TEXTO PARA DISCUSSÃO N° 1157

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* I am grateful to Marco Bonomo and Maria Cristina Terra for useful comments.

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TEXTO PARA DISCUSSÃO

SUMMARY

SINOPSE

O trabalho desenvolve uma versão para dois países do modelo em Gali & Monacelli (2005), o qual estende para uma pequena economia aberta o modelo de equilíbrio geral dinâmico e estocástico novo-keynesiano usado como ferramenta para análise de política monetária em economias fechadas. Uma importante característica do modelo é que os termos de troca entram diretamente na curva de Phillips novo-keynesiana, como uma nova variável pressionando os custos e alimentando a inflação. Além do mais, a hipótese de *home bias* nas preferências dos consumidores permite a flutuação da taxa de câmbio real, criando um canal alternativo de transmissão da política monetária. Diferente da maior parte da literatura, o modelo deriva a estrutura da pequena economia aberta como um caso limite do modelo para dois países, em vez de supor que as variáveis externas seguem processos exógenos. Esse procedimento preserva o papel das fricções nominais externas na forma como choques monetários internacionais são transmitidos para dentro da pequena economia doméstica.

Two-Country New Keynesian DSGE Model: a Small Open Economy as a Limit Case*

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Abstract

We build a two-country version of the model in Gali & Monacelli (2005), which extends for a small open economy the new Keynesain DSGE model used as tool for monetary policy analysis in closed economies. A distinctive feature of the model is that the terms of trade enters directly into the new Keynesian Phillips curve as a new pushing-cost variable feeding the inflation. Furthermore, home bias in households' preferences allows for real exchange rate fluctuation, giving rise to alternative channels of monetary transmission. Unlike most part of the literature, the small domestic open economy is derived as a limit case of the two-coutry model, rather than assuming exogenous processes for the foreign variables. This procedure preserves the role played by foreign nominal frictions in the way as international monetary policy shocks are conveyed into the small domestic economy.

Key Words: new Keynesian Phillips curve, sticky prices, home bias, open economy

JEL Classification: E32, E52, F41

1 Introduction

Under the New Open Macroeconomic literature, dynamic sthocastic general equilibrium (DSGE) models with imperfect competition and nominal stick-

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iness have been developed for monetary policy analysis in open economies. Built from *first principles,* these models give rise to a macroeconomic dynamics lead by fundamental shocks, at the same time that they preserve the analytical tractability of the traditional Mundell-Fleming approach.

However, a serious failure of the first models is that monetary policy analysis was limited to examine the effects of exogenous monetary shocks on the aggregate macroeconomic variables, so that the interest rate was endogenously determined. More recently, a new generation of models sought to deal more realistically with the way how monetary policy is conducted. These models assume a reaction function for the monetary authority in which the interest rate is the intrumental variable. In this sense, this part of the literature can be regarded as an extension for open economies of the new Keynesian models used to monetary policy analisys in closed economies.

This paper follows this second line of research and builds a two-country version of the model by Gali & Monacelli (2005), which extends for a small open economy the new Keynesian DSGM with Calvo-type staggered price-setting developed initially for closed economies. A distinctive feature of the model is that the terms of trade enters directly into the new Keynesian Phillips curve as a second pushing-cost variable in addition to the output gap, creating in this way a new source of inflationary pressure. Furthermore, real exchange rate fluctuation is embedded into the model by assuming a home bias in households' preferences, so that alternative channels of monetary transmission are in effect.

Most part of the literature built small-open economy models by assuming that foreign variables follow exogenous processes. This paper takes an alternative route and derive the small open economy directly as a limit-case of a two-country model. The advantage of this procedure is that we derive rigorously the small economy as part of a integrated world economy, preserving all international linkages and without taking the risk of setting aside relevant international channels of monetary transmission.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 builds impulse-response functions for the small-country case using calibrated parameters for the Brazilian economy. Section 4 concludes.

2 Model

The world is inhabited by a continuum of infinite-lived households, indexed by $j \in [0,1]$. Each household lives in one of two countries: households on the interval $[0,n)$ live in the Home country, while households on the interval $[n,1]$ live in the Foreign country. The parameter n measures the relative size of the Home country. The small Home country case can be derived by taking the limit of the two-country model as $n \longrightarrow 0$.

Each household owns a competitive-monopolistic firm producing a differentiated good. Thus, there is also a continuum of firms indexed by $i \in [0,1]$, such that firms on the interval $[0,n)$ are located in the Home country, while firms on the interval $[n,1]$ are located in the Foreign country. Firms use only labor for production and there is no investment. Households' labor supply reacts elastically to real wage. Labor market is competitive and internationally segmented.

All Home and Foreign goods are tradable and the law of one price (LOP) holds for all of them. Therefore, the model sets aside the effects of non-tradability and international market segmentation on the real exchange rate fluctuation. In this sense, the only reason for PPP (purchase power parity) violation is the introduction of a *home bias* in households' preferences. In addition, prices are set in the producer's currency, so that we have complete pass-through for the a limit case of a small economy.

There is a set of states of nature S_t for each period t, where $s_t \in S_t$. We denote by H_t the set of all histories $h_t \equiv (s_0, s_1, ..., s_t)$ between the initial period 0 and period t. In particular, $H_0 = S_0 = \{s_0\}$. Let $\Pr_t(h_{t+s})$ be the probability of history h_{t+s} conditioned on period t being reached through some history h_t. We denote by $H_{t,t+s}$ the set of all histories $h_{t+s} \in H_{t+s}$ such that $\Pr_t(h_{t+s}) > 0$. The international financial market structure is complete: every period t, there is market for any Arrow-Debrew security paying a unit of home currency at period t+s in the contingency of some history $h_{t+s} \in H_{t,t+s}$. We denote by $X(h_{t+s})$ the realization of the random variable X_{t+s} at period t+s if history h_{t+s} takes place.

We derive the general equilibrium solution for the log-linearized form of the model around the steady state, in which all shocks get constant in their long-run equilibrium levels. For convenience and without loss of generality, the log-linearization is carried out for the particular case of a symmetric two-country world. This procedure is common in the literature as it makes the log-linearization of the model much easier.

Throughout the paper, we derive the equilibrium conditions just for the Home country's agents, while the corresponding ones for the Foreign country are shown only if necessary. Starred variables refer to the Foreign country and lowercase variables are in log.

2.1 Households

This section describes the optimazing behavior of households.

2.1.1 Preferences

All households living in a same country have identical preferences and initial wealth. With complete financial markets, this assumption allows us to focus on the problem of a representative household for each country, no matter how the ownership of the firms located in a same country is shared among its households. In this sense, a typical Home household j maximizes the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{1}{1-\sigma} \left(C_t^j \right)^{1-\sigma} - \frac{1}{1+\varphi} \left(L_t^{sj} \right)^{1+\varphi} \right], \tag{1}$$

where β is the intertemporal discount factor, φ is the inverse of the wage-elasticity of the labor supply L_t^{sj} and σ is the inverse of the elasticity of intertemporal substitution of consumption C_t^j. As usual, we assume that $0 < \beta < 1$, $\varphi > 1$ and $\sigma > 0$.

The variable C_t^j is defined as the CES composite consumption index

$$C_t^j = \left[(1-\alpha)^{\frac{1}{\mu}} C_{H,t}^{j\,\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} C_{F,t}^{j\,\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \tag{2}$$

where μ is the elasticity of intratemporal substitution between a *bundle* of Home goods $C_{H,t}^j$ and a *bundle* of Foreign goods $C_{F,t}^j$, while α determines the share of the imported (Foreign) goods on the Home household j's consumption expenditure and, as we will see below, is inversely related to the degree of home bias. We assume that $0 < \alpha < 1$ and $\mu > 1$.[1]

[1] With μ very close to 1, the parameter α is exactly equal to the share of the imported (Foreign) goods in the Home household j's consumption expenditure.

The variables $C^j_{H,t}$ and $C^j_{F,t}$ are defined respectively by the CES composite consumption indexes

$$C^j_{H,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n C^j_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

$$C^j_{F,t} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 C^j_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where $C^j_{H,t}(i)$ and $C^j_{F,t}(i)$ are respectively the Home household j's consumption levels of Home good i, with $i \in [0, n)$, and Foreign good i, with $i \in [n, 1]$. The parameter ε is the elasticity of intratemporal substitution among goods produced in a same country. We assume that $\varepsilon > 0$.

The Foreign households' preferences are the same, except for eq.(2), which assumes the form

$$C^{j*}_t = \left[\alpha^{*\frac{1}{\mu}} C^{j*\frac{\mu-1}{\mu}}_{H,t} + (1-\alpha^*)^{\frac{1}{\mu}} C^{j*\frac{\mu-1}{\mu}}_{F,t} \right]^{\frac{\mu}{\mu-1}},$$

where α^* is the share of the imported (Home) goods on the Foreign household j's consumption expenditure, with $\alpha^* \neq \alpha$ in general.

2.1.2 Intratemporal Consumption Choice

The Home household j takes as given the Home-currency market price of all Home and Foreign goods, denoted respectively by $P_{H,t}(i)$, with $i \in [0, n)$, and $P_{F,t}(i)$, with $i \in [n, 1]$. Thus, for any fixed levels of $C^j_{H,t}$ and $C^j_{F,t}$, the optimal $C^j_{H,t}(i)$ and $C^j_{F,t}(i)$ are given respectively by

$$C^j_{H,t}(i) = \frac{1}{n} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C^j_{H,t}, \tag{5}$$

$$C^j_{F,t}(i) = \frac{1}{1-n} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C^j_{F,t}, \tag{6}$$

where $P_{H,t}$ and $P_{F,t}$ are the Home-currency price indexes of the goods produced in Home country (a domestic price index) and $P_{F,t}$ is the Home-currency price index of the goods imported from the Foreign country, given respectively by[2]

$$P_{H,t} \equiv \left(\frac{1}{n}\int_0^n P_{H,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}, \quad (7)$$

$$P_{F,t} \equiv \left(\frac{1}{1-n}\int_n^1 P_{F,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}. \quad (8)$$

How are $C_{H,t}^j$ and $C_{F,t}^j$ chosen? Given $P_{H,t}$ and $P_{F,t}$ derived in the problem above and given also the Home household j's choice of C_t^j derived in the intertemporal problem below, the optimal consumption allocation between Home and Foreign goods is given by

$$C_{H,t}^j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t^j, \quad (9)$$
$$C_{F,t}^j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t^j,$$

where P_t is the Home consumer price index (CPI), given by[3]

$$P_t = \left[(1-\alpha)\,P_{H,t}^{1-\mu} + \alpha P_{F,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}. \quad (10)$$

Aggregating intratemporal choice Combining eqs.(5) and (6) with the definitions below, the Home aggregate demand for the Home good i, with

[2]The optimal choice (5) is the solution of the optimization problem $\min_{C_{H,t}^j(i)} \int_0^n P_{H,t}(i)\,C_{H,t}^j(i)\,di$ subject to the constraint given by eq.(3). In addition, by substituting (5) into the function minimized above, we get $\int_0^n P_{H,t}(i)\,C_{H,t}^j(i) = P_{H,t}C_{H,t}^j$. An analogous result holds the eq.(6).

[3]The optimal choices in (9) and (10) are the solution of the optimization problem $\min_{C_{H,t}^j, C_{F,t}^j} P_{H,t}C_{H,t}^j + P_{F,t}C_{F,t}^j$ subject to the constraint given by eq.(2). In addition, by substituting (9) and (10) into the function minimized above, we get $P_{H,t}C_{H,t}^j + P_{F,t}C_{F,t}^j = P_tC_t^j$.

$i \in [0,n)$, and Foreign good i, with $i \in [n,1]$, denoted respectively by $C_{H,t}(i)$ and $C_{F,t}(i)$, are given by

$$C_{H,t}(i) \equiv \int_0^n C_{H,t}^j(i)\,dj = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, \tag{11}$$

$$C_{F,t}(i) \equiv \int_0^n C_{F,t}^j(i)\,dj = \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}, \tag{12}$$

whereas, by substituting eqs.(9) and (10) into the definitions below for $C_{H,t}$ and $C_{F,t}$, we have that

$$C_{H,t} \equiv \int_0^n C_{H,t}^j dj = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t, \tag{13}$$

$$C_{F,t} \equiv \int_0^n C_{F,t}^j dj = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t, \tag{14}$$

where, since there is a representative agent for each country,

$$C_t \equiv \int_0^n C_t^j dj = nC_t^j. \tag{15}$$

In addition, we can prove that $P_tC_t = P_{H,t}C_{H,t}+P_{F,t}C_{F,t}$, where $P_{H,t}C_{H,t} = \int_0^n P_{H,t}(i)\,C_{H,t}(i)\,di$ and $P_{F,t}C_{F,t} = \int_n^1 P_{F,t}(i)\,C_{F,t}(i)\,di$, so that P_tC_t is the Home country's aggregate consumption expenditure, while $P_{H,t}C_{H,t}$ and $P_{F,t}C_{F,t}$ are respectively the Home country's aggregate consumption expenditure with Home goods and Foreign goods.[4] Note that the share of the imported (Foreign) goods in P_tC_t rises with the parameter α and fall with its relative price.

