

**TEXTO PARA DISCUSSÃO Nº 806**

**OPTIMAL RULES FOR MONETARY  
POLICY IN BRAZIL\***

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## RESUMO

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Este trabalho apresenta regras ótimas de política monetária no Brasil derivadas de um modelo que consiste de uma função keynesiana IS e uma Curva de Phillips Aumentada (IS-AS) e com expectativas voltadas para trás. A função IS revela alta sensibilidade da demanda agregada à taxa de juros e a Curva de Phillips é aceleracionista. A regra ótima de política monetária mostra reduzida volatilidade da taxa de juros com coeficientes de reação menores do que os sugeridos por Taylor (1993a e b). Funções de reação estimadas através de modelos ADL e SUR sugerem que a política monetária não tem sido ótima e tem buscado estabilizar o produto em lugar da inflação.

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## ABSTRACT

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This paper presents optimal rules for monetary policy in Brazil derived from a backward looking expectation model consisting of a Keynesian IS function and an Augmented Phillips Curve (IS-AS). The IS function displays a high sensitivity of aggregate demand to the real interest rate and the Phillips Curve is accelerationist. The optimal monetary rules show low interest rate volatility with reaction coefficients lower than the ones suggested by Taylor (1993*a,b*). Reaction functions estimated through ADL and SUR models suggest that monetary policy has not been optimal and has aimed to product rather than inflation stabilization.

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## 1 - INTRODUCTION

Fixed exchange rate regimes, on the one hand, and chronic fiscal disequilibria, on the other, set limits to active monetary policies. Floating exchange rate regimes, even when imperfect, bring back the discussion of monetary policy and inflation targets, in particular. Inflation target policies require monetary authorities reaction functions, that is, linear relations between their control variables, usually the short run interest rate, the deviation of the rate of inflation to the target, and other variables included in the loss function as the output gap.

A pioneer formulation is Taylor (1993*a,b*) that suggests a reaction function notorious for its simplicity and efficacy. According to Taylor's rule, the monetary authorities should raise the interest rate one and half point each time the rate of inflation deviates one point from its target, and should raise the interest rate half point for each increase in one per cent point of the output gap.

The simplicity of Taylor's rule became the reference for the discussion of monetary policy. Several articles [Ball (1997), Woodford (1994, 1999), Walsh (1998)] have shown that the rule is consistent with price and product stabilization but its optimality depends on the parameters of the economy.

Most of the models used to formalize the optimal policy rule are simple Keynesian IS function with an Augmented Phillips Curve — IS-AS. In general they are based on backward looking expectations. The main transmission channel of the monetary policy is the interest rate that affects the aggregate demand and through aggregate demand it controls inflation rate. Another family of models built from micro foundations based upon the representative agent are used to pursue stability and optimality analysis [Rotemberg (1998) and Woodford (1999) are good examples].

This paper aims to derive an optimal rule for monetary policy in the Brazilian economy using the parameters of a backward looking expectations IS-AS type model. The Loss Function of the Monetary Authorities incorporates inflation, product and interest rate smoothing targets.

The paper is organized as follows: Section 2 presents the time series analysis and the estimation of the model, and in Section 3 the optimal rule is computed. An analysis of the results is done in Section 4. Section 5 reviews Brazilian monetary policy using an ADL and alternatively a SUR framework. Finally in Section 6 some important qualifications are made in respect to the feasibility of optimal rules in economies with weak fiscal fundamentals and Section 7 concludes.

## 2 - AN EMPIRICAL MODEL FOR BRAZIL

### 2.1 - Time Series Analysis

The analysis uses monthly data and the period goes from August 1994 to March 1999. The definition of the sample has to do with the regime switch that occurred after the Real Plan. As a matter of fact time series analysis showed a significant structural change in this period.

The data — nominal interest rate ( $i_t$ ), represented by the overselic, inflation rate ( $\pi_t$ ), measured by the National Consumer Price Index (INPC), and the Gross Domestic Product ( $GDP_t$ ) — were obtained from IPEA. The Output gap ( $y_t$ ) was computed as the deviation of the log of the deseasonalized GDP from its linear trend. The real interest rate, on the other hand, corresponds to the differential of the nominal interest rate and the current rate of inflation. It is important to notice that this formulation makes use of backward looking expectations only.<sup>1</sup>

Unit root tests, based on Dickey and Fuller (1979) and Dickey (1981), showed that all the variables followed a stationary process in level. The results are summarized in Table 1:

Table 1  
**Unit Root Test**

Variables	Level	Lags
Inflation	-5.016*	0
Output GAP	-3.633*	0
Real Interest Rate	-5.081*	0

*Note: \* Defines the 1% significance statistical level of the test. The optimal number of lags for each variable was selected from the general to the specific modeling, taking into account the statistical significance of the last coefficient estimated and the value of Akaike and Schwarz information criteria.*

### 2.2 - Estimation of the Parameters

The two basic equations, corresponding to the Augmented Phillips Curve and the IS, are presented below:

$$\pi_t = \alpha_{11}\pi_{t-1} + \alpha_{12}\pi_{t-5} + \alpha_2 y_{t-3} + \varepsilon_t \quad (1)$$

$$y_t = \beta_1 y_{t-1} - \beta_2 (i_{t-6} - \pi_{t-6}) + \eta_t \quad (2)$$

where the variables are defined according to the previous section.

<sup>1</sup> See in this regard Svensson (1997a,b) and Svensson (2000).

The lag structure of each equation was found by the estimation strategy that went from the general to the specific, taking into account the statistical significance of the estimated coefficients, the diagnostic tests and the lowest values of the Akaike and Schwarz information criteria. The estimated parameters are presented in sequence, where the numbers into parenthesis are the respective standard deviations.

$$\pi_t = \underset{(0.067)}{0.749} \pi_{t-1} + \underset{(0.055)}{0.204} \pi_{t-5} + \underset{(0.017)}{0.046} y_{t-3} + S_t + D_t \quad (3)$$

$$\begin{aligned} T = 56 [1994(8) - 1999(3)]; R^2 = 0.9413; \hat{\sigma} = 0.003; \text{RSS} = 0.0003; \\ \text{AR1} - 4 F(4, 37) = 1.325 [0.2787]; \text{ARCH} 4 F(4, 33) = 0.146 [0.9634]; \\ \text{Normality } \chi^2(2) = 2.091 [0.3515]; \chi_i^2 F(13, 27) = 0.555 [0.8673]; \\ \text{Reset } F(1, 40) = 0.802 [0.3760]. \end{aligned}$$

$$y_t = \underset{(0.097)}{0.672} y_{t-1} - \underset{(0.161)}{0.452} (i_{t-6} - \pi_{t-6}) + S_t + D_t \quad (4)$$

$$\begin{aligned} T = 56 [1994(8) - 1999(3)]; R^2 = 0.6554; \hat{\sigma} = 0.015; \text{RSS} = 0.009; \\ \text{AR1} - 4 F(4, 38) = 1.618 [0.1897]; \text{ARCH} 4 F(4, 34) = 0.210 [0.9313]; \\ \text{Normality } \chi^2(2) = 2.16 [0.3403]; \chi_i^2 F(10, 31) = 0.633 [0.7740]; \\ \text{Reset } F(1, 41) = 0.095 [0.7593]. \end{aligned}$$

where  $S_t$  and  $D_t$  are vectors of seasonal and impulse dummies respectively.<sup>2</sup> The diagnostic test suggests that the models are well specified since no computed statistics was significant at 5% level of significance.

