ISSN 1415-4765

TEXTO PARA DISCUSSÃO N° 911

FORECASTING BRAZILIAN OUTPUT IN REAL TIME IN THE PRESENCE OF BREAKS: A COMPARISON OF LINEAR AND NONLINEAR MODELS

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Rio de Janeiro, outubro de 2002

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SUMMARY

SINOPSE

Neste artigo são comparadas as habilidades preditivas de modelos lineares e não-lineares, com quebras estruturais, para a taxa de crescimento do PIB do Brasil. São estimados os modelos com mudança de regime markoviana propostos por Hamilton (1989) e Lam (1990) — que generaliza o modelo de Hamilton — com dados trimestrais de 1975:1 a 2000:2. Na estimação dos modelos são permitidas quebras estruturais durante os Planos Collor I e II.

As probabilidades de recessão dos modelos são utilizadas para se analisar o ciclo de negócios brasileiro. É examinada a capacidade de se prever a taxa de crescimento do PIB fora da amostra e a habilidade preditiva dos dois modelos é comparada com a de modelos lineares.

Os nossos resultados revelam que os modelos não-lineares são os que apresentam o melhor desempenho preditivo e que a inclusão de quebras estruturais é importante para se obter a representação do ciclo de negócios no Brasil.

ABSTRACT

This paper compares the forecasting performance of linear and nonlinear models under the presence of structural breaks for the Brazilian real GDP growth. The Markov switching models proposed by Hamilton (1989) and its generalized version by Lam (1990) are applied to quarterly GDP from 1975:1 to 2000:2 allowing for breaks at the Collor Plans.

The probabilities of recessions are used to analyze the Brazilian business cycle. The ability of each model in forecasting out-of-sample the growth rates of GDP is examined. The forecasting ability of the two models is also compared with linear specifications.

We find that nonlinear models display the best forecasting performance and that specifications including the presence of structural breaks are important in obtaining a representation of the Brazilian business cycle.

1 INTRODUCTION

The increasing global economic integration and intense volatility in emerging market economies in recent years have re-emphasized the importance of forecasting fundamentals in developing countries, and in particular, gauging the potential of future economic recessions. Recently, the currency crisis in Argentina has raised strong interest in the potential economic vulnerability of neighboring countries, especially of its main trading partner, Brazil.

Nevertheless, the task of forecasting emerging market economies has proven to be a special difficult one, given the great instability in these economies. In particular, models that do not take into account changes in the dynamics of these economies in form of structural breaks may perform poorly in real time. This paper examines the performance of several models in forecasting Brazilian output when structural breaks are explicitly taken into account. First, we examine whether nonlinear time series models produce short run and long run forecasts that improve upon linear models. Second, we compare whether there are gains in endogenously modeling structural breaks to produce out-of-sample forecasts. We conduct an examination of various forecasts at the one, two, four and eight-quarter horizons for the rate of growth of real Brazilian GDP. The study partially simulates real time prediction since all forecasts are based solely on revised data through the date of each forecast.

Linear models have been widely applied in earlier forecasting literature. However, these models have been used to generate a forecast of the rate of growth of output rather than to forecast a nonlinear event such as a turning point, that is, the beginning or end of an economic recession. Generally the filters used to extract turning point forecasts from a linear model require the use of ex post data. This paper uses two classes of Markov switching models, which directly provide real time turning point forecasts in addition to predictions of GDP growth.

More recently, a number of studies has examined the forecasting performance of nonlinear and linear models, including Weigand and Gershenfeld (1994), Hess and Iwata (1997), Stock and Watson (1998), and Camacho and Perez-Quiros (2000), among others. These authors detect nonlinearities in several macroeconomic time series with conflicting results with respect to forecasting performance of the models. As Camacho and Perez-Quiros (2000) conclude for the U.S. economy, we find that nonlinear switching specifications that take into account structural breaks in the Brazilian economy yield better forecasts than linear models of GDP growth, especially at longer horizons. In addition, nonlinear models replicate more accurately Brazilian business cycle features.

The remainder of this paper is organized as follows. The forecasting models are presented in Section 2. The algorithm used to estimate the Markov switching models and their differences are described in the Appendix. Section 3 examines the major structural break in the Brazilian economy due to Collor stabilization Plan implemented in 1990-1992. The results are presented and discussed in Section 4, and conclusions are summarized in Section 5.

2 THE MODELS AND THE ESTIMATION METHODS

2.1 HAMILTON'S MARKOV SWITCHING MODEL (MS)

Hamilton (1989) models the log of GDP, y_t, as divided into a trend, n_t, and a Gaussian cyclical component, z_t:

$$y_t = n_t + z_t \tag{1}$$

$$n_t = n_{t-1} + \alpha_0(1 - S_t) + \alpha_1 S_t \tag{2}$$

$$\phi(L)\ (1 - L)z_t = \varepsilon_t \tag{3}$$

where ε_t ~ iid $N(0, \sigma^2)$ and ε_t is independent on n_{t+k} $\forall k$, and S_t is a latent first-order Markov chain. The drift switches between two states: it takes the value of α_0 when the economy is in an expansion (s_t = 0) and α_1 when the economy is in a recession (s_t = 1). The changes in regimes are ruled by the transition probabilities p_{ij} = prob [s_t = $= j|\ s_{t-1} = i$] where $\sum_{j=0}^{1} p_{ij} = 1,\quad i, j = 0, 1.$

In this model both n_t and z_t display unit roots and the roots of $\phi(L) = 0$ lie outside the unity circle. Hence, the cyclical component follows a zero mean ARIMA(r, 1, 0) process:

$$z_t - z_{t-1} = \phi_1(z_{t-1} - z_{t-2}) + \phi_2(z_{t-2} - z_{t-3}) + \ldots + \phi_r(z_{t-r} - z_{t-r-1}) + \varepsilon_t \tag{4}$$

Taking the first difference of (1) we get:

$$\Delta y_t = \mu_{st} + \phi_1(z_{t-1} - z_{t-2}) + \phi_2(z_{t-2} - z_{t-3}) + \ldots + \phi_r(z_{t-r} - z_{t-r-1}) + \varepsilon_t \tag{5}$$

where $\Delta = 1 - L$ and $\mu_{st} = \alpha_0(1 - S_t) + \alpha_1 S_t$.

2.2 LAM'S MARKOV SWITCHING MODEL (MSG)

Lam (1990) suggests a modification of Hamilton's model that has important implications for the characterization of output trend and cycle. In particular, Lam decomposes the log of GDP into a trend n_t and a cyclical component z_t, where only the trend displays a unit root:

$$y_t = n_t + z_t \tag{6}$$

$$n_t = n_{t-1} + \alpha_0(1 - S_t) + \alpha_1 S_t \tag{7}$$

That is, the autoregressive process z_t is now given by:

$$\phi(L)z_t = \varepsilon_t \tag{8}$$

where ε_t ~ iid $N(0, \sigma^2)$. Taking the first difference of (6) we get:

$$\Delta y_t = \mu_{st} + z_t - z_{t-1} \tag{9}$$

where $\mu_{st} = \alpha_0(1 - S_t) + \alpha_1 S_t$. This model allows for both temporary and permanent shocks: the roots of $\phi(L)$=0 are outside the unity circle, which implies that z_t can be interpreted as the transitory deviations of y_t from its long run trend n_t. Therefore, this model captures structural changes in the trend of the Brazilian GDP. On the other hand, since in Hamilton's model both the cyclical component and the trend present unit roots, all shocks to output are permanent.

Both models require different nonlinear filters to be estimated. A detailed description of Hamilton and Lam filter can be found in Hamilton (1989) and in Lam (1990), respectively. The filter used to estimate Lam's model involves substantial more computation than Hamilton's algorithm for two reasons. First, in the calculation of the error, the states for each observation include all the history of the Markov process, which is treated as an additional variable. Second, the initial value of the autoregressive component is treated as an additional free parameter to be estimated. The Appendix presents a brief description of both filters.

3 STRUCTURAL BREAKS AND INTERVENTION

Markov switching models have been extensively used to represent cyclical changes or structural breaks in the economy. Hamilton (1989) applied this model to the quarterly change in the log of U.S. real GNP from 1952:2 to 1984:4, assuming that the cyclical component follows an AR(4) process. The estimated Markov states obtained were closely associated with the U.S. expansions and recessions as dated by the NBER.

More recently, McConnell and Perez-Quiros (2000) have found evidence of a structural break in the volatility of U.S. growth towards stabilization in the first quarter of 1984. They show that one implication of the break is that the smoothed probabilities miss the 1990 U.S. recession when more recent data are used. There are different ways to handle the problem of structural breaks. McConnell and Perez-Quiros suggest augmenting Hamilton's model by allowing the residual variance to switch between two regimes, and letting the mean growth rate vary depending on the state of the variance.[1] The resulting estimated smoothed probabilities of the augmented model capture the 1990-1991 recession. Notice that Hamilton's model decomposes the log of GDP into the sum of a trend and a cycle, each of which presents unit roots processes that are not identifiable from each other. Thus, in the presence of a structural break, both terms capture both the business cycle component and the break jointly.[2] McConnell and Perez-Quiros' model identifies breaks in the variance from breaks in the mean by allowing each to follow different and dependent Markov processes. Thus, while the Markov chain for the variance captures the break in 1984, the Markov states for the mean capture the business cycle component for the full sample.

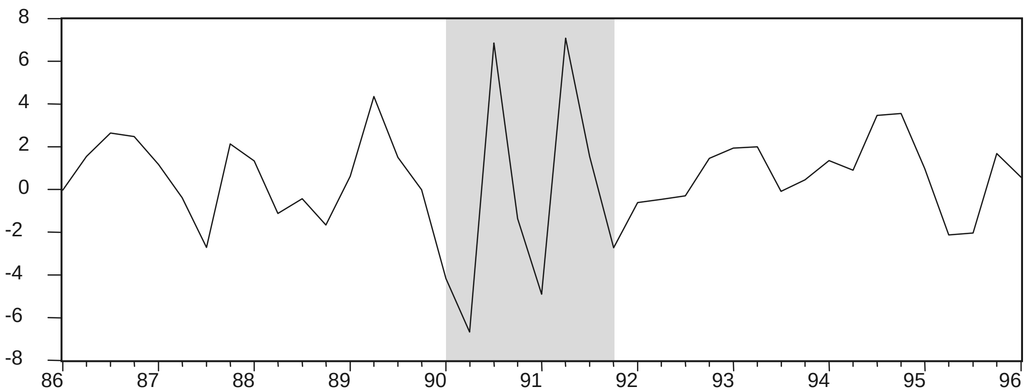
1. This amounts in estimating four mean growth rates: low growth under high and low volatility states, and high growth under high and low volatility states.

2. The smoothed probabilities obtained from a model with switching variance and constant mean captures the break in 1984, while a model with switching mean and constant variance captures the business cycle phases up to the breakpoint [see McConnell and Perez-Quiros (2000)].