Foreign country Analogous results hold for the Foreign country, so that the Foreign aggregate consumption levels of the Home and Foreign good

[4]To derive the expression for $P_{H,t}C_{H,t}$, we multiply both sides of eq.(11) by $P_{H,t}(i)$, sum across all Home households and use eq.(7). An analogous derivation applies to get the expressions for $P_{F,t}C_{F,t}$ and P_tC_t.

i are given by

$$C^*_{H,t}(i) \equiv \int_n^1 C^{j*}_{H,t}(i)\,dj = \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} C^*_{H,t}, \tag{16}$$

$$C^*_{F,t}(i) \equiv \int_n^1 C^{j*}_{F,t}(i)\,dj = \frac{1}{1-n}\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\varepsilon} C^*_{F,t}, \tag{17}$$

where $P^*_{H,t}$ and $P^*_{F,t}$ are the Foreign-currency price indexes of Home and Foreign goods, given by

$$P^*_{H,t} \equiv \left(\frac{1}{n}\int_0^n P^*_{H,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}; \tag{18}$$

$$P^*_{F,t} \equiv \left(\frac{1}{1-n}\int_n^1 P^*_{F,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}, \tag{19}$$

while $C^*_{H,t}$ are $C^*_{F,t}$ are the Foreign composite indexes of Home and Foreign goods, given by

$$C^*_{H,t} \equiv \int_n^1 C^{j*}_{H,t}dj = \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\mu} C^*_t, \tag{20}$$

$$C^*_{F,t} \equiv \int_n^1 C^{j*}_{F,t}dj = (1-\alpha^*)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\mu} C^*_t, \tag{21}$$

where, since there is a representative agent for each country,

$$C^*_t \equiv \int_n^1 C^{j*}_t dj = (1-n)C^{j*}_t \tag{22}$$

and the Foreign CPI index is given by

$$P^*_t = \left[\alpha^* P^{*1-\mu}_{H,t} + (1-\alpha^*)\,P^{*1-\mu}_{F,t}\right]^{\frac{1}{1-\mu}}. \tag{23}$$

Home bias It is crucial to understand how the parameter α (α^*) is related to the degree of home bias in Home (Foreign) households' preferences, since home bias is the *only* source of real exchange rate fluctuation in the model. For that, suppose without loss of generality that $P_{H,t} = P_{F,t}$ $\left(P^*_{H,t} = P^*_{F,t}\right)$. In this case, it follows from (10) ((23)) and (14) ((20)) that

α (α^*) is exactly equal to the share of imported goods in Home (Foreign) consumption expenditure. Thus, it is intuitive that α (α^*) should fall with the relative size of the Home (Foreign) country and with the degree of home bias in Home (Foreign) households' preferences. A tractable way to formalize these ideas is to define $\alpha \equiv \bar{\alpha}(1-n)$ ($\alpha^* \equiv \bar{\alpha}^* n$), where the parameter $\bar{\alpha}$ ($\bar{\alpha}^*$), given exogenously in the model, is inversely related to the degree of home bias in Home (Foreign) households' preferences. For example, if the reason for home bias is international trade barries, $\bar{\alpha}$ ($\bar{\alpha}^*$) can be interpreted as an index of openness for the Home (Foreign) country. The particular case for fully opened countries is with $\bar{\alpha} = \bar{\alpha}^* = 1$, so that $\alpha = n$ ($\alpha^* = 1-n$). There is no home bias in this case, since the share of imported goods for each country is naturally given by its relative size. On the other hand, the particular case for fully closed countries is with $\bar{\alpha} = \bar{\alpha}^* = 0$, so that $\alpha = \alpha^* = 0$. For the reason explained above, we assume the symmetric case with $\bar{\alpha} = \bar{\alpha}^*$.

World aggregate demand The LOP holds for all goods, so that $P_{H,t}(i) = \varepsilon_t\ P^*_{H,t}(i)$ for $i \in [0,n)$ and $P_{F,t}(i) = \varepsilon_t P^*_{F,t}(i)$ for $i \in [n,1]$, where ε_t is the nominal exchange rate (Home-currency price of one unit of the Foreign currency). Substituting these results into eqs.(7) and (8), we get $P_{H,t} = \varepsilon_t\ P^*_{H,t}$ and $P_{F,t} = \varepsilon_t\ P^*_{F,t}$. Therefore, summing eqs.(11) ((12)) and (16) (17) and using again the result (7) ((8)) and the LOP, the Home (Foreign) good i's world aggregate demand, denoted by $Y^d_t(i)$ $\left(Y^{d*}_t(i)\right)$, is given by

$$Y^d_t(i) \equiv C_{H,t}(i) + C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(C_{H,t} + C^*_{H,t}\right), \tag{24}$$

$$Y^{d*}_t(i) \equiv C_{F,t}(i) + C^*_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} \left(C_{F,t} + C^*_{F,t}\right). \tag{25}$$

2.1.3 Intertemporal Consumption Choice

Given the CPI index P_t derived above, the period budget constraint of the Home household j is written as

$$C^j_t + x^j_t \frac{V_t}{P_t} + \sum\nolimits_{s=1}^{\infty} \sum\nolimits_{h_{t+s} \in H_{t,t+s}} \frac{Z_t(h_{t+s})}{P_t} B^j_t(h_{t+s}) \tag{26}$$

$$= \frac{W}{P_t} L^{sj}_t + x^j_{t-1} \frac{DV_t}{P_t} + x^j_{t-1} \frac{V_t}{P_t} + \sum\nolimits_{s=0}^{\infty} \sum\nolimits_{h_{t+s} \in H_{t,t+s}} \frac{Z_t(h_{t+s})}{P_t} B^j_{t-1}(h_{t+s}),$$

where W_t is the Home nominal wage, V_t is the aggregate nominal value of the Home firms, DV_t is total nominal dividends paid out by all Home firms, x_t^j is the Home household j's share on the Home firms's ownership, $B_t^j(h_{t+s})$ is the Home household j's holdings of the history h_{t+s}-contingent claim security and $Z_t(h_{t+s})$ is the nominal price of this asset. All nominal values are measured in Home currency. The corresponding equation to the Foreign household is analogous to the eq.(26), except for the fact that the contingent claims are denominated in Home currency.

Home country's Euler equation and SDF Using eq.(26) to substitute for C_t^j into function (1), the optimal consumption allocation between t and t+s, reached through the history $h_{t+s} \in H_{t,t+s}$, meets the marginal condition with respect to $B_t^j(h_{t+s})$

$$\frac{Z_t(h_{t+s})}{C_t^{j\sigma} P_t} = \beta^s E_t \left[\frac{Z_{t+s}(h_{t+s})}{C_{t+s}^{j\sigma} P_{t+s}} \right] = \frac{\beta^s \Pr_t(h_{t+s})}{C^j(h_{t+s})^{\sigma} P(h_{t+s})}. \tag{27}$$

Now, we derive the s-period stochastic discount factor (SDF) for the Home country, denoted by $D_{t,t+s}$, since this variable allows us to get the Home-currency price X_t of any financial asset with Home-currency pay-off $X(h_{t+s})$ in the contingency of history $h_{t+s} \in H_{t,t+s}$ through the condition $X_t = E_t[D_{t,t+s}X_{t+s}]$. We know from the literature in Finance that complete financial markets imply that it exists a unique and strictly positive SDF given by $D_t(h_{t+s}) = \frac{Z_t(h_{t+s})}{\Pr_t(h_{t+s})}$, where $D_t(h_{t+s})$ is the realization of the random variable $D_{t,t+s}$ if history $h_{t+s} \in H_{t,t+s}$ takes place. As a result, it follows from this definition and from the condition (27) that

$$D_t(h_{t+s}) = \beta^s \left(\frac{C^j(h_{t+s})}{C_t^j} \right)^{-\sigma} \frac{P_t}{P(h_{t+s})}. \tag{28}$$

In the particular case of a one-period zero cupon bond (denominated in Home currency), where s=1 and $X(h_{t+s}) \equiv 1$, we have that its price is given by $\frac{1}{R_t} = \beta E_t \left[\left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]$, where R_t is the one-period nominal spot interest rate. Log-linearizing this equation around the steady state for the symmetric case defined below and using the result (15), we get that the Home aggregate consumption (in log) meets the Euler equation

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(r_t - E_t[\pi_{t+1}] + \ln \beta), \tag{29}$$

where $\pi_t \equiv p_t - p_{t-1}$ is the Home consumer price index (CPI) inflation.

Foreign country's Euler equation Proceeding analogously with the Foreign country, we have that

$$\frac{Z_t(h_{t+s})}{C_t^{j*\sigma} P_t^* \varepsilon_t} = \beta^s E_t \left[\frac{Z_{t+s}(h_{t+s})}{C_{t+s}^{j*\sigma} P_{t+s}^* \varepsilon_{t+s}} \right] = \frac{\beta^s \Pr_t(h_{t+s})}{C^{j*}(h_{t+s})^\sigma P^*(h_{t+s}) \varepsilon(h_{t+s})}. \quad (30)$$

As the contingent claims are denominated in Home currency, the s-period SDF for the Foreign country is given by

$$D_t^*(h_{t+s}) \equiv \frac{Z_t(h_{t+s}) \varepsilon(h_{t+s})}{\varepsilon_t \Pr_t(h_{t+s})} = \beta^s \left(\frac{C^{j*}(h_{t+s})}{C_t^{j*}} \right)^{-\sigma} \frac{P_t^*}{P^*(h_{t+s})},$$

which allows us to get the Foreign-currency price X_t^* of any financial asset with Foreign-currency pay-off $X^*(h_{t+s})$ in the contingency of history $h_{t+s} \in H_{t,t+s}$ through the condition $X_t^* = E_t \left[D_{t,t+s}^* X_{t+s}^* \right]$. As in Home country, the Foreign aggregate consumption (in log) meets the Euler equation

$$c_t^{j*} = E_t \left[c_{t+1}^{j*} \right] - \frac{1}{\sigma} \left(r_t^* - E_t \left[\pi_{t+1}^* \right] + \ln \beta \right), \quad (31)$$

where r_t^* is the log of the one-period spot rate (denominated in Foreign currency) and $\pi_t^* \equiv p_t^* - p_{t-1}^*$ is the Foreign consumer price index (CPI) inflation.

2.1.4 Aggregate Labor Supply

The Home and Foreign j_{th} households' marginal conditions with respect to labor supply are given by $\left(C_t^j\right)^\sigma \left(L_t^{sj}\right)^\varphi = \frac{W_t}{P_t}$, with $j \in [0, n)$, and $\left(C_t^{j*}\right)^\sigma \left(L_t^{sj*}\right)^\varphi = \frac{W_t^*}{P_t^*}$, with $j \in [n, 1]$. Using these conditions to substitute for L_t^{sj} and L_t^{sj*} into the definitions below and using eqs.(15) and (22) and the fact that it exists a representative agent for each country, we have that the Home and Foreign aggregate labor supplies are given by

$$L_t^s \equiv \int_0^n L_t^{sj} dj = n L_t^{sj} = n^{\frac{\sigma}{\varphi}+1} \left(\frac{W_t}{P_t} \right)^{\frac{1}{\varphi}} C_t^{-\frac{\sigma}{\varphi}}, \quad (32)$$

$$L_t^* \equiv \int_n^1 L_t^{sj*} dj = (1-n) L_t^{sj*} = (1-n)^{\frac{\sigma}{\varphi}+1} \left(\frac{W_t^*}{P_t^*} \right)^{\frac{1}{\varphi}} C_t^{*-\frac{\sigma}{\varphi}}. \quad (33)$$

2.1.5 Inflation, Terms of Trade and Real Exchange Rate

This subsection derives the relationship between inflation, terms of trade and real exchange rate. The Home country's terms of trade, defined as $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$, is the relative price of the imported goods (Foreign goods' bundle) in terms of the domestic goods (Home goods' bundle) in the Home market.[5] Dividing the Home (Foreign) CPI index in eq.(10) ((23)) by $P_{H,t}$ $(P^*_{H,t})$ and $P_{F,t}$ $(P^*_{F,t})$ and using the LOP and the definition of terms of trade, we get

$$\frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha S_t^{1-\mu}\right]^{\frac{1}{1-\mu}} \equiv g\left(S_t\right), \tag{34}$$

$$\frac{P_t}{P_{F,t}} = \frac{P_t}{P_{H,t}} \frac{P_{H,t}}{P_{F,t}} = \frac{g\left(S_t\right)}{S_t} \equiv h\left(S_t\right), \tag{35}$$

$$\frac{P^*_t}{P^*_{H,t}} = \left[\alpha^* + \left(1-\alpha^*\right) S_t^{1-\mu}\right]^{\frac{1}{1-\mu}} \equiv g^*\left(S_t\right), \tag{36}$$

$$\frac{P^*_t}{P^*_{F,t}} = \frac{P^*_t}{P^*_{H,t}} \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{g^*\left(S_t\right)}{S_t} \equiv h^*\left(S_t\right), \tag{37}$$

where $g'\left(S_t\right) > 0$, $h'\left(S_t\right) < 0$, $g^{*\prime}\left(S_t\right) > 0$ and $h^{*\prime}\left(S_t\right) < 0$. Log-linearizing eqs.(34) and (37) around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$, we get

$$\pi_t = \pi_{H,t} + \bar{\alpha}(1-n)\Delta s_t, \tag{38}$$

$$\pi^*_t = \pi^*_{F,t} - \bar{\alpha} n \Delta s_t, \tag{39}$$

where the Home and Foreign domestic inflation (i.e., the change of the price index of the goods produced domestically) are given by $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi^*_{F,t} \equiv p^*_{F,t} - p^*_{F,t-1}$. Equation (38) tells us that the effect of a change in the Home country's terms of trade on the gap between the Home CPI and domestic inflation rates increases with the weight of the imported (Foreign) goods in the Home households' preferences, given by $\alpha = \bar{\alpha}(1-n)$, which in turn decreases with the relative size of the Home country and with the degree of home bias, inversely related to $\bar{\alpha}$.[6] The same argument is true for the Foreign country's counterpart in eq.(39). In the particular case of closed

[5] It follow from the LOP that $S^*_t \equiv \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{1}{S_t}$.

