TEXTO PARA DISCUSSÃO N° 1220

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TEXTO PARA DISCUSSÃO

Uma publicação que tem o objetivo de divulgar resultados de estudos desenvolvidos, direta ou indiretamente, pelo Ipea e trabalhos que, por sua relevância, levam informações para profissionais especializados e estabelecem um espaço para sugestões.

SINOPSE

Neste texto, analisamos a produtividade de estabelecimentos agrícolas localizados em 370 municípios da região Centro-Oeste do Brasil. Propomos um modelo de fronteira estocástica de produção com estrutura espacial latente que representa os determinantes não-observados da ineficiência da produtividade da agropecuária. Esse componente espacial condiciona a distribuição da ineficiência. Usamos o paradigma bayesiano para estimar os modelos propostos. Foram exploradas duas distribuições diferentes para este termo, a normal truncada e a exponencial, e utilizamos duas especificações para a variável latente, suposta independente entre os municípios, ou dependente dos municípios vizinhos segundo um modelo auto-regressivo espacial.

O procedimento de inferência considera explicitamente todas as incertezas quando incluímos o termo espacial. Como a distribuição *a posteriori* não tem uma expressão analítica, utilizamos técnicas estocásticas da simulação para obter amostras dessa distribuição. Foram adotados dois critérios que avaliam o desempenho do modelo, e os dois indicaram que o componente espacial latente incorpora informação adicional a um modelo que já contém informação local observada.

ABSTRACT

In this paper, we analyze the productivity of farms across n = 370 municipalities located in the Center-West region of Brazil. We propose a stochastic frontier model with a latent spatial structure to account for possible unknown geographical variation of the outputs. This spatial component is included in the one-sided disturbance term. We explore two different distributions for this term, the exponential and the truncated normal. We use the Bayesian paradigm to fit the proposed models. We also compare between an independent normal prior and a conditional autoregressive prior for these spatial effects. The inference procedure takes explicit account of the uncertainty when considering these spatial effects. As the resultant posterior distribution does not have a closed form, we make use of stochastic simulation techniques to obtain samples from it. Two different model comparison criteria provide support for the importance of including these latent spatial effects, even after considering covariates at the municipal level.

SUMMARY

1 Introduction

Stochastic frontier models have been widely used to describe productivity and firm efficiency. These models were introduced by Aigner et al. (1977) and, Meeusen and van den Broeck (1977). A stochastic frontier production function decomposes output into three components. The first is a deterministic component that includes inputs in the production function and other variables that affect productivity and are correlated with the inputs. The second is an asymmetric stochastic component that captures the inefficiency of each producer (measured as the distance to the frontier). The moments of the distribution of the inefficiency component might depend on a set of variables–not all of which are observed. The third component of the model is a random disturbance. There are different proposals in the literature for the distribution of the inefficiency component: the exponential (Meeusen and van den Broeck, 1977), the half-normal (Aigner et al., 1977), the truncated normal (Stevenson, 1980), the gamma (Greene, 1990) and, the log-normal (Migon, 2006).

In many real world examples of production, local conditions affect productivity. In the context of agriculture, for example, differences across localities in transportation infrastructure, soils and climate, human capital of the local labor force, and other factors, can create systematic variation in the efficiency of agricultural production across localities. Local determinants of efficiency can be estimated with fixed or random effects. Alternatively, when information is available to describe the local determinants, these variables can be included directly in the stochastic frontier model. In the first case, local effects are measured fully but it is not possible to interpret the determinants of the effects, while in the second case interpretation is feasible but the vector of determinants included in the model does not exhaust the list of possible determinants. An additional local effect remains, and failure to account for this will lead to bias in the estimated coefficients on the included variables.

The novelty of our approach lies in the introduction of a municipal level latent effect in the modelling of the inefficiencies. More specifically, we believe that the inefficiency of unit j, $j = 1, \cdots, n_i$ in municipality i, $i = 1, \cdots, n$ depends on which municipality it is located in. Our model combines models from the stochastic frontier and spatial econometric literatures. Inference is conducted following the Bayesian paradigm. See Koop and Steel (2001) and Kumbhakar and Tsionas (2005) for a review of the use of the Bayesian paradigm for this class of models and, Anselin (1988) and Gamerman and Moreira (2004) for a review on spatial econometric models.

A priori, we assume that the inefficiencies, u_{ij}, for $i = 1, \cdots, n$ and $j = 1, \cdots, n_i$ are conditionally exchangeable within each municipality i. More formally, the inefficiency u_{ij} of agent $j = 1, \cdots, n_i$, located in municipality $i = 1, \cdots, n$, is estimated conditioned on the unobserved local characteristics α_i, $u_{ij} \sim g(i)$, where g is a positive and asymmetric probability density function, depending on α_i. This component α_i, which represents the unobserved local characteristics in each municipality, is a common component among the inefficiencies of all agents in each municipality. Therefore, the inefficiencies of units which are in the same municipality are generated from the same distribution. However, these distributions are allowed to differ across the region. This is because we assume that agents are heterogeneous, and that part of this heterogeneity might derive from the location of the agents. One might assume, *a priori*, that these latent effects are either independent or spatially structured. The latter means that $\alpha_i - \alpha_k$, $\forall i \neq k = 1, \cdots, n$ would tend to be smaller for municipalities that are closer together. Therefore, we also discuss the prior distribution of these latent effects.

Frequently, in the stochastic frontier literature, the observations are available in the form of panel data. Here, however, the replicates are obtained across space and not time, and we have many observations within each of the n municipalities. Our aim is to model this spatial heterogeneity even after accounting for a possible set of covariates at the municipality level which might affect the productivity of each unit. Tsionas (2002) proposes stochastic frontier models with random coefficients to separate technical inefficiency from technological differences across firms. On the other hand, Greene (2005) extends fixed effects models in the context of the stochastic frontier literature with a variety of approaches to incorporate firm specific heterogeneity. From a spatial perspective, Druska and Horrace (2004) consider generalized moments estimation of panel data assuming a decomposition of the error term as the sum of two components: one that is spatially structured and the other that is white noise. This spatial component is modelled through a simultaneous autoregressive model. See Anselin (1988) for details. Helfand and Levine (2004) and Sampaio de Souza et al. (2005) employ a similar approach. First, they estimate Data Envelopment Analysis (DEA) technical efficiency scores for municipalities spread over a region. After computing the efficiency scores, they use either a spatial SUR or a simultaneous autoregressive model, respectively, to investigate the determinants of those scores. In other words, both papers mentioned above perform the inference procedure in two steps, taking no explicit account of the uncertainty in the estimation of the efficiencies.

This paper is organized as follows. Section 2 provides the economic motivation for the problem and describes the data set to be analyzed. Section 3 discusses the proposed model, and in Section 4 the inference procedure is described. Section 5 presents the analysis of the data under the proposed model. As we explore different model specifications for the data, two model comparison criteria, one based on that suggested by Gelfand and Ghosh (1998) and the other on that by Spiegelhalter et al. (2002), are described and performed. Finally, Section 6 concludes the paper and describes future avenues for research.