The restriction that the sum of the estimated coefficients of past inflation is not significantly different from one in the Phillips curve can not be rejected according to the Wald test, presented below. This result is important since it confirms that the Phillips curve is vertical in the long run.

$$\text{Restriction } \alpha_{11} + \alpha_{12} = 1: \chi^2(1) = 1.1456 [0.2845]$$

The estimated parameters of the model reveal that an increase of one point percentual in the real interest rate pushes down the output gap by almost half point percentual (0.45), six months later. On the other hand one point percent of the later causes a reduction of 0.046 percents of the rate of inflation three months

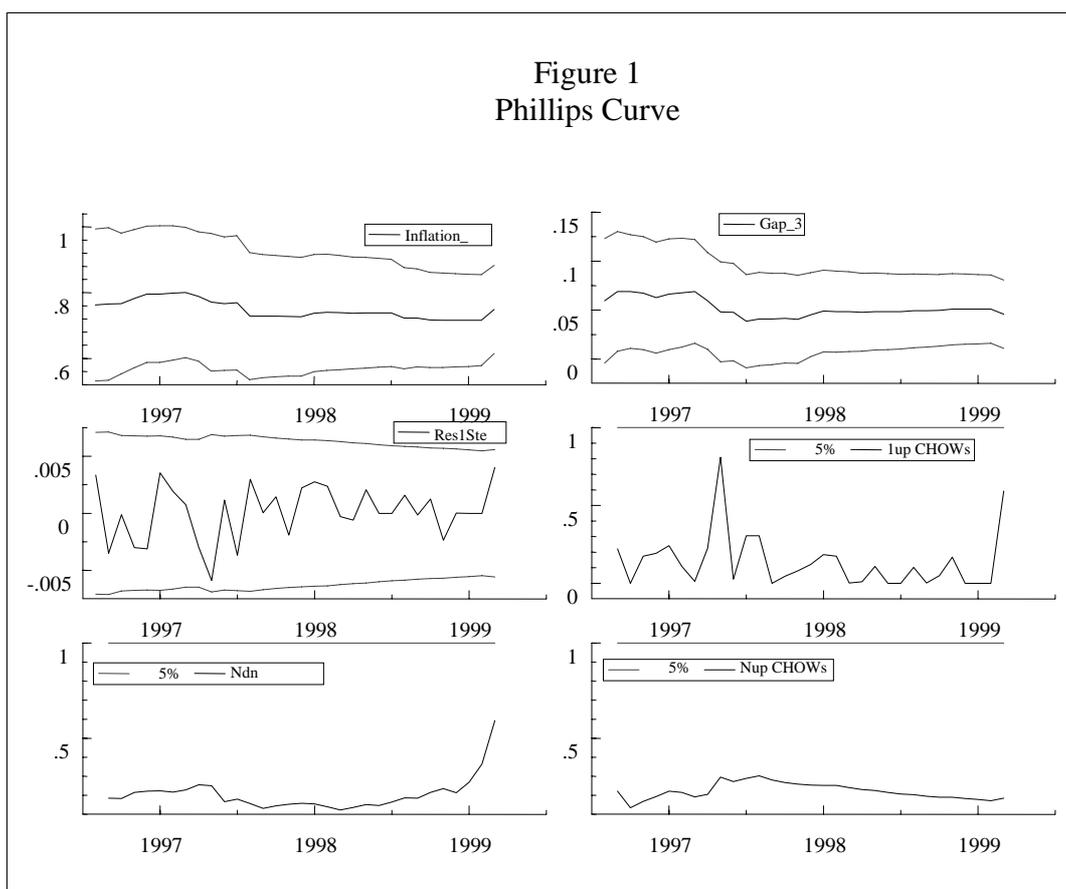
<sup>2</sup> In the Phillips Curve we have considered step dummies for the following months: 98m2, 98m6, 98m7 (Asian and Russian crises), 99m1 and 99m2 (beginning of the floating exchange-rate regime). In addition seasonal dummies for months 5 and 11 were included. In the IS Curve a step dummy for 97m4 and seasonal dummies for the months 1, 4, 6, 8 and 10 were included.

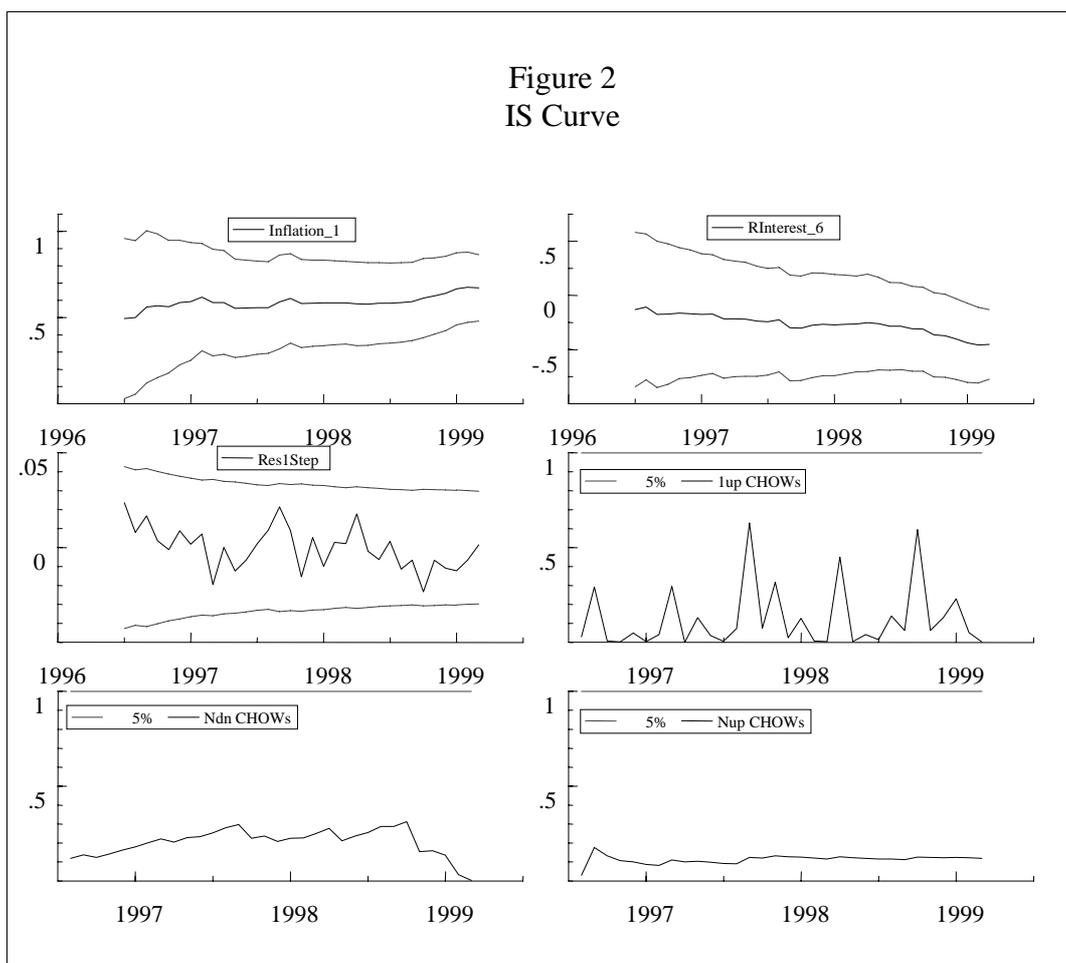
later. As a consequence, a rise of one point percent of the interest rate will reduce the rate of inflation in 0.02 percent point nine months later.