Lima and Domingues (2000) models the change in the log of Brazilian GDP as a hidden Markov chain with an AR(4) component. Alternatively, Chauvet (2002*a*) and (2002*b*) model the change in the log of Brazilian and U.S. GDP, respectively, as a hidden Markov chain with no autoregressive component. This specification captures business cycle features of these economies regardless of the presence of structural breaks in the mean or variance of output. Several authors such as McConnell and Perez-Quiros (2000), Harding and Pagan (2001) or Albert and Chib (1993), among others, have found the GDP growth in the U.S. and other countries is better modeled as a low autoregressive process. In particular, Albert and Chib use Bayesian methods to estimate Hamilton's model and find that the best specification for changes in GDP is an AR(0) process, as the autoregressive coefficients are not statistically significant. This finding is perhaps due to the presence of structural breaks in the stochastic process of GDP.

The Brazilian economy also displays several structural breaks. In particular, the series of stabilization plans and changes in policy regime in the last two decades resulted in several breaks in the Brazilian GDP, especially in the early 1990s due to the Collor Plan. Figure 1 shows the Brazilian GDP[3] around the period of implementation of the Collor stabilization Plan. As it can be observed, the economy faced a period of large swings for five quarters. Upon introduction of the plan in the second quarter of 1990, GDP decreased at a quarterly average rate of –6.7%. In the third quarter GDP experienced an abrupt increase of 6.8%, but in the following two quarters it fell again by 1.4% and 4.9%, respectively. In the second quarter of 1992 the economy again underwent a large increase of 7.1%.

FIGURE 1
Brazilian GDP Growth and the Collor Plan

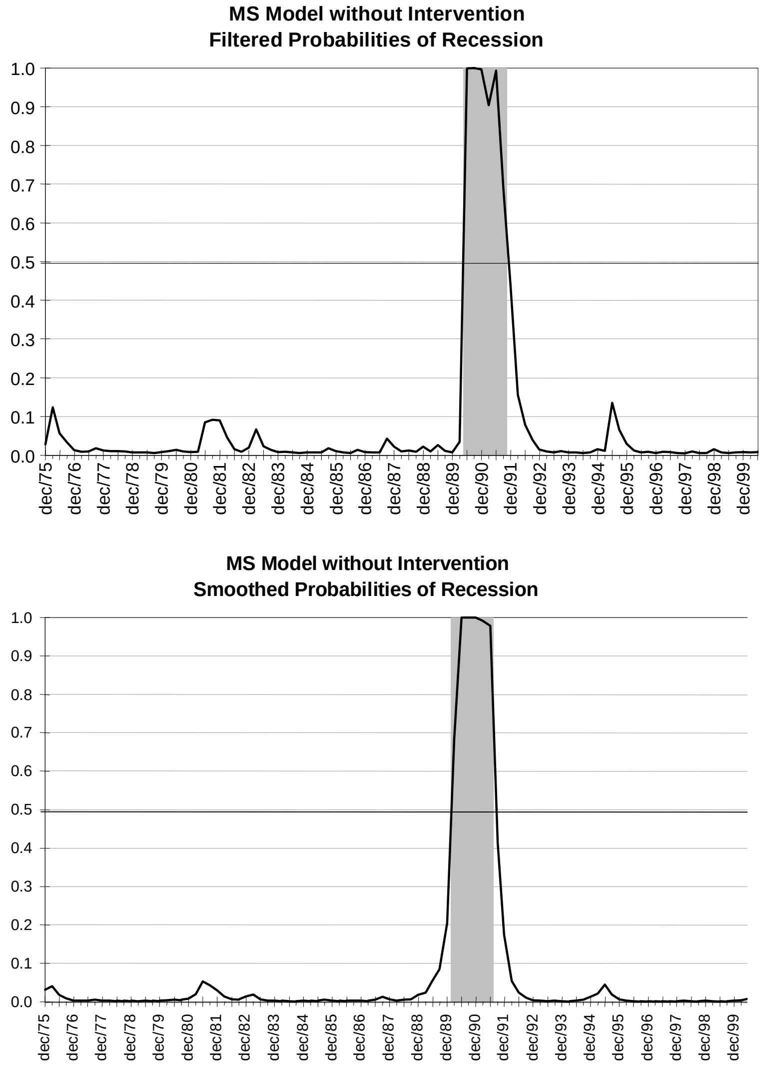


These large pulse-breaks in the Brazilian economy cause estimation problems for standard Markov switching models and the optimization routines frequently

3. The data on real Brazilian GDP were seasonally adjusted using the X-12 method. The series was obtained from IPEA database.

converges to a local maximum.[4] If the autoregressive part is not long enough, or if it does not display a unit root, then the models and probabilities capture solely the pulse breaks due to the Collor Plan. For example, when the MS specification with an AR(1) [MS-AR(1)] or an AR(2) [MS-AR(2)] component and the MSG specification with different autoregressive components [from MSG-AR(1) to MSG-AR(5)] are applied to real Brazilian GDP growth, the filtered and smoothed probabilities of low growth concentrate in the observations between 1990:I to 1991:II (Collor I and Collor II Plans), as illustrated in Figures 2 [MS-AR(2)] and 3 [MSG-AR(3)]. That is, without intervention both models capture solely the abrupt pulse breaks experienced by the Brazilian economy during the Collor Plans instead of cyclical economic expansions and contractions.

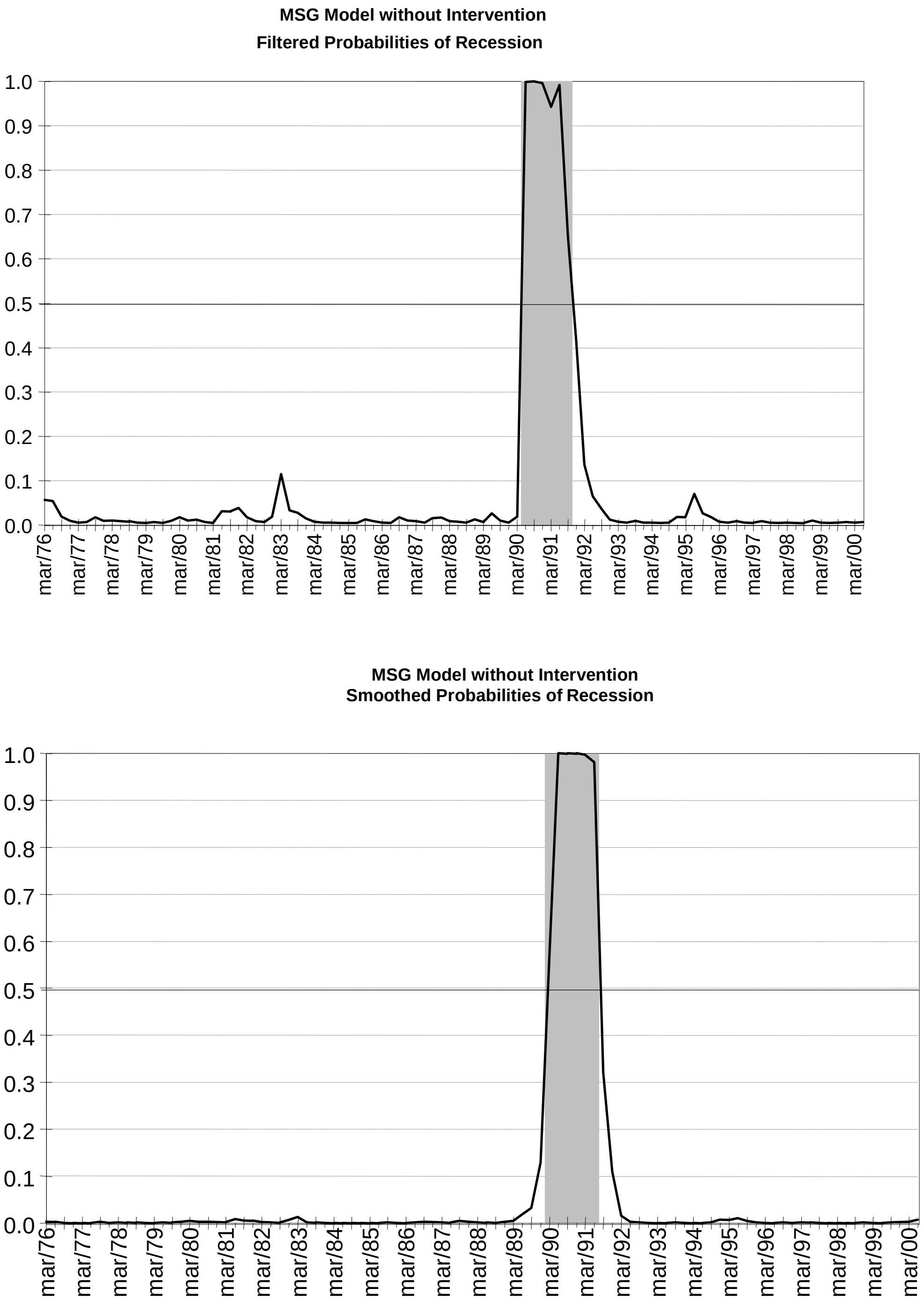
FIGURE 2
Filtered and Smoothed Probabilities of Recessions: MS-AR(2) Model without Intervention



4. The estimation procedure was as follows: first, the MS model was estimated considering an AR(0). Second, the MLE parameters from this model were used to initialize the estimation of the MS-AR(1). Next, the MLE parameters of the MS-AR(1) were used to initialize the MS-AR(2) and so on. The MLE parameters of the MS models were then used to initialize the MSG model.

FIGURE 3

Filtered and Smoothed Probabilities of Recessions: MSG-AR(3) Model without Intervention



The estimation results without i'ntervention of several autoregressive specifications of MS and MSG models are reported in Tables 1 and 2. Notice that these models were estimated allowing both mean and variance to switch regimes. The specifications allowing only the mean to switch between states did not converge.[5] Overall the estimates from Lam's model were more stable as the number of lags increased. On the other hand, Hamilton's model presented instability with respect to the parameters as the number of lags increased. This is not surprising since, as mentioned before, for low order processes there is concentration of recession probabilities during the Collor Plans.

Using the likelihood ratio test, we find that the best specifications without intervention were an AR(4) process for the MS model [MS-AR(4)] and an AR(2) process for the MSG model [MSG-AR(2)]. We have also tested the out-of-sample forecasting performance of several Markov switching models, with autoregressive components, comparing them with linear models and with the MS-AR(0) model. Two linear models were estimated for comparison with the Markov switching models: an AR(3) and an ARMA(1,1) model.[6] All models were estimated from 1976:2 up to 1992:1, and then recursively re-estimated for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2 to generate the out-of-sample forecasts.[7] Table 3 shows the relative mean squared error (MSE) of a selected group of alternative models for the 1992:2-2000:2 period ['No Change,'[8] AR(3), ARMA(1,1), MS-AR(2), MS-AR(4) and MSG-AR(2)]. The relative mean squared errors are computed with respect to three benchmark models: AR(3), ARMA(1,1) and MS-AR(0). Table 3 also reports the heteroscedasticity and autocorrelation consistent (HAC) standard errors of these relative MSE.[9] The MS-AR(4) gives the best short-run forecasts (1 to 2 steps ahead). The linear AR(3) model does better than the other models for longer forecasts.

