Poverty and Public Utilities Pricing

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TEXTO PARA DISCUSSÃO tem o objetivo de divulgar resultados de estudos desenvolvidos no IPEA, informando profissionais especializados e recolhendo sugestões.

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POVERTY AND PUBLIC UTILITIES PRICING

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1. INTRODUCTION

Public utilities' rates in many developing countries are being used as instruments of income redistribution: these firms charge discriminatory prices to users, cross-subsidizing the consumption of poor households. The author of this paper is interesting in examining the theoretical reasoning that defines these price schedules within a distributional context.

The traditional economic literature on the subject is directly to what is well-know as the Ramsey prices (or the inverse elasticity relationship rule) when the firm's objective is profit maximization. Since public utilities in most countries are state-owned enterprises, their pricing policies tend to be related to government's goals other than the efficient allocation of resources. Several studies, including some chapters of the author's doctoral dissertation, analyze how equity consideration can be introduced in the definition of public utility's rates.\(^1\) This author's main contribution is to show how these prices are affected by several variables, including the social welfare weights the government attaches to different users in a welfare maximization exercise.

The present paper examines the problem of finding an optimal set of prices for these public monopolies when the objective function is the minimization of the level of poverty. The author's approach is a normative one, that is, the intention is to derive optimal price schedules that could replace those currently used by several public utilities. The analysis developed in this paper is theoretical. An empirical study would require our access to specific sources of data, not available in the present circumstances, and should be thought as the next stage to which the author would like to commit himself.

There are at least two fundamental analytical differences in the maximization of welfare and the minimization of poverty approaches:

a) in the latter approach our subject-matter is only that segment of the total population considered as poor, while in the former approach the whole population enters in the maximization exercises; and

b) as noted by Atkinson (1989, p.9), in the poverty analysis, the concern is with the household's level of

\(^1\)See, for instance, Feldstein (1972a, 1972b, 1972c) and Le Grand (1975).
resources and not with its sense of well-being, as measured by its own welfare evaluation.

Besides leading to a different pricing policy, the poverty approach avoids the need of defining a social welfare function, rarely made explicit in the governmental programmes, with the additional advantage of general acceptance as an objective to be pursued. However, the replacement of the maximization of social welfare for the minimization of poverty as the objective in the process of deriving prices for public utilities does not necessarily make the task simpler. Actually, this alternative approach introduces us to the additional set of complex questions related to the definition and the measurement of poverty. Both questions have been scrutinized abundantly in the economic literature in order to refine the concept of being poor and to improve the quantification of this social condition. The next section makes a summary of the main points covered by this literature with the purpose of putting the analysis developed in Section 2 within a theoretical framework.

Using poverty as an analytical subject in the definition of governmental policies in developing countries is justified not only by the large number of those that are in deprivation, but also by the fact that the intensity of the phenomenon requires a wider attack against it. Hence, the use of public prices as one of the instruments to tackle the problems of poverty and income inequality in those countries.

Although related phenomena, poverty and income inequality are different characteristics of the income distribution: we can have income inequalities without poverty (there are no poor and individual incomes differ) and no income inequalities and poverty (all individuals earn the same, low level, income). It should be noted that, for this reason, neither minimization of poverty necessarily means minimization of income inequalities, nor the reverse is necessarily true.

2. POVERTY CONCEPTS AND MEASUREMENTS

Poverty can be defined in several different ways, some of them taking into account a more restricted view of the problem, others focusing a wider spectrum of characteristics, including not only economic dimensions, but also social and political aspects. However, these different definitions have in common the idea that poverty is related to the lack of access to some standard of living considered essential or minimal for an adequate life in society. Departing from this
common understanding, the differences in conceptualizing poverty arise from dissimilar views of what a “minimal adequate standard of living” actually means.

One strand of the different forms of specifying the characteristics of such a standard of living is connected with the idea that poverty has both an absolute and a relative dimension. In the absolute case, the definition of poverty makes no reference to other standards of living that exist in that or other societies; its definition is related to what is considered essential for life. This is the common view of poverty that prevails in developing countries, where the concern with this problem is more weightily linked with the idea of individual survival.

We can recognize three different lines of thought of how that standard of living should be defined:

a) the poverty line approach: according to this approach, the poverty line is that amount of total income or expenditure required for an individual or household to survive, by consuming the commodities in the quantities deemed to be essential to this purpose. Implicit in this approach is the idea of a minimum of welfare that can be derived from the consumption of those commodities. This concept of the poverty line can be modified and expanded to measure relative poverty by utilizing the definition of a basket of goods and services that is considered as the normal or minimum in a given society, hence unrelated to individual survival.

b) the basic needs approach: in this approach, being poor is the conditions of those individuals whose consumption falls short of those consumption targets specified in a development strategy aimed at the abolition of absolute poverty. This approach does not necessarily leads to the determination of a minimum level of income or expenditure as in the poverty line case; the failure of satisfying those targets or needs naturally classifies the individual to be among those

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2 Atkinson (1989, p.11-12) calls the attention for the fact that income and expenditure are distinct ways of measuring poverty, leading to different results.
for which specific social programmes are designed for solving it.\footnote{Streeten and Burki (1978) discuss this approach and suggest six areas covering the essential basic needs: nutrition, basic education, health, sanitation, water supply and housing, and related infrastructure.}

c) the participation approach: this approach is due to Townsend (1979); it differs from the two previous ones by relying neither on commodities nor needs to define poverty. The problem of poverty is viewed in terms of the individual lack of resources required for his social participation or interaction, understood in quite a wide sense: that is, access to a level of consumption of goods and enjoyment of activities that conforms to a customary pattern in that individuals' society. In this sense, the participation approach is directly connected with the idea of right to a minimum level of resources, as suggested by Atkinson (1989, p.12), meaning the minimum level of income required for individual participation in a given society.