### 2.3 - Stability of the Model and the Lucas Critique

The stability of the parameters of a model is a necessary condition for its usefulness as a tool for economic policy evaluation. Unless the estimated parameters are structurally stable in relation to changes in economic policy the model predictions cannot be used to assess economic policy changes as shown by Lucas (1976). In particular, the weak exogeneity analysis is a condition for inference and the superexogeneity is a requirement for the use of the model for economic policy simulations.

In order to evaluate the structural stability the equations were estimated recursively and the recursive graphics of the estimated coefficients and residuals of each equation were computed. In addition to that, Chow test was implemented to verify the occurrences of structural breaks. The results are reported in the Figures 1 and 2.





One can see, from Figure 1, that the Phillips curve presented a high stability in the period. The recursively estimated coefficients did not show any break. In addition, the Chow's tests, with increasing and decreasing horizon, did not reveal any significant break at the 5% level of significance in this equation in the period.

Similar analysis can be applied to the IS curve, that also presented great stability in the same period. The recursive estimations of the coefficients and Chow's tests, shown in Figure 2, did not reveal the presence of any significant structural break. This suggests that the parameters of both equations can be considered structurally stable in the period.

As a second step, the exogeneity of the variables in relation to the relevant parameters was tested. The test for weak exogeneity implemented verifies if the conditional model can be used for statistical inferences without loss of relevant information. The test was designed by Engle (1984), and looks for zero covariance between the error terms of the conditional and the marginal models. The marginal models were estimated for the variables real interest rate and output GAP, that are the explanatory variables of the IS and Phillips curve, respectively. The residuals

of these models were stored and inserted as an extra explanatory variable and also as a dynamic process in each of the correspondent conditional equations. The residuals were not statistically significant suggesting that the correspondent variable of the marginal process can be considered weakly exogenous.

The results are presented below, where  $u_{1t}$  e  $u_{2t}$  denote the estimated residuals of the marginal models of real interest rate and output GAP respectively:

a) inclusion of  $\hat{u}_{2,t}$  e  $\hat{u}_{2,t}^2$  in (2.2.3):  $F(2, 38) = 0.624$  [54.11%];

b) inclusion of  $\sum_{j=0}^6 \delta_{2,j} \hat{u}_{2,t-j}$  in (2.2.3):  $F(6, 34) = 0.307$  [94.46%];

c) inclusion of  $\sum_{j=0}^3 \gamma_{2,j} \hat{u}_{2,t-j}^2$  in (2.2.3):  $F(4, 33) = 0.842$  [50.86%];

d) inclusion of  $\hat{u}_{1,t}$  e  $\hat{u}_{1,t}^2$  in (2.2.4):  $F(2, 40) = 0.928$  [40.35%];

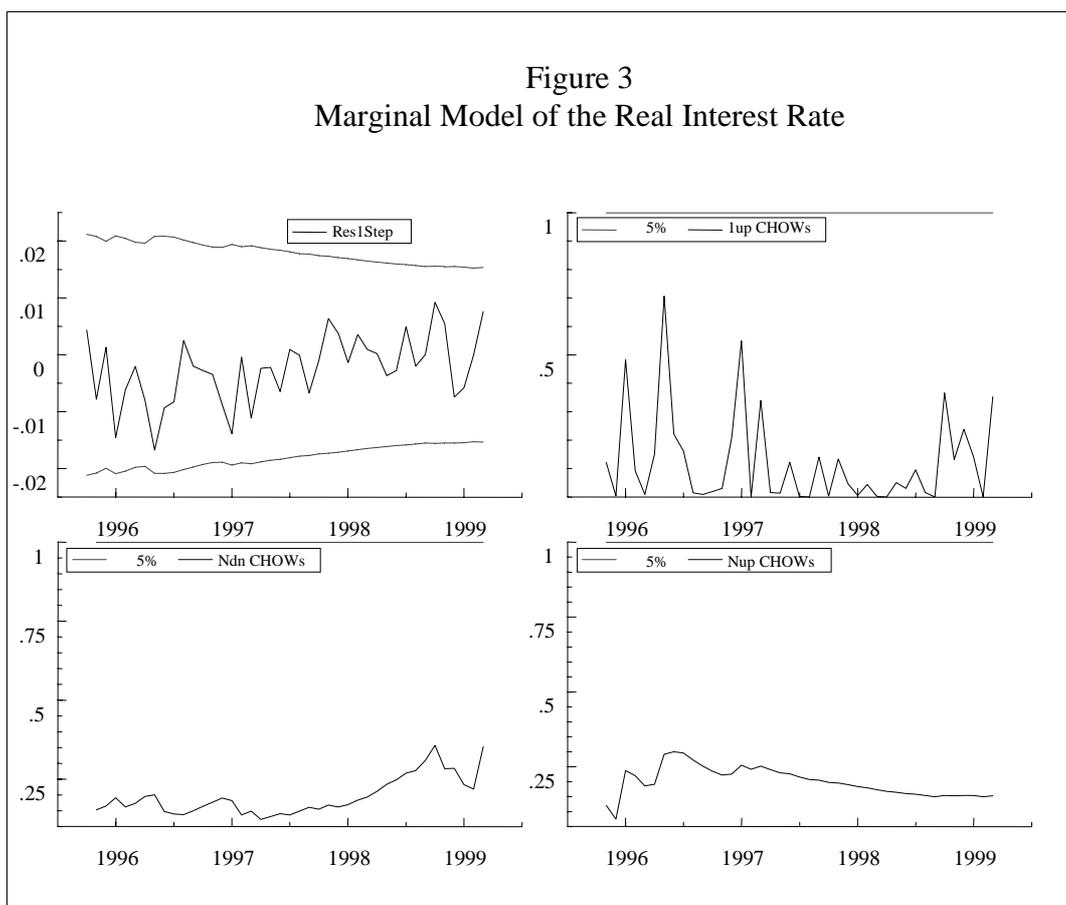
e) inclusion of  $\sum_{j=0}^4 \delta_{1,j} \hat{u}_{1,t-j}$  in (2.2.4):  $F(5, 37) = 1.483$  [21.87%]; and

f) inclusion of  $\sum_{j=0}^3 \gamma_{1,j} \hat{u}_{1,t-j}^2$  in (2.2.4):  $F(4, 38) = 2.39$  [6.71%].

The analysis of superexogeneity of the explanatory variables for the parameters of interest represents a fundamental point for this study in order to validate the use of the model for economic policy assessment overcoming the Lucas critique (1976). The test developed by Hendry, Muellbauer and Murphy (1990) was the first to be implemented. It consists in verifying if there is coincidence between the structural breaks in the conditional and marginal models. If the breaks coincide it means that the parameters are not constant in the sample and the superexogeneity is not verified.

The conditional models, illustrated in Figures 1 and 2 above, do not present any structural break. Figure 2 also shows the marginal model of the output GAP, since it is the only explanatory variable of the Phillips Curve. Since there is not any structural break in these models, the variable output GAP fulfills the definition of superexogeneity in the Phillips Curve. The structural breaks of the marginal model of the real interest rate can be inferred from Figure 3.

As it can be seen in Figure 3, the marginal model of the real interest rate also did not present structural breaks in the period. As a consequence the variable real interest rate can be considered superexogenous.