We introduce interventions in the models for two reasons. First, the Collor Plan has engendered strong real effects in the economy, which influence the specification of the MS and MSG models. Second, without explicitly modeling the breaks the MSG model does not capture the Brazilian business cycle. As it is shown in the next section, the probabilities from the models with interventions characterize recessions and expansions rather than solely the Collor Plan, and increase the forecasting ability of MS and MSG models.

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5. The likelihood function increases as the probability of recessions converges to a very small value, capturing the break instead of expansions and recessions in the Brazilian output.

6. The identification of the ARMA model was implemented using AIC and SBC criteria. In addition, given that structural breaks generally lead to serial correlation in the residuals, Durbin-Watson test was used to test whether the residuals of the selected model are white noise. The identification was implemented considering or not dummies for the period between 1990.1 a 1991.2.

7. Appendix B shows how these forecasts were calculated. For the out-of-sample forecasts, the models were estimated from 1976:2 up to 2000:2, and then were used to predict the annual rate of growth of GDP from 2000:3 to 2001:4.

8. The 'No Change' model refers to the random walk $y_t = y_{t-1} + e_t$, $e_t \sim WN(0, \sigma^2)$.

9. There is an asymptotic justification for this procedure in the case of recursively estimated models, as explained in West (1996).

TABLE 1

Hamilton's Model (MS) under Different Specifications — No Intervention

	AR(0)	AR(0)	AR(1)	AR(0)	AR(1)	AR(2)	AR(0)	AR(1)	AR(2)	AR(3)	AR(0)	AR(1)	AR(2)	AR(3)	AR(4)	AR(0)	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Num. obs.	101	100	100	99	99	99	98	98	98	98	97	97	97	97	97	96	96	96	96	96	96
Log(L(θ))	248.79	246.25	247.66	243.10	244.57	246.49	240.18	241.67	243.72	245.58	240.38	240.40	244.16	245.79	247.99	237.45	238.74	240.92	242.57	244.51	244.60
P_{00}	0.928	0.928	0.986	0.928	0.985	0.986	0.923	0.985	0.986	0.839	0.927	0.924	0.793	0.817	0.787	0.924	0.985	0.986	0.643	0.782	0.780
	(0.054)	(0.056)	(0.015)	(0.059)	(0.015)	(0.014)	(0.063)	(0.016)	(0.015)	(0.104)	(0.051)	(0.053)	(0.072)	(0.066)	(0.068)	(0.053)	(0.016)	(0.015)	(0.118)	(0.070)	(0.067)
P_{11}	0.812	0.807	0.821	0.804	0.821	0.816	0.803	0.821	0.815	0.630	0.824	0.835	0.678	0.644	0.683	0.826	0.821	0.815	0.811	0.677	0.673
	(0.141)	(0.144)	(0.158)	(0.145)	(0.158)	(0.161)	(0.144)	(0.157)	(0.161)	(0.128)	(0.132)	(0.125)	(0.149)	(0.114)	(0.111)	(0.130)	(0.157)	(0.161)	(0.070)	(0.109)	(0.107)
μ_0	0.012	0.012	0.008	0.011	0.008	0.008	0.011	0.008	0.008	0.015	0.011	0.012	0.017	0.016	0.018	0.012	0.008	0.008	0.016	0.018	0.018
	(0.003)	(0.003)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)	(0.001)	(0.003)	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)
μ_1	–0.004	–0.004	0.00004	–0.004	0.00005	–0.007	–0.003	0.0002	–0.006	–0.011	–0.004	–0.003	–0.009	–0.011	–0.008	–0.004	0.000	–0.007	–0.010	–0.009	–0.009
	(0.008)	(0.008)	(0.024)	(0.008)	(0.023)	(0.022)	(0.008)	(0.023)	(0.022)	(0.009)	(0.007)	(0.007)	(0.006)	(0.005)	(0.004)	(0.007)	(0.024)	(0.022)	(0.005)	(0.004)	(0.003)
σ_0	0.015	0.015	0.018	0.015	0.018	0.017	0.015	0.018	0.017	0.012	0.014	0.014	0.010	0.011	0.009	0.014	0.017	0.017	0.011	0.009	0.009
	(0.002)	(0.002)	(0.001)	(0.003)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)
σ_1	0.033	0.033	0.054	0.033	0.054	0.055	0.034	0.054	0.055	0.028	0.032	0.032	0.024	0.025	0.022	0.032	0.055	0.056	0.025	0.022	0.022
	(0.007)	(0.007)	(0.017)	(0.008)	(0.017)	(0.017)	(0.008)	(0.016)	(0.017)	(0.006)	(0.006)	(0.006)	(0.004)	(0.004)	(0.004)	(0.006)	(0.017)	(0.017)	(0.004)	(0.003)	(0.003)
ϕ_1	-	-	0.178	-	0.177	0.200	-	0.176	0.199	–0.063	-	–0.028	–0.387	–0.244	–0.330	-	0.187	0.209	0.190	–0.325	0.375
			(0.101)		(0.102)	(0.101)	-	(0.102)	(0.101)	(0.179)		(0.134)	(0.219)	(0.163)	(0.251)		(0.103)	(0.101)	(0.172)	(0.191)	(0.196)
ϕ_2	-	-	-	-	-	–0.189	-	-	–0.197	–0.222	-	-	–0.447	–0.359	–0.534	-	-	–0.199	0.327	–0.535	0.546
						(0.095)	-		(0.095)	(0.159)			(0.168)	(0.126)	(0.219)			(0.093)	(0.126)	(0.156)	(0.149)
ϕ_3	-	-	-	-	-	-	-	-	-	0.385	-	-	-	0.237	0.001	-	-	-	0.272	0.013	0.057
							-		-	(0.153)				(0.125)	(0.279)				(0.130)	(0.168)	(0.212)
ϕ_4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	–0.244	-	-	-	-	–0.243	0.276
							-		-						(0.142)					(0.106)	(0.131)
ϕ_5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.072
																					(0.171)
Theil-U	0.710	0.714	0.669	0.714	0.668	0.635	0.715	0.667	0.633	0.420	0.709	0.573	0.464	0.421	0.385	0.711	0.668	0.635	0.409	0.387	0.362

Note: Standard deviation in parenthesis.

TABLE 2

Lam's Model (MSG) under Different Specifications — No Intervention

	AR(1)	AR(1)	AR(2)	AR(1)	AR(2)	AR(3)	AR(1)	AR(2)	AR(3)	AR(4)	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
Obs.	100	99	99	98	98	98	97	97	97	97	96	96	96	96	96
Log(L(θ))	250.529	247.623	250.359	244.559	247.287	248.378	243.049	245.551	246.817	247.851	240.334	243.579	244.741	245.956	246.641
P_{00}	0.986	0.986	0.988	0.986	0.988	0.988	0.986	0.988	0.988	0.988	0.933	0.988	0.988	0.988	0.988
	(0.015)	(0.015)	(0.012)	(0.016)	(0.012)	(0.012)	(0.015)	(0.012)	(0.012)	(0.012)	(0.050)	(0.013)	(0.012)	(0.012)	(0.013)
P_{11}	0.818	0.817	0.816	0.816	0.815	0.812	0.818	0.816	0.813	0.811	0.826	0.817	0.814	0.812	0.760
	(0.165)	(0.166)	(0.164)	(0.166)	(0.164)	(0.165)	(0.164)	(0.163)	(0.164)	(0.168)	(0.127)	(0.163)	(0.164)	(0.167)	(0.218)
μ_0	0.007	0.006	0.007	0.006	0.007	0.007	0.007	0.007	0.007	0.007	0.009	0.007	0.007	0.007	0.007
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)
μ_1	−0.001	−0.001	−0.004	−0.001	−0.004	−0.004	−0.002	−0.005	−0.006	−0.006	−0.001	−0.004	−0.004	−0.004	−0.005
	(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.009)	(0.010)	(0.011)	(0.010)	(0.010)	(0.004)	(0.010)	(0.010)	(0.010)	(0.012)
σ_0	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.017	0.016	0.016	0.014	0.017	0.016	0.016	0.016
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
σ_1	0.049	0.048	0.056	0.048	0.056	0.056	0.049	0.056	0.056	0.054	0.032	0.056	0.057	0.054	0.057
	(0.015)	(0.015)	(0.018)	(0.015)	(0.018)	(0.017)	(0.015)	(0.018)	(0.017)	(0.017)	(0.007)	(0.018)	(0.017)	(0.017)	(0.020)
ϕ_1	0.878	0.872	1.076	0.869	1.073	1.102	0.887	1.078	1.110	1.122	0.889	1.107	1.136	1.152	1.177
	(0.046)	(0.047)	(0.098)	(0.048)	(0.098)	(0.100)	(0.049)	(0.097)	(0.098)	(0.099)	(0.056)	(0.099)	(0.100)	(0.101)	(0.102)
ϕ_2	-	-	−0.228	-	−0.229	−0.380	-	−0.217	−0.376	−0.418	-	−0.256	−0.407	−0.455	−0.529
			(0.094)		(0.095)	(0.138)		(0.094)	(0.136)	(0.138)		(0.097)	(0.138)	(0.140)	(0.148)
ϕ_3	-	-	-	-	-	0.138	-	-	0.147	0.308	-	-	0.141	0.312	0.441
						(0.09)			(0.09)	(0.15)			(0.09)	(0.15)	(0.16)
ϕ_4	-	-	-	-	-	-	-	-	-	−0.146	-	-	-	−0.155	−0.349
										(0.107)				(0.105)	(0.158)
ϕ_5	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.125
															(0.094)
Z_0	−0.159	−0.163	−0.140	−0.164	−0.142	−0.151	−0.151	−0.132	−0.140	−0.131	−0.162	−0.143	−0.151	−0.142	−0.154
	(0.050)	(0.049)	(0.037)	0.048	(0.037)	(0.042)	0.054	(0.040)	(0.047)	(0.042)	(0.082)	(0.039)	(0.044)	(0.040)	(0.047)
Theil-U	0.693	0.692	0.694	0.692	0.694	0.685	0.695	0.696	0.687	0.669	0.691	0.697	0.689	0.669	0.656

Note: Standard deviation in parenthesis.