[6] This result is very intuitive when we note that home bias is inversely related to the degree of openness of the countries.

countries, when $\bar{\alpha} = 0$, we get $p_t = p_{H,t}$ and $p_t^* = p_{F,t}^*$. In the particular case of a small Home country, when n is very close to 0, we get $p_t^* = p_{F,t}^*$.

Another important result follows from combining the LOP and the results (34) and (36) with the definition of Home country's real exchange rate, denoted by Q_t, so that

$$Q_t \equiv \frac{\varepsilon_t P_t^*}{P_t} = \frac{g^*(S_t)}{g(S_t)}, \tag{40}$$

where Q_t is a increasing function of S_t. Log-linearizing eq.(40) around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$, we get

$$q_t = (1 - \bar{\alpha}) s_t. \tag{41}$$

Equation (41) tells us that home bias is the only source of PPP violation. Note that $q_t = 0$ every period when there is no home bias ($\bar{\alpha} = 1$). The real exchange rate's volatility increases with the degree of home bias and with the terms of trade's volatility. Although the LOP holds for all goods individually, the real exchange rate q_t is directly related to the terms of trade s_t, which fluctuates over time in response to shocks on both countries. The intuition behind this result is that P_t and P_t^* are consumer-based price indexes, while home bias implies that Foreign (Home) preference puts a higher weight on Foreign (Home) goods than Home (Foreign) preference does.

2.1.6 International Risk Sharing

Setting t=0 and s=t, eqs.(27) and (30) can be rewritten as

$$\frac{\beta^t \Pr_0(h_t) C_0^{j\sigma}}{C^j(h_t)^\sigma} = \frac{Z_0(h_t) P(h_t)}{P_0}, \tag{42}$$

$$\frac{\beta^t \Pr_0(h_t) C_0^{j*\sigma}}{C^{j*}(h_t)^\sigma} = \frac{Z_0(h_t) P^*(h_t) \varepsilon(h_t)}{P_0^* \varepsilon_0} = \frac{Q(h_t)}{Q_0} \frac{Z_0(h_t) P(h_t)}{P_0}. \tag{43}$$

Note that $Q(h_t) \equiv \frac{P^*(h_t)\varepsilon(h_t)}{P(h_t)}$ in eq.(43) by the definition for real exchange rate. Substituting eq.(42) into eq.(43), we get the international risk sharing (IRS) condition

$$C_t^j = \vartheta Q_t^{\frac{1}{\sigma}} C_t^{j*}, \tag{44}$$

where $\vartheta \equiv Q_0^{-\frac{1}{\sigma}} \frac{C_0^j}{C_0^{j*}}$. We omit h_t because this condition holds for any history h_t. Equation (44) tells us that home bias allows for a variable gap between the Home and Foreign households' consumption (per-capita consumption) growth rates, even if international financial market structure is complete. This is because home bias makes shocks on terms of trade to cause real exchange rate depreciation, which in turn, according to eqs.(42) and (43), gives rise to a gap between the Home and Foreign's intertemporal relative price of consumption.

Combining eq.(44) with eqs.(15) and (22), we get an aggregate version of the IRS condition, given by

$$C_t = \frac{n}{1-n} \vartheta Q_t^{\frac{1}{\sigma}} C_t^*. \tag{45}$$

Following a general procedure in the literature, we assume the same initial conditions for Home and Foreign households, so that $\vartheta = 1$. Log-linearizing eq.(45) around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$ and $\vartheta = 1$, we get

$$c_t = \ln \left\{ \frac{n}{1-n} \right\} + \frac{1}{\sigma} q_t + c_t^*. \tag{46}$$

The ratio between the Home and Foreign aggregate consumption levels collapses to zero as Home country becomes a small economy.

2.2 Firms

This section describes the firms' optimazing behavior. There is a continuum of firms indexed by $i \in [0, 1]$, such that firms on the interval $[0, n)$ are located in the Home country, while firms on the interval $[n, 1]$ are located in the Foreign country. When allowed, each monopolistic-competitive firm must set the relative price of its differentiated good, faced with a isoelastic and downward-sloping demand curve and subject to a technological constraint. Firms use only a homogeneous type of labor for production and there is no investment. Labor market is competitive.

2.2.1 Technology and Cost Minimization

All Home firms operate an identical CRS technology

$$Y_t(i) = A_t L_t(i),$$

where $Y_t(i)$ is the Home firm i's output, $L_t(i)$ is the Home firm i's labor demand and A_t is the Home total factor productivity shifter, which follows the AR(1) process

$$a_t \equiv \ln A_t = \rho a_{t-1} + \xi_t, \tag{47}$$

where $0 < \rho < 1$ and ξ_t are i.i.d Gaussian shocks. All Foreign firms operate a similar technology, except for the fact that $\rho \neq \rho^*$ is possible. The shocks ξ_t and ξ_t^* may be correlated.

The technology constraints imply that the Home and Foreign i_{th} firms' labor demands are given respectively by

$$L_t(i) = \frac{Y_t(i)}{A_t}, \tag{48}$$

$$L_t^*(i) = \frac{Y_t^*(i)}{A_t^*}, \tag{49}$$

so that the Home and Foreign nominal marginal costs are given by $MC_t^n = \frac{W_t}{A_t}$ and $MC_t^{n*} = \frac{W_t^*}{A_t^*}$, while the Home and Foreign real marginal costs are defined as

$$MC_t \equiv \frac{MC_t^n}{P_{H,t}} = \frac{W_t}{A_t P_{H,t}}, \tag{50}$$

$$MC_t^* \equiv \frac{MC_t^{n*}}{P_{F,t}^*} = \frac{W_t^*}{A_t^* P_{F,t}^*}. \tag{51}$$

2.2.2 Aggregate Labor Demand

Substituting eqs.(48) and (49) respectively into the definitions below for the Home and Foreign aggregate labor demands, we get

$$L_t \equiv \int_0^n L_t(i)\, di = \frac{Y_t}{A_t} U_t, \tag{52}$$

$$L_t^* \equiv \int_n^1 L_t^*(i)\, di = \frac{Y_t^*}{A_t^*} U_t^*, \tag{53}$$

where Y_t and Y_t^* are the Home and Foreign aggregate output indexes, defined as

$$Y_t \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{54}$$

$$Y_t^* \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 Y_t^*(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{55}$$

while $U_t \equiv \int_0^n \frac{Y_t(i)}{Y_t} di$ and $U_t^* \equiv \int_n^1 \frac{Y_t^*(i)}{Y_t^*} di$ measure respectively the dispersion of the Home and Foreign firms' output. In the flexible-price particular case, in which all firms located in a same country have the same output, we have that $U_t = U_t^* = 1$.

2.2.3 Price-Setting

Firms set prices in a staggering way, as in Calvo (1983): every period, a mesure of $1-\phi$ randomly selected firms set a new price, with an individual firm's probability of readjusting each period being independent of the time elapsed since it last reset its price. A Home firm i adjusting price in period t set a new price $\bar{P}_{H,t}$ in order to maximize the present value of its stream of expected future profits (dividends)[7], which is given by

$$V_t(i) = \sum\nolimits_{s=0}^{\infty} E_t \left[D_{t,t+s} DV_{t+s}(i) \right], \tag{56}$$

where $D_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+s}}$ is the s-period Home SDF at period t, $DV_{t+s}(i) = \left[P_{H,t+s}(i) - MC_{t+s}^n \right] Y_{t+s}^d(i)$ is the profit at period t+s and $Y_{t+s}^d(i)$ is the world demand for Home good i at period t+s, given in eq.(24)[8].

Log-linearizing the solution of this problem around the steady state, in

[7] As managers act in the interest of the owners, this decision is consequence of the optimizing behavior of the households, which have their intertemporal budget constraints relaxed by an increase of the firms' stream of profits present value.

[8] We assume that firms not adjusting prices meet the demand. This decision is optimal if shocks on the productivity shifter a_t are not so adverse to push the marginal cost above the price. Therefore, the markets of all Home goods clear in equilibrium, i.e., $Y_t(i) = Y_t^d(i)$. This result, along with the technology constraint and the definition of nominal marginal cost, is what to allows us to say that the Home firm i's profit at period t+s is given by $DV_{t+s} = P_{H,t+s}(i) Y_{t+s}(i) - W_{t+s} N_{t+s}(i) = \left[P_{H,t+s}(i) - MC_{t+s}^n \right] Y_{t+s}^d(i)$.

which inflation is zero, we get[9]

$$\begin{aligned}\bar{p}_{H,t} &= \psi + (1-\phi\beta)\sum\nolimits_{s=0}^{\infty}(\phi\beta)^s E_t\left[mc_{t+s}^n\right] \\ &= (1-\phi\beta)\sum\nolimits_{s=0}^{\infty}(\phi\beta)^s E_t\left[\psi + mc_{t+s}^n\right] \\ &= (1-\phi\beta)(\psi + mc_t^n) + \phi\beta E_t\left[\bar{p}_{H,t+1}(i)\right], \qquad (57)\end{aligned}$$

where $\psi \equiv \ln\frac{\varepsilon}{\varepsilon-1}$ is the gross markup (in log) in the steady state and in the flexible-price case. The firm sets the new price as a markup over the weighted average of the current and expected future nominal marginal costs. This forward-looking behavior is because the firm recognizes that this price will be effective for a random number of periods. The expected one-period ahead optimal price, given by the term $E_t\left[\bar{p}_{H,t+1}\right]$, embeds all information as to the marginal costs in future periods. As the firm faces a isoelastic demand curve, it does not readjust price in response to a shift in this curve if it is not accompannied by a change in current or expected future marginal costs. This result implies that inflationary pressures must have a cost-pushing origin,

[9]Deriving an expression for $E_t\left[D_{t,t+s}DV_{t+s}(i)\right]$ is not trivial since $DV_{t+s}(i)$ depends on $\bar{P}_{H,t}$ only if there was no other adjustment until t+s. With this purpose, let us define the indicator function g_{t+s} for $s \geqslant 0$, such that $g_{t+s}=1$ if $DV_{t+s}(i)$ does depend on $\bar{P}_t$ and $g_{t+s}=0$ if $DV_{t+s}(i)$ does not. Note that $g_{t+s}=1$ if s=0 or if price has not been adjusted again between t+1 and t+s. Otherwise, $g_{t+s}=0$. Therefore, $\Pr\left[g_{t+s}=1\right]=\phi^s$ and, using familiar results of condicional probability, we get

$$E_t\left[D_{t,t+s}DV_{t+s}(i)\right] = \phi^s E_t\left[D_{t,t+s}DV_{t+s}(i)\,|g_{t+s}=1\right] + (1-\phi^s)E_t\left[D_{t,t+s}DV_{t+s}(i)\,|g_{t+s}=0\right].$$

As only the first term of the right-hand side of the equation above depend on $\bar{P}_{H,t}$, the problem (56) can be rewritten as

$$\max_{\bar{P}_t(i)}\sum\nolimits_{s=0}^{\infty}\phi^s E_t\left[D_{t,t+s}^n DV_{t+s}(i)\,|g_{t+s}=1\right]$$

where $DV_{t+s}(i)$ when $g_{t+s}=1$ is given by

$$DV_{t+s}(i) = \left[\bar{P}_{H,t} - MC_{t+s}^n\right]Y_{t+s}^d(i) = \left[\bar{P}_{H,t} - MC_{t+s}^n\right]\left(\frac{\bar{P}_{H,t}}{P_{H,t+s}}\right)^{-\varepsilon}\left(C_{H,t} + C_{H,t}^*\right).$$