2 Motivation

This article is part of a larger research project which uses microdata from the 1995/96 Agricultural Census in Brazil to study the determinants of agricultural productivity. The project is motivated by the observation that Brazil has one of the most unequal distributions of land holdings in the world, and an extremely high rate of rural poverty. In the past decade, a series of governments have pursued an aggressive policy of land reform. Brazil is also among the largest producers and exporters in the world of many agricultural products. A deeper understanding of the determinants of productivity in Brazilian agriculture has important implications for reducing rural poverty by increasing productivity and income among small farmers, and for relaxing constraints on macroeconomic growth by increasing foreign exchange earnings.

Here we concentrate on a sample of farms in the Center-West region of Brazil. This region contained three states, 426 municipalities, and over 240,000 farms in 1996. From this region, we chose a random sample of 25,494 farms spread over n=370 municipalities. In order to reduce certain types of heterogeneity that are not the principal focus of this paper, we restricted the sample to (a) owner operated farms, (b) farms that hired one or more permanent workers, and (c) farms that used inputs relatively intensively (identified as farms in the upper half of the distribution of input expenditures per hectare)[1]. Because the choice of which outputs to produce is endogenous, and we are interested in comparing the efficiency with which agricultural producers transform inputs into numerous outputs through the Center-West, we follow standard practice of using an output quantity index as our dependent variable (see e.g. Coelli et al. (1998); Koop and Steel (2001)).

3 Proposed Model

Assume observations are available in the form of panel data. However, here, replications are at the municipal level. Let y_{ij} be the logarithm of output for municipality i, $i = 1, 2, \cdots, n$ and unit j, $j = 1, 2, \cdots, n_i$. We consider that observations are being generated through the following structure

$$y_{ij} = f(\mathbf{x}_{ij}, \boldsymbol{\beta}) + \boldsymbol{\gamma}'\mathbf{z}_i + \boldsymbol{\delta}'\mathbf{c}_{ih} - u_{ij}(\alpha_i) + \epsilon_{ij} \quad \epsilon_{ij} \sim N(0, \sigma^2), \tag{1}$$

where $N(0, \sigma^2)$ denotes the zero mean normal distribution with variance σ^2, $f(\mathbf{x}_{ij}, \boldsymbol{\beta})$ represents the production function, with $\mathbf{x}_{ij}$ being a vector of dimension q_1 of farm specific inputs, and $\boldsymbol{\beta}$ a vector of their respective coefficients. The vector $\mathbf{z}_i$ comprises all the covariates, say q_2, which vary at the municipal level, and might influence the outputs. $\mathbf{c}_{ih}$, of dimension q_3, corresponds to dummy variables indicating farms size classes.

The component $u_{ij}(.)$ models the inefficiency of unit j at location i and is independent of ϵ_{ij}. It is described as a function of a latent effect, α_i, which varies across the municipalities. We consider two different distributions for the inefficiency component, $u_{ij}(.)$ in equation (1). More specifically, we either model u_{ij} as

$$(u_{ij} \mid \alpha_i, \tau^2) \sim N^+(\alpha_i, \tau^2) \tag{2}$$

or as

$$u_{ij} \mid \theta_i \sim \exp(1/\theta_i) \text{ with } \log \theta_i = \alpha_i, \tag{3}$$

[1]In our larger research project on agricultural productivity, we examine all farm types and study the determinants of differences in productivity across these types of farms.

where $N^+(a,b)$ denotes the normal distribution truncated at zero, whose associated normal has mean a and variance b, and $\exp(1/\theta)$ denotes the exponential distribution with mean θ. It is worth noting that the distribution in equation (2) states that the u_{ij} are exchangeable at the municipal level, conditioned on α_i and τ^2. On the other hand, in equation (3), they are exchangeable, at the municipal level, conditioned only on θ_i. Notice that these latent effects are introduced in the second level of hierarchy, therefore they do not affect the Y_{ij}'s directly. If the random effects were introduced in the mean of Y_{ij} we might get some correlation between $\mathbf{z}_i$ and α_i. This kind of centering provides more stable computation. See, for example, Gelfand et al. (1996) and Papaspiliopoulos et al. (2003). If there are no municipality level covariates, one might consider introducing α_i directly in the mean structure of Y_{ij}. We discuss this further in Section 5.

As the inference procedure is made through the Bayesian Paradigm, an important issue is the effect of the prior distribution associated to the random effects α_i's. The simplest choice is to assume *a priori* that the α_i's are all independent among themselves, that is $\alpha_i \sim N(0, \psi^2)$, $\forall i = 1, 2, \cdots, n$. However, as the observations are being made across municipalities, we would expect these random effects to be spatially correlated. Due to the geographical characteristics of the data, it is natural to expect that the inefficiencies of units which are located in neighboring municipalities are of similar magnitude. That is, the magnitude of the inefficiencies might vary smoothly across the region. For this reason, following Besag et al. (1991), we assume a conditional autoregressive (CAR) prior for $\boldsymbol{\alpha} = (\alpha_1, \cdots, \alpha_n)'$, that is

$$\begin{aligned} (\alpha_i \,|\, \alpha_k,\, k \neq i) \quad &\sim \quad N(m_i, v_i) \text{ where} \\ m_i = \frac{\sum_{k \in \varsigma_i} W_{ik} \alpha_k}{\sum_{k \in \varsigma_i} W_{ik}} \quad &\text{and} \quad v_i = \frac{\psi^2}{\sum_{k \in \varsigma_i} W_{ik}} \quad (i, k = 1, \cdots, n), \end{aligned} \tag{4}$$

where ς_i is the set containing the neighboring municipalities of i. We denote this prior as $\boldsymbol{\alpha} \sim CAR(\psi^2)$. It is common practice to consider a $0-1$ neighborhood structure, such that $W_{ik} = 1$ if i and k share boundaries and 0 otherwise. In this case, we have that

$$m_i = \frac{\sum_{k \in \varsigma_i} \alpha_k}{n_i^*} \qquad \text{and} \qquad v_i = \frac{\psi^2}{n_i^*},$$

where n_i^* is the number of neighbors of municipality i. Notice that here, ψ^2 is the variance of the conditional distribution of $(\alpha_i \mid \alpha_k,\, k \neq i)$, therefore its interpretation is not straightforward, that is, it is not the same as when we assume that these random effects are independent *a priori*. It is worth mentioning that the conditional distribution in equation (4) leads to an improper joint distribution for $\boldsymbol{\alpha}$. This is not a problem, as we make clear in Section 4.

Assigning a CAR prior to the random effects α_i means that they are correlated *a priori*, therefore, if we model u_{ij} as in equation (2) we are implicitly assuming that the Y_{ij} are independent among themselves conditioned on the inefficiencies, u_{ij}'s.