TABLE 3

Linear and Nonlinear Models: Out-of-Sample Forecasting Performance without Intervention

Steps	RMSE Benchmark model	No change		AR(3)		ARMA(1,1)		MS-AR(0)		MS-AR(2)		MS-AR(4)		MSG-AR(2)	
	AR(3)	MSE of each model relative to the MSE of the AR(3) model													
1	0.01586	1.920	(0.393)	-	-	0.956	(0.043)	1.172	(0.069)	1.001	(0.061)	0.932	(0.085)	1.085	(0.044)
2	0.01597	3.088	(1.664)	-	-	0.994	(0.023)	1.271	(0.095)	1.054	(0.028)	0.967	(0.035)	1.170	(0.089)
3	0.01687	1.836	(0.677)	-	-	1.001	(0.026)	1.058	(0.019)	1.006	(0.051)	1.082	(0.053)	1.112	(0.068)
4	0.01669	1.960	(0.410)	-	-	1.060	(0.018)	1.119	(0.040)	1.049	(0.013)	1.182	(0.104)	1.117	(0.107)
5	0.01671	2.658	(0.888)	-	-	1.042	(0.021)	1.110	(0.038)	1.051	(0.016)	1.097	(0.046)	1.066	(0.062)
6	0.01695	1.944	(0.456)	-	-	1.003	(0.006)	1.024	(0.016)	1.011	(0.006)	1.102	(0.054)	1.070	(0.051)
7	0.01698	1.291	(0.228)	-	-	0.998	(0.005)	1.025	(0.014)	1.004	(0.006)	1.105	(0.046)	1.083	(0.072)
8	0.01746	2.255	(0.505)	-	-	1.003	(0.005)	1.070	(0.038)	1.003	(0.008)	1.029	(0.014)	1.032	(0.048)
	ARMA(1,1)	MSE of each model relative to the MSE of the ARMA(1,1) model													
1	0.01550	2.009	(0.446)	1.047	(0.047)	-	-	1.227	(0.074)	1.047	(0.026)	0.975	(0.075)	1.136	(0.063)
2	0.01592	3.108	(1.700)	1.006	(0.024)	-	-	1.279	(0.112)	1.060	(0.011)	0.973	(0.039)	1.177	(0.108)
3	0.01689	1.834	(0.695)	0.999	(0.026)	-	-	1.056	(0.027)	1.005	(0.028)	1.080	(0.075)	1.110	(0.086)
4	0.01718	1.849	(0.353)	0.943	(0.016)	-	-	1.055	(0.025)	0.989	(0.007)	1.115	(0.084)	1.054	(0.087)
5	0.01707	2.550	(0.806)	0.959	(0.020)	-	-	1.065	(0.033)	1.008	(0.012)	1.053	(0.047)	1.023	(0.064)
6	0.01697	1.938	(0.456)	0.997	(0.006)	-	-	1.021	(0.019)	1.008	(0.005)	1.099	(0.058)	1.067	(0.054)
7	0.01697	1.294	(0.227)	1.002	(0.005)	-	-	1.028	(0.015)	1.007	(0.003)	1.107	(0.048)	1.086	(0.077)
8	0.01749	2.248	(0.501)	0.997	(0.005)	-	-	1.067	(0.042)	1.000	(0.004)	1.025	(0.011)	1.029	(0.052)
	MS-AR(0)	MSE of each model relative to the MSE of the MS-AR(0) model													
1	0.01717	1.638	(0.252)	0.853	(0.050)	0.815	(0.049)	-	-	0.854	(0.045)	0.795	(0.092)	0.926	(0.043)
2	0.01801	2.430	(0.973)	0.787	(0.059)	0.782	(0.068)	-	-	0.829	(0.067)	0.761	(0.076)	0.920	(0.055)
3	0.01736	1.736	(0.609)	0.945	(0.017)	0.947	(0.024)	-	-	0.951	(0.043)	1.022	(0.055)	1.051	(0.071)
4	0.01765	1.752	(0.307)	0.894	(0.032)	0.948	(0.022)	-	-	0.937	(0.023)	1.057	(0.062)	0.998	(0.071)
5	0.01761	2.394	(0.695)	0.901	(0.031)	0.939	(0.029)	-	-	0.947	(0.022)	0.988	(0.019)	0.960	(0.042)
6	0.01715	1.899	(0.426)	0.977	(0.015)	0.980	(0.019)	-	-	0.987	(0.016)	1.076	(0.043)	1.046	(0.048)
7	0.01720	1.259	(0.221)	0.975	(0.013)	0.973	(0.015)	-	-	0.980	(0.016)	1.078	(0.042)	1.056	(0.064)
8	0.01807	2.107	(0.434)	0.934	(0.033)	0.937	(0.037)	-	-	0.937	(0.039)	0.961	(0.039)	0.964	(0.024)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The entries from the second to the last column are the mean squared forecast error (MSE) of the model described in the first line of the table relative to the MSE of the Benchmark model. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

We estimate the models under several alternative interventions in order to overcome the problem of structural breaks in the Brazilian economy. In particular, we estimate alternative specifications in which the drift parameters are allowed to take different values during the Collor I and II stabilization Plans. We also estimate the model treating the observations of Collor I and II Plans as outliers.

Models with Intervention

When the models are estimated without intervention, there is a tendency for the filtered probabilities to concentrate around the 1990:1-1991:2 structural break [for MS-AR(1) and MS-AR(2) models and all estimated MSG specifications]. These results suggest that intervention should be implemented in the 1990:1-1991:2 period. The models were estimated under alternative interventions in the drift term or treating the observations for certain periods as outliers. We report the results for only the two interventions that were successful in characterizing the Brazilian business cycle.[10] The first intervention is modeled as the sum of an additional parameter δ_i during the Collor Plan (intervention type 1):

$$\mu_{st} = \mu_0 (1 - S_t) + \mu_1 S_t + \delta_i \qquad \text{for } i = 1990{:}1, \ldots, 1991{:}2$$

$$\mu_{st} = \mu_0 (1 - S_t) + \mu_1 S_t \qquad \text{otherwise}$$

The second intervention considers the period of the Collor Plans (1990.1 to 1991.2) as outliers (intervention type 2). One advantage of this method is that the intervention capturing the break is not restricted to be only in the trend component.

4 RESULTS

For the models with intervention types 1 and 2, there is no convergence problem and the regime switching parameters are significant at all levels. Compared with the alternative specifications, these interventions are the ones that yield the most reasonable results. The results for the best models are discussed below.

4.1 RESULTS FOR SELECTED MODELS

Based on the likelihood ratio test, Theil-U statistic and the filtered probabilities, the models selected as presenting the best fit to the Brazilian business cycle are a MS-AR(2) and a MSG-AR(2) with intervention of type 1 and 2. Table 4 shows the results for MS and MSG models for the intervention of type 1, while Table 5 reports the results for intervention type 2. Since the results are similar, for both interventions, we choose to report the ones for intervention type 2.

10. The results for the other interventions are available from the authors upon request.

TABLE 4

Hamilton´s Model (MS) and Lam´s Models (MSG) under Different Specifications and Intervention Type 1

Num. obs.	Hamilton's model (MS)										Lam's model (MSG)					
	AR(0) 101	AR(0) 100	AR(1) 100	AR(0) 99	AR(1) 99	AR(2) 99	AR(0) 98	AR(1) 98	AR(2) 98	AR(3) 98	AR(1) 100	AR(1) 99	AR(2) 99	AR(1) 98	AR(2) 98	AR(3) 98
Log(L(θ))	271.113	268.444	268.447	265.130	265.139	269.050	261.864	261.915	265.775	266.401	274.177	272.058	277.074	270.144	275.604	277.066
P_{00}	0.875	0.876	0.877	0.874	0.875	0.853	0.874	0.876	0.854	0.851	0.853	0.853	0.835	0.830	0.833	0.834
	(0.060)	(0.060)	(0.061)	(0.062)	(0.064)	(0.047)	(0.063)	(0.068)	(0.048)	(0.052)	(0.044)	(0.044)	(0.045)	(0.048)	(0.045)	(0.045)
P_{11}	0.503	0.500	0.498	0.499	0.496	0.571	0.498	0.486	0.570	0.521	0.567	0.572	0.552	0.540	0.552	0.545
	(0.143)	(0.145)	(0.147)	(0.145)	(0.151)	(0.107)	(0.147)	(0.159)	(0.109)	(0.122)	(0.109)	(0.107)	(0.102)	(0.108)	(0.102)	(0.101)
$\mu 0$	0.015	0.015	0.015	0.015	0.015	0.017	0.015	0.014	0.017	0.016	0.017	0.016	0.017	0.016	0.017	0.017
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)	(0.003)	(0.001)	(0.002)	(0.0005)	(0.0005)	(0.0004)	(0.0005)	(0.0004)	(0.0004)
$\mu 1$	–0.016	–0.016	–0.016	–0.016	–0.016	–0.014	–0.016	–0.016	–0.014	–0.015	–0.015	–0.015	–0.015	–0.015	–0.015	–0.015
	(0.004)	(0.004)	(0.005)	(0.005)	(0.005)	(0.002)	(0.005)	(0.005)	(0.002)	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma 0$	0.013	0.013	0.013	0.013	0.013	0.011	0.013	0.013	0.011	0.011	0.010	0.010	0.009	0.010	0.009	0.009
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\phi 1$	-	-	0.012	-	0.021	–0.201	-	0.053	–0.184	0.140	0.424	0.391	0.555	0.405	0.595	0.603
			(0.110)		(0.176)	(0.147)		(0.172)	(0.165)	(0.233)	(0.107)	(0.107)	(0.108)	(0.099)	(0.116)	(0.106)
$\phi 2$	-	-	-	-	-	–0.456	-	-	–0.457	–0.358	-	-	–0.337	-	–0.327	–0.437
						(0.127)			(0.133)	(0.142)			(0.090)		(0.092)	(0.108)
$\phi 3$	-	-	-	-	-	-	-	-	-	0.260	-	-	-	-	-	0.189
										(0.166)						(0.102)
Intervention 1	–0.042	–0.042	–0.041	–0.042	–0.041	–0.050	–0.041	–0.041	–0.049	–0.049	–0.046	–0.045	–0.041	–0.045	–0.041	–0.043
	(0.013)	(0.013)	(0.013)	(0.013)	(0.014)	(0.012)	(0.013)	(0.014)	(0.012)	(0.012)	(0.011)	(0.010)	(0.009)	(0.010)	(0.009)	(0.009)
Intervention 2	–0.100	–0.100	–0.100	–0.100	–0.100	–0.109	–0.100	–0.099	–0.109	–0.107	–0.103	–0.102	–0.106	–0.102	–0.106	–0.107
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.010)	(0.011)	(0.010)	(0.010)
Intervention 3	0.059	0.059	0.059	0.059	0.059	0.061	0.059	0.060	0.061	0.063	0.057	0.057	0.054	0.057	0.054	0.057
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.012)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)
Intervention 4	–0.035	–0.034	–0.034	–0.034	–0.034	–0.032	–0.034	–0.034	–0.032	–0.032	–0.037	–0.036	–0.038	–0.036	–0.038	–0.034
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.012)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)
Intervention 5	–0.059	–0.059	–0.059	–0.059	–0.059	–0.064	–0.059	–0.059	–0.064	–0.063	–0.061	–0.061	–0.061	–0.061	–0.061	–0.061
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.012)	(0.012)	(0.012)	(0.012)	(0.010)	(0.011)	(0.010)	(0.010)
Intervention 6	0.047	0.048	0.048	0.048	0.048	0.040	0.048	0.048	0.040	0.042	0.045	0.046	0.048	0.046	0.048	0.045
	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.011)	(0.013)	(0.013)	(0.011)	(0.012)	(0.010)	(0.010)	(0.009)	(0.010)	(0.009)	(0.009)
Z_0	-	-	-	-	-	-	-	-	-	-	–0.033	–0.035	–0.063	–0.068	–0.063	–0.066
											(0.006)	(0.006)	(0.004)	(0.006)	(0.004)	(0.005)
Theil-U	0.525	0.525	0.460	0.525	0.458	0.428	0.523	0.455	0.426	0.407	0.515	0.512	0.497	0.510	0.499	0.488