Solving this problem, the optimal $\bar{P}_{H,t}$ is given by the expression

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon-1}\frac{\sum\nolimits_{s=0}^{\infty}E_t\left[\phi^s D_{t,t+s}Y_{t+s}^d(i)\,MC_{t+s}^n|g_{t+s}=1\right]}{\sum\nolimits_{s=0}^{\infty}E_t\left[\phi^s D_{t,t+s}Y_{t+s}^d(i)\,|g_{t+s}=1\right]}.$$

which is a central property of the new Keynesian models. We can derive a similar equation for the Foreign country, where $\phi^* \neq \phi$ is allowed. The flexible-price case is a particular one with $\phi = \phi^* = 0$, so that all Home and Foreign firms adjust price every period according to the pricing rule

$$\bar{p}_{H,t} = \psi + mc_t^n, \quad (58)$$
$$\bar{p}_{F,t}^* = \psi + mc_t^{n*}. \quad (59)$$

As only a fraction ϕ of firms adjusts price each period, we have that $P_{H,t} = \left[\phi P_{H,t-1}^{1-\varepsilon} + (1-\phi)\bar{P}_{H,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$.[10] Log-linearizing this equation around the steady state, we get $\pi_{H,t} = (1-\phi)(\bar{p}_{H,t} - p_{H,t-1})$, wich combined with (57) yields

$$\pi_{H,t} = \beta E_t[\pi_{H,t+1}] + \lambda \widehat{mc}_t, \quad (60)$$

where $\widehat{mc}_t \equiv mc_t^n - p_{H,t} + \psi$ is the deviation of the real marginal cost from its steady state level and $\lambda \equiv \frac{1-\phi}{\phi}(1-\phi\beta)$. Proceeding in the same way with the Foreign country, we get

$$\pi_{F,t}^* = \beta E_t\left[\pi_{F,t+1}^*\right] + \lambda \widehat{mc}_t^*. \quad (61)$$

where $\widehat{mc}_t^* \equiv mc_t^{n*} - p_{F,t}^* + \psi$.

2.3 Equilibrium

This section derives the general equlibrium solution for the log-linearized version of the model around the steady state in which the exogenous variables (productivity shifter) A_t and A_t^* remain constant in their long-run equilibrium values equal to 1.

2.3.1 Demand Side: Goods Markets' Equilibrium

As explained in footnote (8), the markets of all Home and Foreign goods clear in equilibrium, so that $Y_t(i) = Y_t^d(i)$ for $i \in [0, n)$ and $Y_t^*(i) = Y_t^{d*}(i)$ for $i \in [n, 1]$, where $Y_t(i)$ and $Y_t^*(i)$ are the Home and Foreign i$_{th}$ firms' outputs, while $Y_t^d(i)$ and $Y_t^{d*}(i)$ are the Home and Foreign i$_{th}$ goods' world aggregate demands. Therefore, substituting the expression for $Y_t^d(i)$ $\left(Y_t^{d*}(i)\right)$ in

[10] By the law of large numbers, the avarage price of the firms not adjusting prices is the last period's domestic price index.

eq.(24) ((25)) into the definition for the Home (Foreign) output aggregate index (54) ((55)) and using the results (7) ((8)), (13) ((14)) and (20) ((21)), we get

$$
\begin{aligned}
Y_t &= C_{H,t} + C_{H,t}^* = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t + \alpha^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\mu} C_t^*, \quad (62) \\
Y_t^* &= C_{F,t} + C_{F,t}^* = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t + (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\mu} C_t^*. \quad (63)
\end{aligned}
$$

Using the IRS condition (45) to substitute for C_t^* (C_t) into eq.(62) ((63)) and combining the resulting equation with eq.(40) and eqs.(34) ((35)) and (36) ((37)), we get an equation for Y_t (Y_t^*) in terms of C_t (C_t^*) and S_t. Log-linearizing this equation around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$ and $\vartheta = 1$, we get[11]

$$
\begin{aligned}
y_t &= c_t + \frac{\omega_{\bar{\alpha}} + \bar{\alpha} - 1}{\sigma} s_t, \quad (64) \\
y_t^* &= c_t^* - \frac{\omega_{\bar{\alpha}}^*}{\sigma} s_t, \quad (65)
\end{aligned}
$$

where

$$
\begin{aligned}
\omega_{\bar{\alpha}} &\equiv 1 - \bar{\alpha} n + (1-n)\bar{\alpha}(2-\bar{\alpha})(\sigma\mu - 1) > 0, \\
\omega_{\bar{\alpha}}^* &\equiv \bar{\alpha} n \left[1 + (2-\bar{\alpha})(\sigma\mu - 1)\right] > 0.
\end{aligned}
$$

Equations (64) and (65) give the Home and Foreign aggregate outputs when the markets of all goods are in equilibrium, given the Home country's terms of trade and the Home and Foreign aggregate consumption levels. In the particular case of closed countries, when $\bar{\alpha} = 0$, we have that $\omega_{\bar{\alpha}} = 1$, so that and $c_t = y_t$ and $c_t^* = y_t^*$. We can show that $\omega_{\bar{\alpha}} + \bar{\alpha} - 1 > 0$, so that the deterioration of the Home country's terms of trade - an increase in s_t - reduces the demand for Foreign goods and increases the demand for Home goods. Intuition on how this effect is conveyed can be gained if we rewritte the coefficients for s_t as

$$
\begin{aligned}
\frac{\omega_{\bar{\alpha}} + \bar{\alpha} - 1}{\sigma} &= (1-n)\bar{\alpha}(2-\bar{\alpha})\mu + \frac{(1-n)\bar{\alpha}(\bar{\alpha}-1)}{\sigma} > 0, \quad (66) \\
-\frac{\omega_{\bar{\alpha}}^*}{\sigma} &= -n\bar{\alpha}(2-\bar{\alpha})\mu - \frac{n\bar{\alpha}(\bar{\alpha}-1)}{\sigma} < 0. \quad (67)
\end{aligned}
$$

[11] In defining $\omega_{\bar{\alpha}}$ and $\omega_{\bar{\alpha}}^*$, we preserve the notation in Gali & Monacelli (2005).

Fixed c_t (c_t^*), a higher s_t has a direct and positive (negative) effect on y_t (y_t^*) since this change amounts to an increase of the Foreign goods' relative price. However, as explained in subsection (2.1.6), home bias makes positive shocks on Home country's terms of trade to increase the gap between the Home and Foreign consumption levels. Therefore, as c_t (c_t^*) is fixed, the direct effect discussed above is partially and indirectly compensated by the negative (positive) impact that a higher s_t has on c_t^* (c_t). The size of these direct and indirect effects are captured by the first and second terms of the right-hand side of eq.(66) ((67)) respectively. They increase, in absolute value, with the parameter $\bar{\alpha}$, which is inversely related to the degree of home bias. In addition, also in absolute value, the first one increases with the intratemporal elasticity of substitution μ, while the second one decreases with the intertemporal elasticity of substitution σ.

Using eqs.(64) ((65)) and (38) ((39)) to substitute for c_t (c_t^*) and π_{t+1} (π_{t+1}^*) respectively into eq.(29) ((31)), we get

$$y_t = E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(r_t^e + \ln\beta\right) + \frac{1 - \bar{\alpha}n - \omega_{\bar{\alpha}}}{\sigma} E_t\left[\Delta s_{t+1}\right], \tag{68}$$

$$y_t^* = E_t\left[y_{t+1}^*\right] - \frac{1}{\sigma}\left(r_t^{e*} + \ln\beta\right) + \frac{\omega_{\bar{\alpha}}^* - \bar{\alpha}n}{\sigma} E_t\left[\Delta s_{t+1}\right], \tag{69}$$

where $r_t^e \equiv r_t - E_t\left[\pi_{H,t+1}\right]$ and $r_t^{e*} \equiv r_t^* - E_t\left[\pi_{F,t+1}^*\right]$ are the Home and Foreign expected real (related to domestic inflation) interest rates.

2.3.2 Supply Side: Labor Market Equilibrium

The Home (Foreign) labor market's equilibrium condition is given by $L_t^s = L_t$ ($L_t^{s*} = L_t^*$). Substituting eqs.(32) ((33)) and (52) ((53)) into this condition and solving it for $\frac{W_t}{P_t}$ $\left(\frac{W_t^*}{P_t^*}\right)$, we have that

$$\frac{W_t}{P_t} = n^{-(\sigma+\varphi)} \left(\frac{Y_t U_t}{A_t}\right)^{\varphi} C_t^{\sigma}, \tag{70}$$

$$\frac{W_t^*}{P_t^*} = (1-n)^{-(\sigma+\varphi)} \left(\frac{Y_t^* U_t^*}{A_t^*}\right)^{\varphi} C_t^{*\sigma}. \tag{71}$$

Combining eq.(34) ((37)) with the definition of Home (Foreign) real marginal cost (50) ((51)), we get

$$MC_t = \frac{W_t}{P_t}\frac{g(S_t)}{A_t}, \tag{72}$$

$$MC_t^* = \frac{W_t^*}{P_t^*}\frac{g^*(S_t)}{S_t A_t^*}. \tag{73}$$

Substituting eq.(70) ((71)) into eq.(72) ((73)), log-linearizing around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$ and $\vartheta = 1$, and combining with the result (64) ((65)), we get

$$mc_t = (\sigma + \varphi)(y_t - \ln n) + (1 - \bar{\alpha}n - \omega_{\bar{\alpha}}) s_t - (1 + \varphi)a_t, \tag{74}$$

$$mc_t^* = (\sigma + \varphi)(y_t^* - \ln\{1 - n\}) + (\omega_{\bar{\alpha}}^* - \bar{\alpha}n) s_t - (1 + \varphi)a_t^*, \tag{75}$$

where, as explained in Gali & Monacelli (2005), we use the fact that the deviations of $u_t \equiv \ln U_t$ and $u_t^* \equiv \ln U_t^*$ around the steady state are of second order, so that up to a first order approximation we can set $u_t = u_t^* = 0$.

Equations (74) and (75) show how Home and Foreign real marginal costs are determined, so that they are important to undertand the sources of inflationary pressure. Consider first the effect of a higher y_t (y_t^*) on mc_t (mc_t^*), holding s_t and a_t (a_t^*) fixed. This change pushes the Home (Foreign) aggregate labor demand up and thus increases the Home (Foreign) real wage and real marginal cost. The size of this direct effect is captured by the inverse of the real wage-elasticity of labor supply φ.[12] In addition, as s_t is fixed, it follows from eq.(64) ((65)) that a higher y_t (y_t^*) is possible only if c_t (c_t^*) rises, which in turn pushes the labor supply down and thus increases the real wage and real marginal cost. The size of this indirect effect is given by the parameter σ.

Consider now the effect of a higher s_t on mc_t (mc_t^*), holding y_t (y_t^*) and a_t (a_t^*) fixed. Given the positive (negative) sign of the coeffcient for s_t in eq.(64) ((65)), this increase must be accompannied by a lower c_t (higher c_t^*), which in turn pushes the Home (Foreign) labor supply up (down) and thus decreases (increases) the Home (Foreign) real wage and marginal cost. In addition, as we can see in eq.(72) ((73)), given the real wage, a higher s_t diminishes (increases) the purchase power of the Home (Foreign) nominal wage in terms of the Home (Foreign) domestic goods.

[12] This argument is also used to explain the negative effect of an higher factor poductivity shifter a_t on mc_t, holding y_t and s_t fixed.

2.3.3 Current Account

By definition, the Home current account, measured in Home curency, is given by $CC_t \equiv P_{H,t}Y_t - P_tC_t$. Combining this definition with eq.(34), the Home current account as a proportion of the Home output, denoted by NX_t, is given by

$$NX_t \equiv \frac{CC_t}{P_{H,t}Y_t} = 1 - g\left(S_t\right)\frac{C_t}{Y_t}.$$

Log-linearizing the right-hand side of the second equality above around the steady state for the symmetric case, we get $NX_t = \bar{\alpha}(1-n)s_t + c_t - y_t$. In addition, combining this result with eq.(64) yields

$$NX_t = \frac{\bar{\alpha}(1-n)\Lambda}{\sigma}s_t, \tag{76}$$

where the signal of $\Lambda \equiv (1-\sigma) + (2-\bar{\alpha})(\sigma\mu - 1)$ gives the effect of the Home country's terms of trade on its current account. In the particular case of closed countries, when $\bar{\alpha} = 0$, we get $NX_t = 0$.

