TEXTO PARA DISCUSSÃO Nº 537

LONG-RUN DETERMINANTS OF THE REAL EXCHANGE RATE: BRAZIL — 1947/95*

Antonio Fiorencio**
Ajax R. B. Moreira***

Rio de Janeiro, dezembro de 1997

* Ingreed Valdez provided research assistance to the project. CNPq and Pronex provided financial support.
** Da UFF, do IBMEC e bolsista da Anpec/PNPE na Diretoria de Pesquisa do IPEA.
*** Da Diretoria de Pesquisa do IPEA.

Instituto de Pesquisa Econômica Aplicada

O IPEA é uma fundação pública vinculada ao Ministério do Planejamento e Orçamento, cujas finalidades são: auxiliar o ministro na elaboração e no acompanhamento da política econômica e prover atividades de pesquisa econômica aplicada nas áreas fiscal, financeira, externa e de desenvolvimento setorial.

TEXTO PARA DISCUSSÃO tem o objetivo de divulgar resultados de estudos desenvolvidos direta ou indiretamente pelo IPEA, bem como trabalhos considerados de relevância para disseminação pelo Instituto, para informar profissionais especializados e colher sugestões.

ISSN 1415-4765

SERVIÇO EDITORIAL

Rio de Janeiro – RJ
Av. Presidente Antônio Carlos, 51 – 14º andar – CEP 20020-010
Telefax: (021) 220-5533
E-mail: editrj@ipea.gov.br

Brasília – DF
SBS Q. 1 Bl. J, Ed. BNDES – 10º andar – CEP 70076-900
Telefax: (061) 315-5314
E-mail: editbsb@ipea.gov.br

SUMÁRIO

ABSTRACT

This paper presents a model for the long-run determinants of the Brazilian real exchange rate for the period 1947/95. This is a simple representative agent model that links the exchange rate, external debt and net exports. It is assumed that: **a**) the country pays an interest rate on its debt which is an increasing function of the debt/GDP ratio; **b**) the **real** exchange rate is a control variable. The transitional dynamics of the model following different shocks is analysed.

The model suggests that the relevant variables are the **real** exchange rate, external debt and net exports. A VEC model using these variables shows that the Brazilian data support the existence of one cointegrating relation among the three variables, which we interpret as the empirical counterpart of the long-run conditions of the theoretical model. Finally, we impose restrictions to identify shocks that could be interpreted as the non-observable exogenous variables of the theoretical model. The dynamics of the empirical model is estimated.

1 - INTRODUCTION

This paper tries to identify some of the determinants of the long-run Brazilian real exchange rate for the period 1947/95.

"Long-run exchange rates" is often taken as a synonym for Purchasing Power Parity. But academic confidence on PPP as a theory of exchange rate determination has waned and waxed during the last decades.[1] By the end of the 1970s, the predominant view was that, with the possible exclusion of high inflation countries, PPP did not hold in the short-run and might not hold in the long run either. During the 1980s, some researchers actually started finding out they could not reject the hypothesis that the real exchange rate followed a random walk — and, thus, that PPP did not hold even in the long-run. These results were not robust and, as data sets covering several decades began to be used, results turned towards the conclusion of a slow mean reversion, say, with a half-life of about three to four years. By the beginning of the 1990s, cointegration techniques started being applied.[2] Current results tend to reject both continuous PPP and the random-walk hypothesis in favour of long-run converge either to standard PPP (in high inflation countries), or to some sort of "modified" PPP — one allowing for capital flows, productivity changes **etc**.[3]

This paper is in the spirit of this most recent literature. In the first part of the paper, we present a simple model to analyse the long-run determinants of the exchange rate. It is a representative agent (Ramsey) model that links the exchange rate, external debt and net exports. It is assumed that: **a**) the country produces a fixed quantity of only one good; **b**) the **real** exchange rate determines how much of this good will be exported; **c**) the country has access to international capital markets; and **d**) pays an interest rate on its debt which is an increasing function of the debt/GDP ratio. The steady-state and the transitional dynamics of the model following different shocks are analysed. This simple model is used as a somewhat informal motivation for the empirical analysis.

[1] This paragraph draws on the survey by Froot and Rogoff (1994) on long-run real exchange rates. See also the surveys by Frankel and Rose (1994), MacDonald (1995) and MacDonald and Taylor (1992).

[2] Recent studies using cointegration include: Corbae and Ouliaris (1988), Cheung and Lai (1993), Faruque (1995), Kugler and Lenz (1993), MacDonald (1995), McNown and Wallace (1994), Rossi (1991), Pereira and Duarte (1991), Zini and Cati (1993). Engle (1996) argues that results could be biased towards rejecting the null of a unit root.

[3] "So, what the tests for long-run PPP probably can tell us is that there is some sort of 'mean-reversion' in real exchange rates. What is not yet clear is whether there is convergence to PPP in the long run." [Engle (1996, p. 23)]. "Our summary of the battery of tests that have been used to test for the existence of PPP supports the view that **continuous** PPP has not held for the recent floating period, while the evidence in favour of **long-run** convergence of real exchange rates toward PPP is at present mixed." [MacDonald and Taylor (1992, p. 42)]. "Over the past ten years, research on purchasing power parity has enjoyed a rebirth (...) The main positive result is that there does seem to be long-run convergence to PPP (...)" [Froot and Rogoff (1994, p. 39)].

The second part of the paper presents an empirical analysis of the theoretical model, using annual Brazilian data for 1947/95. The economic model suggests that the relevant variables for the empirical analysis are the real exchange rate, external debt and net exports, both as GDP ratio. How we are interested on the determinants of the real exchange rate and it is difficult to accept that these determinants are exogenous we adopt the structural VAR approach.

The empirical results are mixed. We find that the debt to GDP ratio consistently helps to forecast the real exchange rate. Also, we are not able to reject a cointegration relation among the debt to GDP ratio, the net exports to GDP ratio and the real exchange rate. We interpret this cointegration relation as the empirical counterpart of the long-run solvency restriction of the economic model. This result, however, is not robust, and we also present the results of a model with no cointegration. When they are comparable, both sets of results tend to be qualitatively similar. Finally, we impose identifying restrictions on both models and analyse the impulse response functions.