A more wider approach for poverty is being developed by Desai (1990) building on ideas advanced by Sen (1985). This approach deals with the idea of capability, that is, owning or not the resources that allow an individual to have access to a set of capabilities, such as to survive and have good health, to ensure biological reproduction, to interact socially, to have knowledge and freedom of expression and thought, among others. As we can, this approach incorporates all the others cited before. From the operational point of view, we can anticipate several difficulties in quantifying all the multiple dimensions this approach require for selecting those poor in a society. It is true that the set of capabilities may have several highly correlated attributes, which may make less difficult the task of separating the poor from the non-poor.

Once the poor in a society is identified the next step is to measure the intensity of the problem. Several indices have been used or suggested by the poverty analysis literature. One of the most commonly used index is the headcount ratio; it measures the percentage of poor in a population and it is expressed either as

\[ H = \frac{q}{n} \]  

(1)
where \( q \): number of poor,
\[ n: \text{number of individuals (households) in the population.} \]
or as
\[ H = \int_0^z f(Y) dy \]  
(2)

where \( Y \): individual income.
\( f(Y) \): frequency density function of income \( Y \),
\( z \): income level that identifies poor and non-poor (for instance, the poverty line).\(^4\)

The headcount ratio is not a good poverty index: it is a weak indicator of the intensity of the problem since it only measures the percentage of individuals in a population that lack the resources to be considered as non-poor; it is also important to know how the incomes of the poor are dispersed and how far they are from the poverty line.

To eliminate this above weakness, some authors use the poverty gap to indicate the difference between the individual income and the \( z \) income level and aggregate these differences to calculate a poverty index called the **income gap ratio**:\(^5\)

\(^4\)From now on \( z \) must be understood as this income level; for sake of simplicity, we assume that all those already mentioned ways of defining poverty can be summarized by defining an income level that has the role of identifying the poor and the non-poor. In this sense, \( z \) can be named as the poverty line, although it can have more dimensions than the survival characteristic attached to the poverty line itself.

\(^5\)Sen (1976) shows that the headcount and the income gap ratios violate either the monotonicity or the transfer axioms: the monotonicity axiom states that, all other things being equal, a decrease in the income of an individual considered as poor must increase the poverty index (this does not happen with the headcount ratio); the transfer axiom states that a transfer of income from a person below the poverty line to a less poor one must increase the poverty index (this does not happen for the headcount and for the income gap ratios).
\[ i = \sum_{i} \frac{y_i}{a_i z} \quad \text{for } i \in S(z) \]  

where \( \delta_i = z - y_i \)

\( Y_i \): income of individual \( i \), for \( Y_i \leq z \)

\( S(z) \): set of poor individuals.

Atkinson (1989, p.29) has a list of potential poverty indices:

a) the normalized deficit:
   \[ \zeta = \int_{0}^{z} \left[ 1 - \frac{y}{z} \right] f(y)dy \]  

b) the Watts measure [Watts (1968)]:
   \[ w = - \int_{0}^{z} \log_e \left( \frac{y}{z} \right) f(y)dy \]  

c) the Clark et alii second measure [Clark, Hemming and Ulph (1981)]:
   \[ \xi = \frac{1}{c} \int_{0}^{z} \left[ 1 - \left( \frac{y}{z} \right)^{c} \right] f(y)dy \]  

d) the Foster et alii measure [Foster, Greer and Thorbecke (1984)]:
   \[ f_{\alpha} = \int_{0}^{z} \left[ 1 - \frac{y}{z} \right] f(y)dy \]  

where \( \alpha \geq 0 \), \( \alpha \) is the aversion to poverty parameter.

The poverty index derived by Sen (1976) also uses the income gap, but he weights the differences in incomes by the position of the individual in the poverty rank:

\[ P = H \left[ 1 - (1 - I) (1 - G (q/q^t)) \right] \]  

where \( H \): the headcount ratio,

\( I \): the income gap ratio,

\( G \): the Gini coefficient of the income distribution of the poor.
Several authors discuss specific characteristics that the poverty indices should have and most of the time they make suggestions of improvements in this measurement of poverty. Since our objective is not to make contributions in this field, we will not go into the discussion of the advantages or the disadvantages of those indices.

A desirable property for a poverty index is that on that states that its first partial derivative in terms of the poverty line should be positive, that is, that the measurement of the poverty should change in the same direction of the change of the poverty line. This means that for smaller values of z, the poverty index should measure smaller levels of poverty. All the above indices share this property. For this reason, instead of choosing one specific poverty index, in the next section we will be trying to derive a public utility price so as to minimize z, the poverty line, and thus poverty, as defined by any of the above indices, is also minimized.

3. PRICING AND MINIMIZATION OF POVERTY

The purpose of the present section is to derive a price for a public utility compatible with the government objective of pricing public enterprises in such a way that poverty is minimized.

The choice of using the poverty line as the separator line for identifying poor and non-poor households does not mean that those other approaches listed in the previous section could not be applied, despite their broader dimensions. This option was only made to simplify the analysis.