2.3.4 Uncovered Interest Parity

As explained in subsection (2.1.3), the Home-currency equilibrium prices of the one-period zero-cupon bonds denominated in Home and Foreign currencies are given respectively by $R_t^{-1} = E_t\left[D_{t,t+1}\right]$ and $\varepsilon_t R_t^{*-1} = E_t\left[D_{t,t+1}\varepsilon_{t+1}\right]$, where $D_{t,t+1}$ is the one-period Home SDF. Combining these equations, we get the uncovered interest parity (UIP)

$$E_t\left[D_{t,t+1}\left(R_t - R_t^*\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right] = 0, \tag{77}$$

which is log-linearized around the steady state to yield $r_t - r_t^* = E_t\left[\Delta e_{t+1}\right]$. In addition, combining this equation with the results (38), (39) and (41), we get

$$r_t^e - r_t^{e*} = E_t\left[\Delta s_{t+1}\right], \tag{78}$$

where $r_t^e \equiv r_t - E_t\left[\pi_{H,t+1}\right]$ and $r_t^{e*} \equiv r_t^* - E_t\left[\pi_{F,t+1}^*\right]$. As it should be clear, the UIP condition (77) is not an additional independent equilibrium condition.

Equation (78) can be used to show that the real exchange rate is endogenously determined as a function of the current and future gaps between the

Home and Foreign expected real interest rates. Rewritting this equation as $s_t = E_t[s_{t+1}] + r_t^e - r_t^{e*}$ and solving it forward, we get

$$s_t = \lim_{s \longrightarrow \infty} E_t[s_{t+s}] + \sum_{s=0}^{\infty} E_t\left[r_{t+s}^e - r_{t+s}^{e*}\right],$$

where $r_{t+s}^e \equiv r_{t+s} - E_{t+s}[\pi_{H,t+s+1}]$ and $r_{t+s}^{e*} \equiv r_{t+s}^* - E_{t+s}\left[\pi_{F,t+s+1}^*\right]$. In the Appendix, we show that $s_t = 0$ in the steady state for the symmetric-country case. Given the stationary structure of the productivity shifters, defined in subsection (2.2.1), we have that PPP holds in the long run, i.e., $\lim_{s \longrightarrow \infty} E_t[s_{t+s}] = 0$. Substituting this result into the equation above and using the law of iterated expectations, we get

$$s_t = \sum_{s=0}^{\infty} E_t\left[(r_{t+s} - \pi_{H,t+s+1}) - \left(r_{t+s}^* - \pi_{F,t+s+1}^*\right)\right]. \tag{79}$$

More intuition on this result is gained by looking at an alternative way of deriving it. First, combining the IRS condition (46) with eqs.(64) and (65), we get

$$s_t = \frac{\sigma}{\omega_{\bar{\alpha}} + \omega_{\bar{\alpha}}^*}\left(y_t - y_t^* - \ln\left\{\frac{n}{1-n}\right\}\right). \tag{80}$$

Second, solving eqs.(68) and (69) forward, substituting the results into eq.(80) and using again the stationary properties of the model, we get the result (79).

2.3.5 Equilibrium with Flexible Prices

By definition, the natural level of a variable is the one observed under flexible prices. In this case, as seen in subsection (2.2.3), every Home and Foreign firm reoptimizes its price each period according to the pricing rule (58) and (59) respectively, i.e., as a mark-up over the marginal cost. Furthermore, as firms located in a same country face equal technological and demand constraints, they set the same price, so that $\bar{p}_{H,t}(i) = p_{H,t}$ for all $i \in [0, n)$ and $\bar{p}_{F,t}(i) = p_{F,t}$ for all $i \in [n, 1]$. Combining these results with the pricing rules (58) and (59) under the flexible-price cases, we have that the Home and Foreign real marginal costs are given by $mc_t = mc_t^* = -\psi$, where $mc_t \equiv mc_t^n - p_{H,t}$, $mc_t^* \equiv mc_t^{n*} - p_{F,t}^*$ and $\psi \equiv \ln \frac{\varepsilon}{\varepsilon - 1}$, where ψ is the gross markup (in log) in the flexible-price case.

Substituting the results above into eqs.(74) and (75) and combining them with eq.(80), we have that the natural level of the Home country's terms of trade, denoted by $\bar{s}_t$, is given by

$$\bar{s}_t = \frac{(1+\varphi)}{1+\frac{\varphi}{\sigma}\left(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}}\right)}\left(a_t - a^*_t\right), \tag{81}$$

while the natural level of the Home and Foreign outputs are given by

$$\begin{aligned} \bar{y}_t &= \ln n - \frac{\psi}{\varphi+\sigma} + \left(1-\Theta_y\right)\frac{1+\varphi}{\varphi+\sigma}a_t + \Theta_y \frac{1+\varphi}{\varphi+\sigma}a^*_t, & (82) \\ \bar{y}^*_t &= \ln\{1-n\} - \frac{\psi}{\varphi+\sigma} + \Theta^*_y\frac{1+\varphi}{\varphi+\sigma}a_t + \left(1-\Theta^*_y\right)\frac{1+\varphi}{\varphi+\sigma}a^*_t. & (83) \end{aligned}$$

where $\Theta_y \equiv \frac{\sigma(1-\bar{\alpha}n-\omega_{\bar{\alpha}})}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}})}$ and $\Theta^*_y \equiv \frac{\sigma(\bar{\alpha}n-\omega^*_{\bar{\alpha}})}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}})}$. Substituting the processes for a_t and a^*_t in subsection (2.2.1) into eq.(81), we get

$$E_t\left[\Delta\bar{s}_{t+1}\right] = \frac{1+\varphi}{1+\frac{\varphi}{\sigma}\left(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}}\right)}\left[\left(\rho-1\right)a_t - \left(\rho^*-1\right)a^*_t\right]. \tag{84}$$

Finally, in order to get the Home (Foreign) natural expected real interest rate, denoted by $\bar{r}^e_t$ ($\bar{r}^{e*}_t$), we combine the results (82) ((83)) and (84) with eq.(68) ((69)), so that

$$\begin{aligned} \bar{r}^e_t &\equiv \bar{r}_t - E_t\left[\bar{\pi}_{H,t+1}\right] \\ &= \frac{\left(1-\Theta_r\right)\sigma\left(1+\varphi\right)\left(\rho-1\right)}{\varphi+\sigma}a_t + \frac{\Theta_r\sigma\left(1+\varphi\right)\left(\rho^*-1\right)}{\varphi+\sigma}a^*_t - \ln\beta, & (85) \\ \bar{r}^{e*}_t &\equiv \bar{r}^*_t - E_t\left[\bar{\pi}^*_{F,t+1}\right] \\ &= \frac{\Theta^*_r\sigma\left(1+\varphi\right)\left(\rho-1\right)}{\varphi+\sigma}a_t + \frac{\left(1-\Theta^*_r\right)\sigma\left(1+\varphi\right)\left(\rho^*-1\right)}{\varphi+\sigma}a^*_t - \ln\beta, & (86) \end{aligned}$$

where $\Theta_r \equiv \frac{\varphi(\bar{\alpha}n+\omega_{\bar{\alpha}}-1)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}})}$ and $\Theta^*_r \equiv \frac{\varphi(\omega^*_{\bar{\alpha}}-\bar{\alpha}n)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}})}$. Under flexible prices, all changes in real variables are induced by shocks on productivity shifters, so that there is no scope for monetary policy to affect the output gap.

2.3.6 Equilibrium with Sticky Prices

IS Curve Equations (68) and (69) give the IS curves for both the flexible and sticky-price cases. Hence, the Home and Foreign output gaps $\tilde{y}_t$ and $\tilde{y}^*_t$ move according to the differential equations

$$\begin{aligned} \tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(r_t - E_t\left[\pi_{H,t+1}\right] - \bar{r}^e_t\right) + \frac{1-\bar{\alpha}n-\omega_{\bar{\alpha}}}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right], & (87) \\ \tilde{y}^*_t &= E_t\left[\tilde{y}^*_{t+1}\right] - \frac{1}{\sigma}\left(r^*_t - E_t\left[\pi^*_{F,t+1}\right] - \bar{r}^{e*}_t\right) + \frac{\omega^*_{\bar{\alpha}}-\bar{\alpha}n}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right], & (88) \end{aligned}$$

where $\bar{r}_t^e \equiv \bar{r}_t - E_t[\bar{\pi}_{H,t+1}]$ and $\bar{r}_t^{e*} \equiv \bar{r}_t^* - E_t\left[\bar{\pi}_{F,t+1}^*\right]$ are given by eqs.(85) and (86) respectively.

Furthermore, as eqs.(79) and (80) also hold for both the flexible and sticky-price cases, $\tilde{s}_t$ is given by

$$\begin{aligned}\tilde{s}_t &= \frac{\sigma}{\omega_{\bar{\alpha}} + \omega_{\bar{\alpha}}^*}\left(\tilde{y}_t - \tilde{y}_t^*\right) && (89)\\ &= \sum\nolimits_{s=0}^{\infty} E_t\left[\left(r_{t+s}^e - \bar{r}_{t+s}^e\right) - \left(r_{t+s}^{e*} - \bar{r}_{t+s}^{e*}\right)\right]. && (90)\end{aligned}$$

New Keynesian Phillips Curve Equations (74) and (75) give the Home and Foreign real marginal costs under both the flexible and sticky-price cases. In addition, these variables are given by $mc_t = mc_t^* = -\psi$ in the flexible-price case, which are also their steady state levels. Therefore, the deviations of the Home and Foreign real marginal costs from their steady state values, denoted by $\widehat{mc}_t$ and $\widehat{mc}_t^*$ are given by

$$\begin{aligned}\widehat{mc}_t &= mc_t + \psi = (\varphi + \sigma)\,\tilde{y}_t + (1 - \bar{\alpha}n - \omega_{\bar{\alpha}})\,\tilde{s}_t, && (91)\\ \widehat{mc}_t^* &= mc_t^* + \psi = (\varphi + \sigma)\,\tilde{y}_t^* + (\omega_{\bar{\alpha}}^* - \bar{\alpha}n)\,\tilde{s}_t, && (92)\end{aligned}$$

where the output gaps $\tilde{y}_t \equiv y_t - \bar{y}_t$ and $\tilde{y}_t^* \equiv y_t^* - \bar{y}_t^*$ are the deviations of the Home and Foreign outputs from their natural levels, while the terms of trade gap $\tilde{s}_t \equiv s_t - \bar{s}_t$ is the deviation of the Home country's terms of trade from its natural level. Substituting eq.(91) ((92)) into eq.(60) ((61)), we get

$$\begin{aligned}\pi_{H,t} &= \beta E_t[\pi_{H,t+1}] + \lambda(\varphi + \sigma)\,\tilde{y}_t + \lambda(1 - \bar{\alpha}n - \omega_{\bar{\alpha}})\,\tilde{s}_t, && (93)\\ \pi_{F,t}^* &= \beta E_t\left[\pi_{F,t+1}^*\right] + \lambda(\varphi + \sigma)\,\tilde{y}_t^* + \lambda(\omega_{\bar{\alpha}}^* - \bar{\alpha}n)\,\tilde{s}_t. && (94)\end{aligned}$$

As in closed economies, productivity and monetary policy shocks affect inflation indirectly through their impacts on output gap. However, a new and more direct transmission channel of these shocks is created in open economies as Home country's terms of trade deviation (from its natural level) enter directly into Home and Foreign new Keynesian Phillips curves. Note in eq.(90) that this variable depends on Home and Foreign current and future expected real interest rate deviations (from its natural level), which are in turn affected by both types of shocks.

Monetary Policy For simplicity, we assume that both Home and Foreign Central Banks follow the Taylor-type rules

$$r_t = \delta_\pi \pi_t + \delta_y \tilde{y}_t + \xi_{M,t}, \quad (95)$$

$$r_t^* = \delta_\pi^* \pi_t^* + \delta_y^* \tilde{y}_t^* + \xi_{M,t}^*, \quad (96)$$

where $\xi_{M,t}$ and $\xi_{M,t}^*$ are Gaussian i.i.d monetary policy shocks. We do not derive the optimal monetary rules because they are not necessarily used in practice.

2.4 Canonical Form

The structural model derived in the paper can be summarized by its canonical form, which gives the joint dynamics of the main macroeconomic variables. The canonical form is composed by three sets of equations. The first one, related to the Home country's structure, comprises the eqs.(93), (87), (85), (47), (95) and (47). The second one, related to the Foreign country's structure, comprises the eqs.(94), (88), (86), (96) and the Foreign counterpart of eq.(47). The third one comprises only the eq.(89).

2.4.1 Two-Country Model

First, we describe the canonical form for the two-country case.