2 - THE THEORETICAL MODEL

We assume an open economy with a fixed exogenous endowment of only one internationally traded good that may be either consumed, exported or imported. The representative agent has a standard intertemporal utility function that depends only on this good; thus no other imports occur. Foreigners have access to more goods, and, given the exchange rate, they choose between imports and their domestic goods.

Agents have access to international capital markets, but must pay a spread over the international interest rate that depends positively on the country's debt/GDP ratio This is a short-cut to represent the fact that more indebted countries are more likely to default on debt and thus will have to pay a higher interest rate, and can be regarded as a parametric solution of the supply credit agent problem.

Thus, the real exchange rate determines net exports and matters for domestic agents only insofar as it is a means of transferring purchasing power through time.

Finally, we assume that the country can choose its **real** exchange rate, provided it has access to international capital markets. Of course, countries do not directly control their real exchange rates. But they can control the nominal exchange rate if they have access to international capital markets. And they can control inflation, or the price level, if they are willing to adopt the necessary monetary/fiscal policies. So, we are assuming that the country adopts a set of economic policies that allows it to (indirectly) control its real exchange rate. Access to international capital markets is crucial, of course. To garnet this access, countries can not choose paths of the real exchange rate that will lead to explosive debt, since, in this case, they would find no willing lenders. Provided this long-run solvency condition is met, the real exchange rate will be treated as a control variable.[4]

This specification of the model was suggested by the data. Preliminary analysis shows that the external debt and some measure of demand and/or supply of foreign currency will have explanatory power for the exchange rate.[5] Also, reliable data for many potentially important variables are likely to be annual and not longer than about 50 years. This means that the theoretical model that will support the empirical work must be very parsimonious.

Given these assumptions, the agent's problem is:[6]

$$\max_{\{s(t)\}} \int_0^\infty u(c(t)).e^{-\rho.t}.dt \quad , u'>0, u''<0 \qquad (2.1)$$

subject to:

$$c(t)=1-x(t) \qquad (2.2)$$

$$\dot{b}_t = r(t).b(t) - x(t) \qquad (2.3)$$

$$r(t) = \chi.h(b(t)) \qquad , h'>0 , h''>0^7 \qquad (2.4)$$

$$x(t) = \gamma + g(s(t)) \qquad , g'>0 \qquad (2.5)$$

$$\lim_{t\to\infty} \left\{ b(t).\exp\left[-\int_0^t r(v).dv\right]\right\} \geq 0 \qquad (2.6)$$

(2.1) shows that utility depends on the path of consumption (c(t)) discounted at rate ρ. In (2.2), consumption GDP ratio is the residual between (exogenous) production (y=1) and net exports ratio (x(t)). (2.3) is the dynamical budget constraint relating increases in debt ratio (b(t)) to interest payments and net exports ratio. In (2.4), the interest rate on debt ratio (r(t)) is an increasing country-risk function of the debt ratio adjusted by a exogenous coefficient (χ). (2.5) shows

---

4 "This paper contends that the real exchange rate is a variable that is significantly influenced by policy, not in all cases (...), but in many. (...) To interpret the real exchange rate as a policy-influenced or even policy-dominated variable, the disequilibrium view starts from the proposition that there is some degree of inertia in the level or rate of change of wages and prices." [Dornbusch, Goldfajn and Valdés (1995, p.250-251)].

5 Explanatory power in the sense that these variables will precede exchange rate in the short- or long-run .

6 Consumption, net exports and debt are divided by GDP.

7 In this formulation, a change in χ can also be due to changes in international interest rates. This expression can be formulated as $r(t) = r^*(t) + \chi.h(b(t))$ where $(r^*(t))$ is an international interest rate and h(.) is the spread. In this formulation χ would represent only changes in country risk. Both formulations are equivalent.

that net exports ratio depends on exogenous factors (γ) and on the real exchange rate (s(t)). (2.6) is the transversality condition ruling out explosive debt ratio.

The Hamiltonian for this problem is:

$$H(t) = u\left(1-\gamma-g(s(t))\right)+\mu(t).\left[\left(\chi.h(b(t))\right).b(t)-\gamma-g(s(t))\right] \qquad (2.7)$$

The first order conditions are:

$$\frac{\partial H(t)}{\partial s} = -u'(.)g'(.)-\mu(t)g'(.) = -u'(.)-\mu(t)=0 \quad \Rightarrow$$

$$u'\left(1-\gamma-g(s(t))\right) = -\mu(t) \qquad (2.8)$$

$$\frac{\partial H(t)}{\partial b} = \mu(t).\left[\chi.h(b(t))+b(t).\chi.h'(b(t))\right] = \rho.\mu(t)-\dot{\mu}(t) \quad \Rightarrow$$

$$\dot{\mu}(t) = \mu(t).\left[\rho-\chi.\left(h\left(b(t)\right)+b(t).h'\left(b(t)\right)\right)\right] \qquad (2.9)$$

$$\lim_{t\to\infty} \quad \mu(t)b(t)e^{-\rho t} \geq 0 \qquad (2.10)$$

For later reference, note that (2.8) implies that:

$$\frac{\partial s}{\partial \mu} = \frac{1}{u''.g'} < 0 \qquad (2.11)$$

and that the derivative of the term in brackets in (2.9) is:

$$\frac{\partial}{\partial b}\left[h\left(b(t)\right)+b(t).h'\left(b(t)\right)\right] = 2.h'\left(b(t)\right)+b(t).h''\left(b(t)\right) > 0^{8} \qquad (2.12)$$

Setting (2.3) and (2.9) equal to zero gives the steady-state:

$$\dot{b}(t) = 0 \quad \Rightarrow \quad \left[\chi.h\left(b(t)\right)\right].b(t)-\gamma = g\left(s(t)\right) \qquad (2.13)$$

[8] The derivative is positive provided debt is not “too” negative, which we assume henceforth. If this condition does not hold, equilibrium will be globally unstable.

$$\dot{\mu}(t) = 0 \quad \Rightarrow \quad h\big(b(t)\big) + b(t).h'\big(b(t)\big) = \rho / \chi \qquad (2.14)$$

(2.14) is a vertical line on the (s, b) plane. The higher is impatience (ρ), or the lower is country-risk (χ), the higher will be the steady-state debt. If interest rates are constant, h´(.)=0, this expression is not defined, and we do not have a interior solution.