TABLE 5

Hamilton’s Model (MS) and Lam’s Models (MSG) under Different Specifications and Intervention Type 2

	Hamilton's model (MS)										Lam's model (MSG)					
Num. obs.	AR(0) 101	AR(0) 100	AR(1) 100	AR(0) 99	AR(1) 99	AR(2) 99	AR(0) 98	AR(1) 98	AR(2) 98	AR(3) 98	AR(1) 100	AR(1) 99	AR(2) 99	AR(1) 98	AR(2) 98	AR(3) 98
Log(L(θ))	251.295	248.634	248.660	245.374	245.415	248.116	242.165	242.275	244.941	245.864	251.327	249.011	253.387	247.737	251.914	253.206
P_{00}	0.864	0.865	0.868	0.862	0.865	0.849	0.862	0.863	0.848	0.846	0.779	0.777	0.768	0.770	0.767	0.769
	(0.069)	(0.070)	(0.074)	(0.073)	(0.078)	(0.054)	(0.075)	(0.087)	(0.055)	(0.074)	(0.040)	(0.038)	(0.034)	(0.040)	(0.034)	(0.034)
P_{11}	0.502	0.498	0.491	0.498	0.489	0.554	0.496	0.477	0.551	0.495	0.567	0.584	0.596	0.570	0.592	0.594
	(0.150)	(0.152)	(0.160)	(0.152)	(0.162)	(0.116)	(0.154)	(0.171)	(0.118)	(0.137)	(0.123)	(0.109)	(0.088)	(0.113)	(0.088)	(0.090)
$\mu 0$	0.015	0.015	0.014	0.015	0.014	0.016	0.015	0.014	0.016	0.015	0.017	0.017	0.017	0.017	0.017	0.017
	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)	(0.001)	(0.0005)	(0.0004)	(0.0006)	(0.0004)	(0.0005)
$\mu 1$	–0.015	–0.015	–0.016	–0.015	–0.015	–0.014	–0.015	–0.015	–0.014	–0.016	–0.015	–0.014	–0.015	–0.015	–0.015	–0.015
	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.003)	(0.005)	(0.006)	(0.003)	(0.005)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\sigma 0$	0.013	0.013	0.013	0.013	0.013	0.012	0.013	0.014	0.012	0.012	0.010	0.010	0.009	0.010	0.009	0.009
	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
$\phi 1$	-	-	0.036	-	0.047	–0.123	-	0.080	–0.098	0.193	0.432	0.398	0.546	0.410	0.582	0.596
			(0.158)		(0.165)	(0.164)		(0.174)	(0.175)	(0.232)	(0.116)	(0.123)	(0.112)	(0.109)	(0.120)	(0.113)
$\phi 2$	-	-	-	-	-	–0.383	-	-	–0.384	–0.303	-	-	–0.321	-	–0.309	–0.420
						(0.143)			(0.141)	(0.169)			(0.100)		(0.100)	(0.115)
$\phi 3$	-	-	-	-	-	-	-	-	-	0.275	-	-	-	-	-	0.185
										(0.172)						(0.101)
Z_0	-	-	-	-	-	-	-	-	-	-	–0.064	–0.066	–0.063	–0.067	–0.063	–0.065
											(0.006)	(0.006)	(0.004)	(0.006)	(0.004)	(0.005)
Theil-U	0.628	0.627	0.687	0.624	0.684	0.633	0.622	0.679	0.629	0.594	0.752	0.749	0.729	0.749	0.732	0.717

The estimated parameters from both models are very similar and the sample identifies two significant states for the Brazilian economy. The MS-AR(2) model estimates in state 1 that the economy grows at an average negative rate of around 1.4% per quarter (–5.6% a year) while in state 0 the Brazilian economy grows at an average rate of 1.6% per quarter (6.4% a year). The MSG-AR(2) model estimates that in state 1 the economy grows at an average negative rate of around 1.5% per quarter (6% a year) while in state 0 the Brazilian economy grows at a rate of 1.7% per quarter (6.8% a year). Recessions in Brazil last a short time, averaging between two and three quarters for both models. Expansions last twice as long. The MS model estimates that periods of positive growth last on average between six and seven quarters (p_{00}=0.85), while for the MSG model the duration of expansions is around four and five quarters (p_{00}=0.77). Table 6 shows a summary of these results.

TABLE 6
Business Cycle Features for Selected Models

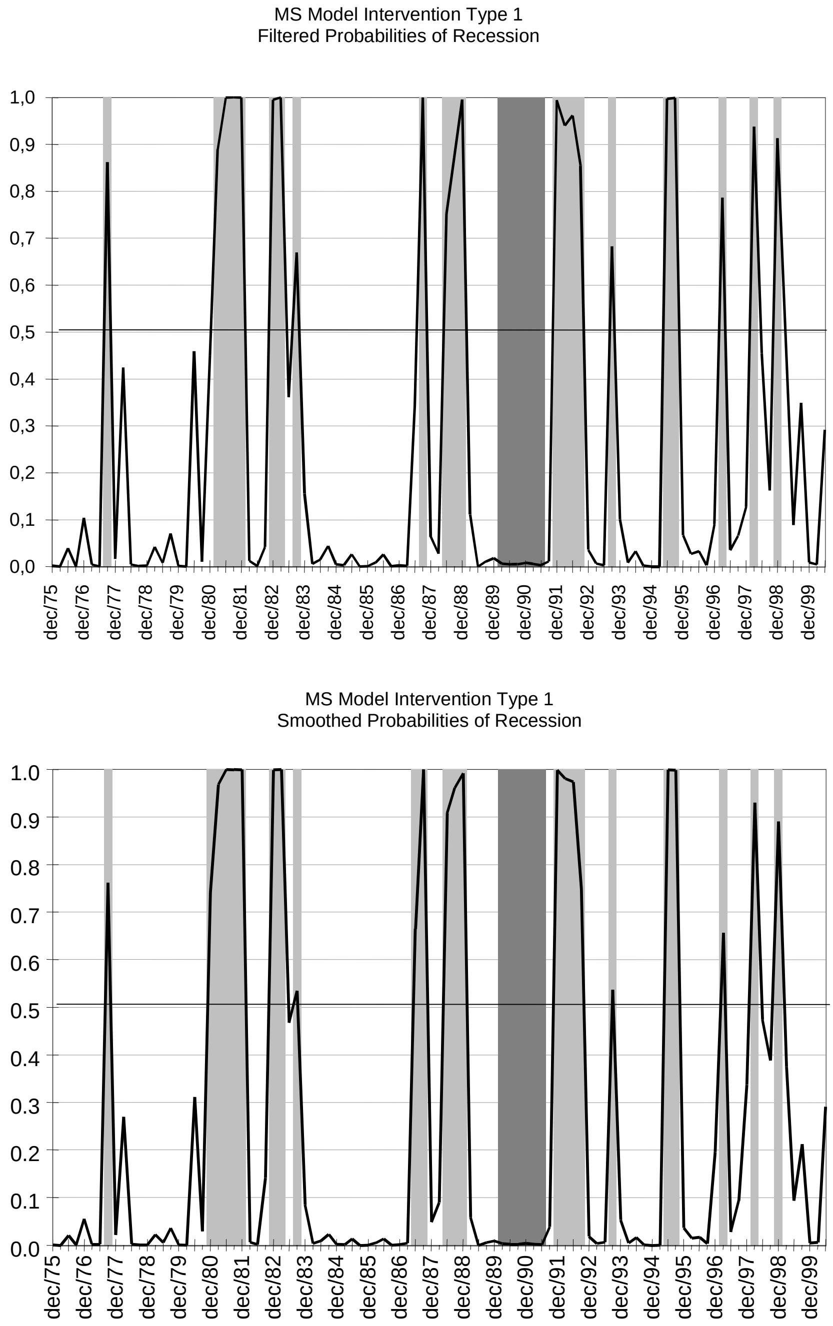
		Type 1		Type 2		
		MS-AR(2)	MSG-AR(2)	MS-AR(0)	MS-AR(2)	MSG-AR(2)
Recession	Mean growth rate (%)	–1.4	–1.5	–1.5	–1.4	–1.5
	Duration in quarters	2-3	2-3	1-2	2-3	2-3
Expansion	Mean growth rate (%)	1.7	1.7	1.5	1.6	1.7
	Duration in quarters	6-7	6-7	7-8	6-7	4-5

Thus, these models predict that the length of the Brazilian business cycle is between two and three years. This short duration of the Brazilian business cycle is a consequence of the economic instability and turbulence due to the hyperinflationary process in the 1980s and the implementation of several stabilization plans in the last two decades. These results are very similar to those obtained for Brazil in Chauvet (2002*a*), Lima and Domingues (2000) and Mejia-Reyes (1999). In addition, Mejia-Reyes finds that several other Latin American countries present these same business cycle features.

The filtered and smoothed probabilities for the selected models are plotted in Figures 4 to 7. Several results stand out from these inferences. First, the filtered and smoothed probabilities models are very similar, which points out to the stability of the recursive one-step-ahead estimation (filtered probabilities) compared to the estimation using the whole sample (smoothed probabilities). Second, the probabilities from the MS and the MSG models are also very similar, capturing the same features and phases of the Brazilian business cycles.

Using the criteria that a turning point occurs if the smoothed probabilities of a state are greater or equal than the probability of the other state, the Brazilian economy experienced ten downturns between 1980 and 2000. However, some of these contractions were very short-lived, lasting only one quarter (e.g., the low growth phase in 1984 and the expansion in 1998). If we consider recessions as periods of negative growth with a minimum duration of six months, the downturns in 1982-1983, 1983-1984 would be considered as one longer recession rather than a double dip. This is also the case for the downturns in 1997-1998. Under this minimum duration rule for business cycle phases, the Brazilian economy experienced eight recessions in the last two decades. These results are corroborated by the findings in Mejia-Reyes (1999),[11] Lima and Domingues (2000) and Chauvet (2002*a*).

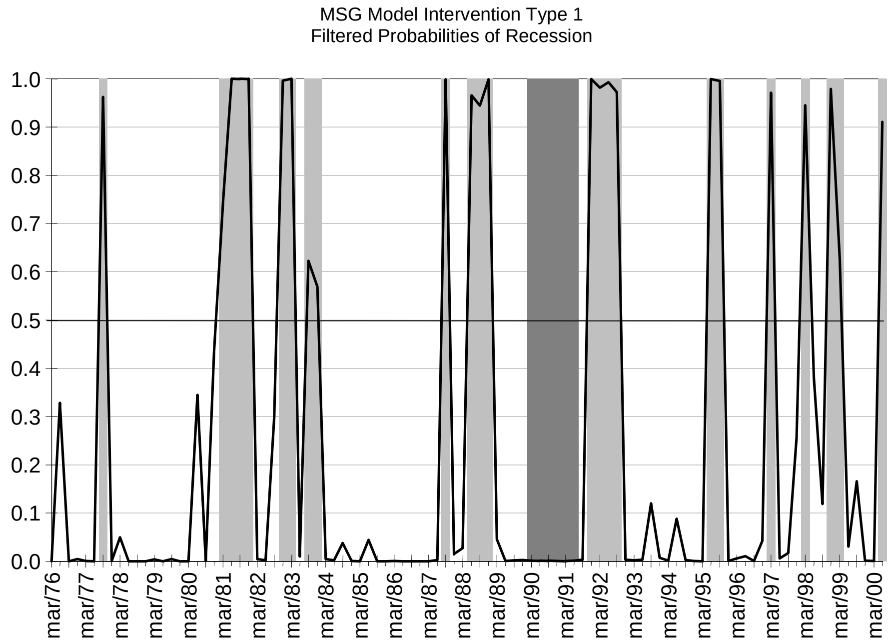
FIGURE 4
Filtered and Smoothed Probabilities of Recessions: MS AR(2) Model (Intervention Type 1)



11. The results are consistent with the ones obtained by this author up to the last year of its estimation for Brazil (1995).