There are two ways of solving this problem of minimizing poverty through the price to be charged by the public utility to its customers. One of them is optimizing an objective function subject to relevant constraints.

As usually, we have to make some assumptions, similar to those made in previous sections, to reduce the

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6See, for instance, Subramanian (1990) criticizing the Sen and Foster, Greer and Thobecke indices and deriving another index; Thon (1979), contrary to Sen's views, thinks that the weighting of the income gap should take in to account not the individual income position in the poverty rank, but in relation to the whole population income distribution; Vaughan (1987) is interested in the welfare aspects of the poverty indices.
complexities of the functioning of the economy. These assumption are:

**Assumption 1:** Two goods produced in the economy, good 1 (produced by the public utility) and good 2 (a composite good).

**Assumption 2:** A poverty line \( z = P_1 x_1^2 + P_2 x_2^2 \), where the \( x_i \)'s are, respectively, the quantities considered as the minimum amount of goods 1 and 2 that should be consumed in a poor household for the survival of its components. \( P_2 \) is the price of good 2 and \( P_1 \) is the price of good 1 whose value will be derived minimizing the poverty index subject to a deficit constraint: total cost \( (TC) \) minus revenue equals the deficit \( (D) \) the government is willing to finance.

**Assumption 3:** The population is composed of \( n \) households.

**Assumption 4:** A given distribution of household incomes, where \( f(Y) \) is the frequency density function of income \( Y \) and \( F(Y) \) is its distribution function.

**Assumption 5:** The demand function for good 1 is: \( X_{1j} = x_1(P_1, P_2, Y_j) \), where \( Y_j \) is the income of household \( j \) \((j=1,...,n)\), \( P_2 \) is the price index for the composite good 2, and \( X_{1j} \) is the quantity it demands, given \( P_1 \), \( P_2 \) and \( Y_j \). Its demand price elasticity is \( e_{1j} = -\frac{P_2}{X_{1j}} \cdot \frac{dx_{1j}}{dP_2} \).

**Assumption 6:** The public utility total cost of production \( (TC) \) is a positive function of the total quantity of units produced, that is, \( TC = C(\sum x_j) \). The derivative of \( TC \) in terms of \( P_1 \) can be expressed as

\[
\frac{\partial TC}{\partial P_1} = \sum_{j=1}^{n} \frac{\partial TC}{\partial x_{1j}} \cdot \frac{\partial x_{1j}}{\partial P_1} = m \cdot \sum_{j=1}^{n} \frac{\partial x_{1j}}{\partial P_1} \tag{9}
\]

where \( m \) is the marginal cost.

The poverty index is defined as \( \int_0^z f(Y) dY \), that is, the percentage of households that earn an income equal or less than the poverty line \( z \).

Figure 1 shows that we can write that

\[
\int_0^z f(Y) dY = \int_0^{P_2 x_2^2} f(Y) dY + \int_{P_2 x_2^2}^{z} f(Y) dY \tag{10}
\]
Since \( \int_0^z \frac{X^2}{Y} f(Y) dY \) is a constant (it does not depend on \( P_1 \)), then, to minimize that poverty index, we must minimize \( \int_{z_1}^z \frac{z^2}{z} f(z) dY \) that is, to minimize \( F(z) \). It is important to note that \( \frac{\partial F(z)}{\partial P_1} = \frac{\partial F(z)}{\partial z} \frac{\partial z}{\partial P_1} = f(z) \frac{X^2}{z} \).

The function to be minimized is:

\[
L = F(z) + \mu \left( C(X) - \sum_{j=1}^n P_j X_{1j} \cdot - \bar{D} \right) \tag{11}
\]

where \( \sum_{j=1}^n P_j X_{1j} \) is the public utility revenue.

**Figure 1**

Calculating the first-order derivatives of expression (1) in terms of \( P_1 \) and \( \mu \) and equating them to zero, we have the necessary conditions:

\[
\frac{\partial L}{\partial P_1} = f(z) \frac{X^2}{z} + \mu \left[ \sum_{j=1}^n \frac{\partial X_{1j}}{\partial P_1} - \sum_{j=1}^n (X_{1j} - P_j \frac{\partial X_{1j}}{\partial P_1}) \right] = 0 \tag{12}
\]

\[
\frac{\partial L}{\partial \mu} = C(X) - \sum_{j=1}^n P_j X_{1j} - \bar{D} = 0 \tag{13}
\]

and solving expression (12) in terms of \( P_1 \), we have:
\[ P_i = \frac{m}{f(z)X^z - \mu X_n} - 1 \] (14)

The use of expression (14) to calculate the optimal price \( P_i \) would require us the specification of both the income distribution and the commodity \( 1 \)'s functions. There is an alternative procedure, a method that shows the same result as that produced by expression (14). Let us explain it.

Since the objective is to minimize the value of the poverty line \( z \), this is dependent on how low the public utility can charge for an unit of the commodity it produces. Figure 2 shows that as the price range for \( P_i \) varies from zero to \( +\infty \), the poverty index varies from a minimum of \( \frac{p}{z^2} \) to a value that tends to 1; the smaller \( P_i \), the less will be poverty.