Let $\mathbf{y} = (y_{11}, \cdots, y_{1n_1}, \cdots, y_{n1}, \cdots, y_{nn_n})'$ represent a random sample of the logarithm of the outputs at location $i = 1, 2, \cdots, n$ and unit $j = 1, 2, \cdots, n_i$. Under the specification in (2) or in (3), the likelihood is given by

$$p(\mathbf{y} \mid \mathbf{x}_{ij}, \boldsymbol{\beta}, u_{ij}, \tau^2, \sigma^2) \propto \left(\sigma^2\right)^{-n/2} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\sum_{j=1}^{n_i}\left(y_{ij} - f(\mathbf{x}_{ij}, \boldsymbol{\beta}) - \boldsymbol{\gamma}'\mathbf{z}_i - \boldsymbol{\delta}'\mathbf{c}_{ij} + u_{ij}\right)^2\right\}. \quad (5)$$

Assuming u_{ij} is distributed as in (2), Stevenson (1980) and Broeck et al. (1994) show that when integrating the likelihood above with respect to u_{ij} one obtains

$$p(\mathbf{y} \mid \mathbf{x}_{ij}, \boldsymbol{\beta}, \alpha_i, \sigma^2) \propto \prod_{i=1}^{n}\prod_{j=1}^{n_i} \frac{1}{\sqrt{\sigma^2+\tau^2}}\phi\left(\frac{y_{ij} - f(\mathbf{x}_{ij}, \boldsymbol{\beta}) - \boldsymbol{\gamma}'\mathbf{z}_i - \boldsymbol{\delta}'\mathbf{c}_{ij} + \alpha_i}{\sqrt{\sigma^2+\tau^2}}\right)$$
$$\times\Phi\left(\frac{\alpha_i\sqrt{\sigma^2+\tau^2}}{\sigma\tau} - \frac{\tau(y_{ij} - f(\mathbf{x}_{ij}, \boldsymbol{\beta}) - \boldsymbol{\gamma}'\mathbf{z}_i - \boldsymbol{\delta}'\mathbf{c}_{ij} + \alpha_i)}{\sigma\sqrt{\sigma^2+\tau^2}}\right)\Phi^{-1}\left(\frac{\alpha_i}{\tau}\right), \quad (6)$$

which is a skew normal distribution as described in Domínguez-Molina and González-Farías (2003). On the other hand, following the specification for u_{ij} as in (3) and, marginalizing the likelihood with respect to u_{ij}, Stevenson (1980) and Broeck et al. (1994) show that

$$p(\mathbf{y} \mid \mathbf{x}_{ij}, \boldsymbol{\beta}, \alpha_i, \sigma^2) \quad \propto \quad \prod_{i=1}^{n}\prod_{j=1}^{n_i}\theta_{ij}^{-1}\exp\left\{\frac{-m_{ij}}{\theta_{ij}} - \frac{\sigma^2}{2\theta_{ij}^2}\right\}\Phi\left(\frac{m_{ij}}{\sigma}\right), \quad (7)$$

where $m_{ij} = f(\mathbf{x}_{ij}, \boldsymbol{\beta}) + \boldsymbol{\gamma}'\mathbf{z}_i + \boldsymbol{\delta}'\mathbf{c}_{ij} - y_{ij} - \frac{\sigma^2}{\theta_{ij}}$, $\Phi(x)$ is the cumulative distribution and $\phi(x)$ is the probability density function of the standard normal distribution evaluated at point x. These marginalizations with respect to u_{ij} will prove useful in the simulation methods which will be described in the next Section.

4 Inference Procedure

Following the Bayesian paradigm the model specification is complete after assigning a prior distribution to all unknowns (parameters) in the model. In the previous Section we only discussed the prior distribution of the latent random effects α_i, and of the one-sided disturbance term. Here we will discuss the prior distribution of the remaining parameters in the model. Then, following Bayes' theorem, we will obtain the kernel of the posterior distribution of the parameter vector. Lastly, we will describe how to make inference from the resultant posterior distribution.

Let $\boldsymbol{\theta}$ comprise the parameter vector, such that $\boldsymbol{\theta} = (\beta_0, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\alpha}, \psi^2, \tau^2, \sigma^2)'$. Initially, we assume that all components of $\boldsymbol{\theta}$ are independent *a priori*. Therefore, the joint density prior distribution for $\boldsymbol{\theta}$ is given by

$$p(\boldsymbol{\theta}) = \prod_{i=0}^{q_1} p(\beta_i) \prod_{j=1}^{q_2} p(\gamma_j) \prod_{k=1}^{q_3} p(\delta_k)\, p(\boldsymbol{\alpha} \mid \psi^2)\, p(\psi^2)\, p(\tau^2)\, p(\sigma^2).$$

Following standard procedures, for the coefficients β_i, γ_j, and δ_k $i = 1, \cdots, q_1$, $j = 1, \cdots, q_2$, $k = 1, \cdots, q_3$ we assign a zero mean normal prior distributions with variance (σ_β^2) fixed at some big value. For the variances τ^2 and σ^2 we assign inverse gamma prior distributions with both parameters equal to ε, i.e. $\tau^2 \sim IG(\varepsilon, \varepsilon)$, where ε is fixed at some small number, say 0.1, to describe our prior ignorance about these parameters. Some care must be taken when assigning the prior distribution for ψ^2, the variance of the CAR distribution. This prior cannot be too uninformative, as this is a non-identifiable parameter in the sense of Dawid (1979). We assign an inverse gamma prior, $IG(a_\psi, b_\psi)$, whose mean is equal to the OLS estimate based on an independent stochastic frontier model, with some fixed variance. Once the prior distribution has been assigned, following Bayes' theorem, the posterior distribution of $\boldsymbol{\theta}$ is proportional to the likelihood function times the prior, which considering, for example, the specification in (2), results in