(2.13) is positively sloped on the (s, b) plane, if debt is not “too” negative:[9]

$$\left.\frac{ds}{db}\right|_{\dot{b}=0} = \frac{\chi.\big(h(b(t)) + b(t).h'(b(t))\big)}{g'(s)} > 0 \qquad (2.15)$$

For a given level of debt, the lower is exogenous country-risk factor (χ), or exogenous exports (γ), the more appreciated will be the steady-state exchange rate.

2.1 - Equilibrium Conditions

The ($\dot{b}$) curve represents equation (2.13), that is, combinations of debt and the real exchange rate that keep debt constant. It is positively sloped since an increase in debt raises interest payments for two reasons. First, at a given interest rate, higher debt means higher payments. Second, an increase in debt GDP ratio is assumed to increase country risk and to raise the interest rate itself. The increase in interest payments requires more exports and thus a higher exchange rate to keep debt constant.

The ($\dot{s}$) curve represents equations (2.14) and (2.11), combinations of debt and exchange rate that keep the real exchange rate constant. The reason it is vertical is the following. Suppose, first, that the interest rate did not depend on debt. Then the path of the marginal disutility of debt ($\mu(t)$) would only depend on the ratio between the rate of time preference (ρ) and the country-risk (χ). We have, however, assumed that the “effective” interest rate also depends on the debt/GDP ratio, but it does not depend on exports — and, hence, does not depend on the exchange-rate. Thus, the curve is vertical.

To the right of the ($\dot{s}$) curve the effective interest rate is lower than the rate of time preference; (2.9) shows that (μ) is rising and, thus, according to (2.11), the

[9] Even if it exists, the negatively sloped portion of (2.13) is of no interest: the steady-state will always lie on the positively sloped region, as comparison of (2.14) and (2.15) shows.

exchange rate is falling.[10] Above the ($\dot{b}$) curve, exports are high and debt is falling — see (2.3). The equilibrium is a saddle-path, represented by the curve labelled XX.

Figure I illustrates the behaviour of this model economy.

2.2 - Transitional Dynamics[11]

An exogenous reduction in country risk ($\Delta\chi<0$) — Equations (2.13) and (2.14) show that the ($\dot{s}$) curve moves to the right and the ($\dot{b}$) curve moves downwards, in Figure II. The steady-state changes from point A to point C. The saddle-path associated with point C is shown as the dotted line labelled XX. The exchange rate appreciates on impact, falling from A to B, then it continuously depreciates along XX. In the new steady-state, debt is higher than in the original one, since the interest rate is lower. The new steady-state exchange rate may be more appreciated (since the interest rate fell) or more depreciated (since debt increased) than the previous one.

Even though this is a real model and we can not analyse such matters as inflation and stabilization, it is interesting to informally compare this result with one stylised fact of exchange rate based stabilizations in high inflation countries.

[10] Remember (2.8) implies $\mu < 0$. If the derivative in (2.12) were negative, the dynamics would be reversed and the equilibrium would be globally unstable.

[11] All exercises in this section refer to Figure II.

It has been suggested that exchange rate based stabilizations lead to an initial appreciation followed by a depreciation of the exchange rate. Of course, the real exchange rate appreciation that follows stabilization is a consequence of the fact that the nominal exchange rate is frozen — or nearly so — and domestic inflation does not instantaneously drop to zero. This initial appreciation produces endless dispute on whether the exchange rate is "overvalued" or not. Defendants of the stabilization program will usually say that stabilization brings forth productivity gains that justify the appreciation. If this were true, however, the exchange rate should not depreciate later on during the program, as it seems to do so often.

The point we wish to make is that this model suggests another reason why the exchange rate should initially appreciate — and then depreciate. It may be argued that a stabilization program that produces a sharp drop in inflation may reduce economic uncertainty and, thus, country risk. If this is true, the exchange rate will react to the stabilization program in the way just described.

An exogenous stimulus to net exports ($\Delta\gamma > 0$) — Equations (2.13) and (2.14) show that the $(\dot{s})$ curve does not move and the $(\dot{b})$ curve moves downwards in reaction to an increase in net exports. The exchange rate immediately appreciates to its new steady-state value, falling from A to D. Since debt and interest rates are unchanged, exports also do not change. The currency appreciates exactly enough to cancel the exogenous stimulus. There is no transitional dynamics in this case.

An increase in impatience ($\Delta\rho>0$) — A change in the intertemporal rate of preference does not affect the dynamic budget constraint and, thus, the $(\dot{b})$ curve does not move. The $(\dot{s})$ curve will move to the right, since more impatient agents will have more debt in equilibrium. The exchange rate appreciates on impact, thus allowing individuals to immediately consume more. It then continuously depreciates along the saddle-path that leads to point E (not shown). In the new steady-state debt is higher, the exchange rate is more depreciated and consumption is lower than in the original position.

3 - EMPIRICAL ANALYSIS

The previous section presented a simplified economic model to discuss the determinants of the long-run real exchange rate. This section applies the VEC approach to confront the results of the economic model with Brazilian data for 1947/95. Before presenting the results of the empirical analyses, it may be useful to consider the relation between the economic and the econometric models employed.

