TEXTO PARA DISCUSSÃO Nº 1177

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TEXTO PARA DISCUSSÃO

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SUMÁRIO

SINOPSE

Como o investimento público afeta a economia? A literatura econômica tem enfatizado o impacto na produtividade e no crescimento econômico. Neste artigo, adota-se uma abordagem diferente, considera-se que o investimento público pode reduzir o risco enfrentado pelo setor, estimulando novos investimentos privados. Estende-se o modelo de Pindyck (1988), onde o capital público entra na função de produção como mais um insumo. O governo tem um orçamento intertemporal equilibrado, de modo que o capital público fornecido no primeiro período é lastreado na receita futura de impostos. Mostra-se que a alíquota ótima é constante e independe da incerteza.

ABSTRACT

In this paper, we extend the usual models of irreversible investment under uncertainty by introducing the stock of public capital as an input for the private sector. Public investment takes place in a stochastic environment. Public capital then increases the productivity of private capital which is assumed to be fully irreversible. In our model, the government has an intertemporal budget constraint, i.e. taxes are collected each period to fund the public debt. We provide a partial equilibrium analysis, as it is standard in models of irreversible investment under uncertainty. Even under uncertainty, the optimal tax rate is then constant and does not depend on the size of uncertainty, it is exactly the same as the one that would prevail in a deterministic world. We show that the government has an insurance role since it removes part of the uncertainty faced by the firm.

1 INTRODUCTON

How does public investment affect the economic performance ? Such a question has yet received a large attention especially since it has provided a way to explain the productivity slowdown, Aschauer [1989] shows that an increase of 1% in the public-private capital ratio would increase by 0.39% the total productivity in the economy. This has given rise to a number of empirical works as well as to debates about the productivity of public spending (see Gramlich [1994] and Shioji [2001]). Barro [1990] proposes a theoretical approach (following Arrow and Kurz [1970]) in which public capital appears as a productive factor. He considers an endogenous growth model with public expenditures that enter the production function as flows of services and a balanced government budget for each period. Growth is then maximum when the tax rate equals the elasticity of public capital with respect to output. Many extensions have been proposed: for instance, Glomm and Ravikumar [1994] introduces congestion, Cashin [1995] considers the productivity of the stock of public capital and not of the flow of its services. Moreover, Burguet and Fernàndez-Ruiz [1998] relax the assumption on the government budget, allowing for borrowing and analyzing the possibility of poverty traps. All this literature about public investment affecting private production remains in a deterministic framework.

And yet Arrow and Kurz (1970) have highlighted in their seminal work the importance of public investment as a risk-bearing activity to the private sector in the presence of some uninsurable risks:

"If the government adopts an expected value criterion, while private industry does not, then a government investment may indeed displace a private investment of higher expected value; however, this is correct in the context [incomplete markets], because the government is supplying a valuable complementary activity of risk bearing which is not being supplied by the private sector"

(Arrow, K. and Kurz, M. (1970) Public Investment the rate of return and optimal fiscal policy; page XXVII)

Dixit and Pindyck (1994) also claim that the presence of incomplete markets might justify the use the fiscal policy, as a risk sharing strategy:

"Various kind of taxation do provide indirect risk sharing. In an open economy, trade taxes are such an instrument."

(Dixit, A. and Pindyck, R. (1994) Investment Under Uncertainty, page 295-6)

Moreover, in the US, large public expenditures are currently undertaken which bear a large part of the macroeconomic risk while in the European Union the "stability pact" which constrains public expenditures becomes strongly criticized. For instance, Blanchard and Giavazzi (2004) suggests that the "stability pact" should be reinterpreted allowing the Eurozone Countries to use public investment as an important source of demand management and implicitly as a risk bearing instrument.

What is then the optimal tax rate if the productivity of the public good is stochastic as it would be the case in an uncertain environment? The literature on "optimal fiscal policy"deals with some these aspects. The focus of this line of research is to define tax rate and debt structure, given an exogenous stochastic sequence of

public expenditure using a dynamic Ramsey model. There are three main results in this literature: time-inconsistency and indeterminacy of fiscal policy (see for instance Zhu[1992]), tax and public debt being state-contingent (Lucas and Stockey [1983]) Random walk taxes (Barro [1979] and Aiyagari et. al. [2002]).

Following a broader approach, Lasing (1998) studies the impact of the introduction of public investment in such a framework. Fiscal expenditures (consumption and investment) are then endogenous. Using numerical methods, the author claims that the model reproduces some features of the American Economy - the higher volatility of private investment with respect to public one, the lack of correlation between output and income tax rate, higher volatility of public debt with respect to output. Finally, Turnovsky [1999] includes stochastic features into an endogenous growth model with productive public spending.

However, the existing literature neglects one important characteristic of investment decision: the irreversible nature of capital expenditure. It is now largely admitted (see Dixit and Pindyck [1994]) that the assumption of a perfectly flexible private capital is no longer realistic in a stochastic world. Investment decisions are for sure affected by the joined facts that investment is irreversible and generates returns that are uncertain. In fact, a firm that may face bad news and which cannot easily sell its capital may prefer to postpone some investment projects, that is, to accumulate less capital for a given state of nature. This significantly alters the productivity of the private sector and one may wonder how the public sector is in turn affected: what happens for public investment, that is, for the optimal tax rate and the public capital provision?

In this paper, we extend the usual models of irreversible investment under uncertainty by introducing the stock of public capital as an input for the private sector. This approach seems more realistic than considering the flow of public spending or the services provided by the government. Public investment takes place in a stochastic environment. Public capital then increases the productivity of private capital which is assumed to be fully irreversible. In our model, the government has an intertemporal budget constraint, i.e. taxes are collected each period to fund the public debt. We provide a partial equilibrium analysis, as it is standard in models of irreversible investment under uncertainty. Even under uncertainty, the optimal tax rate is then constant and does not depend on the size of uncertainty, it is exactly the same as the one that would prevail in a deterministic world. Nevertheless the optimal provision of public capital is negatively affected by uncertainty. We show that the government has an insurance role since it removes part of the uncertainty faced by the firm. Such a role for the government has already been suggested by Rodrik (1998): observing that the positive correlation between openness of economies and the government size of these economies is stronger when terms-of-trade risk is higher, he deduces that government spending may play a risk-reducing role. Our paper can therefore be considered as an attempt to show how the public sector uses public investment as a risk-reducing strategy. Moss (2004) provides a historical view of a set of strategies that US government has used either to spread risk or transfer it. For instance, the author highlights the bankruptcy laws, deposit insurance, social security among others. However, he does not emphasize the possible role public investment in infrastructure as a possible risk reducing strategy.