An additional test originally proposed by Engle and Hendry (1993) was applied. This test verifies if the interventions that affect the marginal models appear also in the conditional models. It uses dummy to model the interventions that occurred in the marginal models and test the significance of these dummies in the respective conditional models. The results are presented below:

a) Inclusion of the *dummy* variables of the marginal model of the output GAP in the Phillips equation:  $F(6, 35) = 1.009$  [43.51%]; and

b) Inclusion of the *dummy* variables of the marginal model of the real rate of interest in the IS equation:  $F(2, 40) = 0.857$  [43.21%].

These results suggest that the interventions that occurred in the marginal models of output GAP and real rate of interest did not affect the conditional model of the Phillips Curve and IS equation. The dummy variables used to model the interventions in the marginal models were not significant in the respective conditional models. In this way, the variables output GAP and real interest rate can be considered as superexogenous in its respective equations. This result is so important because it validates the application of this model to evaluate monetary policy.

### 3 - COMPUTATION OF THE OPTIMAL RULE<sup>3</sup>

The policy of pursuing an inflationary target, can be represented in this context by the choice, by the Central Bank, of the sequence of the control variable, interest rate, that minimizes the following intertemporal loss function:

$$E_t \sum_{\tau=t}^{\infty} \delta^{\tau-t} L_{t+\tau}, \quad (5)$$

where  $E_t$  is the operator of expectations conditional to the information set of time  $t$  and the rate of discount  $\delta$  fulfills the condition  $0 < \delta < 1$ . The loss function can be represented by the quadratic function:

$$L(\pi_t) = \frac{1}{2} [\pi_t - \bar{\pi}]^2 \quad (6)$$

Let us assume that the monetary authorities consider as target not only inflation but also stability of the product around its natural level and also tries to smooth the interest rate. In that case the loss function can be represented as follows:

$$L_t = (\bar{\pi}_t)^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 \quad (7)$$

where  $\bar{\pi}_t$  is the deviation of inflation in relation to the inflation target in period  $t$ ,  $\lambda$  is the weight given to the stabilization of the GAP *vis-à-vis* inflation and  $\nu$  is the weight given to interest rate smoothing.

The model has a convenient state-space representation,

$$X_{t+1} = A X_t + B i_t + v_t \quad (8)$$

The  $14 \times 1$  vector  $X_t$  of state variables, the  $14 \times 14$  matrix  $A$ , the  $14 \times 1$  column vector  $B$  and the  $14 \times 1$  column disturbance vector  $v_t$  are given by

<sup>3</sup> This section follows closely Svensson (1997a,b). See also Chow (1970) and Sargent (1987).

$$X_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \\ \pi_{t-5} \\ y_t \\ y_{t-1} \\ y_{t-2} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \\ i_{t-4} \\ i_{t-5} \end{bmatrix}; A = \begin{bmatrix} 0.749 & 0 & 0 & 0 & 0.204 & 0 & 0 & 0 & 0.046 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.45 & 0.64 & 0 & 0 & 0 & 0 & 0 & 0 & -0.45 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \eta_t \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It is convenient to express the vector of target variables as a function of the state variables and the control variable according to:

$$Y_t = C_X X_t + C_i i_t \quad (9)$$

where the  $3 \times 1$  vector  $Y_t$ , the  $3 \times 14$  matrix  $C_X$  and the  $3 \times 1$  column vector  $C_i$  are given by

$$Y_t = \begin{bmatrix} \bar{\pi}_t \\ y_t \\ i_t - i_{t-1} \end{bmatrix} \quad C_X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad C_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

This problem can be seen as a stochastic discounted optimal linear regulator problem:

$$\max_{\{i_t\}_{t=0}^{\infty}} E_0 \sum_{i=0}^{\infty} \beta^i \{ Y_t' K Y_t \} = \max_{\{i_t\}_{t=0}^{\infty}} E_0 \sum_{i=0}^{\infty} \beta^i \{ X_t' R X_t + 2i_t W X_t + i_t' Q i_t \} \quad \beta \in (0,1) \quad (10)$$

subject to

$$X_{t+1} = A X_t + B i_t + v_t$$

where

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \nu \end{bmatrix}$$

and

$$R = C_x' K C_x \quad W = C_i' K C_x \quad Q = C_i' K C_i$$

To solve this problem we have to start with the Bellman's equation and guess the value-function is quadratic,

$$V(X) = X' P X + d \quad (11)$$

where

$$d = \beta(1 - \beta)^{-1} tr P \sum_{vv}$$

$tr$  is the trace of the matrix  $P$  times the covariance matrix of disturbance vector  $v$ .  $P$  is a 14x14 dimensional negative semidefinite symmetric matrix which is the solution to the algebraic matrix Riccati equation defined below.

The first order condition corresponds to the optimal policy function:

$$F = \frac{(W + \beta B' P A)}{Q + \beta B' P B} \quad (12)$$

The optimal instrument rule is a linear function of the state vector and is increasing in  $X_t$ .

$$i_t = F X_t$$

The 14x14 matrix  $P$  satisfies the algebraic matrix Riccati equation

$$P = R + \beta A' P A - (W' + \beta A' P B)(Q + \beta B' P B)^{-1}(W + \beta B' P A) \quad (13)$$

where all matrices in the matrix Riccati difference equation are given by their steady-state values of the state variables.

It can be shown that in steady state

$$V(X) = tr P \sum_{vv} \text{ when } \bullet=1 \quad (14)$$

The results of the optimal rule under different weights (K matrices) are reported in Table 2. It is noticeable that the reaction coefficient to current inflation, under the assumption of very low interest rate smoothing and equal weights for inflation and output stabilization, are substantially lower than the ones suggested by Taylor (1993a,b).<sup>4</sup>

<sup>4</sup> According to Taylor (1993a,b) the monetary policy should accord to:  $i_t = 1.5(\bar{\pi}_t) + 0.5 y_t$ .

Table 2

*Optimal Rule — Vector F Under Different Matrices K*

	0.1	0.1	0.5	0.5	1	1	5	5	100	100
	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01	0.1
$\bullet_t$	1.8700	0.934	1.3490	0.896	1.204	0.902	1.048	0.951	1.0030	0.996
$\bullet_{t-1}$	0.6460	0.271	0.6330	0.335	0.635	0.389	0.638	0.530	0.6400	0.632
$\bullet_{t-2}$	0.5287	0.237	0.4500	0.243	0.433	0.265	0.415	0.343	0.4100	0.405
$\bullet_{t-3}$	0.4552	0.223	0.3290	0.196	0.299	0.195	0.270	0.222	0.2630	0.259
$\bullet_{t-4}$	0.4435	0.229	0.2800	0.198	0.234	0.188	0.183	0.174	0.1690	0.168
$\bullet_{t-5}$	0.1255	0.053	0.1130	0.060	0.111	0.068	0.108	0.090	0.1070	0.106
$y_t$	0.2360	0.106	0.1830	0.099	0.170	0.105	0.157	0.130	0.1530	0.151
$y_{t-1}$	0.0570	0.032	0.0210	0.016	0.122	0.010	0.003	0.002	0.0001	0.000
$y_{t-2}$	0.0650	0.037	0.0270	0.026	0.016	0.020	0.004	0.008	0.0002	0.001
$i_{t-1}$	-0.1770	0.338	-0.4510	0.021	-0.528	-0.133	-0.613	-0.438	-0.6390	-0.626
$i_{t-2}$	-0.2730	-0.105	-0.3500	-0.183	-0.374	-0.229	-0.401	-0.333	-0.4090	-0.404
$i_{t-3}$	-0.2010	-0.080	-0.2350	-0.123	-0.245	-0.151	-0.258	-0.214	-0.2620	-0.259
$i_{t-4}$	-0.1550	-0.063	-0.1600	-0.085	-0.163	-0.100	-0.167	-0.138	-0.1680	-0.166
$i_{t-5}$	-0.1250	-0.053	-0.1130	-0.060	-0.111	-0.068	-0.108	0.090	-0.1070	-0.106

On the other hand any interest rate smoothing leads the monetary policy to accommodation and dangerously so in the sense that the model may become unstable [see Leeper (1991)]. It is interesting also to observe that the sensitivity of the reaction coefficients to changes in the weight of output stabilization becomes very small for big values of  $\lambda$ .