The economic problem we are discussing is, of course, a stochastic problem, but we have used a deterministic model for the sake of simplicity. The cost of the simplification is that the correspondence between the non-linear deterministic economic model and the linear stochastic empirical model can only be informal.[12] To move from the economic to the econometric model note that:

a) the theoretical model suggests the variables to include in the empirical model and the identification of its structural form — i.e., of the exogenous shocks;

b) quantity variables are divided by GDP to be consistent with the theoretical model that assumes constant domestic production;

c) the impatience parameter (ρ) is taken as a constant and will be ignored in what follows;

d) we will assume that the two other parameters, (χ, γ), correspond to the exogenous shocks of the structural VAR/VEC model. Thus, on the one hand, we associate changes in (χ, γ), to changes in the steady-state and the short-run dynamics of the economic model. And, on the other hand, we associate (χ, γ) to the non-observable stochastic variables (structural shocks) that have permanent effects in the VEC model;

e) at each instant t, the economic model forecasts the paths of the endogenous variables, $z_{t+h}=(b_{t+h},s_{t+h} \mid \chi,\gamma)$ given the exogenous variables. These paths could be compared to the impulse-response functions of the VAR/VEC model; and

f) as (χ, γ) are not observable, we investigate if observable **proxy** variables could

[12] Besides linearisation, the empirical model also assumes away the second order terms — related to uncertainty — that would result from a theoretical model that explicitly allowed for stochastic variables.

incorporate more information to the model. We consider international interest rates, net exports/GDP, and exports/GDP.[13]

3.1 - Estimation

The data set consists of:[14] **a**) GDP (y);[15] **b**) net external debt (B);[16] **c**) the real exchange rate (s);[17] **d**) an international interest rate, (r);[18] and **e**) either exports of goods (exp)[19] or net exports of goods and services, (expl).[20] Debt, exports and net exports are normalised by GDP:[21] (b=B/y), (x=expl/y), (γ=exp/y),[22] respectively. All the variables in logs. Which variables in this preliminary list should be included in the preferred model is an empirical question that will be assessed by checking whether they incorporate information for the forecasts of z=(b,s).

Table 1 shows that: (s,b) does not lead net exports (x); the interest rate (r) does not lead (s,b); exports (x) and net exports (γ) leads (s,b); and (s,b) lead exports (x). Even when the variables are integrated, not rejection of the null is sufficient[23] to not reject that it does not add information to the model, and eliminate the interest rate (r). We will consider three alternative formulations: $u_t=(b_t,s_t)'$, $v_t=(b_t,s_t,x_t)'$, or $w_t=(b_t,s_t,\gamma_t)'$.[24]

[13] The inclusion of international interest rates and net exports/GDP is directly suggested by the economic model — see footnote 7. The reason to consider exports/GDP is that we use this variable as a proxy to net autonomous exports/GDP, since Table 1 shows that exports/GDP does not depend on the exchange rate. The finding that the exchange rate does not lead exports will be commented upon later on.
[14] Data up to 1970 is from "Séries Históricas para o Brasil", IBGE. After 1970, data come from various sources. Data is annual for 1947/95.
[15] After 1980: IBGE, Contas Nacionais.
[16] Net external debt is the difference between registered and non-registered external debt and international reserves (international liquidity concept) deflated by USA WPI. Source: BACEN. Up to 1974 reserves were calculated integrating the balance of payments results.
[17] Before 1974 the real exchange rate is the annual rate and after 1974 it is the average of the monthly rates. The real exchange rate is the nominal exchange rate times the USA WPI divided by the Brazilian GDP deflator.
[18] Prime Rate deflated by the USA CPI. Source: FED data bank
[19] Exports of goods, deflated by the USA WPI, includes net unilateral transfers.
[20] Exports less Imports of goods and non factor services, deflated by the USA WPI.
[21] The theoretical model assumes that GDP does not depend on the external sector variables. A partial test, focusing on precedence, did not reject this hypothesis (H_0:B(L)=0) for $pib_t=A(L)pib_{t-1}+B(L)(c,exp,B,r)_{t-1}$.
[22] We abuse notation here. See footnote below.
[23] In this case, even if we correct the test for the nuisance parameter we would not reject the null. Todda and Phillips (1994).
[24] In model v=(b,s,x)´, γ represents the (non-observable) shock to net exports. Abusing notation, in model w=(b,s,γ)´, γ represents both the observable part of autonomous exports and the non-observable shock to it.

Table 1
P-Value (%) of preliminary tests

Alternative Models	**(H_0:B(L)=0)**
$x_t = A(L)x_{t-1} + B(L)(s,b)_{t-1}$	0
$\gamma_t = A(L)\gamma_{t-1} + B(L)(s,b)_{t-1}$	26
$z_t = A(L)z_{t-1} + B(L)r_t$	68
$z_t = A(L)z_{t-1} + B(L)x_{t-1}$	1
$z_t = A(L)z_{t-1} + B(L)\gamma_{t-1}$	2

The steady-state conditions of the theoretical model can be associated with the long-run equilibrium concept of VEC models, (3.1).

$$\Delta z_t = B(L)\,\Delta z_{t-1} + \alpha\beta' z_{t-1} + \varepsilon_t \;,\; \varepsilon_t \approx N(0,\Sigma) \qquad (3.1)$$

The three versions of the model (u, w, v) have two shocks with permanent effects: shocks to country-risk and to autonomous exports. The u=(b,s)' version has two variables and, thus, we expect no cointegrating relation. The w=(b,s,γ)' and v=(b,s,x)' versions have three variables, and, thus, we expect one cointegrating relation.

We comment briefly on the results of model (v).[25] Testing for cointegration,[26] we could not reject (at 95%) the hypothesis of no cointegration relation (r=0). A less demanding criterion would allow us to reject (r=0) and not to reject (r=1), but the coefficients of the cointegrating relation would not be sensible.[27] Thus, we rejected this model.

Let's turn to models (u) and (w). The number of lags was chosen so as to minimise the Hanna-Quin (H-Q) criterion, and to eliminate serial correlation of the residuals. Table 2 shows the results.