Barrios, Bertinelli and Strobl (2003) shows that the decrease in rainfall since the 1960's has a significant explicative power in the GDP growth in Sub-Saharan African countries. The authors claim that about 60% of the farmers could not insure themselves against these shocks. Supposing the government can provide a once for all public capital, say public irrigation, our model could be applied to the African case. During droughts, there would be a provision of public capital, but part of the irreversible investment made by the private sector, would still be profitable thanks to public irrigation. Fluctuations in GDP would be lower. As a result, the realization of the stochastic process (like the rainfall) needed to perform a new investment would be lower. To sum up, the private sector that bears an uninsurable risk would receive a total amount of public irrigation that would reduce the risk by lowering the fluctuations in output; furthermore the probability of a new investment would then be higher, since the threshold value required for a new investment would be lower. The public good could be funded by future taxes. Grant and Quiggin (2003) highlights the insurance role of the State, analyzing the impact of the public investment in equities. They provide a micro-foundations for market incompleteness. Their results are related to ours in the sense that during recessions the government runs a deficit, whereas in booms there is a surplus to finance the possibility public debt. The US government has proposed that Social Security Funds invests in equities, maintaining the benefits defined. The authors show that this can be a welfare-enhancing strategy.

2 PROVIDING PUBLIC CAPITAL IN A STOCHASTIC WORLD

2.1 THE PROGRAM OF THE FIRM

We consider public capital as another input provided by the government to the firms. Following the literature, an increase in the amount of public capital raises private productivity. The production function has a Cobb-Douglas form[1]:

$$y(t) = A(t)K(t)^{\alpha} Kg^{\beta} \tag{1}$$

with $\alpha + \beta < 1$. Kg is the amount of public capital ; production is continuously perturbed by shocks since parameter A(t) is stochastic and moves according to a geometric Brownian motion[2]:

$$\frac{dA(t)}{A(t)} = \mu dt + \sigma dz(t) \tag{2}$$

with $dz = \varepsilon \sqrt{dt}$ where $\varepsilon \sim N(0,1), E(\varepsilon_i, \varepsilon_j) = 0 \ \forall \ \varepsilon$ i,j with i j.

The private sector must deal with irreversibility in the installed private capital; once investment is implemented there is a sunk cost, which does not allow them to reduce the total stock. The problem of the firm is to maximize its value (the

---

1. Labor can be introduced in the production function, one could interpret the production function in per capita terms.
2. Output price could also be modelled as a geometric Brownian motion, measuring some demand shocks. Such an assumption would alter neither the methodology nor the nature of the results.

discounted sum of its cash flows) by choosing the optimal stock of private capital, given that there are uncertainty and irreversibility. We analyze three cases, the first one supposing that the private sector can indeed buy the public capital in the market. Then, we analyze the provision of the public capital by the government when it levies a non-distortive tax on profit and a third case where the assumption of perfect competition is relaxed. The impact of the public capital can be assessed by the comparison of the provision in the market and the provision by the government.

We assume that the government and the private sector pay the same constant price k for the capital. r is the discount rate of both government and private firms 3

2.1.1 Deriving the desired stock of capital when the public capital is provided in the market

Supposing that the private sector can indeed acquire in the market the public capital, being this input irreversible as well. The problem to the firm is defined as:

$$
\begin{cases}
\max_{I(t)I_g(t)} v(0) = E_0 \int_0^{+\infty} \left[A(t) K_{pr}(t)^\alpha (K g_{pr})^\beta - \kappa I(t) - \kappa I_g(t) \right] e^{-rt} dt \\
\\
s.t. \quad \begin{vmatrix} d.A(t) = \mu A(t) dt + \sigma A(t) dz(t) \\ I(t) = dK(t) \geq 0 \\ I_g(t) = dKg(t) \geq 0 \\ K(t) = 0 \ t = 0 \\ Kg_{pr}(t) = 0 \ t = 0 \end{vmatrix}
\end{cases}
\tag{3}
$$

The Bellman Equation of this problem is defined as follows (see Dixit [1997]):

$$
rv(t) = \alpha A(t) K_{pr}(t)^{\alpha-1} (Kg_{pr})^\beta + \frac{E(dv)}{dt}
\tag{4}
$$

$$
rv(t) = \beta A(t) K_{pr}(t)^\alpha (Kg_{pr})^{\beta-1} + \frac{E(dv)}{dt}
$$

with v(t) being the marginal value of the firm. Note that the problem for each input will be analogous. Deriving the problem for the private capital:

$$
v(t) = \underbrace{\frac{A(t) \alpha K_{pr}(t)^{\alpha-1} (Kg_{pr})^\beta}{r - \mu}}_{\substack{Discounted\,value\,of \\ future\,cash-flows}} + \underbrace{Z_k (K_{pr}(t) A(t)^\lambda)}_{Option\,to\,invest}
\tag{5}
$$

where $Z_k < 0$ is the derivative of $Z(K_{pr})$ with respect to K_{pr} (t) and

$\lambda = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{r}{\sigma^2}} > 1$ thus, $\partial\lambda/\partial\sigma^2 < 0$. The value of the last

---

3 In a different setting, Arrow and Lind (1970) show that uncertainty should not prevent from using the same discount riskless rate for private firms and government.