![Figure 2](image)

The minimum price the public utility can charge its consumers depends on its budgetary constraints, that is, on its total cost, on the revenue collected by the sales of its commodity, and on the amount of deficit the government is prepared to finance; in other words, the public utility has to satisfy the constraint total

\[ \text{We assume that at this price the household consumes at least the quantity } X^z \text{ ; see the discussion of this assumption in Section 6.} \]
cost minus total revenue should be at least equal to its financed deficit. The lowest price is that one that satisfies the equality in this constraint, that is,

$$C(\sum_{j \in S_1} x_{1j} - \sum_{j \in S_1} p_j x_{1j} = \bar{D}$$

The above expression cannot be solved unless both the demand and the cost functions are specified. For simplicity, let us assume that demand function for this service is \( x_{1j} = \alpha y_j^{\gamma} \), where \( y_j \) is the income of the jth household. This demand function is that derived from a Cobb-Douglas utility function of the form \( u_j = x_{1j}^\alpha x_{2j}^{1-\alpha} \)

where \( U_j \) is the utility enjoyed by the jth household in consuming the commodities 1 and 2 in the quantities \( x_{1j} \) and \( x_{2j} \), respectively, and \( \alpha \) is a functional parameter.

Let us also assume that the total cost function is

$$C(\sum_{j \in S_1} x_{1j}) = F + k(\sum_{j \in S_1} x_{1j})^\theta$$

where \( F \) is the public utility fixed cost of production, \( k \) is a positive \((k>0)\) constant and \( \theta \) is a returns to scale parameter, for \( \theta \leq 1 \).

Using these functions in expression (15), we can write

$$P_1 = \left[ \frac{k}{\bar{D} - F + \sum_{j \in S_1} y_j^\theta + \frac{1}{\alpha \sum_{j \in S_1} y_j^{\theta - 1}}} \right]^{1/\theta} \tag{16}$$

This price is the lowest that the public utility can charge. As such, this is the price that minimizes poverty in this society since it is the public utility tariff that makes \( z \) reach its smaller possible value. As expected, this price is inversely related to the amount of deficit financed by the government, the only instrument it has, in the present case, to make this
price smaller and, consequently, decrease the level of poverty. 8

An alternative way of reaching the same result is to induce the implementation of programmes that increase productivity in this public utility, thereby reducing the cost component contribution in the minimum price determination.

Inspection of expression (16) shows that the public utility price in the present case cannot be zero; it tends to zero (without being equal to zero) when $p$ tends to $\infty$. This means that the lowest level of poverty that could be reached using this tariff would be higher than the area below the income distribution curve calculated between incomes $0$ and $p_k$. This impossibility of charging a zero price, however, derives from the particular case of demand and cost functions used in the analysis; it is obvious that without these functions, one can imagine a situation by which the public utility could distribute freely its production, that is, without charge, as long as the government covers the total cost of implementing such a policy.

4. DISCRIMINATORY PRICES AND MINIMIZATION OF POVERTY

In the preceding section we were interested in deriving a public utility price that minimizes the poverty level in a society. This price would be unique, indifferently charged to all households, irrespective of their social condition. As we saw, this price would be as low as the deficit financing by the government allowed.

One could discuss how adequate such a policy would be from a social point of view: in reality, the government, trying to minimizing poverty by charging a low tariff to consumers, would be extending this benefit to households that do not need such a protection. It does not make sense the use of government scarce resources to finance consumption of non-poor households. In other words, this type of pricing policy suffers from the same problem of targeting diagnosed in poverty-alleviation programmes, in which part of the financial transfers applied to

8Since this deficit ($D$) is financed by transferences of resources out of government revenues, care should be taken that the increased amount required to lower the public utility tariff will be raised by additional taxation or other ways that have a perverse net effect upon poverty.
implement them leaks to the non-poor, checking their efficacy.9

Instead of charging the same low price to all customers, we can now think in deriving two different prices, the lower one to be paid by the poor households (as we are interested in decreasing the level of poverty), and the other by the non-poor.

Let us change the demand assumption: let us assume two different demand functions for commodity 1, one for the poor and the other for the non-poor:

\[ x_{1j}^p = x_{1j}^r \left( P_{11}, P_r, \frac{\nu_j}{1} \right) \]

for \( i=P(\text{poor}), R(\text{rich}) \)

where \( x_{1j}^p \): quantity demanded of commodity 1 by household \( j \) earning and income \( \frac{\nu_j}{1} \),

\( P_{1i} \): price to be paid by household with social condition \( i \) for a unit of commodity 1,

\( P_r \): price of the composite commodity 2.

We assume that commodity 1 is a normal good for both types of households, then \( \frac{\partial x_{1i}^p}{\partial P_{1i}} < 0 \).

Let us order the households' incomes from the lowest one (\( Y_1 \)) to the highest (\( Y_n \)) and assume that those households earnings incomes in the range \( Y_1 \) to \( Y_n \) are considered poor and those with incomes between \( Y_{n+1} \) to \( Y_n \) are non-poor.

The total quantity demanded of commodity 1(\( X_1 \)) can be written as \( \sum_{j=1}^{n} x_{1j}^p + \sum_{j=n+1}^{n} x_{1j}^r = X_1 \), or \( x_{11}^p + x_{1}^r = X_1 \). The public utility total revenue(\( TR \)) is \( TR = P_{1p} x_{1p} + P_{1r} x_{1r} \) and its total cost of production (\( TC \)) is \( F + k x_{1}^r \), where \( F \) is its fixed cost, \( k \) is a constant and \( \theta \) is a returns to scale parameter.