[25] Since this model was rejected, we do not present the results of the tests to save on space.
[26] This is always the Johansen (1991) methodology, and the program is PCFIML.
[27] The cointegration relation is (β=-c+.17x-.49dl). To identify the permanent shocks, the vector of long-run effects (ξ) must be orthogonal to the cointegration relation (β). One of these permanent shocks must be the exports shock that, as the theoretical model indicates, does not affect the long run debt. The orthogonal condition and this property would imply that an export increase will result in an exchange rate devaluation, which is not reasonable.

Table 2
Number of Lags

Lags	u/H-Q	u/AR	w/H-Q	w/AR
2	-11.26	1	-6.915	0.3
3	-11.57*	43	-7.184	0.3
4	-11.25	20	-7.106*	9.8

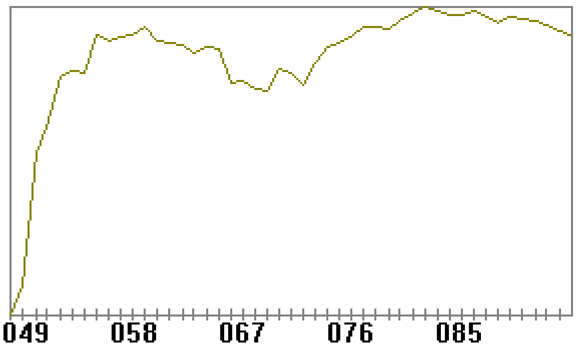
Testing for cointegration, we could not reject (r=0) for model (u) and (r=1) for model (w), as expected (see Table 3).

Table 3
Number of Cointegrating Relations

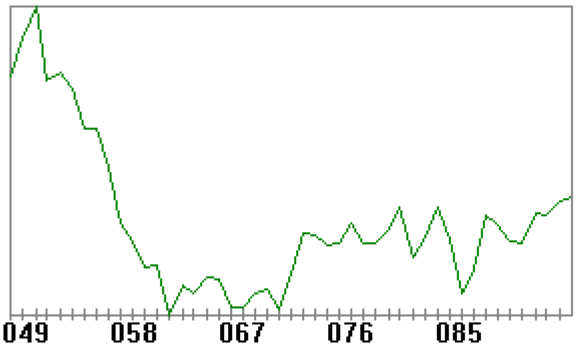
Model		Largest Eigenvalue	Critical Values (95%)	Trace	Critical Value (95%)
u	**0**	4.18	14.1	5.7	15.4
	1	1.5	3.8	1.8	3.8
w	**0**	51.0	21.0	61.1	29.7
	1	8.8	14.1	10.0	15.4
	2	1.2	3.8	1.2	3.8

The graphics of the variables and of the cointegration relation for model (w) are shown next.

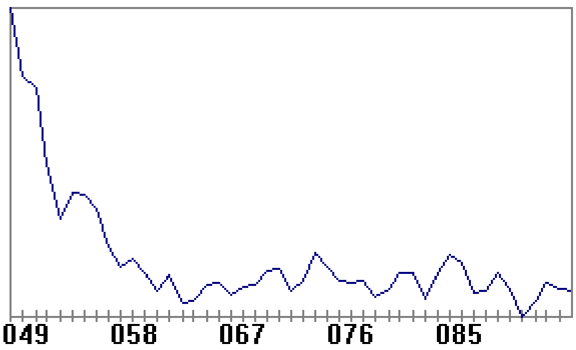
Net debt/GDP



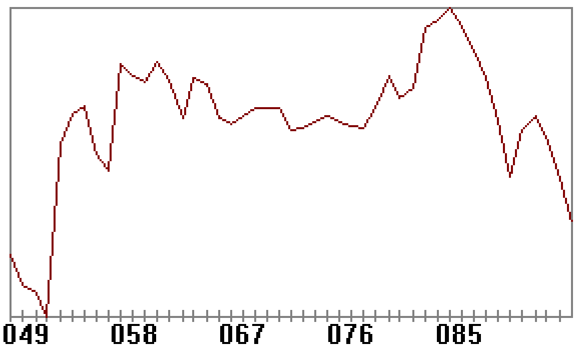
Exports/GDP



Cointegration Relation



Exchange rate



For model (w) we found one cointegration relation (β=-c-1.334γ-.549dl), and this result is robust concerning marginal changes in the data set.[28] We checked the statistical significance of the long-run elements of the VEC model — (α) and (β)

[28] The data set was reduced by eliminating up to the first three observations.

— testing whether each coefficient is zero. The p-value (%) of the tests is in Table 4, that shows that all variables belong in the cointegrating relation and, except for net exports, depend on this relation.

Table 4
Coefficients of the Cointegration Relation

	Net Exports	Debt	Exchange rate
α	27.0	0.0	0.2
β	0.0	0.0	0.0

We then applied a multivariate cointegration test[29] were the null is that the variable is stationary. This test check whether all the coefficients in the cointegration relation are null, except the coefficient of the own variable.[30] The results of Table 5 show that the hypothesis of stationarity is rejected for all series.

Table 5
Multivariate Integration Test

	Net Exports	Debt	Exchange rate
I(0)	0.0	0.0	0.0

We also checked for short-run relations, by testing, for each equation, the hypothesis that the temporary effect of the other variables is null.[31] Taken together, the results of Table 6 show that: we can not reject the hypothesis that net exports do not depend on the other variables, both in the short — and in the long-run — which is one of the conditions of the theoretical model; net exports and debt impact the exchange rate, both in the short — and in the long-run; debt depends on the other variables in the long-run, but not in the short-run.

Table 6
Short Run Tests

	Exports	Debt	Exchange rate
Model (w)	44.5	9.1	0.0
Model (u)	-	66	0.0

[29] As suggested by Juselius and Johansen in the CATS manual. This test logically should have preceded the cointegration test, but in this context we believed that these variables are integrated.
[30] Non-rejection of this hypothesis implies that the cointegration relation only depends on a single variable, which is, thus, stationary.
[31] For model (w), the test is conditional on the cointegrated vector.