unit of private capital is given by equation (5). It encompasses the discounted present value of future cash-flows given A(t), less the option to invest this unit later since having one more marginal unit of capital implies to give up this option (and thus prevents to wait for better realizations of the stochastic variable). We will have to solve for Z_k and we will show it is actually negative. Note that the higher the uncertainty, the larger the value of the option the firm has to give up to invest in the marginal unit and thus, the smaller the value of the marginal unit. This is the main conclusion of models of irreversibility and uncertainty, and for plausible values of the model's parameters, this option value may be non-negligible, and may thus significantly affect the optimal stock of capital (again, see Dixit [1997] and Eberly and Van Mieghem [1997]). Dixit (1997) considers the case of the "partially irreversible". We consider the extreme case of completely irreversible inputs, that negative investment is never possible. In our case, the "quadrilateral" region, that is the space in which investment can be negative, does not exist. As pointed out by Dixit (1997), it shrinks to a point. Furthermore, since both inputs have the same stochastic process and the price, the decision to invest will take place simultaneous for both inputs.

The desired capital stock is obtained through "value matching" and "smooth pasting" conditions that are standard in the irreversible investment under uncertainty literature (see Dixit and Pindyck [1994]) :

$$Z_\kappa(K_{pr}(t)A(t)^\lambda + \frac{A(t)\alpha K_{pr}(t)^{\alpha-1}(kg_{pr})^\beta}{r-\mu}=k \ for \ A(t)=A^*(t) \tag{6}$$

$$\lambda Z_\kappa(K_{pr}(t)A(t)^{\lambda-1} + \frac{\alpha K_{pr}(t)^{\alpha-1}(kg_{pr})^\beta}{r-\mu}=0 \ for \ A(t)=A^*(t) \tag{7}$$

These optimality conditions allow to derive the value of the desired stock of capital K^d as well as Z_k, for a given value of A(t). Firms observe the value of the parameter A(t) and then can choose the desired stock of capital as follows :

$$K^d(t) = \left[\frac{\lambda-1}{\lambda} \frac{A(t)\alpha Kg^\beta_{pr}}{(r-\mu)} \right]^{\frac{1}{1-\alpha}}$$

$$\tag{8}$$

$$\lambda Z\kappa(K_{pr}(t)A(t)^{\lambda-1} + \frac{\alpha K_{pr}(t)^{\alpha-1}(kg_{pr})^\beta}{r-\mu}=0 \ for \ A(t)=A^*(t) \tag{9}$$

$$\frac{\partial K_{pr}^{d}(t)}{\partial Kg} > 0$$

$$\frac{\partial Z\kappa_{pr}}{\partial Kg} > 0 \tag{10}$$

Another way to study the investment behavior of firms is to focus on the level of the stochastic variable required to install a new capital unit given an installed stock of private capital, $K_{pr}(t)$. Such a threshold may also be derived from the value matching and smooth pasting conditions:

$$A*(t) = \frac{k(r-\mu)}{\alpha K_{pr}(t)^{\alpha-1}(kg_{pr})^{\beta}} \left(\frac{\lambda}{\lambda-1} \right) \tag{11}$$

Note that the higher is Kg, the lower is A*.

Analogously, it is possible to find the solution for the other input, Kg, bought in the market. The solution of these two equations yield the following system of equations:

$$\begin{cases} \dfrac{A(t)\alpha K_{pr}(t)^{\alpha-1}(Kg_{pr}(t))^{\beta}}{(r-\mu)k} = \dfrac{\lambda}{\lambda-1} \\[4mm] \dfrac{A(t)\beta\alpha K_{pr}(t)^{a}(Kg_{pr}(t)^{\beta-1})}{(r-\mu)k} = \dfrac{\lambda}{\lambda-1} \end{cases} \tag{12}$$

where $\lambda = 0.5 - \dfrac{\mu}{\sigma^{2}} + \sqrt{\left(\dfrac{\mu}{\sigma^{2}} - 0.5\right)^{2} + \dfrac{2r}{\sigma^{2}}} > 1$ and $\partial\lambda / \partial\sigma^{2} < 0.$ \tag{13}

Hence:

$$K_{pr}(t) = \frac{\alpha}{\beta} Kg_{pr}(t)$$

$$Kg_{pr}(t)^{1-\alpha-\beta} = \frac{\lambda-1}{\lambda} \frac{A(t)}{(r-\mu)\kappa} \beta^{1-a} \alpha^{a} \tag{14}$$

The private sector would adjust the total capital, respecting the two optimality conditions. Next section derives the result when the government provides an once for all public capital, that cannot be acquired in the market.

2.2 THE PROGRAM OF THE GOVERNMENT

Until now, public capital has been considering as another input to the private sector. In this section we show how the government intervention changes the allocation. The problem of the government will be: how should the level of taxes and the stock of public capital be determined ? Note that the model dealt here is a partial

equilibrium analysis. Therefore the objective of the government will be to maximize the value of the firm subject to its intertemporal budget constraint.

2.2.1 Deriving the public capital stock provided by the government with non - distortive taxes

The private sector must still deal with irreversibility and uncertainty, but in this section, we consider the case in which the public capital cannot be bought in the markets. It is only through the public provision that firms can dispose of the input. Note that in this case the risk input public capital is owned by the State and not by the firms. As stressed by Eberly and Van Mieghem (1997) the presence of more than one irreversible input makes firms more caution to invest. For both inputs, the threshold value increases. In the case where the State does provide public capital, the private sector has just to deal with one input, therefore leading a reduction in the total risk for the private sector.