$$
\begin{aligned}
& p(\boldsymbol{\theta} \mid \mathbf{y}) \propto p(\mathbf{y} \mid \mathbf{x}_{ij}, \boldsymbol{\theta})\, p(\boldsymbol{\theta}) \\
\propto \; & \prod_{i=1}^{n} \prod_{j=1}^{n_i} \frac{1}{\sqrt{\sigma^2+\tau^2}} \phi\left(\frac{y_{ij} - f(\mathbf{x}_{ij}, \boldsymbol{\beta}) - \boldsymbol{\gamma}'\mathbf{z}_i - \boldsymbol{\delta}'\mathbf{c}_{ij} + \alpha_i}{\sqrt{\sigma^2+\tau^2}}\right) \\
& \times \Phi\left(\frac{\alpha_i\sqrt{\sigma^2+\tau^2}}{\sigma\tau} - \frac{\tau(y_{ij} - f(\mathbf{x}_{ij}, \boldsymbol{\beta}) - \boldsymbol{\gamma}'\mathbf{z}_i - \boldsymbol{\delta}'\mathbf{c}_{ij} + \alpha_i)}{\sigma\sqrt{\sigma^2+\tau^2}}\right) \Phi^{-1}\left(\frac{\alpha_i}{\tau}\right) \\
& \times \prod_{i=0}^{q_1} \exp\left\{-\frac{1}{2\sigma_\beta^2}\beta_i^2\right\} \prod_{i=1}^{q_2} \exp\left\{-\frac{1}{2\sigma_\beta^2}\gamma_i^2\right\} \prod_{i=1}^{q_3} \exp\left\{-\frac{1}{2\sigma_\beta^2}\delta_i^2\right\} \qquad (8) \\
& \times \left(\psi^2\right)^{-n/2} \exp\left\{-\frac{1}{2\psi^2}\sum_{i=1}^{n}\sum_{j<i} W_{ij}(\alpha_i - \alpha_j)^2\right\} \left(\psi^2\right)^{-(a_\psi+1)} \exp\left\{-\frac{b_\psi}{\psi^2}\right\} \\
& \times \left(\tau^2\right)^{-(\varepsilon+1)} \exp\left\{-\frac{\varepsilon}{\tau^2}\right\} \left(\sigma^2\right)^{-(\varepsilon+1)} \exp\left\{-\frac{\varepsilon}{\sigma^2}\right\}.
\end{aligned}
$$

The equation above does not have an analytical closed form. Therefore, simulation methods are needed to make inference about $p(\boldsymbol{\theta} \mid \mathbf{y})$. We make use of Markov chain Monte Carlo (MCMC) methods, in particular the Gibbs sampling algorithm (Gelfand and Smith, 1990) with some steps of the Metropolis-Hastings (Hastings, 1970) and the slice sampling (Neal, 2003) algorithms. Gamerman and Lopes (2006) give more details about these sampling schemes.

As described in Gamerman and Lopes (2006), the Gibbs sampler is a MCMC scheme where the transition kernel of the Markov chain is based on the full conditional posterior distributions, $\pi(\theta_i \mid \theta_{-i})$, where $\theta_{-i} = (\theta_1, \cdots, \theta_{i-1}, \theta_{i+1}, \cdots, \theta_p)$, and p is the dimension of the parameter vector. Because of the marginalization in (6) or in (7), the full conditional distributions of the elements of $\boldsymbol{\beta}$ do not follow any known distribution. We make use of a Metropolis-Hastings random walk to sample from this full conditional. From our experience, the speed of convergence of the chains might be sensitive to the starting values of the elements of $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\delta}$. We suggest to start the chain from their OLS estimates based on a multiple regression without the inefficiency component. In the case of the variances, σ^2 and τ^2, their full conditionals do not have an analytical closed form. Here we also suggest the use of a Metropolis-Hastings random walk step for $\log \sigma^2$ and $\log \tau^2$, respectively.

On the other hand, ψ^2 has an inverse gamma posterior full conditional distribution regardless of whether an independent normal or a CAR prior is assigned to $\boldsymbol{\alpha}$.

Our main challenge when implementing the algorithm is the sampling of each α_i. Notice, that we have as many α_i's as the number of municipalities in the sample. As the use of a random walk Metropolis-Hastings step requires the tuning of the variance of the proposal distribution, we prefer to use the slice sampling algorithm (Neal, 2003) to sample from $\pi(\alpha_i \mid \boldsymbol{\theta}_{-\alpha_i}, \mathbf{y})$. The slice sampler is an auxiliary variable MCMC algorithm. It is based on the idea of slicing the target distribution (the full conditional of α_i in our case) horizontally at the contour level of a uniformly distributed variable $U(0, \pi(\alpha_i \mid .))$. See Gamerman and Lopes (2006) for more details. Recall that the CAR prior as defined in (4) results in an improper joint distribution for the spatial effects. Following Besag et al. (1995) as a remedy we impose the constraint $\sum_i \alpha_i = 0$ after each iteration of the Gibbs sampler.

It is worth mentioning the computational advantage of marginalizing the likelihood with respect to u_{ij}. While running the MCMC, we avoid updating these parameters and the algorithm becomes faster. After convergence has been reached, we can use the samples from the elements of $\boldsymbol{\theta}$ to obtain posterior samples of the efficiencies, $\exp(-u_{ij}(.))$, through the posterior of u_{ij}.

5 Empirical Analysis

In this Section we fit the proposed model to the sample described in Section 2. We assume that the relationship between inputs and outputs along the frontier can be described by a constant returns to scale translog production function[2]. This is augmented by a number of variables that are relevant for explaining efficiency. More specifically, an output quantity index (y) is specified as a function of production inputs (x), a farm size fixed effect (c), the conditions (z) of each locality (i), the existence of local public goods and institutions specific to each farm size group and locality (ih), an asymmetric stochastic term $u_{ij}(.)$, and a random error. We clarify below the different model structures fitted to the data.

The factors of production (x) include a) area of the establishment, b) the quantity of family labor, c) the value of machines, d) the value of other forms of capital (trees, stock of animals), and e) expenditure on variable inputs such as fertilizer, pesticides, seeds, and hired labor. With the exception of the last variable, all inputs are determined prior to the production cycle and endogeneity is not a major concern. Variable inputs for each farm, in contrast, are clearly endogenous and were instrumented for by using the mean value from each locality and farm size group (ih). Finally, input and output quantity indices were constructed using constant prices in order to eliminate the effect of spatial variation in the prices of these goods and services.

[2] Deviations from constant returns to scale are captured in the farm size fixed effect (c) described below. We adopt this specification in order to be consistent with the model used in our larger project on agricultural productivity, and because the restriction does not affect the spatial focus of the analysis here.

Local characteristics are described by a vector of variables that includes: a) four indices of temperature and rainfall that capture the suitability of each location for temporary and/or permanent crop production, b) five variables that capture soil, slope, and other physical attributes, c) transport costs to the city of São Paulo, the dominant consumption center of Brazil, d) distance to the sea, and to the relevant state capital, and e) average years of schooling used as a measure of human capital in each municipality.

Initially, we propose six different models to fit the data. The models differ in terms of the inefficiency term (truncated normal and exponential distributions); the inclusion or not of the latent effect α_i; and, the specification of the prior distribution of α_i. Therefore, assuming the model in (1), we propose to fit the following specifications:

$M1$: $u_{ij} \sim N^+(0, \tau^2)$;

$M2$: $u_{ij} \sim N^+(\alpha_i, \tau^2)$ and, *a priori*, $\alpha_i \sim N(0, \psi^2)$, independent, $\forall i = 1, \cdots, n$;

$M3$: $u_{ij} \sim N^+(\alpha_i, \tau^2)$ and, *a priori*, $\boldsymbol{\alpha} \sim CAR(\psi^2)$;

$M4$: $u_{ij} \sim \exp(1/\tau^2)$ with $\tau^2 \sim IG(\varepsilon, \varepsilon)$.