Home Country's Structure

$\pi_{H,t} = \beta E_t [\pi_{H,t+1}] + \lambda (\varphi + \sigma) \tilde{y}_t + \lambda (1 - \bar{\alpha} n - \omega_{\bar{\alpha}}) \tilde{s}_t;\ \lambda \equiv \frac{1-\phi}{\phi} (1 - \phi\beta)$
$\tilde{y}_t = E_t [\tilde{y}_{t+1}] - \frac{1}{\sigma} (r_t - E_t [\pi_{H,t+1}] - \bar{r}_t^e) + \frac{1-\bar{\alpha}n-\omega_{\bar{\alpha}}}{\sigma} E_t [\Delta \tilde{s}_{t+1}]$
$\bar{r}_t^e = (1 - \Theta_r) \frac{\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma} a_t + \Theta_r \frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma} a_t^* - \ln \beta$
$a_t = \rho a_{t-1} + \xi_t$
$r_t = \delta_\pi \pi_t + \delta_y \tilde{y}_t + \xi_{M,t}$
$\omega_{\bar{\alpha}} \equiv 1 - \bar{\alpha} n + (1 - n) \bar{\alpha} (2 - \bar{\alpha}) (\sigma\mu - 1)$
$\Theta_r \equiv \frac{\varphi(\bar{\alpha}n+\omega_{\bar{\alpha}}-1)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$

Foreign Country's Structure

$\pi^*_{F,t} = \beta E_t\left[\pi^*_{F,t+1}\right] + \lambda\left(\varphi+\sigma\right)\tilde{y}^*_t + \lambda\left(\omega^*_{\bar{\alpha}} - \bar{\alpha}n\right)\tilde{s}_t$
$\tilde{y}^*_t = E_t\left[\tilde{y}^*_{t+1}\right] - \frac{1}{\sigma}\left(r^*_t - E_t\left[\pi^*_{F,t+1}\right] - \bar{r}^{e*}_t\right) + \frac{\omega^*_{\bar{\alpha}}-\bar{\alpha}n}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right]$
$\bar{r}^{e*}_t = \Theta^*_r\frac{\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma}a_t + \left(1-\Theta^*_r\right)\frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma}a^*_t - \ln\beta$
$a^*_t = \rho^* a^*_{t-1} + \xi^*_t$
$r^*_t = \delta^*_\pi\pi^*_t + \delta^*_y\tilde{y}^*_t + \xi^*_{M,t}$
$\omega^*_{\bar{\alpha}} \equiv \bar{\alpha}n\left[1+(2-\bar{\alpha})(\sigma\mu-1)\right]$
$\Theta^*_r \equiv \frac{\varphi(\omega^*_{\bar{\alpha}}-\bar{\alpha}n)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega^*_{\bar{\alpha}})}$

Terms of Trade and Output Gaps

$$\tilde{s}_t = \frac{\sigma}{\omega_{\bar{\alpha}} + \omega^*_{\bar{\alpha}}}\left(\tilde{y}_t - \tilde{y}^*_t\right)$$

2.4.2 Small-Home Country Model

Now, we describe the canonical form for the small Home country case, in which $n = \omega^*_{\bar{\alpha}} = 0$ and $\Theta^*_r = 0$ in the tables above.In this case, although the Foreign country's is open, its structure is identical to the closed economy's one, regardless of its own home bias degree. Note that $\pi^*_t = \pi^*_{F,t}$.

Home Country's Structure

$\pi_{H,t} = \beta E_t\left[\pi_{H,t+1}\right] + \lambda\left(\varphi+\sigma\right)\tilde{y}_t + \lambda\left(1-\omega_{\bar{\alpha}}\right)\tilde{s}_t;\ \lambda \equiv \frac{1-\phi}{\phi}\left(1-\phi\beta\right)$
$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(r_t - E_t\left[\pi_{H,t+1}\right] - \bar{r}^e_t\right) + \frac{1-\omega_{\bar{\alpha}}}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right]$
$\bar{r}^e_t = \left(1-\Theta_r\right)\frac{\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma}a_t + \Theta_r\frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma}a^*_t - \ln\beta$
$a_t = \rho a_{t-1} + \xi_t$
$r_t = \delta_\pi\pi_t + \delta_y\tilde{y}_t + \xi_{M,t}$
$\omega_{\bar{\alpha}} \equiv 1 + \bar{\alpha}(2-\bar{\alpha})(\sigma\mu-1)$
$\Theta_r \equiv \frac{\varphi(\omega_{\bar{\alpha}}-1)}{\sigma+\varphi\omega_{\bar{\alpha}}}$

Foreign Country's Structure

$\pi^*_t = \beta E_t\left[\pi^*_{t+1}\right] + \lambda\left(\varphi+\sigma\right)\tilde{y}^*_t$
$\tilde{y}^*_t = E_t\left[\tilde{y}^*_{t+1}\right] - \frac{1}{\sigma}\left(r^*_t - E_t\left[\pi^*_{t+1}\right] - \bar{r}^{e*}_t\right)$
$\bar{r}^{e*}_t = \frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma}a^*_t - \ln\beta$
$a^*_t = \rho^* a^*_{t-1} + \xi^*_t$
$r^*_t = \delta^*_\pi\pi^*_t + \delta^*_y\tilde{y}^*_t + \xi^*_{M,t}$

Terms of Trade and Output Gaps

$$\tilde{s}_t = \frac{\sigma}{\omega_{\bar{\alpha}}} \left(\tilde{y}_t - \tilde{y}_t^* \right)$$

3 Numerical Analysis

An interesting application of the model developed in the previous section is to simulate the dynamical effects of domestic and foreign shocks on the Brazilian economy. For that, we can use the small country version of the model, in which Brazil is the small Home country, while the rest of the world or some large country is the Foreign one.

3.1 Calibration

We leave for future research the task of estimating realistic parameters for the Brazilian economy. In this work, we just follow the usual practice of calibrating the model by either taking values commonly used in the international literature, among with some estimated with U.S. or Brazilian data, or setting values consistent with Brazilian data moments. We use values for quarterly data. In this sense, we set $\sigma = 1$, $\mu = 1,5$, $\varphi = 2,5$, $\rho = \rho^* = 0.95$, $\delta_\pi = \delta_\pi^* = 1.5$ and $\delta_y = \delta_y^* = 0.5$. We also set $\beta = 0,98$, which is very close to some estimatives for the Brazilian economy. For the parameter giving the degree of price-stickiness, works for developed countries commonly set $\phi = 0.75$, which corresponds to an average period of one-year between price adjustments.

In the small Home country version of the model, the parameter $\bar{\alpha}$ gives the share of imported (Foreign) goods on the Home country's consumption. Therefore, we set $\bar{\alpha} = 0,13$, which is close to the ratio between the Brazilian imports and total domestic expenditure in the period after the exchange rate liberalization.

3.2 Impulse-Response Functions

Now, we calculate impulse-response (IR) functions for Home and Foreign variables to unit-size positive productivity and monetary policy shocks. As Home country is small, Home-originated shocks do not affect Foreign country's economy and then the related IR functions are omited. As expected, the stationarity of the model implies that all variables converge to their steady-state levels in the long-run. Note still that, unlike AR (1) productivity shocks, monetary policy shocks are white-noise and then their effects are not persistent.

3.2.1 Productivity Shocks

Figure (A.1) shows IR functions for Home variables to a Home country's productivity shock. Under price-stickiness, the immediate fall of the natural interest rate puts a positive gap between the expected real interest rate and its natural level, which pushes the output and terms of trade gaps down through the IS curve and UIP equation respectively. These two last effects in turn affect negatively the domestic inflation, so that the shock has a direct effect on inflation via terms of trade in addition to the indirect effect via output gap change. However, CPI inflation increases as a result of the deterioration of the terms of trade, since the positive impact of the shock on the natural terms of trade exceeds, in magnitude, the fall of the terms of trade gap.

Figure (A.2) shows IR functions for Home variables to a Foreign country's productivity shock. On a hand, the immediate positive gap between the Foreign expected real interest rate and its natural level causes a deterioration of the Home terms of trade via UIP equation, which in turn affects directly and negatively the Home domestic inflation. On the other hand, despite the positive impact of the shock on the Home natural interest rate, the net result of this effect when combined with the change in terms of trade gap is to push the Home output gap down, which in turn curbs the domestic inflation.

Figure (A.3) shows IR functions for Foreign variables to a Foreign country's productivity shock, which are consistent to the literature for closed countries.

3.2.2 Monetary Policy Shocks

Figure (B.1) shows IR functions for Home variables to a Home country's monetary policy shock. The higher nominal interest rate gives rise to a positive gap between the expected real interest rate and its natural level, which in turn pushes the output and terms of trade gaps down. This in turn has a negative effect on domestic and CPI inflation.

Figure (B.2) shows IR functions for Home variables to a Foreign country's monetary policy shock. The resulting fall in the Foreign ouptut gap leads to a deterioration of the Home country's terms of trade, which in turn has a positive effect on domestic inflation and Home output gap. In a consequence, the Home interest rate rises.

Figure (B.3) shows IR functions for Foreign variables to a Foreign country's monetary policy shock, which are consistent to the literature for closed countries.

4 Conclusion

We build a two-country new Keynesian DSGE model with Calvo-type staggered price setting, which is an extension of the standard model largely used for monetary policy analisys in closed economies. The small country version of the model follows naturally as a limit case of the world economy. This procedure has two advantages relative to the usual way as most part of the literature models a small open economy, which simply assumes that foreign variables follow exogenous processes. First, we do not take the risk of setting aside important channels of international monetary transmissions. In this sense, our model takes into account the effects of foreign frictions, such as price-stickiness, on the way how domestic and foreign real and monetary shocks are conveyed into the small country's economy. Second, we can build inpulse-response functions to see how these shocks affect simultaneously both economies in an integrated way.

The new Keynesian Phillips curve for open economies embeds the terms of trade as an additional pushing-cost variable feeding the inflation Therefore, monetary and real shocks affects inflation not only indirectly through their effects on output gap, but also directly through their effects on terms of trade.

Although the law of one price holds for all goods, the assumption of home bias in households' preferences allows for real exchange rate fluctuation, even if international financial market structure is complete. This in turn gives rise a variable gap between the consumption across counries, so that a new monetary transmission channel arises. Furthermore, as real exchange rate flutuation is an important empirical evidence, a promising avenue for future research would consist in enriching the model with other sources of PPP violation, such as nontradability and international segmentation in the goods' market. The latter assumption would allow the model to incorporate the imperfect pass-through observed in actual data.

More generally, other interesting extensions would consist in introducing nominal and real frictions required to reproduce important empirical regularities, such as price indexation to create persistence in inflation and habit information in consumption and/or adjustment costs to captal.to create persistence in output.

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6 Appendix: Steady State

This appendix characterizes the steady-state equilibrium under the sticky and flexible-price cases, in which the productivity shifters remain at their long-run equilibrium values, i.e., $A_t = A_t^* = 1$. With flexible prices, we know from subsection (2.3.5) that the Home and Foreign real marginal costs are time-invariant and given by $MC_t = MC_t^* = \frac{\varepsilon-1}{\varepsilon}$. We also know from subsection (2.2.2) that $U_t = U_t^* = 1$ under this case. Using these results and the IRS condition (45), we can substitute eq.(70) ((71)) into (72) ((73)) to get

$$Y_t = n^{\frac{\sigma}{\varphi}+1} A_t^{1+\frac{1}{\varphi}} \left(\frac{\varepsilon - 1}{\varepsilon} \frac{1}{g^*(S_t)} \right)^{\frac{1}{\varphi}} \left(\frac{n}{1-n} \vartheta C_t^* \right)^{-\frac{\sigma}{\varphi}}, \tag{97}$$

$$Y_t^* = (1-n)^{\frac{\sigma}{\varphi}+1} A_t^{*1+\frac{1}{\varphi}} \left(\frac{\varepsilon - 1}{\varepsilon} \frac{S_t}{g^*(S_t)} \right)^{\frac{1}{\varphi}} C_t^{*-\frac{\sigma}{\varphi}}. \tag{98}$$