The problem of the firm is to maximize its after-tax value by choosing the optimal stock of private capital, taking as given the provision of public capital and the taxes. The public capital cannot be bought in the market and the government will provide it once for all:

$$
\left\{
\begin{array}{l}
\max\limits_{I(t)I_g(t)} v(0) = E_0 \int_0^{+\infty} \left[A(t) K_{pr}(t)^\alpha \, (K_{g\,pr})^\beta - \kappa I(t) - \kappa I_g(t) \right] e^{-rt} dt \\[2ex]
s.t. \quad
\left|
\begin{array}{l}
d.A(t) = \mu A(t) dt + \sigma A(t) dz(t) \\
I(t) = dK(t) \geq 0 \\
I_g(t) = dKg(t) \geq 0 \\
K(0) = 0 \; \forall t = 0 \\
Kg_{pr}(t) = 0 \;\; t = 0
\end{array}
\right.
\end{array}
\right.
\tag{15}
$$

It is clear that this problem can be rewritten and will yield the same result for the private capital as in the "decentralized economy", where the public capital can be bought by private sector. Nevertheless, The provision of public capital is done by the government once for all, and it is supposed that the government maximizes the value of the firm. The government commits to a tax schedule defined by the initial debt needed to finance the provision of public capital. Indeed, the government should have a very high amount of information to be able to adjust the provision of public capital. Moreover, for technological reasons it would not be easy to the government to adjust the public capital stock, as it is the case of infrastructure. So, we suppose that the public capital is provided once for all.

2.2.2 Deriving the initial value of the firm

Given the desired capital stock derived before, we can express $V^d(t)$, the value of the firm when the installed stock of capital is the desired one, before taxes. Since we know the expression for $v^d(t)$ (the marginal value of the firm when the installed capital is the desired one), $V^d(t)$ may be computed as follows:

$$V^d(t) = \int_0^{+\infty} v^d(K)dK = \int_0^{\kappa^d} \frac{\alpha A(t)K(t)^{a-1}(Kg)^\beta}{r - \mu}dK(t)$$

$$+ \int_{\kappa^d}^{+\infty} Z\kappa(K(t))A(t)^\lambda dK(t) \tag{16}$$

$$\Leftrightarrow V^d(t) = \frac{A(t)K^d(t)^a Kg^\beta}{r - \mu} \tag{17}$$

$$+ \left(\frac{1}{\lambda(1-\alpha)+1}\right)\left(\frac{\kappa}{(\lambda-1)}\right)^{1-\lambda}\left(\frac{\alpha A(t)Kg^\beta}{(r-\mu)\lambda}\right)^\lambda K^d(t)^{\lambda(\alpha-1)+1} \tag{18}$$

assuming $\lambda(1-\alpha) > 1$ to ensure the convergence of the integral.

We assume that the firm has initially no capital: it only starts to invest at the time t=0 when the government installs the capital Kg. Due to the specification of the cash flows, that are positive whatever the realization of the stochastic variable, at time t=0, the firm will then jump for sure to its desired capital stock $K^d(0)>0$. Thus, the initial value of the firm is such that given the amount of public capital and the realization of the stochastic variable at time t=0, the installed capital stock K(0) corresponds to the desired stock $K^d(0)$. Replacing $K^d(0)$ by its expression given by equation (8) the initial value of the firm is thus :

$$V(0) = V^d(0) = \left(\frac{A(0)}{r-\mu}\right)^{\frac{1}{1-a}}\left(\frac{(\lambda-1)\alpha}{\lambda\kappa}\right)^{\frac{a}{1-a}}Kg^{\frac{\beta}{1-a}}\left[1 + \frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right] \tag{19}$$

Not surprisingly, this value is an increasing function of the stock of public capital and a decreasing function of the tax rate. The effect of uncertainty on this initial value is given by:

$$\frac{\partial V(0)}{\partial \sigma^2} = \underbrace{\frac{\partial V(0)}{\partial \lambda}}_{>0}\underbrace{\frac{\partial \lambda}{\partial \sigma^2}}_{<0} < 0 \tag{20}$$

There exists two opposite effects of uncertainty on V(0). On the one hand, more uncertainty induces the firms to install less capital ($K^d(0)$ is smaller) thus reducing the current cash-flow ; on the other hand, a larger uncertainty increases the option value part of V(0) which relates to future cash flows. Clearly here, the first effect prevails and more uncertainty reduces the initial value of the firm.

2.2.3 The government budget constraint

At the beginning of the program, the government defines Kg, the optimal level of public capital to be provided once for all[4]. The commitment to the provision of the public capital once for all can be viewed as a signal to the private sector that the government will stick to the same level of taxes; therefore problems of consistency of the public policy are then eliminated. Indeed, since the private sector cannot reduce its stock of capital, the fear that taxes could be higher in the future would alter the private investment decision; in turn (through the initial value of the firm) the provision of public capital would be affected, raising, questions about the consistency of the public policy. Expenses are then kKg with k being the unit price of capital. This public debt is completely funded in future taxes on the instantaneous profits of the firms. Therefore, we are supposing that there is a "Ricardian equivalence" in the sense that the public debt is completely funded in future revenues[5]. Since the model is stochastic, future tax revenues are subject to uncertainty and the budget constraint is such that the expected present value of the revenues must be equal to the expenses in terms of public capital (see for instance Pennings(2000)). Moreover, the expected present value of the revenue derives from the tax rate applied to the expected value of the future cash flows of the firm. This latter stream is given by V(0), which is the value of the firm at period 0. The government budget is thus:

$$\tau V(0) = \kappa \mathrm{K}g \qquad (21)$$

Note that this implies a precise time schedule in the realization of private and public investments : at time t=0 the government observes the realization of the stochastic variable. It can then deduce the amount of capital the firm wants to install and the initial value of the firm, depending on the levels of tax and public capital. Using this information the government decides how much to tax and how much public goods to provide, which implies (given k) the initial amount of debt.

The program of the government is to choose the levels of tax and of public capital, that maximize the after taxes value of the firm:

$$\begin{cases} Max_{\{\tau,kg\}} V(0) = \psi \mathrm{K}g^{\frac{\beta}{1-\alpha}} - \tau V(0) \\ st. \ \tau V(0) = \kappa \mathrm{K}g \end{cases} \qquad (22)$$

where

$$\psi = \left(\frac{A(0)}{r-\mu}\right)^{\frac{1}{1-a}} \left(\frac{(\lambda-1)\alpha}{\lambda\kappa}\right)^{\frac{a}{1-a}} \left[1+\frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right] \qquad (23)$$

Eliminating τ, the problem becomes:

---

4 Lansing (1998) claims that the private investment is much more volatile than public one. We have here an extreme case, where public investment occurs only once.