$M5$: $u_{ij} \sim \exp(1/\theta_i)$ with $\log \theta_i = \alpha_i$ and, *a priori*, $\alpha_i \sim N(0, \psi^2)$, independent, $\forall i = 1, \cdots, n$;

$M6$: $u_{ij} \sim \exp(1/\theta_i)$ with $\log \theta_i = \alpha_i$ and $\boldsymbol{\alpha} \sim CAR(\psi^2)$.

Notice that models $M1$ and $M4$ are the standard stochastic frontier models. Models $M2$ and $M5$ assume that the α_i's are independent *a priori*, with the inefficiencies following a truncated normal or exponential distribution, respectively. Finally, $M3$ and $M6$ assume that the α_i's follow a CAR prior as specified in (4), again with the u_{ij}'s following, respectively, a truncated normal or an exponential distribution. In order to explore all the possibilities with regard to the spatial heterogeneity present in the data, we also fit the 6 specifications above removing the municipal level covariates, $\mathbf{z}_i$, in equation (1). We will label these specifications as $Mk_{\overline{z}}$, $k = 1, \cdots, 6$. When removing the $\mathbf{z}_i$ one might also consider fitting the model with municipal level random effects, that is, assuming $y_{ij} = f(\mathbf{x}_{ij}, \boldsymbol{\beta}) + \boldsymbol{\delta}'\mathbf{c}_{ij} + \alpha_i - u_{ij} + \epsilon_{ij}$. In this case, one might consider fitting 4 other specifications, which are:

$M7$: *a priori*, $\alpha_i \sim N(0, \psi^2)$, independent, $\forall i = 1, \cdots, n$ and $u_{ij} \sim N^+(0, \tau^2)$;

$M8$: *a priori*, $\boldsymbol{\alpha} \sim CAR(\psi^2)$ and $u_{ij} \sim N^+(0, \tau^2)$;

$M9$: *a priori*, $\alpha_i \sim N(0, \psi^2)$, independent, $\forall i = 1, \cdots, n$, and $u_{ij} \sim \exp(1/\tau^2)$ with $\tau^2 \sim IG(\varepsilon, \varepsilon)$;

$M10$: $\boldsymbol{\alpha} \sim CAR(\psi^2)$, and $u_{ij} \sim \exp(1/\tau^2)$ with $\tau^2 \sim IG(\varepsilon, \varepsilon)$.

Therefore, we are considering 16 different model specifications. The aim is to check which model fits the data best. In the next Subsection we describe the two criteria we use to make the model comparisons.

5.1 Model comparison criteria

As we are proposing different model specifications, model choice techniques emerge as an important tool to indicate which model, among those proposed, fits the data best. A well known model comparison criterion in the Bayesian literature is the Bayes factor. However, its limitation lies in the use of noninformative priors, as in this case, they are not well defined. As we are using the CAR prior, which is improper, we compute two other criteria, the posterior predictive loss (EPD) which was introduced by Gelfand and Ghosh (1998) and the deviance information criterion (DIC), proposed by Spiegelhalter et al. (2002). They are briefly described below.

5.1.1 Posterior predictive loss

This measurement is based on replicates of the observed data, $Y_{i,j}^{rep}$, $i = 1, \cdots, n$, $j = 1, \cdots, n_i$, and the selected models are those that perform well under a loss function. This loss function penalizes actions both for departure from the corresponding observed value as well as for departure from what we expect the replicate to be (Banerjee et al., 2004). Assuming a normal model as in (5), and a squared loss function, the criterion is computed as

$$D_\nu = \frac{\nu}{\nu+1} G + P, \text{ where}$$
$$G = \sum_{i=1}^{n} \sum_{j=1}^{n_i} (\mu_{i,j} - y_{i,j}^{obs})^2 \text{ and } P = \sum_{i=1}^{n} \sum_{j=1}^{n_i} \sigma_{i,j}^2.$$

In the equation above, $\mu_{i,j} = E(Y_{i,j}^{rep} \mid \mathbf{y})$ and $\sigma_{i,j}^2 = Var(Y_{i,j}^{rep} \mid \mathbf{y})$, the mean and variance of the predictive distribution of $Y_{i,j}^{rep}$ given the observed data $\mathbf{y}$. Banerjee et al. (2004) mention that ordering of models is typically insensitive to the choice of ν, therefore we fix $\nu = 1$. Smaller values of D indicate better models. Notice that, after convergence has been reached, at each iteration of the MCMC we can obtain replicates of the observations given the sampled values of the parameters. Then we approximate the expected values above via Monte Carlo integration.

5.1.2 Deviance Information Criterion

Spiegelhalter et al. (2002) propose a generalization of the AIC based on the posterior distribution of the deviance, $D(\boldsymbol{\theta}) = -2 \log L(\mathbf{y}; \boldsymbol{\theta})$. The Deviance Information Criterion (DIC) is defined as

$$DIC = \overline{D} + p_D = 2\overline{D} - D(\overline{\boldsymbol{\theta}}),$$

where $\overline{D}$ defines the posterior expectation of the deviance, $\overline{D} = E_{\boldsymbol{\theta}|\mathbf{y}}(D)$, and p_D is the effective number of parameters, $p_D = \overline{D} - D(\overline{\boldsymbol{\theta}})$ and here $\overline{\boldsymbol{\theta}}$ represents the posterior mean of the parameters. Smaller values of DIC indicate a better fitting model. Notice that computation of DIC is easily achieved through MCMC methods.

Banerjee et al. (2004) do not recommend a choice between EPD and DIC. They claim that both involve summing a goodness of fit term and a complexity penalty.

They go on to say that the fundamental difference is that the latter works in the parameter space with the likelihood, while the former works in the predictive space with posterior predictive distributions. They suggest that if the objective is to use the model for explanation, DIC should be preferred; whereas if the objective is prediction, EPD should be used.

5.2 Results

The implementation of the MCMC algorithm was made in `Ox`[3]. For each of the models above, we run two parallel chains, each of length $L = 60,000$. We discarded the first $10,000$ iterations and kept one at each $50th$ iteration, in order to avoid autocorrelation among the sampled values. Convergence was checked following standard procedures such as those described in Gamerman and Lopes (2006).

Table 1 shows the values of EPD and DIC, and their respective components, under each of the sixteen fitted models. Many different comparisons emerge from this Table. First, it is clear that the standard models, $M1$, $M1_{\overline{Z}}$ and $M4$, $M4_{\overline{Z}}$, which do not assume the inclusion of a latent effect α_i provide the worst results, both for the truncated normal and the exponential distributions for the inefficiencies. Recall that models $M1_{\overline{Z}}$ and $M4_{\overline{Z}}$ do not consider municipal level covariates, and they result in the highest values of both EPD and DIC. In other words, both criteria are pointing to the fact that the spatial information must be taken into account. When we compare between models Mi and $Mi_{\overline{Z}}$, $i = 1, \cdots, 6$, those which have both the spatial latent effect and spatial level covariates (Mi's) perform better under both EPD and DIC. This suggests that (a) the municipal level covariates belong in the model, and (b) even when they are in included, there is still some structure left at the municipal level which is being captured by the latent effects $\boldsymbol{\alpha}$. On the other hand, depending on the distribution we assume for the inefficiencies, both criteria choose either the CAR prior (in the truncated normal case) or the independent prior (in the exponential case) for α_i. When we compare between the distribution of the inefficiencies (u_{ij}), both criteria have smaller values when the truncated normal is assumed. This might be an indication that we need more flexibility in the model that is, another parameter is needed to describe the variance of the inefficiencies. Among all fitted models, if we had to choose one, this would be $M3$, which results in the smallest values of both EPD and DIC. From now on, we will present results only for models $M1, M2, \cdots, M6$, as they were the ones that performed best under both model comparison criteria.

