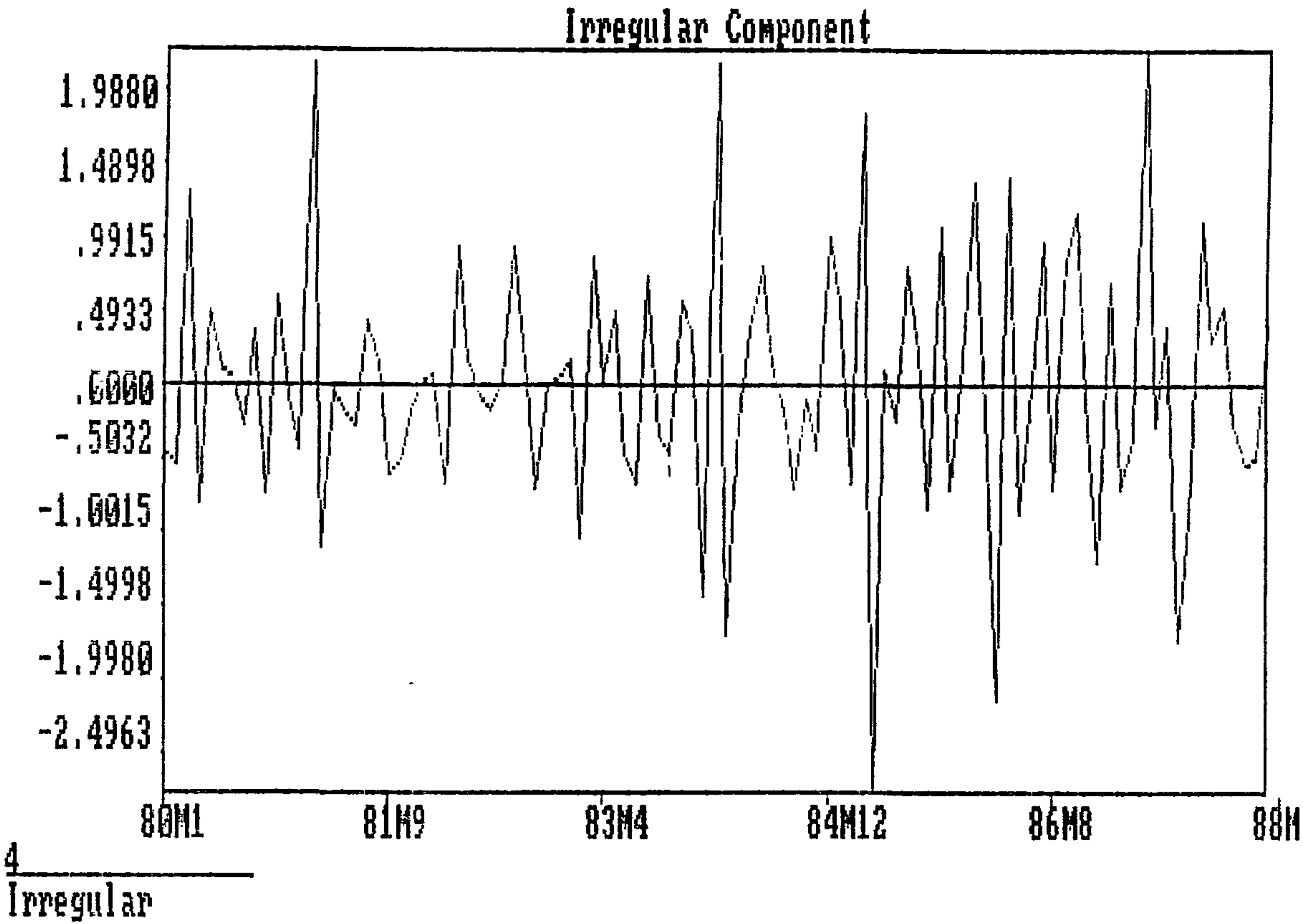

FIGURE 8 - IRREGULAR FOR BRAZILIAN INDUSTRIAL PRODUCTION

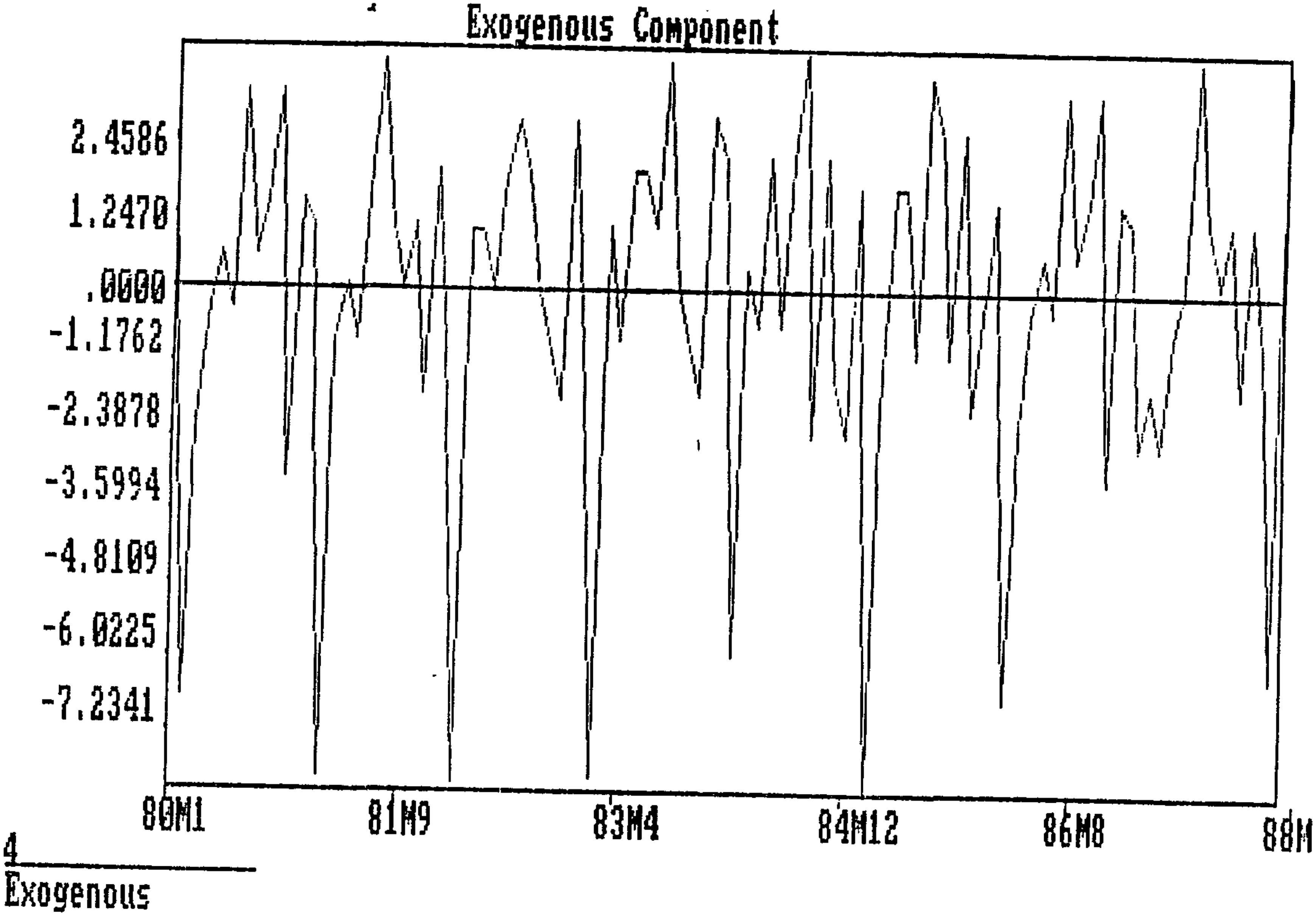