5 However, the term "Ricardian Equivalence" exactly refers to the public debt not being regarded by the agents as wealth. Here, it is used in the sense of Walsh (1998) and Sargent (1982).

$$Max_{\{kg\}}V(0) = \psi \mathrm{K}g^{\frac{\beta}{1-\alpha}} - \kappa \mathrm{K}g \qquad (24)$$

The value of the public capital provided by the government in an once for all basis:

$$Kg^{1-\alpha-\beta} = \left(\frac{A(0)^{\beta 1-\alpha}}{k(r-\mu)}\right)\alpha^a \left(\frac{(\lambda-1)^\alpha}{\lambda\kappa}\right)(1-\alpha)^\alpha \left[1+\frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right]^{1-\alpha} \qquad (25)$$

From equation 10:

$$\mathrm{K}g = \mathrm{K}g_{pr}(0)c \qquad (26)$$

where

$$c = \left\{\left(\frac{1}{(1-\alpha)(\lambda-1)}\right)\left[\lambda+\frac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right]\right\}^{1-\alpha} > 1 \qquad (27)$$

The public capital provided by the government will always be greater than the public capital bought in the market, provided that $A(t) < cA(0) \ \forall \ t$. Furthermore, the parameter c is increasing with the variance of $A(t)$.

This statement can be seen in the following steps: Taking equation 26, the optimal police in the decentralized regime is invest, when the sequence $A(t)$ is higher than the threshold value (equation 11) . Kg_{pub} will be greater than $Kg_{pr}(t)$ if $Kg_{pr}(0)c > Kg_{pr}(t) \Rightarrow A(0) \ c > A^*(t)$. Due to irreversibility the optimal for the stock of capital Kg_{pr} will be higher than $Kg_{pub,}$ if the observed $A(t) = max[A(t)]$ is higher than the threshold $cA(0)$, for $t[0,T]$, therefore we should ensure that the whole sequence is lower than $A(0)c$.[6]

Given that $\partial\lambda/\partial\sigma^2 < 0$ and $\partial c/\partial\lambda < 0$, $\partial c/\partial\sigma^2 = (\partial c/\partial\lambda)(\partial\lambda/\partial\sigma^2) > 0$.

The result says that the public provision of public capital will conduct the economy produces more. Inputs are complementaries due to the Cobb-Douglas production function, implying a higher output. The production decision will not be affected by taxes, since taxes do not affect relative prices. If the shocks is below the threshold $A(0)c$, then the total output and private capital will be higher under the provision of public sector. Note that from the proposition, the public capital will be higher than the level on the decentralized case. If there are bad shocks with respect to the initial shock, so from equation 6, we can conclude, as well, that the private capital will be higher than in the model with private provision of public capital. Being both inputs at a higher level the total production is higher. Furthermore, this range $A(0)c$ is increasing with the variance of the shock. In other words, if the

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6 Note that the model can easily be generalized, opening the possibility to the public sector to increase public capital when A(t)>cA(t).

environment is more uncertain than the role of the government providing insurance will be more important.[7].

2.2.4 Deriving the optimal tax rate

The optimal tax rate can be obtained directly from the problem solved for the public capital. It can also be restated to check that the indeed the result holds.

The program of the government is to choose the levels of tax and of public capital, that maximize the after taxes the net value of the firm:

$$
\begin{cases}
Max_{\{\tau, Kg\}} V(0) = \left(\dfrac{A(0)}{r-\mu}\right)^{\frac{1}{1-a}} \left(\dfrac{(\lambda-1)\alpha}{\lambda\kappa}\right)^{\frac{a}{1-a}} Kg^{\frac{\beta}{1-a}} \left[1+\dfrac{\alpha}{\lambda[(1-\alpha)\lambda+1]}\right] \\
st.\ \tau V(0) = \kappa Kg
\end{cases}
\tag{28}
$$

$$
\begin{cases}
Max_{\{\tau, Kg\}} V(0) = \left(\dfrac{A(0)}{r-\mu}\right)^{\frac{1}{1-a}} \left(\dfrac{(\lambda-1)\alpha}{\lambda\kappa}\right)^{\frac{a}{1-a}} Kg^{\frac{\beta}{1-a}} \left[1+\dfrac{\alpha}{\lambda[(1-\alpha)\lambda+1]}-\dfrac{\alpha(\lambda-1)}{\lambda}\right] \\
st.\ \tau V(0) = \kappa Kg
\end{cases}
\tag{29}
$$

Substituting for Kg using the budget constraint, the problem of the public sector can be restated as:

$$
Max_{\tau} V(0) = \psi(1-\tau)^{\frac{1-\alpha}{1-\alpha-\beta}} \tau^{\frac{\beta}{1-\alpha-\chi}}
\tag{30}
$$

$$
with\ \psi = \left(\dfrac{A(0)}{(r-\mu)\kappa^{(\alpha+\beta)}}\right)^{\frac{1}{1-a-\beta}} \left(\dfrac{(\lambda-1)\alpha}{\lambda\kappa}\right)^{\frac{a}{1-a-\beta}} \left[1+\dfrac{\alpha}{\lambda[(1-\alpha)\lambda+1]}-\dfrac{\alpha(\lambda-1)}{\lambda}\right]^{\frac{1-a}{1-a-\beta}}
\tag{31}
$$

The first order condition of the problem is then (note that ψ does not depend upon τ):

$$
\frac{\partial V}{\partial r} = \frac{(1-\alpha)\psi}{1-\alpha-\beta}(1-\tau)^{\frac{1-\alpha}{1-\alpha-\beta}-1}\tau^{\frac{a}{1-a-\beta}} + \frac{\beta\psi}{1-\alpha-\beta}(1-\tau)^{\frac{1-a}{1-a-\beta}}\tau^{\frac{\beta}{1-\alpha-\beta}} - 1 = 0
\tag{32}
$$