In Table 2, we present the posterior summary of the sources of variability for the different components of the model in (1). That is of σ^2, τ^2, the ratio $\kappa = \tau^2/(\tau^2+\sigma^2)$ (only when the inneficiencies follow a truncated normal distribution) and ψ^2. As described in Section 3, ψ^2, the variance of α_i, does not have a straight forward interpretation when the CAR prior is assumed. We notice that σ^2, the variance of the white noise, is the smallest for $M3$, which is in agreement with EPD and DIC. We also notice that κ is relatively big, supporting the inclusion of the inefficiency term

[3]See `http://www.doornik.com/ox/` for more details.

to model this dataset. It is worth mentioning that under $M4$, τ^2, as reported there, is the mean of the exponential distribution of u_{ij}. We do not show the summary of τ^2 for M_5 and M_6 as in these cases, the mean of the exponential varies across the municipalities.

In Figure 1 we present the comparison of the posterior median of the α_i's, $i = 1, \cdots, n$ amongst the six models. This Figure shows that the magnitude of these effects vary according to the prior, as well as the distribution of the inefficiencies we assume. Figure 2, in contrast, presents the box plots of the posterior samples obtained for each of the coefficients of the covariates $\mathbf{Z}_i$, which carry information at the municipal level. Even though they are present in the model, both, EPD and DIC, indicate M_3 as the best model, suggesting the importance of the inclusion of the latent effect α_i with a prior spatial structure. In other words, this is an indication that there is still some spatial structure left even after the inclusion of $\mathbf{Z}_i$ in the model. We also notice from this Figure that the coefficients do not change much across the different models. As expected, for some coefficients, the inclusion of α_i increases the uncertainty in their estimation.

In Figure 3, we present the boxplots of the posterior of those coefficients whose covariates are related to farm size ($\boldsymbol{\delta}$). Apparently, the distribution assumed for the inefficiencies does not influence much the resultant distributions (compare the panels in the first row with those in the second) whereas, the inclusion or not of the latent effect at the municipal level, seems to affect the significance of these coefficients.

Figure 4 presents the description of the variation of the spatial effects α_i, $i = 1, \cdots, n$ under models M_2 and M_3, across the region under study. We notice that under the CAR prior (M_3) the change of the magnitude in these effects is slightly smoother than under the independent prior (M_2).

Figure 5 illustrates the strength of our proposed model. Under model M3, in Panel (a), we have the distribution of the posterior mean of all (approx. 25 thousand) inefficiencies (u_{ij}'s) in the sample. Panel (b) shows the posterior mean of the inefficiencies of all farms (103 in total) located in the municipality of Matupá. This municipality was the one which resulted in the smallest posterior mean of the latent effect α_i. Finally, Panel (c) presents the posterior mean of the inefficiencies of all farms (22 in all) located in the municipality of Cuiabá, the capital of one of the states in the region. Cuiabá had the biggest value of the posterior mean of α_i. The histograms clearly show the effect of the α_i's in the estimation of the inefficiencies and how they allow the distributions to differ across the region. It is worth mentioning that Cuiabá and Matupá are not neighboring municipalities.

6 Concluding Remarks and Future Work

This paper proposed a stochastic frontier model with latent spatial components in the one-sided disturbance term. The model is of particular interest for units which are observed across a geographical region. Inference procedure was conducted under the Bayesian paradigm. Therefore, it was performed in a single framework, taking explicit account of the uncertainty involved in the estimation procedure of the parameters. We also discussed the prior distributions one can consider for these latent effects. We proposed either the assignment of independent zero mean normal distributions or of a nearest neighbour Markov random field model, which carries spatial information and imposes a prior spatial structure on the inefficiencies.

A sample of the outputs of farms spread over the Center West region of Brazil was analyzed. Different model structures were fit to this data set and following two model comparison criteria, the EPD and DIC, both indicated the importance of including the latent effect at the municipal level, even after the inclusion of covariates which vary at the municipal level. In other words, some spatial structure remained that was not completely explained by the covariates we had available for this study.

As an extension of the model proposed here, and following Tsionas (2002) and Gamerman et al. (2002), one might consider models which allow the frontier to vary at the municipal level. In this context we could assign a prior distribution for $\boldsymbol{\beta}$ with a spatial structure. This might be done following the multivariate CAR prior proposed by Gelfand and Vounatsou (2003). This is our current topic of research.

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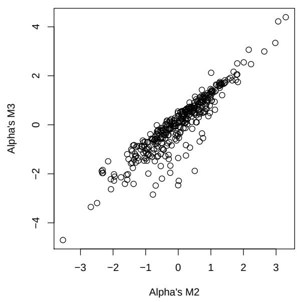
Models with municipality level covariates ($\mathbf{z}_i$)								
			EPD			DIC		
Model	Dist. of u	α Prior	P	G	D	p_D	$\overline{D}$	DIC
M1	N^+	-	13556.87	3115.28	15114.51	15929.65	43814.68	59744.33
M2	N^+	Indep.	12660.47	3070.08	14195.51	15295.18	42499.92	57795.10
M3	N^+	CAR	12529.30	2992.75	14025.68	15630.59	41975.33	57605.92
M4	exp	-	14683.84	4872.77	17120.22	12604.94	47789.63	60394.58
M5	exp	Indep.	13286.06	3824.67	15198.40	13934.53	44391.28	58325.81
M6	exp	CAR	13810.93	4476.45	16049.15	12830.83	46081.97	58912.80
Models without municipality level covariates ($\mathbf{z}_i$)								
			EPD			DIC		
Model	Dist. of u	α Prior	P	G	D	p_D	$\overline{D}$	DIC
$M1_{\overline{z}}$	N^+	-	13888.16	3243.67	15510.00	15795.33	44503.38	60298.70
$M2_{\overline{z}}$	N^+	Indep.	13322.49	3402.87	15023.93	15065.05	43878.90	58943.95
$M3_{\overline{z}}$	N^+	CAR	13325.81	3437.99	15044.81	14979.33	43941.91	58921.24
$M4_{\overline{z}}$	exp	-	14938.05	5006.07	17441.09	12502.50	48291.04	60793.55
$M5_{\overline{z}}$	exp	Indep.	13513.27	3835.90	15431.21	14029.91	44741.48	58771.38
$M6_{\overline{z}}$	exp	CAR	14135.30	4592.52	16431.56	12726.93	46630.87	59357.80
Models without municipality level covariates ($\mathbf{z}_i$) and with random effects								
			EPD			DIC		
Model	Dist. of u	α Prior	P	G	D	p_D	$\overline{D}$	DIC
M7	N^+	Indep.	12857.26	4455.67	15085.10	16482.82	42555.55	59038.36
M8	N^+	CAR	12858.75	5605.43	15661.46	16496.43	42526.44	59022.87
M9	exp	Indep.	13810.81	6028.05	16824.83	13081.53	46245.44	59326.97
M10	exp	CAR	13809.80	7041.22	17330.41	13096.66	46214.57	59311.24