In addition, we can substitute the IRS condition (45) and the results (34) ((35)) and (36) ((37)) into (62) ((63)) to get

$$Y_t = \frac{1}{1-n} \left[(1 - \bar{\alpha} + \bar{\alpha} n) \vartheta g(S_t)^{\mu - \frac{1}{\sigma}} g^*(S_t)^{\frac{1}{\sigma}} + \bar{\alpha}^* (1-n) g^*(S_t)^{\mu} \right] C_t^*, \tag{99}$$

$$Y_t^* = \left[n\bar{\alpha}\vartheta g(S_t)^{\mu - \frac{1}{\sigma}} g^*(S_t)^{\frac{1}{\sigma}} + (1 - \bar{\alpha}^* n) g^*(S_t)^{\mu} \right] S_t^{-\mu} C_t^*, \tag{100}$$

where $g(S_t)$ and $g^*(S_t)$ are defined in eqs.(34) and (36) respectively. For any levels of A_t and A_t^*, the flexible-price equilibrium values of S_t and C_t^* are the solution of the system formed by combining eqs.(97) and (99), relative to the Home country, and eqs.(98) and (100), relative to the Foreign country. Therefore, the steady-state levels for S_t and C_t^*, denoted by S and C^*, are the solution of this system for the particular case with $A_t = A_t^* = 1$, which is given by

$$
\begin{aligned}
& n^{\frac{\sigma}{\varphi}+1}\left(\frac{\varepsilon-1}{\varepsilon}\frac{1}{g^{*}\left(S\right)}\right)^{\frac{1}{\varphi}}\left(\frac{n}{1-n}\vartheta C^{*}\right)^{-\frac{\sigma}{\varphi}} \qquad (101)\\
= & \frac{1}{1-n}\left[\left(1-\bar{\alpha}+\bar{\alpha}n\right)\vartheta g\left(S\right)^{\mu-\frac{1}{\sigma}}g^{*}\left(S\right)^{\frac{1}{\sigma}}+\bar{\alpha}^{*}\left(1-n\right)g^{*}\left(S\right)^{\mu}\right]C^{*},\\
& \left(1-n\right)^{\frac{\sigma}{\varphi}+1}\left(\frac{\varepsilon-1}{\varepsilon}\frac{S}{g^{*}\left(S\right)}\right)^{\frac{1}{\varphi}}C^{*-\frac{\sigma}{\varphi}} \qquad (102)\\
= & \left[n\bar{\alpha}\vartheta g\left(S\right)^{\mu-\frac{1}{\sigma}}g^{*}\left(S\right)^{\frac{1}{\sigma}}+\left(1-\bar{\alpha}^{*}n\right)g^{*}\left(S\right)^{\mu}\right]S^{-\mu}C^{*}.
\end{aligned}
$$

The steady state equilibrium exists if the system above has a solution, which in turn depends on the parameters of the model. For the symmetric case, in which $\bar{\alpha}=\bar{\alpha}^{*}$ and $\vartheta=1$, we can easily show that, for any n, there is a steady state solution with $S=1$. For that, it is enough to substitute these values into both eqs.(101) and (102) and verify that both provide the same value for C^{*}.