Finally it is worthy to focus on the limiting cases for  $\bullet = 0.1$  and  $\bullet = 100$ . The reaction coefficient to current inflation maybe quite high when the inflation stabilization is the sole objective, approximately 1.9, and on the contrary, when output stabilization is the sole objective, the reaction coefficient tends to 1. In addition, it is important to notice that the coefficients suggested by Taylor do not fall into the optimal subset of reaction coefficients even if we take account of the lags.

The optimal policy can be summarized, as a reaction function in which one point of deviation in inflation will imply an increase of 1.2 percentual points in the nominal interest rate, and 1 point of deviation of the GAP will lead to a 0.17 percentual points of increase in the nominal interest rate, assuming equal weights to inflation and output stabilization. Optimal monetary policy in this case implies a very low volatility of the interest rate.

## 4 - ANALYSIS OF THE OPTIMAL RULE

### 4.1 - Impulse Response Functions

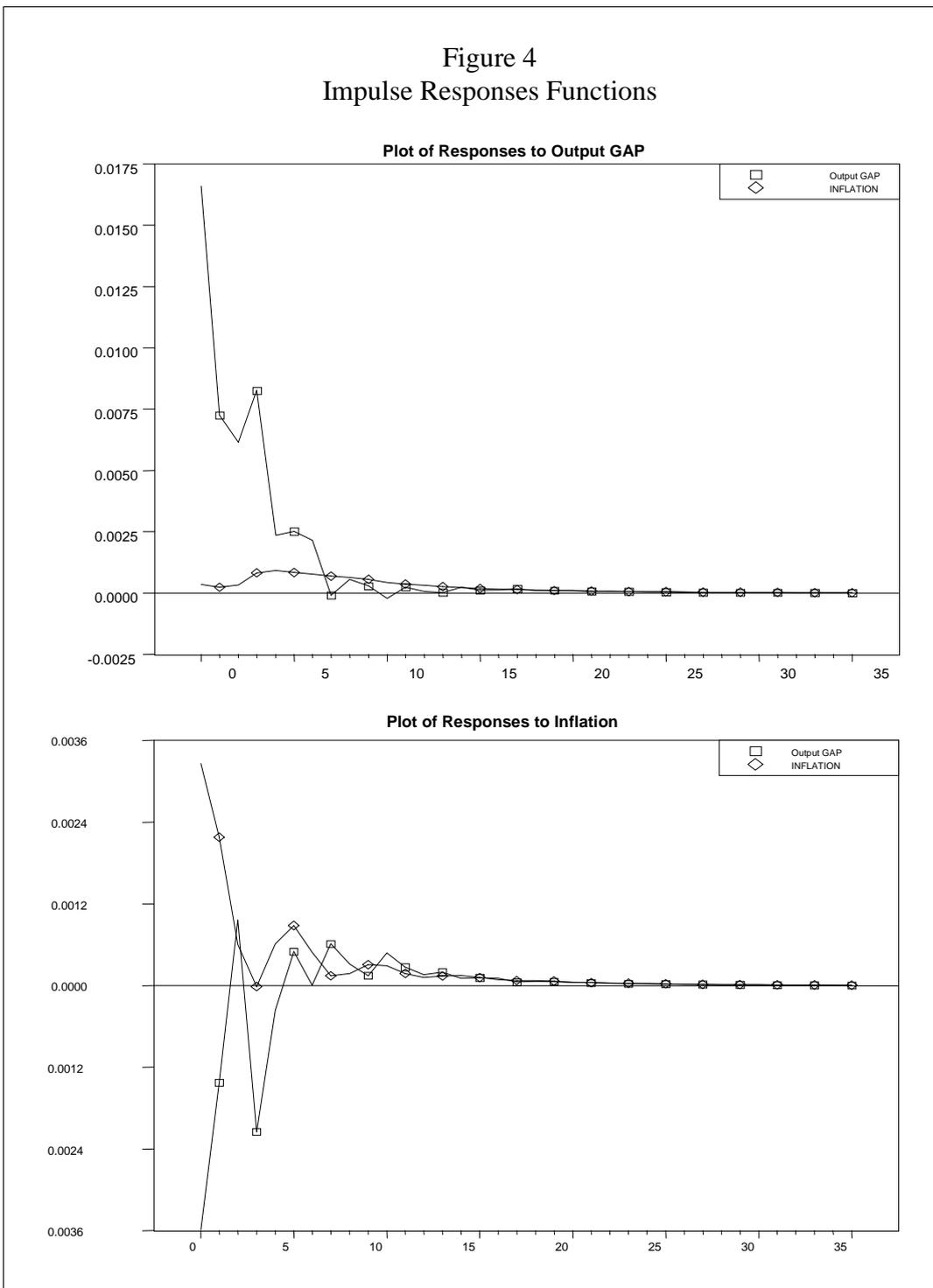
To understand the workings of the monetary rule we propose to compare the behavior of the inflation and output gap in response to shocks without interest rate adjustment, with the optimal monetary policy and finally with the Taylor's rule of thumb monetary policy.

#### **Exogenous Interest Rate — SUR approach**

The impulse response functions, relating output gap, inflation and interest rate, were computed to show the response of the model to different shocks. These functions were obtained from a system of equations estimated by SUR. The reason for applying SUR is due to the fact that the interest rate is exogenous in the two equations of the system. In this case, there are gains of efficiency in the use of SUR *vis-à-vis* VAR, as shown by Greene (1993). To establish the optimal number of lags to be used in the system the modeling strategy is to go from the general to the specific. The choice among rival models was done by the lowest Akaike and Schwarz selection criteria. Using this methodology, four lags were identified for each variable of the system.

The impulse response functions depicted in Figure 4 show a quick convergence of both variables. In the case of inflation, however, the convergence is reached following an initial period of oscillation. This suggests that inflation presents a higher volatility than output GAP. These results should be compared with the responses of these variables when there is a monetary policy either as a rule of thumb as in Taylor's rule or as an optimal policy rule as the one presented in Section 3.