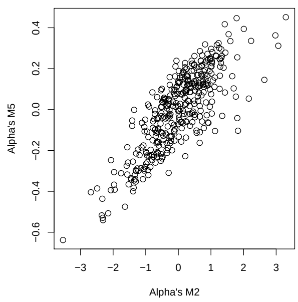
Table 1: Model comparison criteria for each of the sixteen model specifications fitted to the data (see text for details).

Model	σ^2	τ^2	κ	ψ^2
M1	0.3275 (0.3132;0.3411)	1.9597 (1.9006;2.030)	0.8567 (0.8486;0.8651)	-
M2	0.3086 (0.2938;0.3248)	1.9645 (1.8797;2.0526)	0.8640 (0.8540;0.8735)	1.0921 (0.8947;1.3442)
M3	0.3050 (0.2887;0.3214)	1.9607 (1.8556;2.0678)	0.8654 (0.8547;0.8761)	12.7935 (10.1537;15.6325)
M4	0.3853 (0.3725;0.3998)	1.2446 (1.2161;1.2720)	-	-
M5	0.3260 (0.3260;0.3483)	-	-	0.0576 (0.0444;0.0745)
M6	0.3612 (0.3483;0.3735)	-	-	2.0579 (1.6241;2.613)

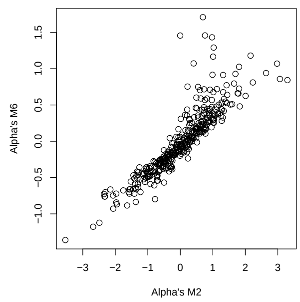
Table 2: Posterior summary, median and 95% credible intervals (in brackets), for σ^2, τ^2, $\kappa = \tau^2/(\tau^2 + \sigma^2)$ and ψ^2 for all fitted models.



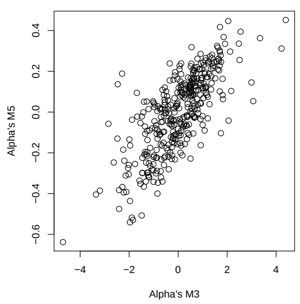
(a) M2 vs M3



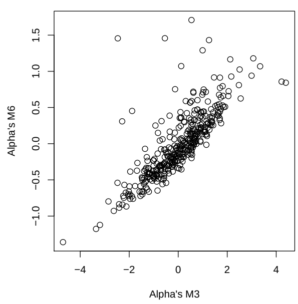
(b) M2 vs M5



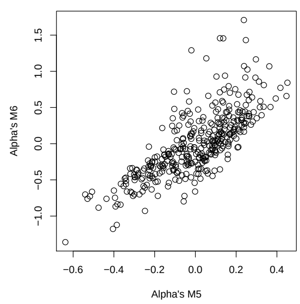
(c) M2 vs M6



(d) M3 vs M5

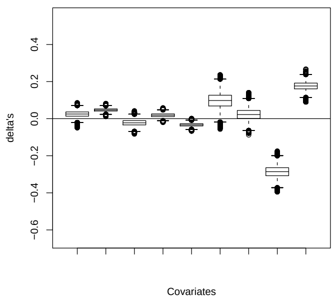


(e) M3 vs M6

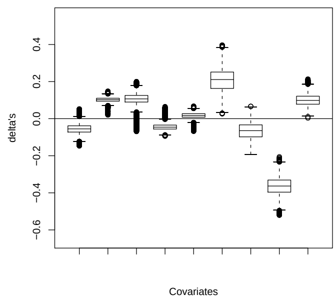


(f) M5 vs M6

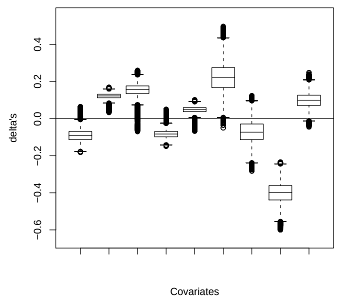
Figure 1: Comparison of the posterior median of the latent effects α_i's among the different fitted models.



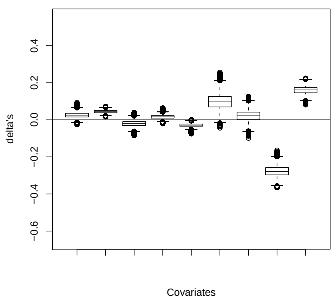
(a) M1



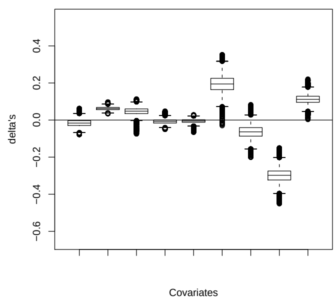
(b) M2



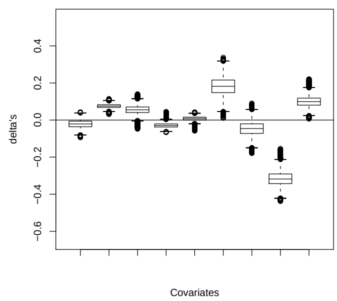
(c) M3



(d) M4

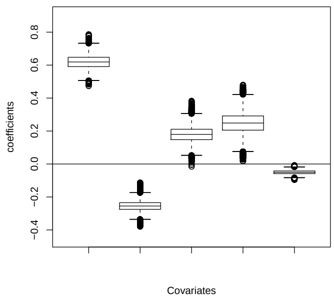


(e) M5

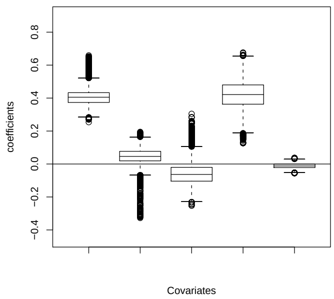


(f) M6

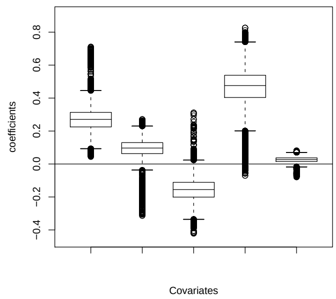
Figure 2: Sample of the posterior of the coefficients, δ's, of the covariates which enter in the model in the municipality level ($\mathbf{Z}_i$).