Since the public utility must balance its revenue with its cost, that is, \( TC - TR = D \), we can write that

9Kanbur (1987) discussed the issue of targeting in relation to the transfers made by the social security programmes in the United Kingdom and their impact on poverty.
\[(F + k (x_1^p)^\theta) - (P_{1p} x_1^p + P_{1r} x_1^r) = \tilde{D}\]  

or

\[P_{1p} x_1^p = F - \tilde{D} + k (x_1^p)^\theta P_{1r} x_1^r\]

Expression (17) shows the interrelationship between prices \(P_p\) and \(P_r\). In the Appendix we calculate the derivative \(\partial P_r / \partial P_p\) and we show the values it can take. This derivative is the expression

\[
\frac{\partial P}{\partial P_r} = -\frac{x_1^p (1 - \varepsilon_1^r) - k \theta x_1^p \partial x_1^r / \partial P_r}{x_1^p (1 - \varepsilon_1^p) - k \theta x_1^p \partial x_1^r / \partial P_p}
\]

Expression (18) indicates that the sign of that derivative is dependent upon the relative net effect of changes that simultaneously occur both in the cost of production and in the total revenue. In other words, a decrease in the price \(P_p\) can be allowed if the changes in the revenues and in total cost of production is sanctioned by a increase in \(P_r\); however, it may be the case that an increase in \(P_r\) will require and increase in \(P_p\) to cover the gap in costs and revenues. Then, as shown in the Appendix, assuming that the demand price elasticities are constant, the form of association between these two prices in trade-off curve with the following shapes:

a) monotonically decreasing, that is, the price that could be charged to the poor household could be as lower as that allowed by a higher price paid by the rich; mathematically, \(P_p\) could be zero as long \(P_r\) could be raised to \(\infty\). This is the case, for instance, for demand functions derived from a Cobb-Douglas utility function. Although mathematically possible, \(P_r\) could not be equal to \(\infty\) and the minimum value the price \(P_p\) can take (the value that minimizes poverty) will be higher than zero. Other cases of a decreasing curve is illustrated by cells 1, 2, 3, 8, 9, 11, 13, 14 and 15 in Table 1 in the Appendix;

b) monotonically increasing, when the negative net effect of the decrease in the revenue predominates over the cost of production and requires a rise in \(P_p\) when \(P_r\) is increased; this is the case of cells 4, 5, 6, 7, 12, and 16. For these cases, a public utility pricing policy for poverty alleviation would require a lowering of the price paid by the rich household, that is, a decrease in the price they pay would generate a net revenue that would allow a lower price to be charged to the poor.
Let us adopt a more realistic view, that is, let us assume that the household's demand for commodity 1 has a variable price elasticity and that $\frac{\partial x_1}{\partial P_1} \geq 0$, being inelastic at a lower price and very elastic at a higher prices. In this case, the $(P_{1p}, P_{1h})$ trade-off curve show an U form, $P_{1p}$ decreasing in value for an increasing value of $P_{1h}$, reaching a minimum and, after this point, increasing as $P_{1h}$ continues to increase; this case can be identified in Table 1, in the Appendix, by cells 1, 10 and 16, when $P_{1h}$ increases from a lower to an upper value.

Figure 3 illustrates the trade-off between prices $P_{1p}$ and $P_{1h}$ when the curve has an U form.

The descending section of that curve (section AC) is, as mentioned before, explained by the fact that the increased revenue generated by a higher price charged to the rich household (since its demand is assumed to be inelastic at those prices) exceeds the additional cost of producing an increased quantity of commodity 1 sold at a lower price to the poor (whose demand is elastic at that prices). Section CD in that curve shows the reverse: higher $P_{1h}$ prices are not sufficient to generate enough revenue (the rich's demand elasticity is now price elastic and the poor's is inelastic) to overcome a higher cost of production and the price $P_{1p}$ must increase to balance the public utility's accounts.
elastic at that prices). Section CD in that curve shows the reverse: higher $P_{1R}$ prices are not sufficient to generate enough revenue (the rich’s demand elasticity is now price elastic and the poor’s is inelastic) to overcome a higher cost of production and the price $P_{1p}$ must increase to balance the public utility’s accounts.

The line of 45 degrees in Figure 3 shows the points of identical prices $P_{1p}$ and $P_{1R}$. Let us assume that point B in the price trade-off curve marks the minimum price the public utility bears to charge in a system of an unique price, the price $P_1$ we derived earlier. Let us also assume that point C in the same curve shows the combination of the minimum price that can be charged to the poor ($P_{1p}$) and the respective price to be paid by the rich ($P_{1R}$) in a discriminatory price system that subsidizes the consumption of the poor, since $P_{1p} < P_1$. We can see in that figure that the arc BC is the relevant section of that trade-off curve for a discriminatory pricing policy in favour of the poor: the choice of the prices the poor and the rich should pay is constrained by the intervals ($P_1 > P_{1p} \geq P_{1R}$) and ($P_1 < P_{1R} \leq P_{1p}$).

Since the objective is to minimize poverty, the public utility should choose the minimum value the price $P_{1p}$ can take, that is, $P_{1p}$. This price is that one for which the derivative $\partial P_{1p}/\partial P_{1R}$ is equal to zero; this minimum $P_{1p}$ is reached when

$$x^R_{1} (1 - \varepsilon_{1R}) = k \theta x^D_{1} \partial x^R_{1}/\partial P_{1R}$$  \hspace{1cm} (19)

The derivation of the minimum price $P_{1p}$ and the compatible price $P_{1R}$, and the respective quantities demanded at these prices (four unknowns) requires the solution of a system of four simultaneous equations comprising expressions (17) and (19) and the two demand equations for commodity 1. Those four unknowns will be function of the exogenous variables $F$, $D$, $y_{j1}^{*}$, $P_{z}^{*}$, $h$ and $n$ and the parameters $k$, $\theta$ and $\varepsilon_{1R}$.