Before closing this subsection, we briefly discuss the finding that we can not reject the hypothesis that the real exchange rate does not lead net exports ratio, given our data set.[32] A possible explanation would be that Brazil exports mainly commodities with prices given in international markets, so that changes in exchange rates do not affect demand. There are two shortcomings with this explanation. The first is that it is not true: manufactured goods are a non-negligible share of Brazilian exports. The second problem is that, even if it were true, one would expect supply effects to be discernible in a data set spanning almost half a century.

We suspect the real problem is that the real exchange rate has not been a good measure of the remuneration of the Brazilian export sector in the last decades. The economic model we use for the determination of the real exchange rate obviously oversimplifies the trade-offs the government faces when "choosing" the real exchange rate. We have not considered the effects of changes in the exchange rate on inflation, public finance, prices of key imported consumption goods (say, wheat) **etc**. Since changing the exchange rate has so many implications, the Brazilian government has systematically used other instruments to affect the remuneration of the export sector, specially tax and credit subsidies.[33]

Even if these subsidies were uncorrelated with the exchange rate, we would have an omitted variables problem. But the case is likely to be more serious, since it is highly probable that subsidies will be correlated with the exchange rate: the government will probably increase subsidies when the real exchange rate appreciates and **vice-versa**. If this negative correlation does exist and if no good **proxy** for subsidies can be found, it may be not difficult to accept that the real exchange rate does not affect exports.[34]

3.2 - Identification

The theoretical model and the hypothesis that (χ, γ) correspond to the stochastic shocks with permanent effects suggest an identifying restriction: that the shock associated with net exports (γ) will have no long-run effect on debt, as indicated

[32] Note that we did not test causality from exchange rate to net exports; this result is not inconsistent with theoretical model formulation.

[33] Lamounier, Carneiro and Abreu (1994) comment the economic reforms of the 1967/73 period as follows: "Although the reorientation of economic policy intended to be modernising, it was not possible to entirely preserve the exchange rate as the central element of the remuneration of exporters and of the definition of costs for importers. It looked as if a more elaborate system of multiple exchange rates returned by the back door, notwithstanding policy-makers' explicit concerns as to the perversity of the distortions introduced by extra-market criteria of resource allocation." [Lamounier, Carneiro and Abreu (1994, p. 181), our translation].
Recently, after meeting the Brazilian Finance Minister, the president of the Brazilian Association of External Trade, Pratini de Moraes, declared to the press: "I came here to ask for a reduction of an export tax. But if the minister does not agree, he will have to change [devalue] the exchange rate."[Gazeta Mercantil (3/3/97, p. A-4)].

[34] Fiorencio and Moreira (1996**a**) show that the real exchange rate leads Brazilian exports of manufactured goods.

by the steady-state equations (2.13) and (2.14). This restriction will be maintained in both versions of the empirical model.

To impose this identifying restriction, the estimated VEC model is reparameterized in the moving average representation (3.2) that decomposes the effects of the shocks into their permanent and transitory components. (3.2) can be reparemeterized again, using the matrix of contemporaneous relations (A) to obtain the structural form of the model, (3.3, 3.3a).[35] This representation shows the common trends (τ), the long-run relation (β), and the temporary effects of the identified shocks R*(L).

$$z_t = \delta_t + C(1)\sum_{i=1}^{t}\varepsilon_i + C^*(L)\varepsilon_t \qquad \varepsilon_t \approx N(0,\Sigma) \qquad (3.2)$$

$$z_t = \delta_t + C(1)A^{-1}\sum_{i=1}^{t}A\varepsilon_i + C^*(L)A^{-1}A\varepsilon_t \quad A^{-1}\varepsilon_t = \nu_t \approx N(0,I) \quad (3.3)$$

$$z_t = R(1)\tau_t + R^*(L)\nu_t + \delta_t \qquad \tau_t = \sum_{i=1}^{t}\nu_i\,, \ \ \nu_t \approx N(0,I) \quad (3.3a)$$

In this representation, the C(1) matrix measures the long-run effects of the reduced form shocks. The $C(1)A^{-1}$ matrix measures the long-run effects of the structural shocks and allows the identification of the model. Matrix (A) will be characterised by the restrictions[36] imposed upon $R(1)=C(1)A^{-1}$.

In the case of model (u), the two variables present no cointegration relation, and, thus, C(1) has full rank and matrix (A) must be such that R(1) is recursive: one of the shocks can not impact the exchange rate.

Model (w) has a cointegration relation and, thus, C(1) is of rank 2,[37] and, without loss of generality, we can set $C(1)A^{-1}{}_{3X3}=(\Gamma_{3X2},0_{3X1})$. This representation isolates the two shocks that have permanent effects from the last shock, that only has transitory effects. The orthogonality conditions impose three restrictions on Γ, and the condition that $\beta'\Gamma=0$[38] imposes another two restrictions. We need one more restriction to get an unique estimate of the six coefficients of the Γ matrix. We impose the same restriction as before: the exports shock will not affect long-run debt. Tables 7 and 8 show the R(1) matrices for both models.[39]

The long-run effects of a reduction in risk and of an increase in exports are as expected, in both models. The increase in exports appreciates the exchange rate;

[35] King **et alii** (1991) and Moreira, Fiorencio and Lopes (1997).
[36] These restrictions allow the estimation of the lines of this matrix that correspond to the shocks that have permanent effects, and is estimated under the condition that the shocks are not correlated.
[37] If β are the cointegration relations, $\beta'y$ is stationary and $\beta'C(1)=0$.
[38] This is necessary to guarantee that $\beta'y$ is stationary.
[39] Where IRF and VarDec stand for Impulse-Response Function and forecast variance decomposition, respectively.

the reduction in risk increases debt and exports,[40] and depreciates the exchange rate.[41] Both models suggest that about 3/4 of the variation in the long-run exchange rate is due to changes in net autonomous exports and about 1/4 is due to changes in country-risk.