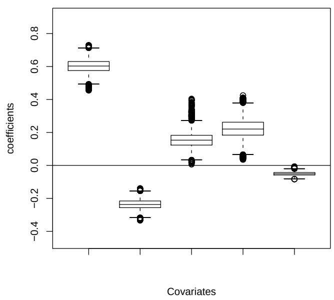
(a) M1



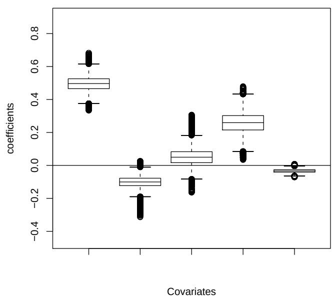
(b) M2



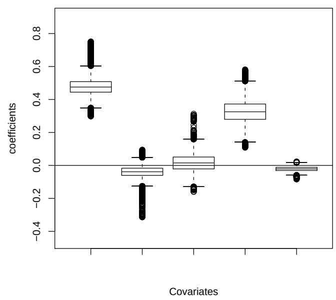
(c) M3



(d) M4

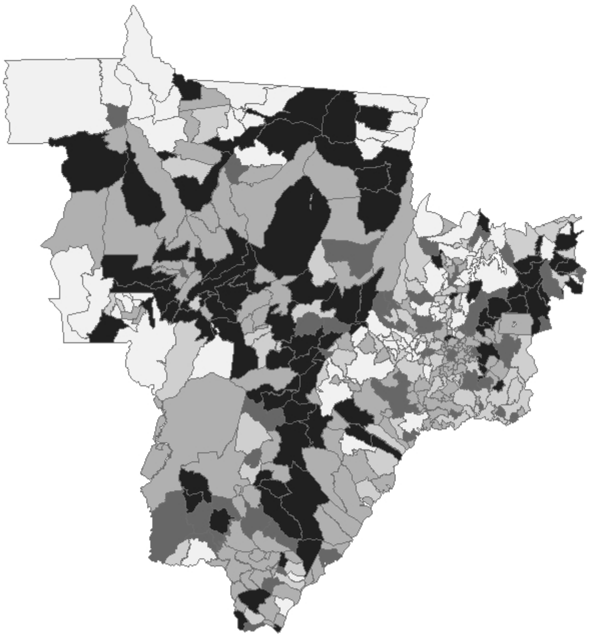


(e) M5

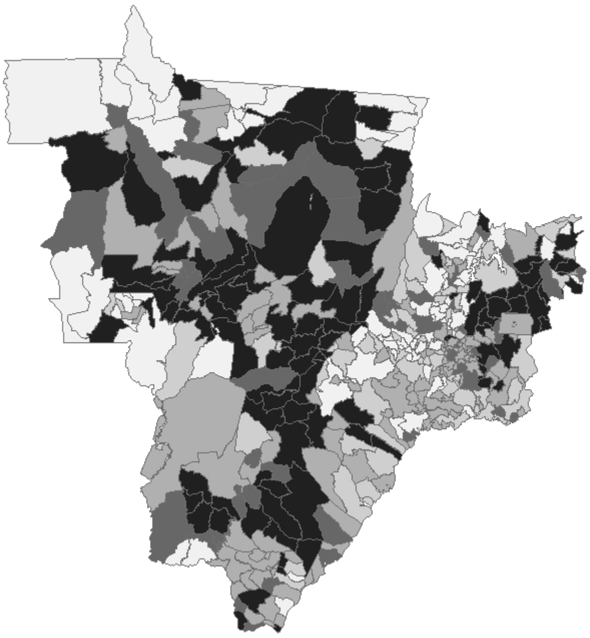


(f) M6

Figure 3: Sample of the posterior of the coefficients of the covariates which define farm's sizes.

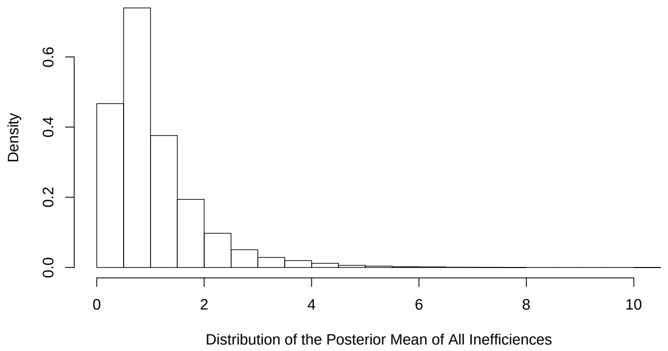


(a) M2

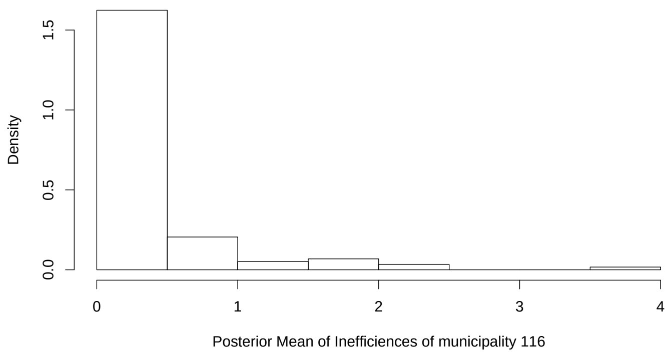


(b) M3

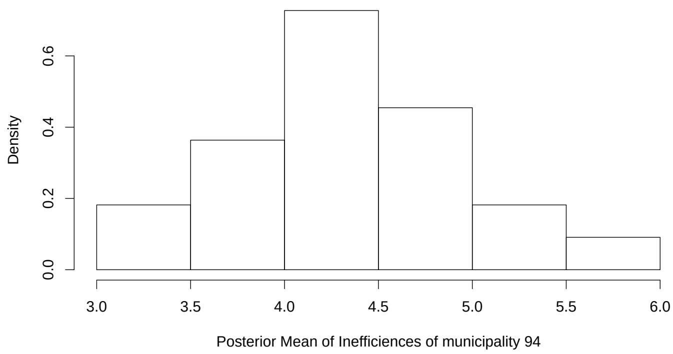
Figure 4: Geographical display of the significance of the spatial effects α_i, $i = 1, \cdots, n$, under models (a) M2 and (b) M3. Darker color indicates positive significance (lower 2 sd credible limit above zero); lighter color: negative significance (higher 2 sd credible limits below zero). The intermediate categories are: lower 1 sd credible limit above zero, credible interval including zero and higher 1 sd credible limit below zero.



(a)



(b)



(c)

Figure 5: Distribution of the posterior mean of the inefficiencies considering: (a) all farms, (b) only farms located in the municipality of Matupá, which resulted in the smallest posterior mean of the latent effect α, (c) only farms located in the municipality of Cuiabá, which resulted in the biggest posterior mean of the latent effect α. These results are under model $M3$.

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