FIGURE 5

Filtered and Smoothed Probabilities of Recessions: MSG AR(2) Model with Intervention Type 1



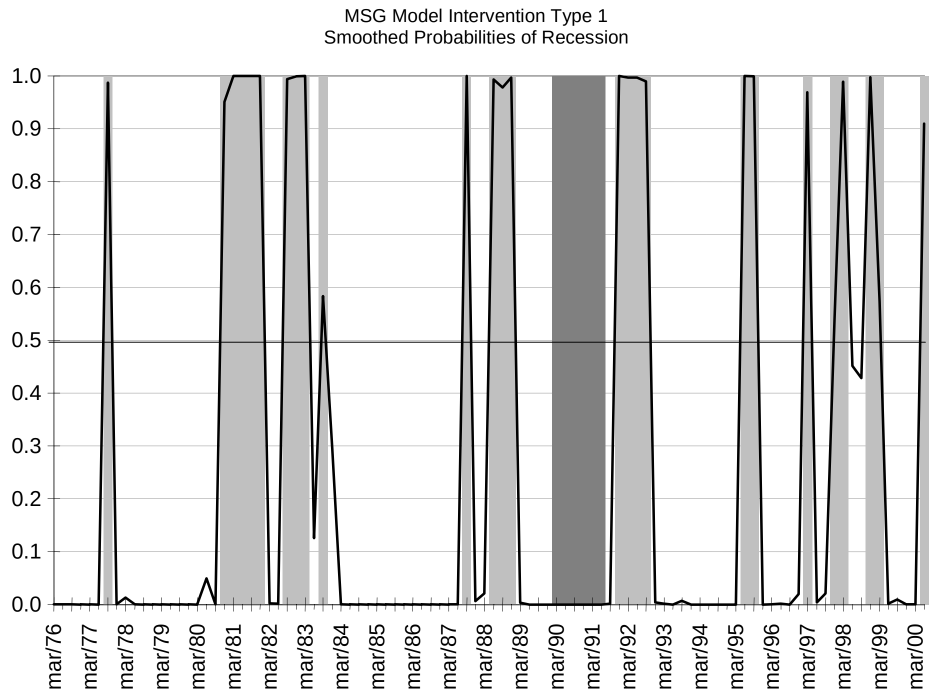
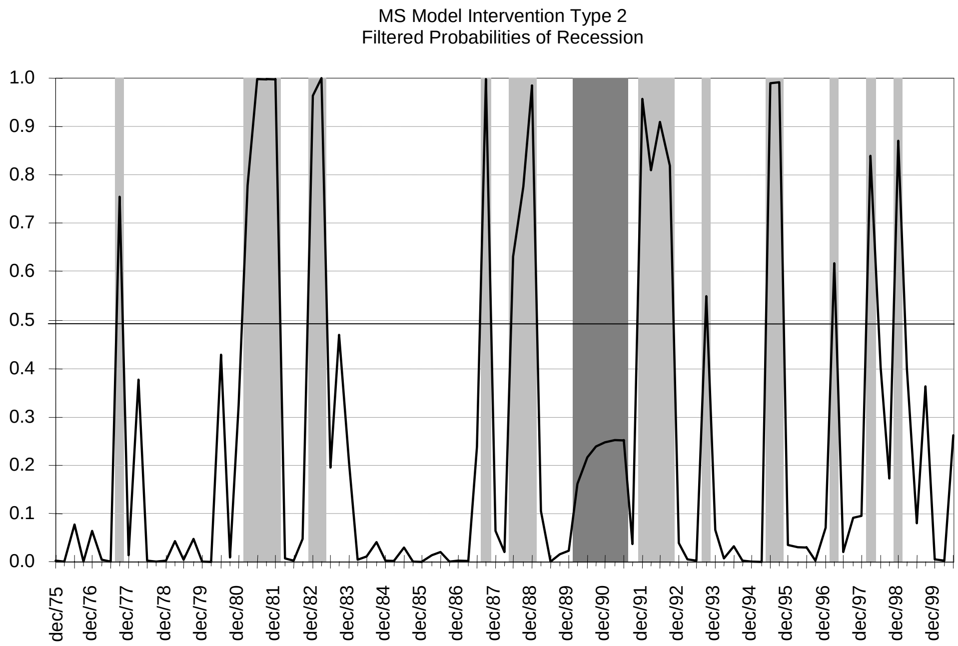


FIGURE 6
Filtered and Smoothed Probabilities of Recessions: MS AR(2) Model Intervention Type 2



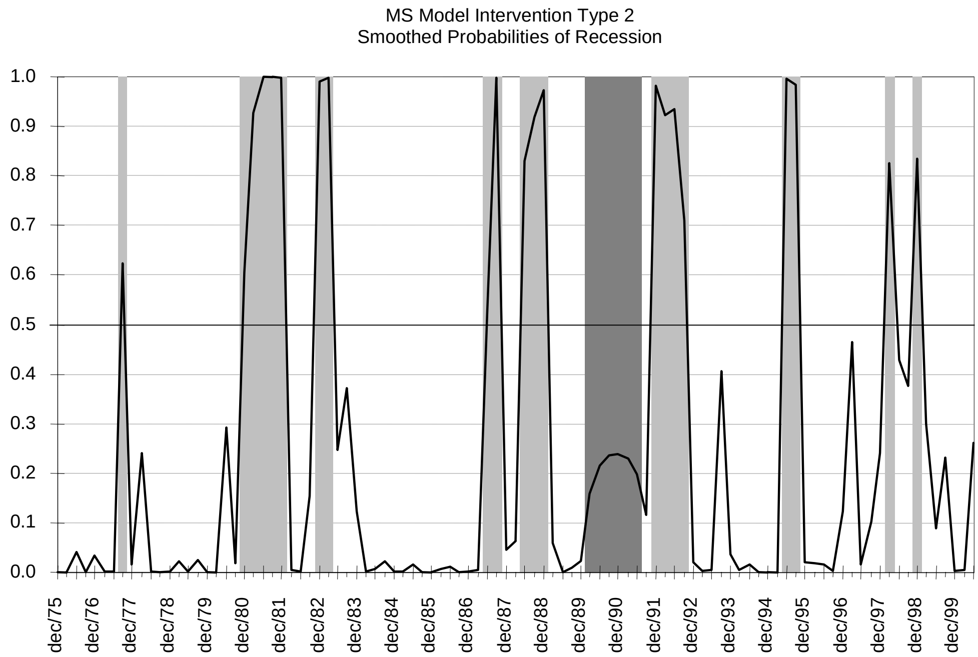
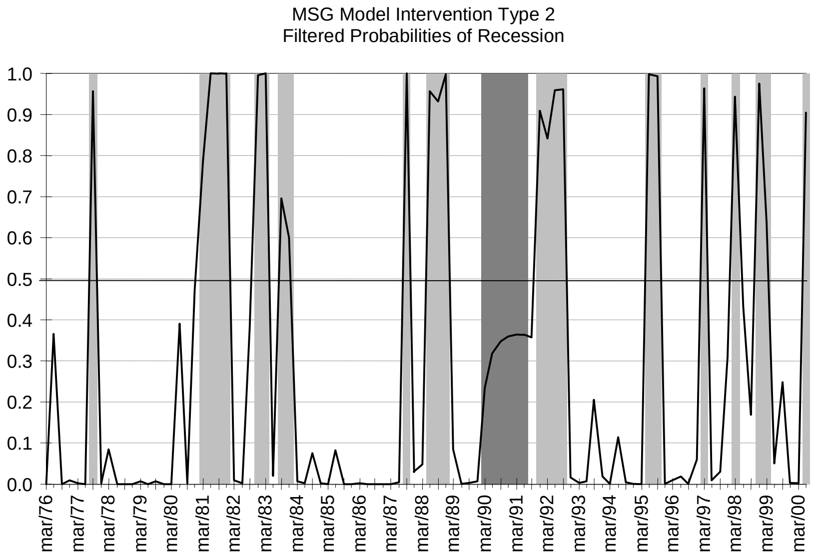
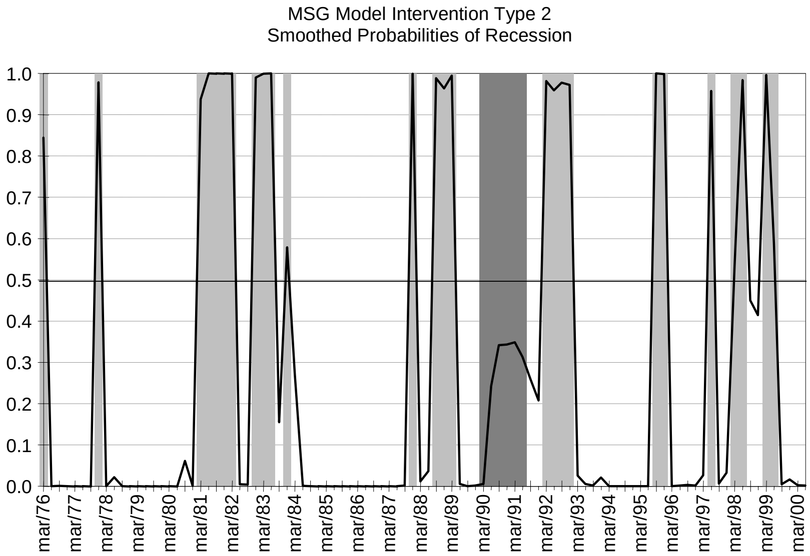


FIGURE 7

Filtered and Smoothed Probabilities of Recessions: MSG Model AR(2) with Intervention Type 2



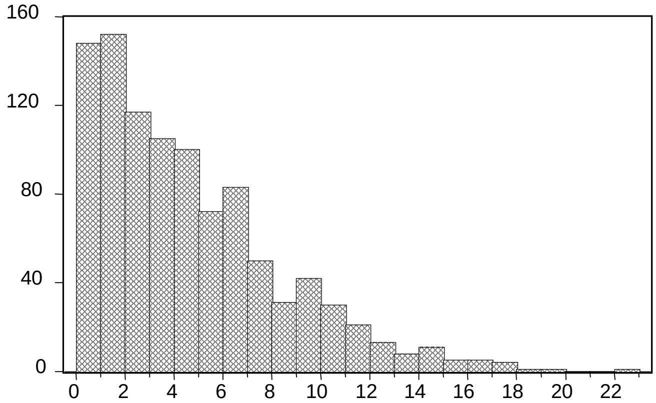


4.2 COMPARISON BETWEEN THE MS AND MSG MODELS

The MSG-AR(3) model nests the models selected as presenting the best fit to the Brazilian business cycle: the MS-AR(2) and the MSG-AR(2). The likelihood ratio used to test the MSG-AR(2) model against the MSG-AR(3) model has a standard asymptotic distribution, $\chi^2(1)$, and can be easily calculated using the likelihood values presented in Table 5. The likelihood ratio is equal to 2.584 and, therefore, we cannot reject that the MSG-AR(2) model fits the data better than the MSG-AR(3) model. If we can reject the MS-AR(2) model when compared to the MSG-AR(3) model than we can say that the MSG-AR(2) model fits the data better than the MS-AR(2) model. The likelihood ratio of this last test does not have a standard distribution and we report below the Monte Carlo simulations used to implement the test.