Fig. A.1: IR Functions for Home Variables to a Home Productivity Shock

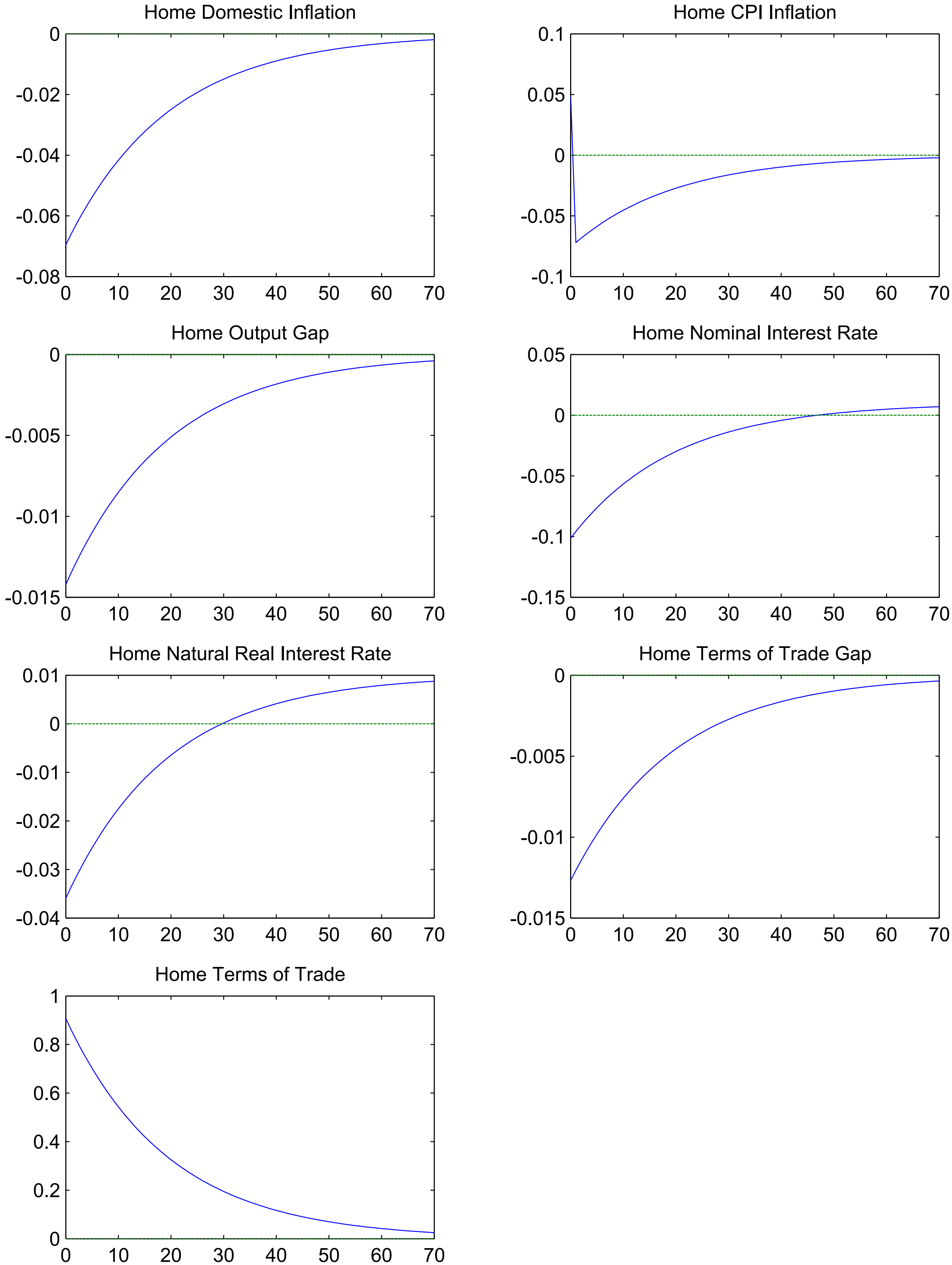


Fig. A.2: IR Functions for Home Variables to a Foreign Productivity Shock

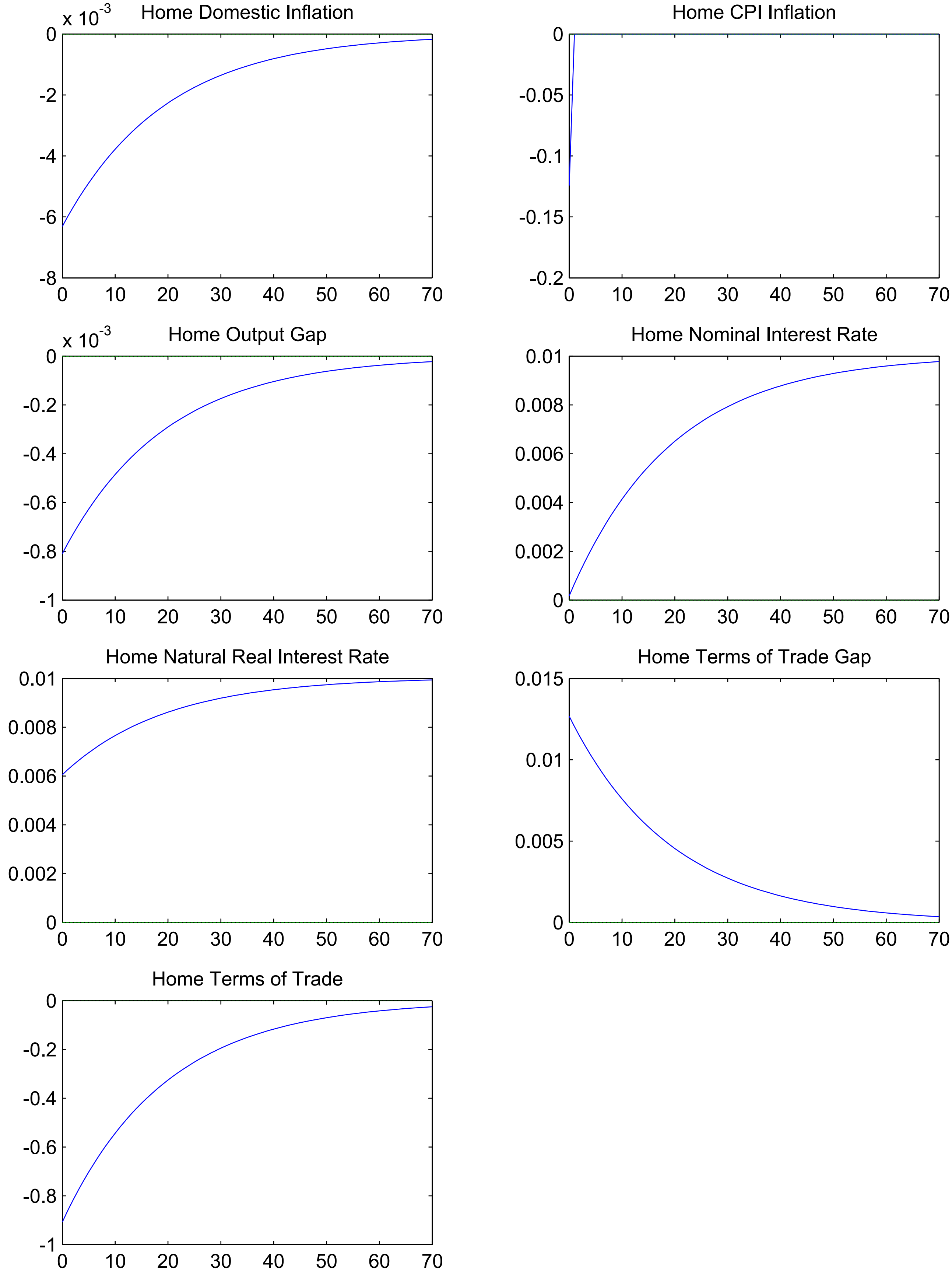


Fig. A.3: IR Functions for Foreign Variables to a Foreign Productivity Shock

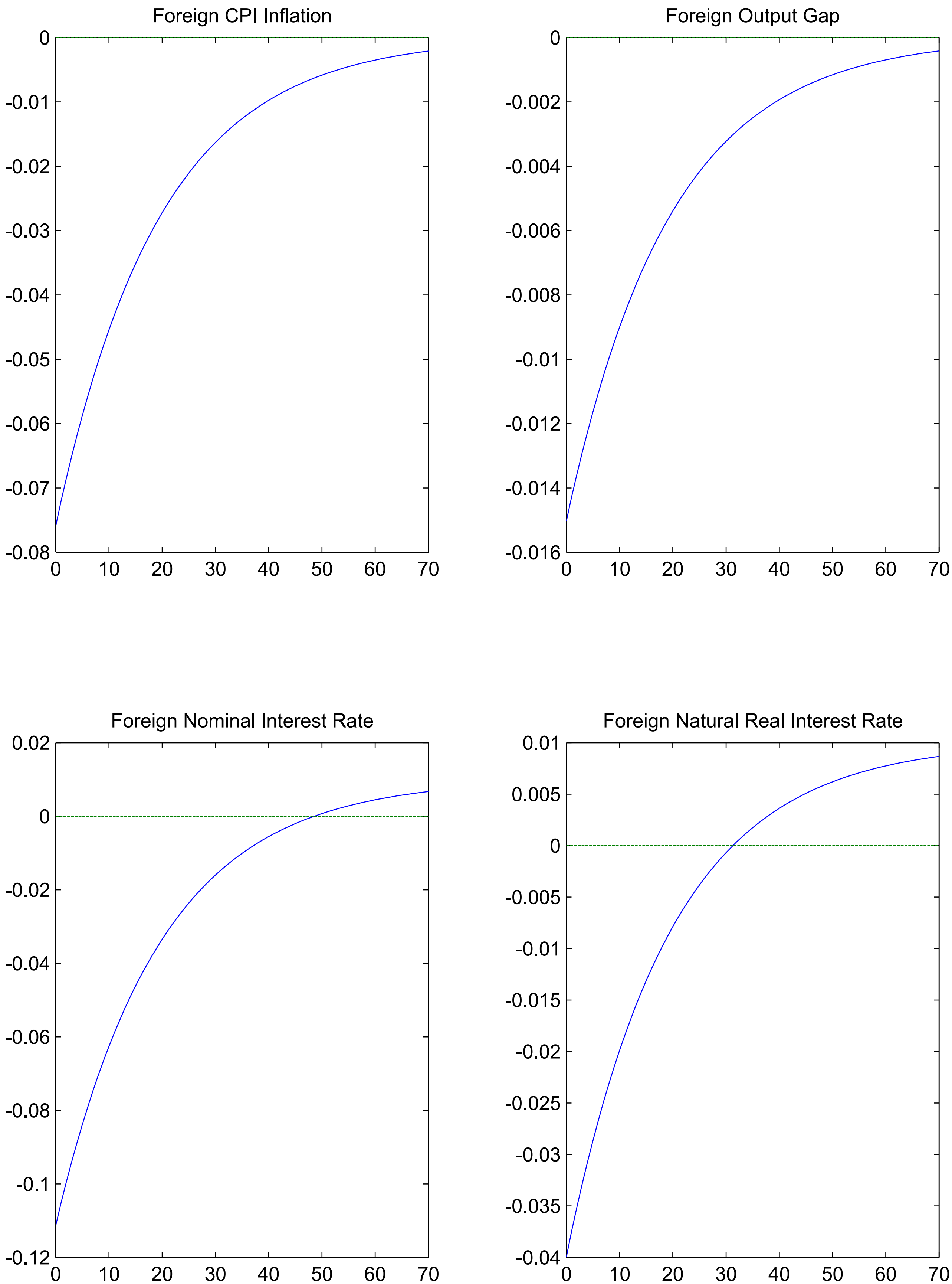


Fig. B.1: IR Functions for Home Variables to a Home Monetary Policy Shock

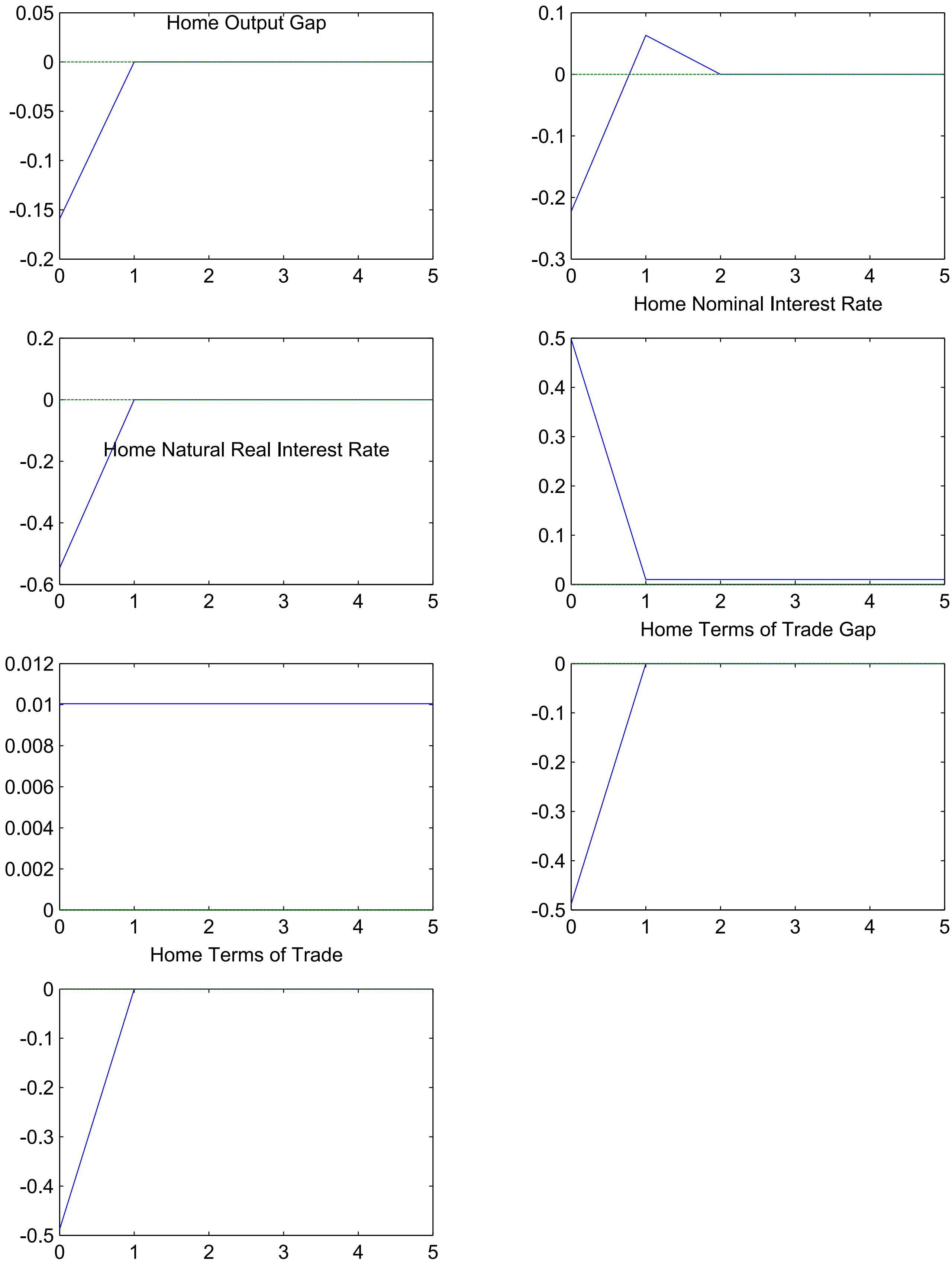


Fig. B.2: IR Functions for Home Variables to a Foreign Monetary Policy Shock

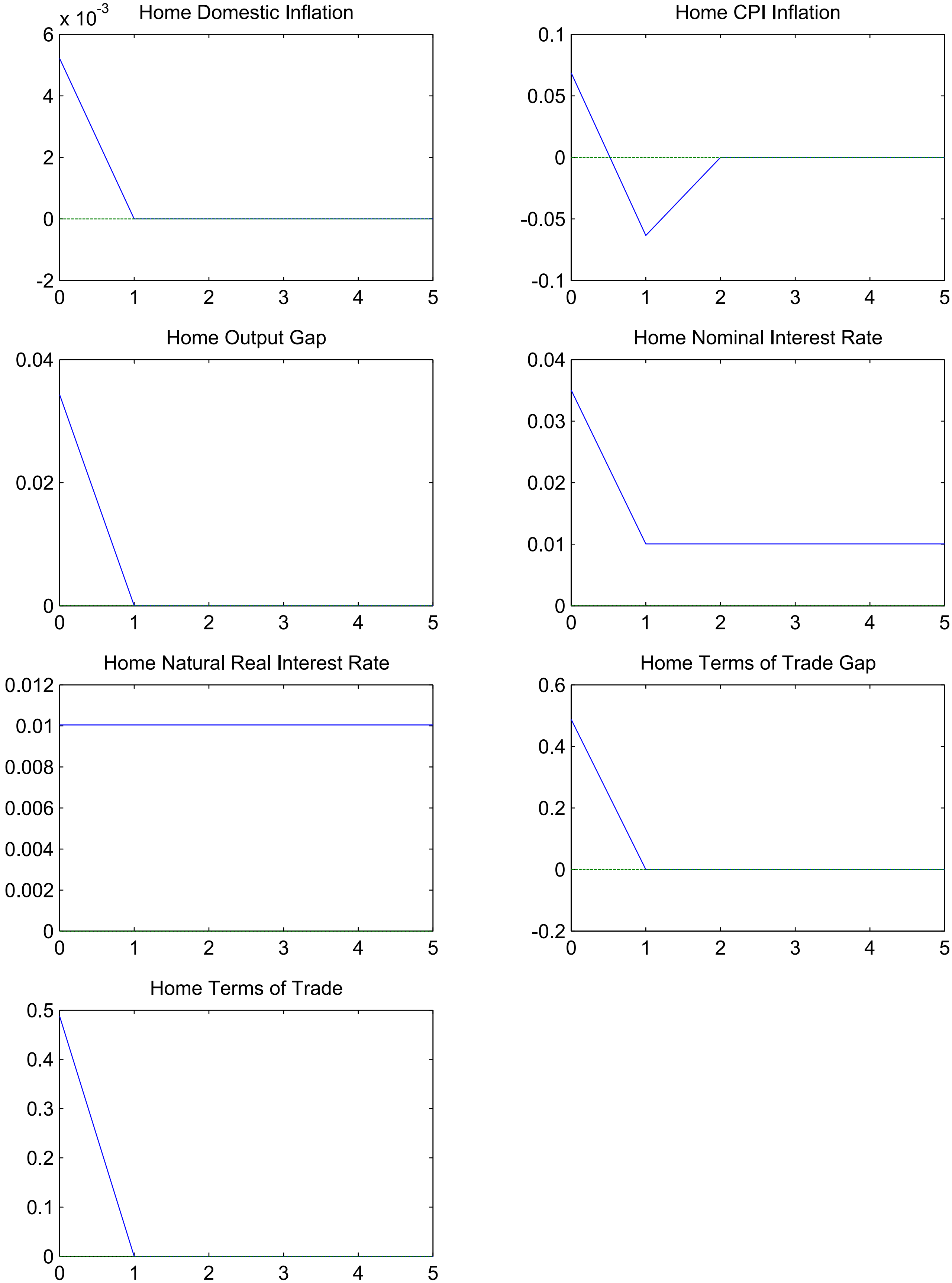
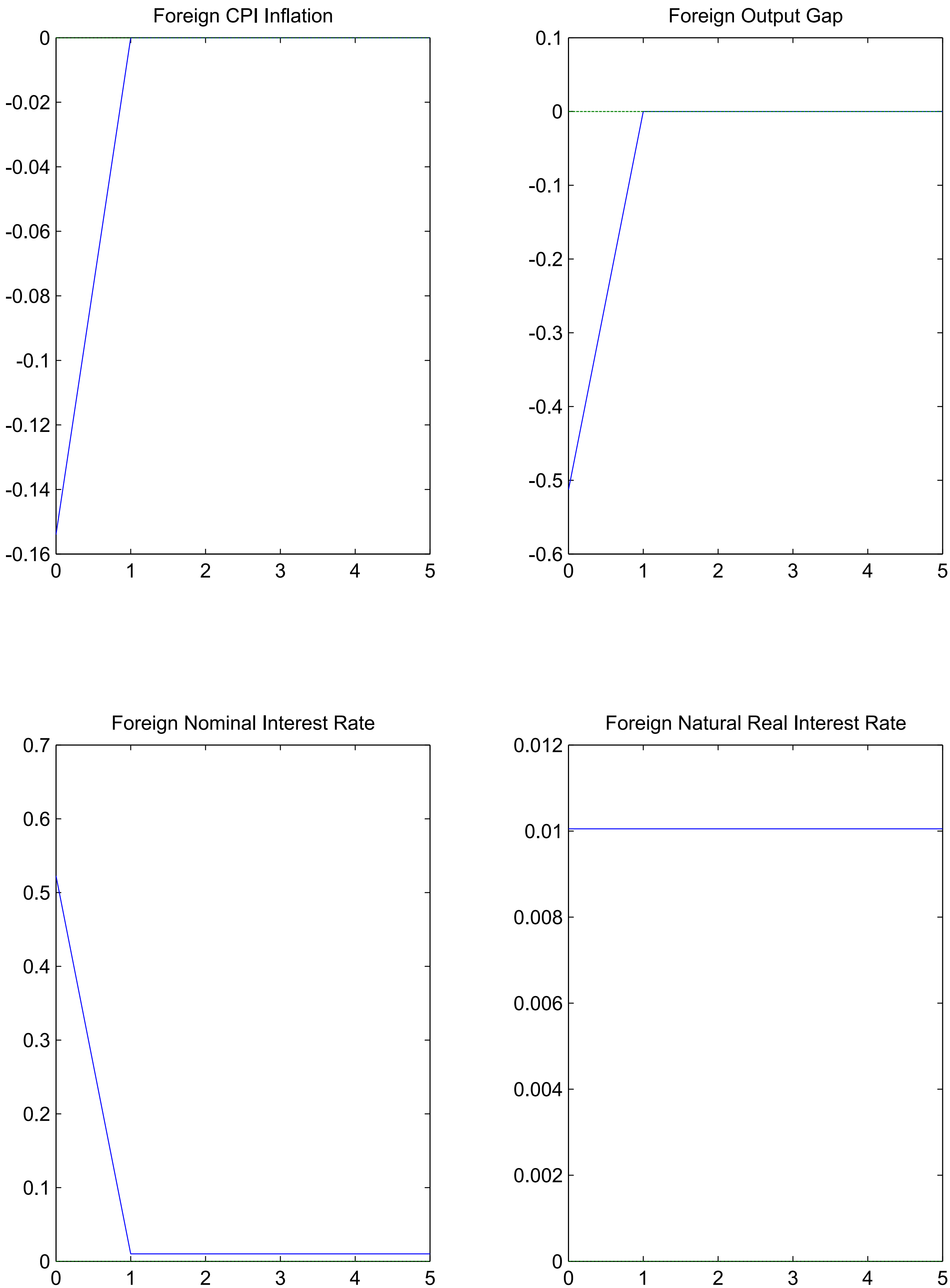


Fig. B.3: IR Functions for Foreign Variables to a Foreign Monetary Policy Shock



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