Moreover, it can be easily be checked that

$$
\frac{\partial^2 V}{\partial r^2} < 0
\tag{33}
$$

The optimal tax rate $\tau^*{}_p$ is thus :

$$\tau_p^* = \frac{\beta}{1 - \alpha - \beta} \tag{30}$$

2.2.5 Effect of uncertainty

2.2.5.1 Effect uncertainty on the optimal tax rate

The effect of uncertainty on the optimal tax rate is given by:

$$\frac{\partial \tau_p^*}{\partial \sigma^2} = 0 \tag{31}$$

Hence an increase in uncertainty has no effect on the tax rate, which is quite remarkable. This happens because the impact of the tax (either positive, through the amount of public good it generates, or negative since reducing the current profit) on the current cash flow is the same as that on the option value since the tax is levied on current cash flow as well as on future ones. Its optimal level has therefore nothing to do with the size of uncertainty. Comparing with the deterministic counterpart of this model, it is clear that the introduction of uncertainty does not change the optimal level of tax since the tax rate only depends on how productive is the public capital.[8]

FIGURE 1

Average Tax Rate and the Standard Deviation Error of the Total Factor Productivity

Average Effective Tax rate and the Standard Deviation Error of the Total Factor Productivity
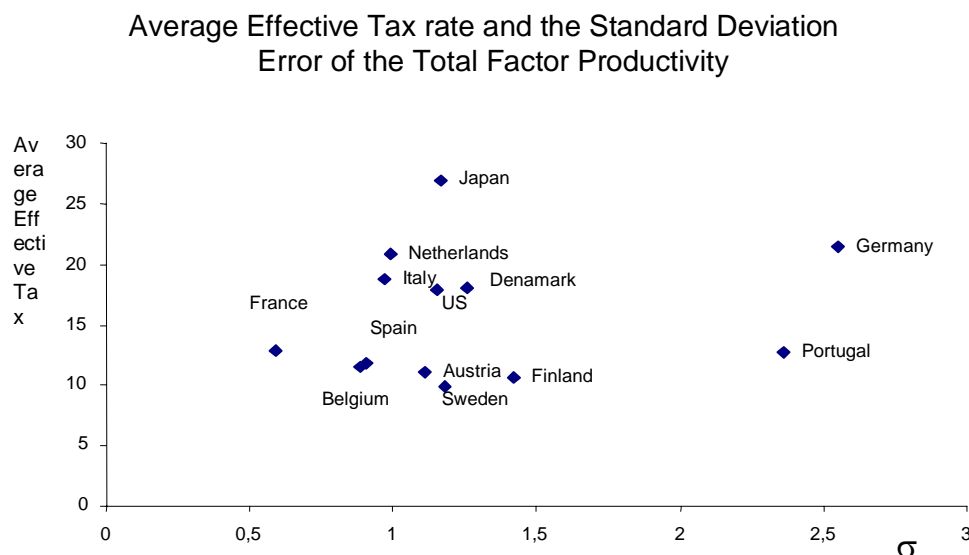
Source: OCDE Perspectives Économiques, juin 2001
European Comission BACH - Bank for the Accounts of Companies Harmonised

Some empirical data should be used to support more strongly our theoretical statements, therefore we provide a scatterplot diagram, using the BACH - Bank for

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[8] For the US economy, Lansing (1998) claims that there is a almost no correlation with the tax rate and the output.

the Accounts of Companies Harmonized- available at the European Commission to calculate the average effective tax rate for 13 countries (11 Europeans countries + Japan and US), which is a ratio of the tax on profits over the value added in the manufacturing industry .The correlation of the average effective tax rate appears to be unrelated to the standard deviation errors of the total factor productivity[9]: indeed, the slightly positive trend line is not significant (t-test=0.533 and R²=0.0252). Due to the very restrictive sample, a more careful empirical test should be conducted ; however it does not reject our result of no relationship between the tax rate and the uncertainty parameter.

2.2.5.2 Effect of Uncertainty on the Optimal Provision of Public Capital

Contrary to the optimal tax rate, the corresponding optimal provision of public capital is affected by uncertainty. It arises from the fact that the amount of public good the government can provide, given any tax rate, directly depends on the future tax revenues which are determined by the future cash flows of the firm (see equation (14)).

$$\frac{\partial g_p^*}{\partial \sigma^2} = \frac{\tau}{\kappa} \underbrace{\frac{\partial V(0)}{\partial \sigma^2}}_{<0} + \frac{V(0)}{\kappa} \underbrace{\frac{\partial \tau}{\partial \alpha 2}}_{0} < 0 \tag{32}$$

This results from the two effects of uncertainty on the initial value: as we have seen below, the negative effect of uncertainty (which applies through the current cash flow) prevails.

2.2.5.3 Uncertainty and the role of the government

The government has an interesting role in this model since it bears the risk of stochastic tax revenues while offering as a counterpart a deterministic initial amount Kg to the private sector.

First, the government offers less public good in the stochastic world when uncertainty is larger (see equation (32)) is due to the fact that uncertainty generates an option value which reduces the marginal productivity of private capital which in turn negatively affects the productivity of the public good. Second, it can be shown that any smaller government intervention would reduce the desired stock of private capital for any realization of the stochastic variable, or symmetrically, for a given installed stock of private capital, the level of the stochastic variable required to invest would be higher. As the government decides the provision of public good once for all on the basis of the expected value of its revenue, the private sector benefits from a kind of insurance scheme: during bad realizations ($A(t)<A(0)e^{\mu t}$) of the stochastic process, the government is providing more public good than the expected revenues, but in good times ($A(t)>A(0)e^{\mu t}$) the reverse applies, this mechanism keeping the expected intertemporal budget balanced. From another point of view, since both the amount of public capital and the tax rate are constant, the representative firm gets a

---

9 Nidodème (2001) provides a good discussion about the methodologies on effective tax rates; we use here the methodology called "microbackward". For the standard deviation errors of TFP, the values come from the estimated standard deviation of errors in an autoregressive model.

deterministic amount of public capital and in return it is paying small amounts of tax in bad times and larger amount of taxes in good time, therefore stabilizing its cash-flow. Indeed, "the tax may also reduce the degree of risk" as already pointed out by Domar and Musgrave [1944]. The government has therefore an insurance role.