We have generated 1.000 trials — simulating the MS-AR(2) model under intervention type 2 — each with the same number of observations of our sample size. For each trial both models [MS-AR(2) and MSG-AR(3)] were estimated and the likelihood ratio statistic was computed. Figure 8 shows the histogram of the likelihood ratio statistic obtained for these 1.000 trials. The null hypothesis of the test is the MS-AR(2), estimated under intervention type 2, and the alternative hypothesis is the MSG-AR(3) specification.

FIGURE 8
Histogram of the Likelihood Ratio [Null:MS-AR(2), Alternative:MSG-AR(3)]



Number of trials =	1000
Mean	4.670350
Median	3.820503
Maximum	22.21226
Minimum	0.018592
Std. dev.	3.775536
Skewness	1.148845
Kurtosis	4.194765
Jarque-Bera	279.4517
Probability	0.000000

In the Monte Carlo simulations the likelihood ratio statistic computed at each trial is less or equal to 11.94 for 95% of the trials, whereas the estimated likelihood ratio computed using the likelihood values of Table 5 is equal to 16.53. The results indicate that the null is rejected at a level of significance smaller than 5%.[12] Therefore, we can conclude that the MSG-AR(3) model fits the data better.

We also test the MS-AR(0) model against the MSG-AR(3) model. The likelihood ratio statistic of the test has a standard asymptotic distribution, $\chi^2(4)$, and can be computed using the likelihood values presented in Table 5. The estimated likelihood ratio statistic is equal to 22.082. Therefore, the MS-AR(0) specification is rejected at a level of significance smaller than 1%.

Despite of the result that the MSG-AR(2) model is the one that best fits the data in-sample, this conclusion does not hold out-of–sample. The out-of-sample forecasting ability of several Markov switching models is presented in the next Subsection 4.3.

4.3 OUT-OF-SAMPLE FORECASTING

This subsection compares the out-of-sample forecasting performance of several Markov switching models with autoregressive components with linear models and the MS-AR(0) model. Two linear models for changes in GDP were estimated for comparison with the Markov switching models: an AR(3) and an ARMA(1,1)

12. Note that the MSG-AR(3) model has two more parameters than the MS-AR(2) model. If we were to apply the standard critical value it would have been equal to 5.99 ($\chi^2(2)$) instead of 11.94.

model.[13] All models were estimated from 1976:2 up to 1992:1, and then recursively re-estimated for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2 to generate the out-of-sample forecasts. Appendix B shows how these forecasts were calculated.

Results

We use the following statistic to compare any two models: the mean squared forecast error (MSE) of one of the models divided by the MSE of the other model. We also report standard errors for these relative MSE.[14] The standard errors are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors. There is an asymptotic justification of the procedure adopted to calculate the standard errors, for recursively estimated models, in West (1996).

Table 7 shows the root mean squared forecast error (RMSE) of the linear AR(3) model and the relative MSE [relative to the AR(3) model] of several Markov switching models, with interventions type 1 and 2, for forecasts from one to eight quarters ahead. The model with the smallest relative MSE, for forecasts from two to seven quarters ahead and for both types of intervention is the MS-AR(2). Almost all the relative MSE of the MS-AR(2) model are smaller than one with the exception of the eight-quarter-ahead forecast. Nevertheless, they are significantly smaller than one only for intervention type 2 and for forecasts from four to six quarters ahead. The ARMA(1,1) model beats the AR(3) model for forecasts from one to two steps-ahead. The 'No Change' model, has the worst forecasting ability for all steps-ahead.

Table 8 compares the same models with the ARMA(1,1) model. It shows that the relative MSE of the MS-AR(2) model is smaller than one for forecasts from three steps-ahead and on. Nevertheless, they are significantly smaller than one for forecasts four and six steps-ahead and for intervention type 2. The AR(3) model forecasts significantly better than the ARMA(1,1) only four quarters ahead and for both types of intervention.

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13. The identification of the ARMA model was implemented using AIC and SBC criteria. In addition, given that structural breaks generally lead to serial correlation in the residuals, Durbin-Watson test was used to test whether the residuals of the selected model are white noise. The identification was implemented considering or not dummies for the period between 1990.1 a 1991.2.

14. The standard errors were calculated using the Gauss routine made available by Mark W. Watson in his web site http://www.wws.princeton.edu/~mwatson/

TABLE 7

Linear and Nonlinear Models: Out-of-Sample Forecasting Performance
MSE of each Model Relative to the MSE of the AR(3) Model

		Linear AR(3)	No change		ARMA (1,1)		MS-AR(0)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
							Intervention type 1							
Steps-ahead	1	0.01573	1.952	(0.412)	0.923	(0.033)	1.186	(0.070)	0.955	(0.108)	1.042	(0.126)	0.963	(0.125)
	2	0.01644	2.916	(1.476)	0.923	(0.035)	1.142	(0.043)	0.922	(0.056)	0.915	(0.089)	0.829	(0.100)
	3	0.01676	1.862	(0.733)	1.014	(0.016)	1.004	(0.011)	0.980	(0.017)	1.033	(0.012)	1.002	(0.009)
	4	0.01686	1.919	(0.391)	1.034	(0.011)	1.019	(0.017)	0.974	(0.025)	1.009	(0.006)	0.994	(0.009)
	5	0.01701	2.566	(0.820)	1.010	(0.011)	1.016	(0.014)	0.991	(0.030)	1.020	(0.007)	1.028	(0.012)
	6	0.01709	1.910	(0.440)	0.992	(0.010)	1.012	(0.012)	0.980	(0.020)	1.017	(0.009)	1.031	(0.015)
	7	0.01700	1.288	(0.220)	0.993	(0.008)	1.006	(0.019)	0.971	(0.029)	1.023	(0.009)	1.023	(0.009)
	8	0.01745	2.259	(0.506)	1.003	(0.007)	1.008	(0.018)	1.000	(0.022)	0.999	(0.009)	1.001	(0.010)
							Intervention type 2							
Steps-ahead	1	0.01574	1.949	(0.422)	0.926	(0.018)	1.173	(0.070)	0.954	(0.084)	1.051	(0.060)	0.986	(0.064)
	2	0.01636	2.946	(1.527)	0.934	(0.015)	1.141	(0.061)	0.929	(0.037)	0.951	(0.059)	0.911	(0.071)
	3	0.01677	1.860	(0.752)	1.013	(0.032)	1.000	(0.022)	0.978	(0.022)	1.064	(0.074)	1.041	(0.066)
	4	0.01694	1.903	(0.385)	1.025	(0.011)	1.008	(0.004)	0.971	(0.014)	1.078	(0.079)	1.037	(0.059)
	5	0.01711	2.536	(0.801)	0.998	(0.006)	0.999	(0.002)	0.980	(0.017)	1.027	(0.050)	1.014	(0.040)
	6	0.01719	1.889	(0.431)	0.982	(0.008)	0.995	(0.002)	0.978	(0.010)	1.007	(0.045)	1.017	(0.049)
	7	0.01704	1.282	(0.214)	0.989	(0.006)	0.997	(0.002)	0.977	(0.013)	1.030	(0.061)	1.060	(0.079)
	8	0.01745	2.259	(0.507)	1.003	(0.008)	1.004	(0.002)	1.001	(0.004)	0.982	(0.041)	1.003	(0.047)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No change" (martingale) model forecast a constant rate of growth for GDP. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the AR(3) model. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 8

Linear and Nonlinear Models: Out-of-Sample Forecasting Performance
MSE of each Model Relative to the MSE of the ARMA(1,1) Model

		ARMA(1,1)	No change		Linear AR(3)		MS-AR(0)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
							Intervention type 1							
Steps-ahead	1	0.01511	2.115	(0.515)	1.083	(0.038)	1.285	(0.101)	1.034	(0.099)	1.129	(0.118)	1.043	(0.115)
	2	0.01579	3.160	(1.770)	1.084	(0.041)	1.237	(0.087)	1.000	(0.030)	0.992	(0.071)	0.898	(0.082)
	3	0.01688	1.836	(0.702)	0.986	(0.015)	0.990	(0.018)	0.966	(0.013)	1.018	(0.022)	0.988	(0.020)
	4	0.01715	1.856	(0.361)	0.967	(0.011)	0.986	(0.014)	0.942	(0.027)	0.976	(0.009)	0.962	(0.009)
	5	0.01710	2.540	(0.803)	0.990	(0.010)	1.005	(0.009)	0.981	(0.022)	1.010	(0.010)	1.018	(0.011)
	6	0.01703	1.925	(0.454)	1.008	(0.010)	1.020	(0.010)	0.988	(0.012)	1.025	(0.016)	1.039	(0.022)
	7	0.01694	1.297	(0.219)	1.007	(0.008)	1.013	(0.011)	0.978	(0.022)	1.030	(0.012)	1.030	(0.012)
	8	0.01747	2.253	(0.504)	0.997	(0.006)	1.005	(0.012)	0.997	(0.017)	0.997	(0.005)	0.998	(0.005)
							Intervention type 2							
Steps-ahead	1	0.01514	2.105	(0.509)	1.080	(0.022)	1.267	(0.095)	1.030	(0.086)	1.135	(0.058)	1.064	(0.060)
	2	0.01581	3.152	(1.761)	1.070	(0.018)	1.221	(0.083)	0.995	(0.029)	1.018	(0.064)	0.975	(0.080)
	3	0.01687	1.837	(0.703)	0.988	(0.032)	0.988	(0.016)	0.966	(0.015)	1.050	(0.049)	1.028	(0.042)
	4	0.01714	1.857	(0.361)	0.976	(0.011)	0.984	(0.009)	0.947	(0.023)	1.052	(0.066)	1.012	(0.047)
	5	0.01710	2.541	(0.803)	1.002	(0.006)	1.001	(0.006)	0.982	(0.017)	1.029	(0.051)	1.016	(0.040)
	6	0.01703	1.924	(0.453)	1.019	(0.008)	1.013	(0.008)	0.997	(0.007)	1.026	(0.048)	1.036	(0.053)
	7	0.01695	1.297	(0.219)	1.011	(0.006)	1.009	(0.005)	0.988	(0.016)	1.042	(0.058)	1.072	(0.077)
	8	0.01747	2.253	(0.504)	0.997	(0.008)	1.001	(0.006)	0.998	(0.012)	0.979	(0.034)	1.000	(0.040)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No change" (martingale) model forecast a constant rate of growth for GDP. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the ARMA(1,1) model. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

Table 9 reports the MSE of the models relative to the MSE of the MS-AR(0) model. It shows that the MS-AR(2) model has a relative MSE significantly smaller than one for almost all steps-ahead and for both types of intervention. The same is true for the AR(3) and ARMA(1,1) models for short run forecasts, one to two quarters ahead.