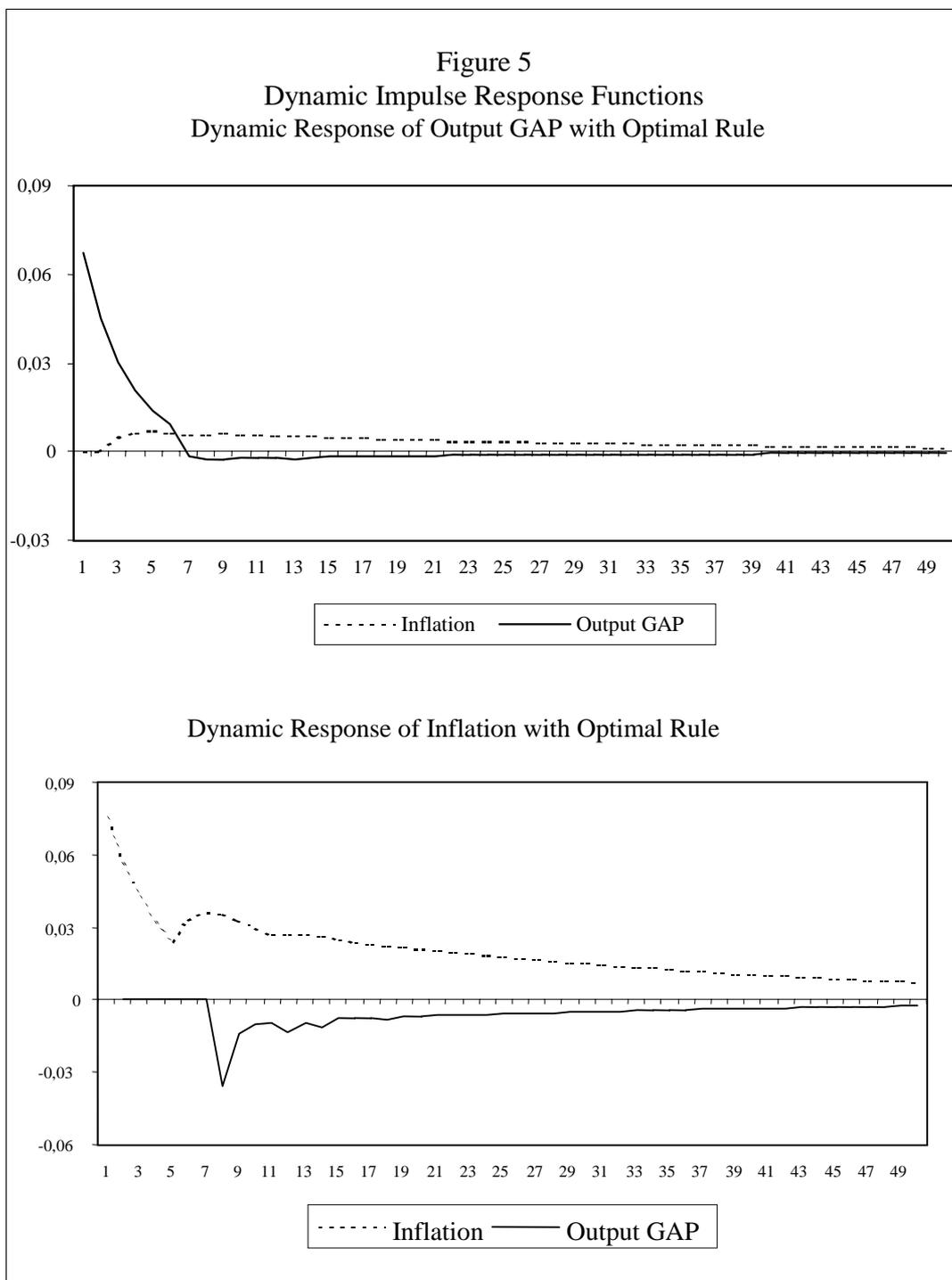
Figure 4  
Impulse Responses Functions



**Dynamic Responses — Optimal Rule**

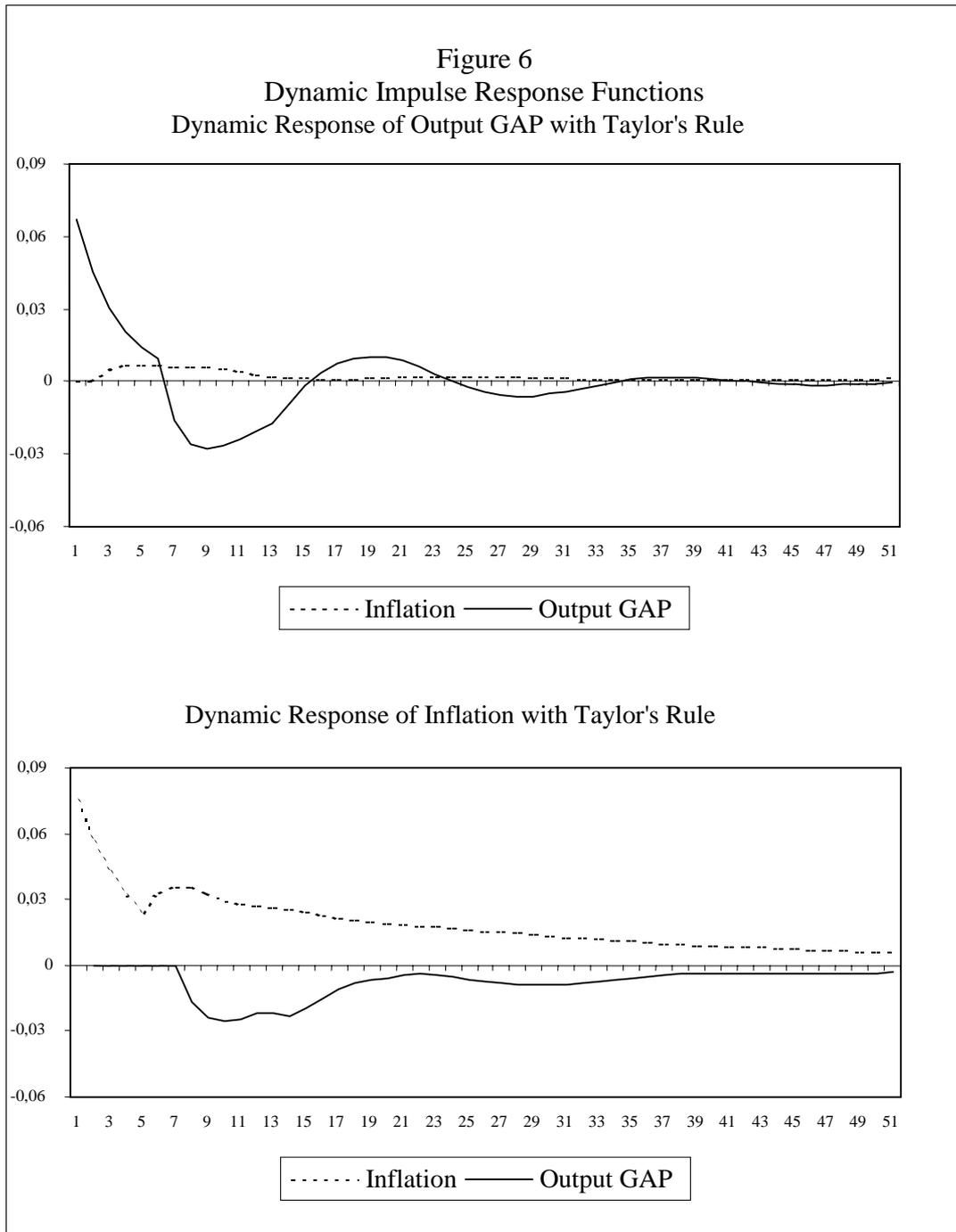
The dynamic responses of the system to unitary shocks in the variables output GAP and inflation, assuming the optimal monetary policy rule is being implemented, with  $\alpha = 1$  and  $\beta = 0.01$ , are presented in Figure 5. It is noticeable

that after the shocks the system converges without any cyclical behavior to the long run equilibrium, particularly in the case of the output GAP. The endogenous optimal behavior of the interest rate leads to a smooth transition of the output GAP and the inflation.