Table 7
Long-run Effects — Model (u)

	IRF		VarDec	
	Risk	Exports	Risk	Exports
Exchange rate	0.105	-1	28	72
Debt	1	0	100	0

Table 8
Long-run Effects — Model (w)

	IRF		VarDec	
	Risk	Exports	Risk	Exports
Exports	0.358	1	25	75
Exchange rate	0.523	-1.334	29	71
Debt	1.823	0	100	0

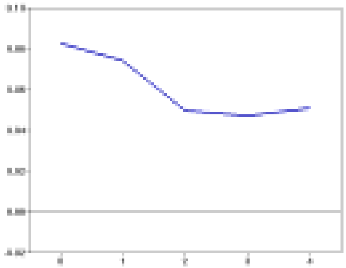
Consider a reduction in country-risk. Figure II showed that a reduction in country-risk will lead to an initial appreciation of the exchange rate followed by a continuous depreciation. The final position may be either higher or lower than the initial one. The path of the exchange rate in model (u) conforms with this pattern, but not in model (u). Figure II also showed that debt should not change on impact and then should continuously increase. The IRFs show that debt jumps up initially and then continuously increases.[42] The initial jump in debt is not incompatible with the results of the economic model, however. Remember that the economic model is set in continuous time — so that the impact change on the stock of debt is zero —, but our data are annual.[43]

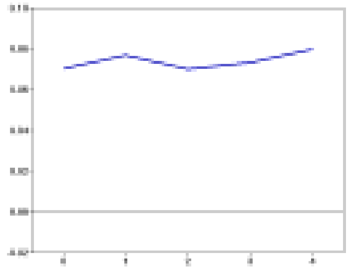
[40] Actually, the economic model predicts that the reduction in risk will not change exports. But it is straightforward to include country-risk in the exports equation, so that exports would rise.
[41] Note that this effect is theoretically ambiguous.
[42] In model (w) there is a temporary fall in period 1.
[43] Figure II also showed that the economic model predicts that an increase in autonomous exports will lead to an appreciation of the exchange rate and no change in debt, but due to the highly simplified nature of the model, there is no transitional dynamics in this case. The impulse-response functions for this shock are nevertheless presented.

Model (W)

Exports reaction in model (w)

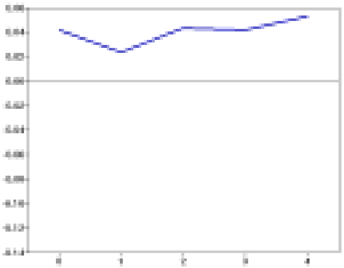
Reduction in Risk **Increase in autonomous exports**

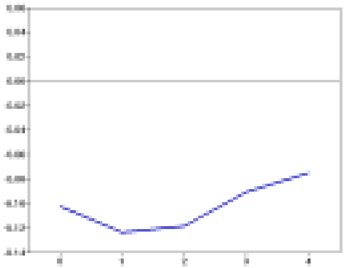




Exchange rate reaction in model (w)

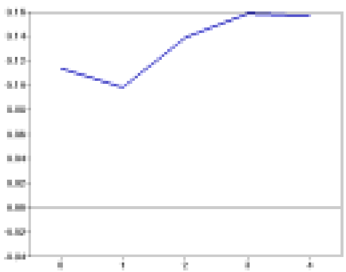
Reduction in risk **Increase in autonomous exports**

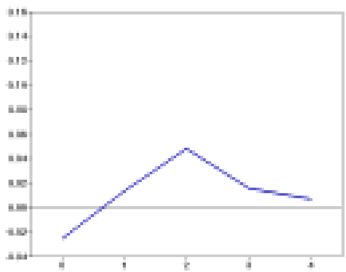




Debt /GDP reaction in model (w)

Reduction in risk **Increase in autonomous exports**

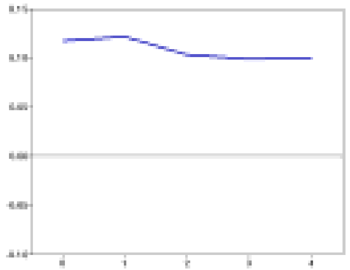


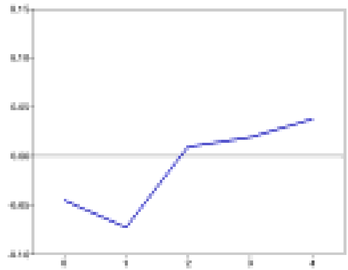


Model (U)

Reaction of the exchange rate in model (u)

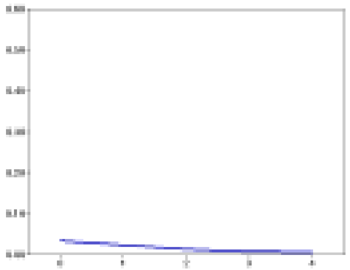
Reduction in autonomous exports **Reduction in risk**

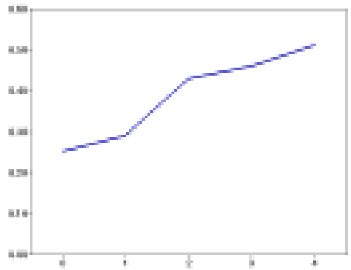




Reaction of Debt (modelo (u)

Reduction in autonomous exports **Reduction in risk**





4 - CONCLUSION

The theoretical model adopted rationalises the determinants of the real exchange rate emphasising that the cost of debt depends on the debt/GDP ratio. The theoretical model suggested the variables and the long-run identification of an empirical model. The empirical model was then used to estimate the long-run effects of exogenous changes in country-risk and net exports for Brazil, 1947/95.

The main result of the paper is that Brazilian data for the 1947/95 period did not allow the rejection of a very parsimonious model relating the external debt, net exports, both as GDP ratio, and the real exchange rate. In this model, the real exchange rate is regarded as a choice variable for the government, provided that the choice is within the limits that do not imply explosive debt. The paper, thus, brings additional evidence to the recent literature that finds that, in the long-run, the real exchange rate converges to some form of modified PPP that depends on international goods and credit market conditions.