Having a discriminatory price system as outlined above is not a sufficient condition for solving the targeting problem: the deficit financed by the government may still be used to subsidize the price paid by the non-
One way of avoiding this problem is to restrict the choice of \( p \) among those prices equal to or greater than the marginal cost. Doing this will spare the subsidy given by the government only to those that are considered to deserve it, making the price they pay as low as possible and minimizing the number of households in poverty. Another solution would be to charge the same price (derived from expression (7), assuming that \( p = 0 \)) to both poor and non-poor households and to give vouchers to the poor (totaling the real amount of \( D \)) so they can use them to pay their public utility bills. This discriminatory price system would be revealed by the existence of two prices, the one derived from expression (6) paid by the non-poor and the smaller effective price paid by the poor.

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10. Actually, the use of a discriminatory price introduces a new type of problem, the poverty trap, a problem frequently examined in studies related to tax and social benefits systems; see, for instance, Dilnot and Star (1986) and Kanbur (1987). The poverty trap occurs for those households whose income are close to the limit at which the price increases: for those, their income is "levied" at a very tax rate, leaving the households with a net income (net of the price paid) smaller than the incomes of some of those who pay a lower price.

11. A system of non-marketable vouchers could be used to achieve both a price reduction for commodity 1 for the poor and the attainment of a target level of consumption of the subsidized commodity; in Section 6 we discuss the question of setting a price that is compatible with a minimum consumption requirement.

12. The entitlement to vouchers could be guaranteed to those households that fulfill a means-tested benefit regulation as done in the United Kingdom for social security benefits and in the United States for welfare assistance. The correct targeting in this case would be assured by the assessment of the household’s income and resources. This system, however, presents some problems: a) it has an administrative cost that should be considered and b) not all eligible poor households would claim the benefit because of the social stigmatization that means-tested programmes produce. For the importance of the welfare stigma in US and UK in this context, see the references made by Kanbur (1987, p.133).
5. MINIMIZATION OF POVERTY AND MINIMUM CONSUMPTION REQUIREMENT

In Section 3 we derived the minimum price \( P \); the public utility can charge to its consumers: this price is that allowed by the balance of its revenues and its costs. The public utility minimum price is that one that minimizes \( z \), the poverty line, and consequently, that one that minimizes poverty. In doing that we have not taken into consideration the quantity of commodity 1 a poor household would buy at that price, that is, we left aside the important aspect of defining a pricing policy that ensures the consumption by the poor household of a minimum quantity considered desirable.

We defined \( x_1^z \) and \( x_2^z \) as the quantities of commodities 1 and 2 required for the household’s survival. Let us assume that the household’s income is greater or equal to \( P x_1^z + P x_2^z \) that is, \( \gamma - P x_1^z - P x_2^z \geq 0 \). This means the household earns enough money for consuming at least the required survival quantities. In order to make this assumption to prevail, the maximum value the price of commodity 1 can take (let us call it \( P_1^* \)) is

\[
P_1^* = \frac{\gamma - P x_2^z}{x_1^z}.
\]

If commodity 1’s price is equal to \( P_1^* \), the household’s consumption of it will be equal to the survival quantity; if less than \( P_1^* \), the household will consume a larger quantity of commodity 1 or of commodity 2 than those survival quantities, or consume more of both commodities.

Let us examine how \( P_1^* \) effects the pricing policy in discussion in this section, that is a policy that intends to reduce poverty. First, we are going to examine its influence when an unique price is used by the public utility and secondly, in the context of discriminatory prices:

A - Unique price system

Expression (16) gave us the minimum price (\( P \)) the public utility can charge for commodity 1 and the above expression tells us the maximum price (\( P_1^* \)) that should be charged to guarantee the consumption \( x_1^z \) at least the survival quantity. The comparison of both prices brings about one of the following outcomes:
a) \( P_1 > P^*_1 \), and the short-run solution for this incompatibility is to increase \( \delta \), the subsidy given by the government, otherwise some households would not consume commodity 1 in the quantity considered desirable for their survival;

b) \( P_1 \leq P^*_1 \), then the consumption requirement will be satisfied by all households and in this sense the constraint is redundant.

B - Discriminatory price system

In Section 4 we examined the case of setting a discriminatory price system, that is, the public utility charging a lower price \( P_1^* \) to poor households in order to minimize the poverty level and a higher price \( P_1^\star \) to rich consumers. We saw that these two prices are interdependent, given the public utility constraint of balancing its costs and revenues; we also saw that there is a range of possible prices \( P_1^* \) to choose from; let us assume that this range is the interval \( [P_1^{l*} - P_1^{h*}] \), for the lowest and the highest values for \( P_1^* \). For the purpose of minimizing poverty the choice should be the lowest value \( (P_1^{l*}) \) when there is no constraint in terms of the quantity \( X_1^* \) of commodity 1 the households consume. However, the price choice should be confronted with price we derived above, \( P_1 \), the price that allows the poor household to consume at least the quantity \( X_1^* \), if a minimum consumption requirement must be observed.