Note that in Barro [1990] public capital positively affects the economy since it is financed by a tax paid by n firms, so each firm only bears a small part of the cost while the public capital entirely benefits to any of them. Here, the public capital is financed by one representative firm. So the role of the government is not to provide a good which exhibits the special characteristics of a public good but rather to remove a part of uncertainty from the firm towards the government. [10]

One very simple example can illustrate the impact of public intervention. In figure 2, it is shown a process with $\mu = 0$ and $\sigma^2 = 0.09$, the parameters are defined as r=0.05, α =0.3, β =0.2 and the price of capital equal to 1. It is plotted the initial value of A*$_{pub}$, that the threshold value under the public provision of Kg, and the same initial value A*$_{pr}$ when the public capital can be bought in the market. As it can be seen, under the public provision the firms invest earlier (firms invest whenever A(t)>A*, since the threshold is lower than the "decentralized case". Of course, it is dependent on the particular sample of A(t). Furthermore, the parameter c is equal to 6.45, in other words when the stochastic process is higher than 13,06, that the public capital under the regime of private provision will be higher than the public provision. [11]

FIGURE 2

The Threshold to invest under the two regimes

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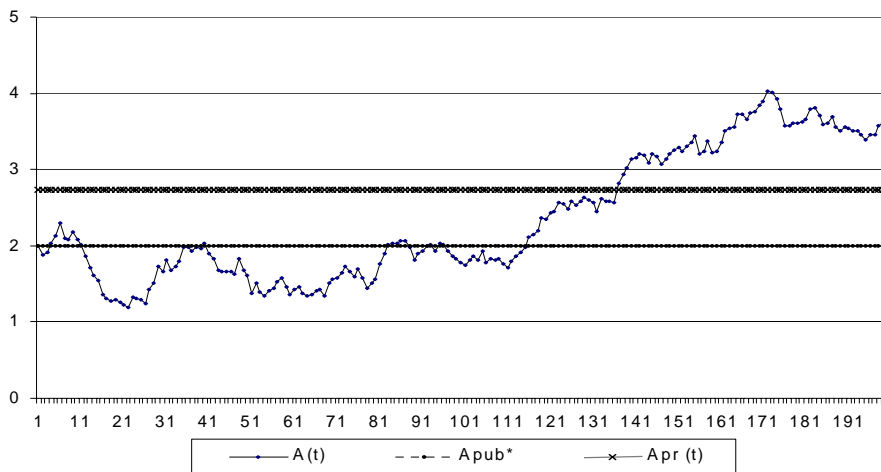
10 This fact can be introduced in a model with heterogeneity, and specific shocks could be insured among tax-payers and common shocks through time. Here, we deal just with the latter.

11 One can imagine that the same effect is reproduced, if there was some operation costs to the private capital, in this case, the firm stops some units. The higher provision of public capital by the government would prevent firms to stop earlier the machines, the lower bound to stop some machines A**, will be lower for the public provision of Kg. The firm will tend to use more frequently full capacity and output will fluctuate less. Indeed, A**/Kg<0, given the fact that Kg is provided once for all, the government will not reduce the provision of Kg, due to bad realizations of the stochastic process, as it will do the private sector. We abstract these cost to keep the model simpler.

3 CONCLUSION

In this paper, we have studied the issue of the optimal provision of public capital under uncertainty. The tax rate will not depend on the degree of uncertainty but only on technological parameters and market power. Nevertheless, the optimal stock of public capital will be negatively affected by uncertainty. The government has an insurance role since it collects taxes from future cash-flows that are stochastic and provides an initial amount of public capital.

A more realistic modelling with heterogenous firms should now be considered in order to avoid periods with no investment in the country ; nevertheless, there is no reason for the insurance role of the government to disappear in such a framework. Following steps would then be to close the model and to allow for successive public investments; this could probably be achieved at the cost of giving up analytical resolution.

APPENDIX

DERIVING THE OPTIMAL CAPITAL STOCK

We follow the presentation in Dixit and Pindyck (1994), chapter 3 and 4, where the solution to the Bellman equation, like (4) is given. First of all, we apply Ito's Lemma to expand the term, and keeping just the terms with dt:

$$\frac{E(dv)}{dt} = \mu A v_A + \frac{1}{2}\sigma 2 A 2 v_{AA} \tag{33}$$

where v_A indicates the derivative of v with respect to A. Replacing in the equation, we end up with the following second order differential equation:

$$\alpha A(t)K_{pr}(t)^{\alpha-1}(Kg_{pr})^{\beta} - rv(t) + \mu A v_A(t) - \frac{1}{2}\sigma^2 A^2 v_{AA}(t) = 0 \tag{34}$$

To solve this differential equation is possible to make an initial guess like:

$$v(t) = \frac{A(t)\alpha K_{pr}(t)^{\alpha-1}(Kg_{pr})^{\beta}}{r-\mu} + Z\kappa\left(K_{pr}(t)\right) + A(t)^{\lambda_1} + J\kappa\left(K_{pr}(t)\right)A(t)^{\lambda_2} \tag{35}$$

the characteristic equation is given by:

$$\frac{1}{2}\sigma^2\lambda(\lambda-1) + (r-\mu)\lambda - r = 0 \tag{36}$$

which yields the following values for λ:

$$
\begin{aligned}
\lambda_1 &= 0.5 - \mu/\sigma^2 + \sqrt{\left(\mu/\sigma^2 - 0.5\right)^2 + 2r/\sigma^2} > 1 \\
\lambda_2 &= 0.5 - \mu/\sigma^2 - \sqrt{\left(\mu/\sigma^2 - 0.5\right)^2 + 2r/\sigma^2} > 0
\end{aligned}
\tag{37}
$$

given the roots, to respect the boundaries conditions, it should be imposed that $J_K=0$, so the solution is given by:

$$v(t) = \frac{A(t)\alpha Kpr(t)^{\alpha-1}\left(Kg_{pr}\right)^{\beta}}{r-\mu} + Z\kappa\left(K_{pr}(t)\right)A(t)^{\lambda} \tag{38}$$

REFERENCES

Aiyagari, S.; Marcet, A.; Sargent, T. and Seppälä, J. (2002) Optimal Taxation without State-Contingent Debt. *Journal of Political Economy* Vol. 110p, No. 6, 1220-1254.