FIGURE 9 - WORKING DAYS EFFECT FOR BRAZILIAN INDUSTRIAL PRODUCTION

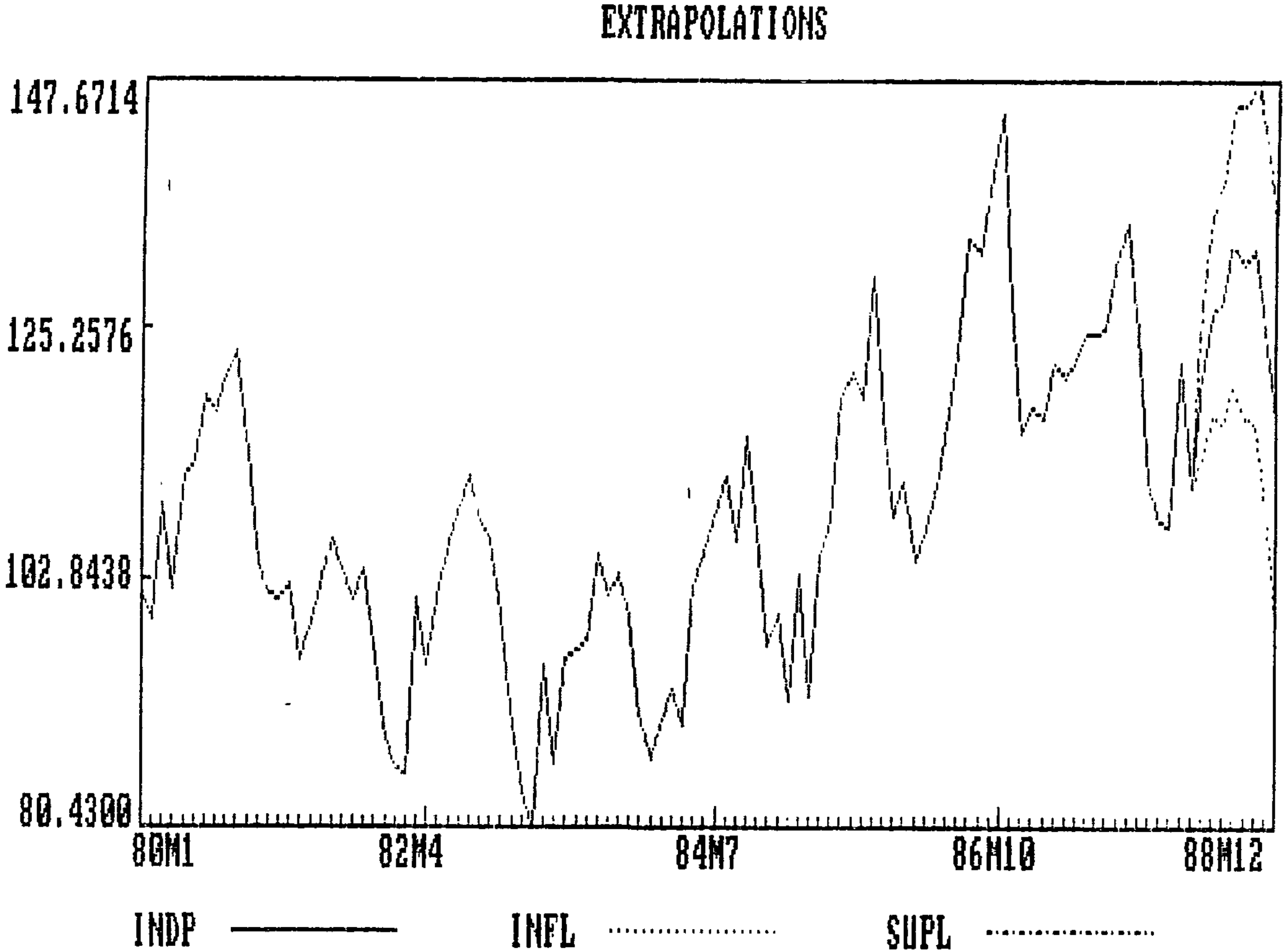

FIGURE 10 - PREDICTIONS FOR BRAZILIAN INDUSTRIAL PRODUCTION

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