Comparing the price that assures the consumption of the minimum requirement (the survival quantity \( X_1^* \)) with those in the interval \( [P_1^{l*} - P_1^{h*}] \), our analysis has to take into consideration the three following forms the price trade-off curve may take:

B.1 - The price trade-off curve has a U form: ¹³

a) \( P_1^* \) is inside that interval, that is, it is one of those prices; then there is no incompatibility between the price the poor household can pay to consume at least \( X_1^* \) and the price the public utility can charge it. The policy of minimizing poverty would prescribe choosing the price \( P_1^* \) since this price is smaller or equal to price \( P_1^* \), what satisfy both the public utility financial balance and the household budget constraint

¹³In this case we are referring to Figure 3 and to the intervals \([P_1 - P_1^*]\) for \( P_1^* \) and \([P_1 - P_1^\star]\) for \( P_1^\star \).
that allows its consumption of at least the survival quantity of commodity 1;

b) $P^*_{i}$ is greater than $P^{H}_{1P}$; this means that all those prices that satisfy the public utility financial balance constraint are compatible with the household affordability of consuming at least $X^*_i$ and there will be no problem in choosing the lowest one of $P^*_{1P}$ to minimize poverty;

c) $P^*_{i}$ is lower than $P^{L}_{1P}$, what means that the prices allowed by the financial balance constraint are higher than the price the poor household can pay to consume the survival quantity $X^*_i$; in case of this incompatibility the only solution in the short-run is an increased government subsidy to allow the poor to pay the price $P^*_{i}$.

B.2 - The price trade-off curve is monotonically decreasing from the left to the right, with $P^*_{1P}$ tending assintotically to $P^{L}_{1P}$, for $P^{L}_{1P} > 0$:

a) If $P^*_{i}$ is greater than that lower limit, it is possible to choose a price that satisfy both the intention of allowing the poor household to consume at least the survival quantity of commodity 1 and the minimization of the poverty level;

b) If $P^*_{i}$ is lower than $P^{L}_{1P}$, there is no price for commodity 1 that can satisfy both the public utility financial constraint and the minimum consumption requirement: the lowest possible price is too high to allow the poor household to buy the quantity $X^*_i$; the short-run solution for this conflict is a higher subsidies given by the government.

B.3 - The price trade-off curve is the same mentioned in B.2 but its lowest limit is zero, that is, $P^{L}_{1P} = 0$;

If this is the case, it is possible to decrease the price of commodity 1 to be charged to a poor household as low as wanted, not only to minimize poverty, but also to allow the household to have access to the quantity $X^*_i$; although this a theoretical possibility, one should worry with the political feasibility of such a low price to be paid by the poor if it implies a too high price to be charged to the rich.\footnote{Actually, this same consideration must be made for case B.2.a.}

As we saw, when an incompatibility of prices happens in any of the two systems (unique price or discriminatory prices) for minimizing poverty, the possibility of
adjustment rests upon the feasibility of an increased transference of resources to finance the subsidy given to the tariffs charged by a public utility. This is the only instrument that can be used in the short-run to solve this kind of conflict; other solutions, such as investments and labour training programmes to lower production costs by increasing the public utility's productivity, are also possible, although demanding a long-run horizon to produce their effects.

6. EFFECTS OF THE POPULATION GROWTH ON POVERTY MINIMIZING PRICES

It is important to note that the U curve depicted in Figure 3 refers to the given numbers of poor and rich households implicit in expression (17). Since a growing of poor households is a common phenomenon observed in large urban centres in countries of the Third World, it is important to examine the consequences it brings to a pricing policy that intends to lower the level of poverty in these countries.

Assuming the poor is paying the subsidized price \( P'_{IP} \), shown in Figure 3, an increased number of poor households means that someone should be called to finance the required additional sum of total subsidy. It can be financed either by cuts made in other government expenditures or by the taxpayer, through additional taxation. It can also be financed by the households themselves, paying higher prices. It should be noted that both groups of households, the poor and the rich, will be affected in this case: charging a higher price only to the rich is not sufficient since \( P'_{IP} \) is the highest price it can pay without generating a smaller total revenue; then, the poor will also be called to contribute, paying also a higher price to complement the required public utility's total revenue.

In the case of the subsidized price \( P^* \) to allow the consumption of a minimum quantity of the commodity, the situation is similar to that just seen. The growing number of poor households will certainly be paying this lower price and the deficit constraint will be affected. The solutions in the short-run are either expanding \( D \) or increasing the prices to consumers or both. In this last case, it is possible that even the poor will be affected since, as we mentioned before, the additional revenue obtained from the rich could not be enough to cover costs.

The accelerated population growth seen in urban centres of Third World countries causes another type of problem to public utilities, with consequences on its prices:
capacity of production is reached more rapidly and funds are required to expand this capacity. This may mean that the discriminatory prices examined in this paper should be reexamined to allow the additional constraint of generating enough financial resources to pay for the costs of the expansion. Additional analysis is required to examine how these funds should be generated by poor and non-poor households through higher tariffs in case the government decides users should bear the full costs of the capacity expansion.