Arrow, K. and Kurz, M. (1970), *Public Investment, the Rate of Return and Optimal Fiscal* Policy, John Hopkins Press, 1970.

Arrow, K. and Lind R., Uncertainty and the Evaluation of Public Investment Decisions, *American Economic Review* Vol. 60 No. 3, 364-378.

Aschauer, D. (1989), Is Public Expenditure Productive?, *Journal of Monetary Economics* Vol. 23, 177-200.

Barrios, S.; Bertinelli, L. and Strobl, E. (2003) Dry Times in Africa: Rainfall and Africa's Growth Performance. *CORE Discussion Paper/2003-61*

Barro, R. (1990), Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy* Vol. 98, s103-s125.

Barro, R. On the Determination of the Public Debt. *Journal of Political Economy* Vol. 87, 940-971.

Bajo-Rubio, O. (2000). A Further Generalization of the Solow Growth Model: The Role of the Public Sector, *Economics Letters*, 68 (1), p. 79-84.

Bentolila, S. and Bertola, G.(1990) Firing Costs and Labour Demand: How Bad is Eurosclerosis? *The Review of Economic Studies*, Vol. 57, No. 3. 381-402.

Bertola G. (1998), Irreversible Investment, *Research in Economics* Vol. 52, 3-37.

Blanchard, O. and Giavazzi, F. (2004) Improving the SGP through a proper accounting of public investment. *CEPR discussion paper* No. 4220.

Burguet, R.and Fernández-Ruiz, J. (1998), Growth Through Taxes or Borrowing? A Model of Development Traps with Public Capital, European *Journal of Political Economy*, Vol. 14 No. 2, 327-344.

Cashin, P. (1995), Government Spending, Taxes and Economic Growth, *IMF Staff Papers*, 42 (2), p. 237-268.

Domar E.D and Musgrave R., (1944), "Proportional Income Taxation and Risk-Taking", *Quarterly Journal of Economics*, vol 58, No. 3, 388-422.

Drèze, J.(1997) Public Economics, Public Projects and their funding. *Asian Development Review*, vol. 15 No. 1, 1-17

Dixit, A. (1997) Investment and Employment Dynamics in the Short Run and the Long Run. *Oxford Economic Papers* vol 49, 1-20.

Dixit, A. and Pindyck, R. (1994) *Investment under Uncertainty*, Princeton University Press.

Dixit, A. Stiglitz J. (1977), Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, 67, 297-308.

Eberly, J. and Van Mieghem, J. (1997) Multi-Factor Dynamic Investment under Uncertainty. *Journal of Economic Theory* Vol. 75, 345-387.

Glomm, G.and B. Ravikumar. (1994) Public Investment in Infrastructure in a Simple Growth Endogenous Model, *Journal of Economic Dynamics and Control* Vol 18, 1173-1118.

Gramlich E., (1994), Infrastructure Investment: A Review Essay, *Journal of Economic Literature*, 32(3), 1176-1196.

Grant,S. and Quiggin, J. (2003) Public Investment and the Risk Premium for Equity. *Economica* 70, 1-18.

Jones, L.E., Manuelli, R. and Rossi, P., (1997), On the Optimal Taxation of Capital Income, *Journal of Economic Theory* 173(1), 93-117.

Judd , K.L. (1997), "The Optimal Tax on Capital Income is Negative", *NBER Working Paper* 6004.

Lansing, K. (1998) Optimal Fiscal Policy in a Business Cycle Model with Public Capital. *Canadian Journal of Economics*. Vol. 31 no. 2, 337-364.

Lucas Jr., R. and Stockey, N. (1984) Optimal Fiscal and Monetary Policy in an Economy without Capital. *Journal of Monetary Economics* Vol. 2, 55-93.

Metcalf, G. and Hassett, K., (1994). Investment with Uncertain Tax Policy, *Working paper no. 4780, NBER*, Cambridge, MA.

Moss, D. (2004). *When All Else Fails: Government As the Ultimate Risk Manager*. Harvard University Press, 464 pages.

Nicodème, G. (2001) Computing Effective Corporate Tax Rate: Comparisons and Results, *Discussion Paper European Commission*, no. 153

Pennings, E. (2000), Taxes and Stimuli of Investment Under Uncertainty, *European Economic Review*, 44 (2) p. 383-391.

Pindyck R.(1988), Irreversible Investment; Capacity Choice and the Value of the Firm, *American Economic Review*, p. 969-985.

Ramsey, F. (1927) A contribution of the Theory of Taxation. *Economic Journal* Vol 37, 47-61.

Rodrick, D. (1989) Why do More Open Economies Have Bigger Governments? *Journal of Political Economy* vol. 106, 997-1032.

Sargent, T. (1982) Beyond the Supply and Demand Curves in Macroeconomics *American Economic Review*, 72, no. 2 (May):382-389.

Shoiji, E. Public Capital and Economic Growth:A convergence Approach, *Journal of Economic Growth*, 6, p.205-227.

Turnovsky, S. (1999) Productive Government Expenditure in a Stochastically Growing Economy, *Macroeconomic Dynamics*, 3, 544-570.

Walsh, C. (2000) *Monetary Theory and Policy*. MIT Press.

Zhu, X. (1992) Optimal Fiscal Policy in A Stochastic Growth Model. *Journal of Economic Theory* Vol. 58, 250-289.