### Dynamic Responses — Taylor's Rule

The dynamic responses of the economy to unitary shocks on inflation and output gap when the Taylor's rule is applied are presented in Figure 6. Its resemblance to the one that was verified under the optimal rule does not come as a surprise given that the optimal rule is somewhat close to the Taylor's rule. Nonetheless it should be noted that the convergence process to a shock on the output gap present notorious cycles contrasting to the smooth one that comes from the optimal rule.



## 4.2 - Efficiency Frontier<sup>5</sup>

According to Svensson (1997a) when the discount rate  $\delta$  gets close to 1, despite the fact that the sum correspondent to (5) becomes unbounded, its deterministic component converges. When  $\delta \rightarrow 1$  the value of the Loss Function tends to the infinite sum of the unconditioned means of the period Loss Functions:

$$\lim_{\delta \rightarrow 1} (1 - \delta) E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} = E[L_t] \quad (15)$$

This allows us to compute the value of the minimum value of the Loss Function as:

$$E[L_t] = \text{Var}[\pi_t] + \lambda \text{Var}[y_t] + \upsilon \text{Var}[i_t - i_{t-1}] \quad (16)$$

By the same token it is possible to assess the variance of the target variables.

The covariance matrix is:

$$\sum_{YY} \equiv E[Y_t Y_t'] = C \sum_{XX} C' \quad (17)$$

where  $\sum_{XX}$  is the unconditional covariance matrix of the state variables and satisfies the matrix equation:

$$\sum_{XX} \equiv E[X_t X_t'] = M \sum_{XX} M' + \sum_{vv} \quad (18)$$

This property will be used to compute the variance of the state variables under different  $K$  matrices.

Table 3 presents the values of the Loss Function and the standard deviation of inflation, output gap and interest rate under different weights for  $\bullet$  and  $\bullet$ .

The efficiency frontier displays clearly the trade-off between inflation stabilization and output stabilization.

It should be noticed, however, that the gain for output stabilization that comes from increasing  $\bullet$  are very substantial up to  $\bullet = 2$ , while the losses for inflation stabilization are very small in terms of standard deviation.

Making the supposition that the loss function weights are the same for inflation and output stabilization we used the same expression to calculate the covariance of the target variables under the Taylor's rule. As it can be seen the variance of

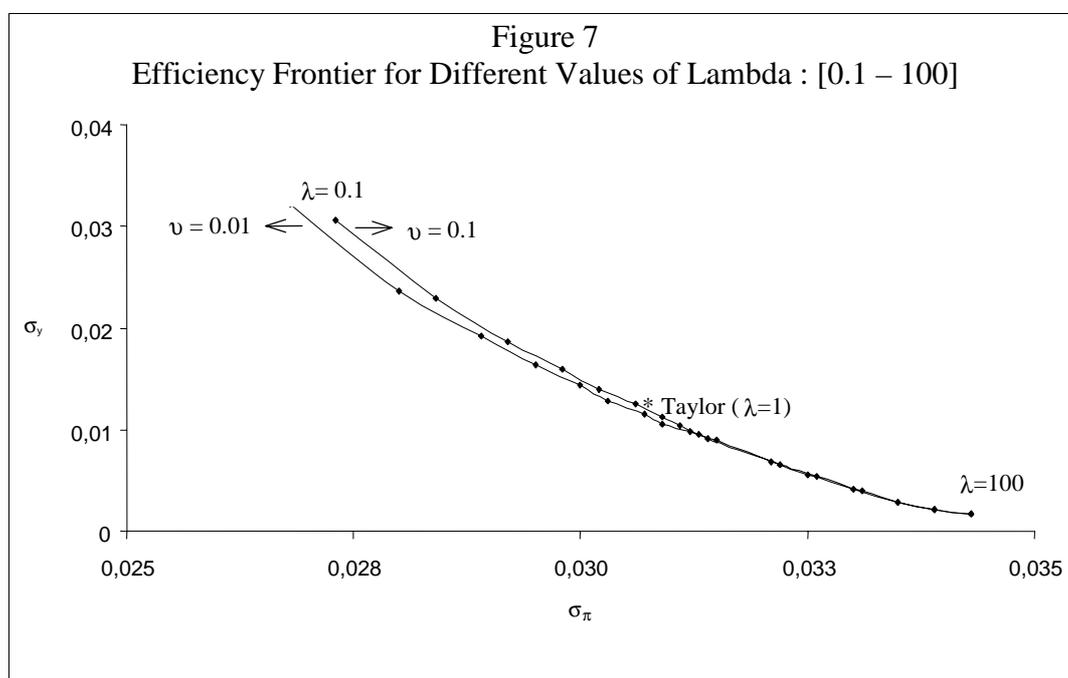
<sup>5</sup> This section is based in Rudebusch and Svensson (1998).

inflation and output stabilization falls out of the mark for the correspondent weights. These results are also shown in Figure 7.

Table 3

**Simulation of the Dynamic Responses of the Target Variables under Optimal Rule for Different Values of  $\lambda$  and  $\nu$**

$\lambda$	$\nu = 0.01$				$\nu = 0.1$			
	$\sigma_\pi$	$\sigma_y$	$\sigma_i$	$L$	$\sigma_\pi$	$\sigma_y$	$\sigma_i$	$L$
0.1	0.0270	0.0321	0.0300	0.00083	0.0273	0.0306	0.0159	0.0009
0.3	0.0289	0.0192	0.0240	0.00095	0.0292	0.0187	0.0147	0.0010
0.5	0.0300	0.0144	0.0215	0.00100	0.0302	0.0140	0.0144	0.0010
0.7	0.0307	0.0116	0.0201	0.00100	0.0309	0.0113	0.0142	0.0011
1	0.0314	0.0091	0.0188	0.00110	0.0315	0.0089	0.0141	0.0011
2	0.0325	0.0055	0.0170	0.00110	0.0326	0.0054	0.0140	0.0011
3	0.0330	0.0041	0.0163	0.00110	0.0331	0.0040	0.0141	0.0012
5	0.0335	0.0029	0.0156	0.00120	0.0335	0.0029	0.0141	0.0012
10	0.0339	0.0021	0.0151	0.00120	0.0339	0.0021	0.0143	0.0012
100	0.0343	0.0017	0.0146	0.00150	0.0343	0.0017	0.0145	0.0015
Taylor ( $\lambda=1$ )	0.0306	0.0125		0.00110				



## 5 - MONETARY POLICY IN BRAZIL

The next step is to compare the optimal monetary policy in Brazil, under target inflation with the monetary policy implemented in the recent past.

### 5.1 - Reaction Function

The empirical evidence of Taylor's rule type reaction function for the Brazilian economy, for the period 1994/99 was tested by the estimation of a long run model relating nominal interest rate, inflation and GAP according to the original Taylor's formulation, Taylor (1993a,b). The representation of the model can be done with the variables in level because all of them have been identified stationary for the period. We have estimated an ADL for the interest rate, whose static solution will be the Taylor's rule for the period.