7. CONCLUSIONS

This paper shows that it is possible to set public utilities prices to attend objectives of minimization of poverty. However, the choice of the price to be paid by the poor is conditioned by several constraints. First of all, the amount of resources the government is inclined to transfer to the public utility: of course, if the government decides to finance the whole cost of production, the public utility could distribute its production free of charge and the level of poverty would be minimized. Since this is not the efficient way of allocating resources, we should expect users to pay a positive price for the services provided by these public utilities. Other factors, such as cost considerations and the possibility of the non-poor paying a higher price to cross-subsidize the poor, are constraints to be observed by a pricing policy that intends to minimize poverty; the level of price subsidy given to the poor and, consequently, the level at which poverty is alleviated depends on how public utilities can combine the rates. Finally, the growing number of poor puts another limitation on the level of subsidy that can be offered to them: cross-subsidization limits by the non-poor and the need to expand the public utility's capacity of production may require a higher price to the charged to the poor.
APPENDIX

ANALYSIS OF THE FUNCTION THAT RELATES $P_{1P}$ TO $P_{1R}$

We saw that prices $P_{1P}$ and $P_{1R}$ are interrelated and that the expression that shows this relationship is:

$$P_{1P} X_{1P}^p = F - D + k (X_{1P}^p) \theta - P_{1R} X_{1R}^r$$  

(1)

We can study the form of this relationship by analyzing the sign of $\partial P_{1P}/\partial P_{1R}$. Before calculating that derivative, let us calculate $\partial(P_{1P} X_{1P}^p)/\partial P_{1P}^{1P}$, $\partial X_{1P}^p/\partial P_{1R}^{1P}$, and $\partial(P_{1P} X_{1P}^p)/\partial P_{1R}^{1R}$ as intermediary steps:

$$\frac{\partial(P_{1P} X_{1P}^p)}{\partial P_{1R}^{1P}} = P_{1P} \frac{\partial X_{1P}^p}{\partial P_{1P}^{1P}} + P_{1P} \frac{\partial P_{1P}}{\partial P_{1R}^{1R}}$$  

(2)

or dividing and multiplying it by $X_{1P}^p$:

$$\frac{\partial(P_{1P} X_{1P}^p)}{\partial P_{1R}^{1R}} = \frac{\partial P_{1P}}{\partial P_{1R}^{1R}} \left[ 1 - \frac{\partial P_{1P}}{\partial P_{1R}^{1R}} \right]$$  

(3)

where $\epsilon_{1P} = \frac{P_{1P}}{X_{1P}^p} \cdot \frac{\partial X_{1P}^p}{\partial P_{1P}^{1P}}$, the poor household's demand price elasticity for commodity 1.

By definition, $X_{1P} = X_{1P}^p + X_{1P}^r$; then,

$$\frac{\partial X_{1R}}{\partial P_{1R}^{1R}} = \frac{\partial (X_{1P}^p + X_{1R}^r)}{\partial P_{1R}^{1R}} = \frac{\partial X_{1P}^p}{\partial P_{1R}^{1R}} + \frac{\partial X_{1R}^r}{\partial P_{1R}^{1R}}$$  

(4)

and

$$\frac{\partial(P_{1R} X_{1R}^r)}{\partial P_{1R}^{1R}} = X_{1R}^r + \frac{\partial X_{1R}^r}{\partial P_{1R}^{1R}} = X_{1R}^r \left( 1 - \epsilon_{1R} \right)$$  

(5)

where $\epsilon_{1R}$ is the rich household's demand price elasticity for commodity 1.
Since \( \frac{\partial (x_1^p)}{\partial P_1^p} = \theta (x_1^p)^{\theta - 1} \frac{\partial x_1^p}{\partial P_1^p} \), we can now use the above intermediary results to express \( \frac{\partial P_1^p}{\partial P_1^r} \) as

\[
\frac{\partial P_1^p}{\partial P_1^r} = - \frac{x_1^p (1 - \varepsilon_1^p) - k \theta x_1^p \theta - 1}{x_1^p (1 - \varepsilon_1^p)} \frac{\partial x_1^p}{\partial P_1^r} \frac{\partial x_1^p}{\partial P_1^r}
\]

(6)

The derivatives that appear in the numerator and in the denominator of expression (6) are negative since commodity 1 is assumed to be a normal good for both the poor and the rich households. To simplify the analysis of the sign of that expression, let us write it as

\[
\frac{\partial P_1^p}{\partial P_1^r} = - \frac{a + b}{c + d} \quad \text{b} > 0 \text{ and } d > 0
\]

Table 1 lists the signs this derivative can take for selected values for the price elasticities.

<table>
<thead>
<tr>
<th>VALUES FOR ( \varepsilon_1^p ) AND ( \varepsilon_1^r )</th>
<th>( a &gt; 0 ) ( (0 &lt; \varepsilon_1^r &lt; 1) )</th>
<th>( a = 0 ) ( (\varepsilon_1^r = 1) )</th>
<th>( a &lt; 0 ) ( (\varepsilon_1^r &gt; 1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c &lt; 0 ) ( (\varepsilon_1^p &gt; 1) )</td>
<td>positive denominator</td>
<td>( 1 )</td>
<td>( 2 )</td>
</tr>
<tr>
<td>negative denominator</td>
<td>( 5 )</td>
<td>( 6 )</td>
<td>( 7 )</td>
</tr>
<tr>
<td>( c = 0 ) ( (\varepsilon_1^p = 1) )</td>
<td>( 9 )</td>
<td>( 10 )</td>
<td>( 11 )</td>
</tr>
<tr>
<td>( c &gt; 0 ) ( (0 &lt; \varepsilon_1^p &lt; 1) )</td>
<td>( 13 )</td>
<td>( 14 )</td>
<td>( 15 )</td>
</tr>
</tbody>
</table>

(\#) for expression(5) or its equivalent \( \frac{\partial P_1^p}{\partial P_1^r} = \frac{a + b}{c + d} \)
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