BIBLIOGRAPHY

ABUAF, N., JORION, P. Purchasing power parity in the long-run. **The Journal of Finance**, v.45, n.1, p.157-174, 1990.

CHEUNG, Y. W., LAI, K. S. Long-run purchasing power parity during the recent float. **Journal of International Economics**, v.34, n.1/2, p.181-192, 1993.

CORBAE, D., OULIARIS, S. Cointegration and tests of PPP. **The Review of Economics and Statistics**, v.70, n.3, p.508-511, 1988.

DIEBOLD, F. X., HUSTED, S., RUSH, M. Real exchange rates under the gold standard. **Journal of Political Economy**, v.99, n.6, p.1.252-1.271, 1991.

DORNBUSCH, R., GOLDFAJN, I., VALDÉS, R. O. Currency crises and collapses. **Brookings Papers on Economic Activity**, v.2, p.219-293, 1995.

EICHENBAUM, M., EVANS, C. **Some empirical evidence on the effects of monetary policy shocks on exchange rates**. 1993 (NBER WP 4271).

ENGLE, C. **Long-run PPP may not hold after all**. 1996 (NBER WP 5646).

FACKLER, P. L. Vector autoregressive techniques for structural analysis. **Revista de Análisis Económico**, v.3, n.2, p.119-134, 1988.

FARUQUE, H. Long-run determinants of the real exchange rate. **IMF Staff Papers**, v.42, n.1, p.80-107, 1995.

FIORENCIO, A., MOREIRA, A. R. B. As importações deslocam a produção nacional? **Anais do XXIV Encontro Nacional de Economia**, p.282-298, 1996**a**.

______. Determinantes de longo prazo da taxa de câmbio real: Brasil 1947-1995. **Anais do XVIII Encontro Brasileiro de Econometria**, v.1, p.255-268, 1996**b**.

FRANKEL, J. A., ROSE, A. K. **A survey of empirical research on nominal exchange rates**. 1994 (NBER WP 4865).

_______. **A panel project on purchasing power parity: mean reversion within and between countries**. 1995 (NBER WP 5006).

FROOT, K. A., ROGOFF, K. **Perspectives on PPP and long-run real exchange rates**. 1994 (NBER WP 4952).

GONÇALVES, R. G., BARROS, A. C. Tendências dos termos de troca: a tese de Prebisch e a economia brasileira — 1850/1979. **Pesquisa e Planejamento Econômico**, v.12, p.109-131, 1982.

JOHANSEN, S. Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. **Econometrica**, v.59, n.6, p.1.551-1.580, 1991.

JOHANSEN, S., JUSELIUS, K. Testing structural hypotheses in a multivariate cointegration analysis of the PPP and the UIP for UK. **Journal of Econometrics**, v.53, 1992.

KING, R. G., PLOSSER, C. I., STOCK, J. H., WATSON, M. H. Stochastic trends and economic fluctuations. **The American Economic Review**, 81, 819-840, 1991.

KUGLER, P., LENZ, C. Multivariate cointegration analysis and the long-run validity of PPP. **The Review of Economics and Statistics**, v.75, p.180-184, 1993.

LAMOUNIER, B., CARNEIRO, D. D., ABREU, M. P. **50 anos de Brasil — 50 anos de Fundação Getulio Vargas.** Rio de Janeiro, FGV, 1994.

LIMA, E. C. LOPES, H. F., MOREIRA, A. R. B., PEREIRA, P. L. V. Tendência estocástica do produto no Brasil: efeitos das flutuações da taxa de crescimento da produtividade e da taxa de juro real. **Pesquisa e Planejamento Econômico**, v.25, n.2, p.249-278, 1995.

MacDONALD, R. Long-run exchange rate modeling. **IMF Staff Papers**, v.42, n.3, p.437-489, 1995.

MacDONALD, R., TAYLOR, M. P. Exchange rate economics: a survey. **IMF Staff Papers**, v.39, n.1, p.1-57, 1992.

McNOWN, R., WALLACE, M. S. Cointegration tests of the monetary exchange rate model for three high-inflation economies. **Journal of Money, Credit and Banking**, v.26, n.3, p.96-411, 1994.

MEESE, R. A., ROGOFF, K. Empirical exchange rate models of the seventies: do they fit out of sample? **Journal of International Economics**, v.14, n.1/2, p.3-24, 1983.

MOREIRA, A. R. B., FIORENCIO, A., LOPES, H. F. Um modelo para a previsão conjunta do PIB, inflação e liquidez. **Revista de Econometria**, v.7, n.1, maio 1997.

MUSSA, M. Empirical regularities in the behavior of exchange rates and theories of the foreign exchange market. In: BRUNNER, K., MELTZER, A.H. (eds.). **Policies for employment, prices, and exchange rates.** North-Holland, 1979.

PEREIRA, P. L. V., DUARTE, A. R. Paridade do poder de compra e paridade da taxa de juros para o Brasil: uma abordagem via co-integração multivariada. **Anais do XIII Encontro Brasileiro de Econometria**. 1991.

REINHART, C. M. Devaluation, relative prices, and international trade. **IMF Staff Papers**, v.42, n.2, p.290-312, 1995.

ROSE, A. K. Exchange rates and the trade balance: some evidence from developing countries. **Economic Letters**, v.34, p.271-275, 1990.

______. The role of exchange rates in popular models of international trade: does the Marshall-Lerner condition hold? **Journal of International Economics**, v.30, p.301-316, 1991.

ROSSI, J. W. Determinação da taxa de câmbio: testes empíricos para o Brasil. **Pesquisa e Planejamento Econômico**, v.21, p.397-412, 1991.

TODDA, H. Y., PHILLIPS, P. C. B. Vector autoregression and causality: a theoretical overview and simulation study. **Econometric Reviews**, v.13, n.2, p.259-285, 1994.

ZINI, A. A., CATI, R. C. Co-integração e taxa de câmbio: testes sobre a PPP e os termos de troca no Brasil de 1855 a 1990. **Pesquisa e Planejamento Econômico**, v.23, p.349-374, 1993.