The ADL model was estimated from a recursive search of the optimal number of lags through the Akaike and Schwarz information criteria and from the diagnostic statistics, as before. The adequate specification was ADL (4), with some intermediate lags omitted due to non statistical significance. Dummy variables correspondent to the contagious effects of Asian and Russian currency crises and recent Brazilian crisis as well were introduced. The results of the long run static solution are presented below, where the values in parenthesis below each coefficient are the respective standard deviations. The constant was not presented since it was not significant.

$$i_t = 1.106 \pi_t + 0.454 y_t + D_t + S_t \quad (19)$$

(0.2759)                      (0.1384)

$$T = 56 [1994(8) - 1999(3)]; R^2 = 0.9962; \hat{\sigma} = 0.002; RSS = 0.0001; DW = 1.98;$$

$$AR1 - 4 \quad F(4, 38) = 0.777; ARCH 4 \quad F(4, 34) = 0.676; Norm. \chi^2(2) = 26.123;$$

$$\chi_i^2 F(13, 28) = 1.684; Reset F(1, 41) = 0.177.$$

where  $S_t$  and  $D_t$  are vectors of seasonal and impulse dummies respectively.<sup>6</sup>

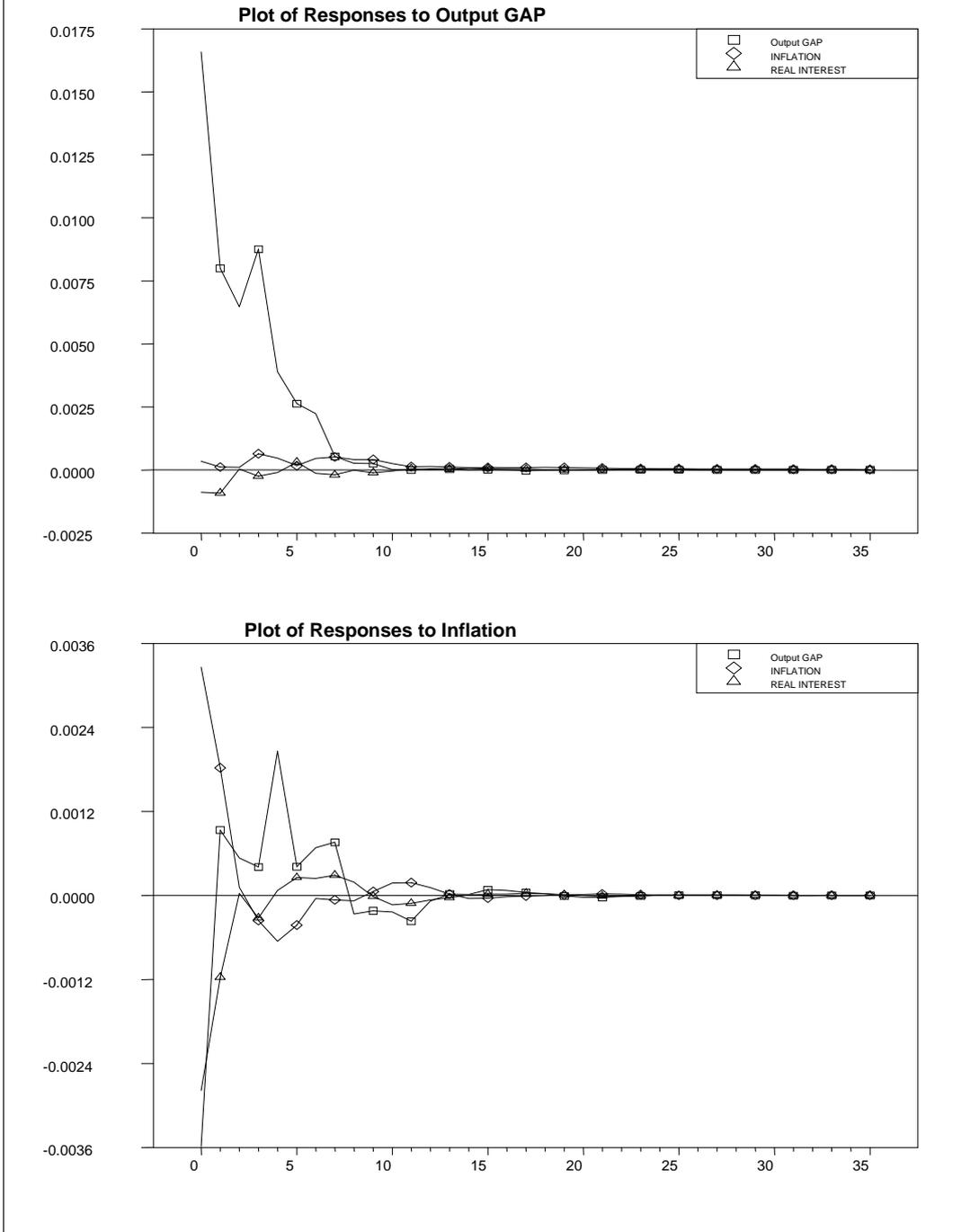
The reaction coefficient of interest rate to current inflation shows a low degree of activism of monetary policy in the same direction of the optimal rule. On the other hand the reaction coefficient to output stabilization is very close to Taylor's 0.5. The results suggest that monetary policy maybe very close to efficiency in its reaction to inflation but overshoots the stabilization output target.

### 5.2 - Reaction Function — SUR Approach

Following Clarida and Gertler (1996) and Chinn and Dooley (1997) we analyzed the recent monetary policy through a constrained SUR. The restrictions imposed on the structure of the SUR followed choleski decomposition procedure where the ordering was: output GAP, inflation and real interest rate. The results of the SUR can be summarized by the impulse reaction functions as shown in Figure 8.

<sup>6</sup> We have considered step dummies for the following months: 97m11, 98m9, 98m10 (Asian and Russian crises), 99m2 and 99m3 (beginning of the floating exchange-rate regime). In addition a seasonal dummy for month 11 was included.

Figure 8  
Dynamic Impulse Response Functions



It is noteworthy that the convergence of the output gap, when the interest rate was considered endogenous, presented in Figure 8, is faster than the one shown in Figure 4, where the interest rate was considered exogenous. On the other hand the path of the inflation after the shock, displayed in Figure 8, seems to be more

cyclical and the convergence slower than the path presented in the Figure 4. This suggests that the monetary policy was output GAP stabilizing.

These results are consistent with the estimated reaction function and tend to confirm the product stabilization bias of the current monetary policy.

## **6 - CONCLUSION**

This study analyzed optimal monetary policy in the case of Brazilian Economy during the recent period after the implementation of the Real Stabilization Plan. With the parameters of an estimated IS — Phillips Curve model a set of optimal rules was computed. It is noteworthy that the IS — Phillips Curve model presented stability of the parameters, particularly the superexogeneity analysis suggests that the model is not subject to the Lucas Critique.

The optimal monetary policy, under the assumption of equal weights to inflation and output stabilization, implies very low interest rate volatility: the reaction coefficients are lower than the ones of the Taylor's rule. An one percent of increase in inflation will raise the interest rate in 1.2 percentual points and an one percent point increase in the GAP will raise the interest rate by 0.17 points.

The reaction function of monetary policy of Brazil was estimated for this period. The sensitivity of interest rate to inflation is very low and close to one and the reaction coefficient to output GAP is high and close to 0.5. These coefficients fall out of the set of efficient monetary rules and seem to indicate that monetary policy over reacts to product shocks and under reacts to inflationary shocks taking as reference the optimal rule.

This analysis is complemented by another methodology. In this case the reaction function is embedded in a SUR framework. The resulting impulse response functions also suggest that current monetary policy has been biased towards product stabilization instead of inflation stabilization.

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