Linear versus Nonlinear Models

For one-quarter-ahead forecast, the ARMA (1,1) model presents the lowest relative MSE. On the other hand, the Markov switching models present the best forecasting performance for two-quarter-ahead forecasts and on. In particular, the MS-AR(2) is the best in forecasting two to seven quarter-ahead. Thus, for forecasts of the annual growth of real GDP, the MS-AR(2) model is the one with the most accurate prediction in this out-of-sample forecasting test.

Intervention versus Non-intervention

Tables 10 and 11 show the relative out-of-sample performance of several Markov switching models, for both types of intervention, when compared to their counterparts without intervention. Table 10 shows the results for Hamilton's models [MS-AR(0), MS-AR(2) and MS-AR(4)] and Table 11 for Lam's models [MSG-AR(1), MSG-AR(2) and MSG-AR(3)]. Most of the relative MSE are smaller than one indicating that the interventions have improved forecast ability. The MSG models and the MS-AR(2) model have, overall, the smallest relative MSE. This is not surprising given that these models, without intervention, concentrate the probability of recession at the Collor Plans. Nevertheless, because the standard errors are relatively high for most models, the relative MSE are in general not significantly smaller than one. However, the greatest advantage of introducing interventions is that they characterize the Brazilian business cycle without loss of forecasting ability.

These findings corroborate the evidence obtained by several authors in that modeling nonlinearities underlying GDP growth improves its forecasting performance. This is particularly true for the case of Markov switching models that take into account abrupt changes and asymmetries of business cycle phases.

Recent Forecast Performance

As an illustration of the recent performance in forecasting GDP growth, a second out-of-sample test was performed. The models were estimated from 1976:2 up to 2000:2, and then were used to predict the annual rate of growth of GDP from 2000:3 to 2001:4. Table 12 reports the out-of-sample forecasts of the annual rate of growth of real GDP for 2000:3-2001:4. As it can be observed, in this period the MS-AR(2) and the AR(3) models provided the closest forecast of changes in GDP compared to the alternative models. The best overall model, for intervention type 2, is the MS-AR(2).

TABLE 9

Linear and Nonlinear Models: Out-of-Sample Forecasting Performance
MSE of each Model Relative to the MSE of the MS-AR(0) Model

		MS-AR(0)	No change		AR(3)		ARMA (1,1)		MS-AR(2)		MSG-AR(2)		MSG-AR(3)	
		RMSE	Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE		Relative MSE	
							Intervention type 1							
Steps-ahead	1	0.01713	1.646	(0.257)	0.843	(0.049)	0.778	(0.061)	0.805	(0.082)	0.879	(0.097)	0.812	(0.101)
	2	0.01757	2.554	(1.101)	0.876	(0.033)	0.808	(0.057)	0.808	(0.069)	0.802	(0.096)	0.726	(0.105)
	3	0.01679	1.854	(0.731)	0.996	(0.011)	1.010	(0.018)	0.976	(0.014)	1.028	(0.016)	0.998	(0.012)
	4	0.01702	1.883	(0.376)	0.981	(0.016)	1.015	(0.014)	0.956	(0.016)	0.990	(0.017)	0.976	(0.015)
	5	0.01714	2.527	(0.796)	0.985	(0.014)	0.995	(0.008)	0.976	(0.018)	1.005	(0.012)	1.012	(0.012)
	6	0.01719	1.888	(0.430)	0.988	(0.011)	0.981	(0.009)	0.969	(0.013)	1.005	(0.011)	1.019	(0.017)
	7	0.01705	1.281	(0.210)	0.994	(0.018)	0.988	(0.011)	0.965	(0.014)	1.018	(0.020)	1.017	(0.018)
	8	0.01752	2.241	(0.499)	0.992	(0.017)	0.995	(0.012)	0.992	(0.005)	0.991	(0.010)	0.993	(0.008)
							Intervention type 2							
Steps-ahead	1	0.01704	1.662	(0.263)	0.853	(0.051)	0.790	(0.059)	0.813	(0.077)	0.896	(0.075)	0.840	(0.082)
	2	0.01747	2.583	(1.128)	0.877	(0.047)	0.819	(0.056)	0.815	(0.071)	0.834	(0.076)	0.799	(0.082)
	3	0.01677	1.859	(0.735)	1.000	(0.022)	1.012	(0.017)	0.978	(0.010)	1.063	(0.057)	1.041	(0.049)
	4	0.01700	1.888	(0.377)	0.992	(0.004)	1.017	(0.009)	0.963	(0.015)	1.070	(0.077)	1.029	(0.057)
	5	0.01710	2.539	(0.803)	1.001	(0.002)	0.999	(0.006)	0.981	(0.018)	1.028	(0.050)	1.015	(0.040)
	6	0.01714	1.899	(0.435)	1.006	(0.002)	0.987	(0.008)	0.984	(0.011)	1.013	(0.045)	1.022	(0.049)
	7	0.01702	1.286	(0.214)	1.003	(0.002)	0.992	(0.005)	0.980	(0.012)	1.033	(0.061)	1.063	(0.080)
	8	0.01748	2.250	(0.503)	0.996	(0.002)	0.999	(0.006)	0.997	(0.006)	0.978	(0.039)	0.999	(0.045)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No change" (martingale) model forecast a constant rate of growth for GDP. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the MS-AR(0) model. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 10

Hamilton' s Model with and without Intervention: Out-of-Sample Forecasting Performance
MSE of the Model with Intervention Relative to the MSE of the Model without Intervention

		MS-AR(0)					MS-AR(2)					MS-AR(4)				
		No intervention	Intervention				No intervention	Intervention				No intervention	Intervention			
			Type 1		Type 2			Type 1		Type 2			Type 1		Type 2	
		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE	
Steps-ahead	1	0.0172	0.995	(0.025)	0.985	(0.020)	0.0159	0.938	(0.092)	0.939	(0.080)	0.0153	1.113	(0.120)	1.061	(0.122)
	2	0.0180	0.951	(0.025)	0.941	(0.027)	0.0164	0.927	(0.040)	0.925	(0.039)	0.0157	1.073	(0.022)	1.065	(0.020)
	3	0.0174	0.936	(0.036)	0.934	(0.035)	0.0169	0.961	(0.023)	0.959	(0.018)	0.0175	0.953	(0.050)	0.940	(0.060)
	4	0.0177	0.930	(0.040)	0.928	(0.036)	0.0171	0.949	(0.040)	0.953	(0.036)	0.0181	0.915	(0.070)	0.921	(0.062)
	5	0.0176	0.947	(0.034)	0.943	(0.031)	0.0171	0.976	(0.038)	0.977	(0.033)	0.0175	1.012	(0.031)	1.004	(0.025)
	6	0.0171	1.006	(0.020)	1.000	(0.017)	0.0170	0.987	(0.022)	0.996	(0.018)	0.0178	0.977	(0.028)	0.953	(0.038)
	7	0.0172	0.983	(0.039)	0.979	(0.034)	0.0170	0.969	(0.039)	0.979	(0.033)	0.0179	0.957	(0.038)	0.945	(0.038)
	8	0.0181	0.940	(0.060)	0.936	(0.056)	0.0175	0.995	(0.033)	0.997	(0.029)	0.0177	0.970	(0.029)	0.959	(0.031)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No change" (martingale) model forecast a constant rate of growth for GDP. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the same model without intervention. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 11

Lam´s Model with and without Intervention: Out-of-Sample Forecasting Performance
MSE of the Model with Intervention Relative to the MSE of the Model without Intervention

		MSG-AR(1)					MSG-AR(2)					MSG-AR(3)				
		No intervention	Intervention				No intervention	Intervention				No intervention	Intervention			
			Type 1		Type 2			Type 1		Type 2			Type 1		Type 2	
		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE		RMSE	Relative MSE		Relative MSE	
Steps-ahead	1	0.0166	1.003	(0.161)	1.079	(0.066)	0.017	0.944	(0.142)	0.954	(0.089)	0.016	0.922	(0.142)	0.944	(0.085)
	2	0.0168	1.003	(0.064)	0.994	(0.032)	0.017	0.829	(0.124)	0.853	(0.084)	0.017	0.786	(0.129)	0.855	(0.071)
	3	0.0172	0.977	(0.051)	0.976	(0.028)	0.018	0.916	(0.078)	0.944	(0.043)	0.018	0.884	(0.072)	0.920	(0.039)
	4	0.0173	0.978	(0.074)	0.982	(0.042)	0.018	0.922	(0.093)	0.994	(0.046)	0.018	0.905	(0.086)	0.952	(0.048)
	5	0.0173	0.987	(0.061)	0.976	(0.041)	0.017	0.991	(0.063)	1.009	(0.027)	0.017	0.988	(0.061)	0.986	(0.031)
	6	0.0173	0.986	(0.044)	0.984	(0.019)	0.018	0.967	(0.047)	0.968	(0.020)	0.017	0.986	(0.043)	0.983	(0.013)
	7	0.0175	0.962	(0.070)	0.995	(0.017)	0.018	0.947	(0.072)	0.958	(0.034)	0.018	0.951	(0.071)	0.990	(0.017)
	8	0.0178	0.969	(0.072)	1.003	(0.021)	0.018	0.967	(0.070)	0.950	(0.044)	0.018	0.964	(0.071)	0.966	(0.034)

Note: The models were estimated from 1975:2 up to 1992:1, and then recursively re-estimated out-of-sample for each subsequent quarter from 1992:2 until the last quarter of the sample, 2000:2. The "No change" (martingale) model forecast a constant rate of growth for GDP. The entries "Relative MSE" are the mean squared forecast error (MSE) of the model described in the first line relative to the MSE of the same model without intervention. The standard errors, shown in parenthesis, are HAC robust and were estimated using a Bartlett kernel with the number of lags, for each step-ahead, equal to the number of computed forecast errors.

TABLE 12

Out-of-Sample Forecasting Performance (2000:3-2001:4)

			AR(3)	ARMA (1,1)	MS-AR (0)	MS-AR(2)	MS-AR(4)	MSG-AR(1)	MSG-AR(2)	MSG-AR(3)
		AR(3) RMSE	MSE of the model relative to the MSE of the AR(3) model							
No intervention		0.03013		1.04738	1.006	1.03045	1.11523	1.01906	1.04587	1.04721
Intervention	Type 1	0.03022	-	1.03407	0.870	0.95607	1.06238	1.03477	1.19631	1.12761
	Type 2	0.03031	-	1.03226	1.021	0.95640	1.03538	1.03333	1.18076	1.12016
		ARMA(1,1) RMSE	MSE of the model relative to the MSE of the ARMA(1,1) model							
No intervention		0.03084	0.95476	-	0.960	0.98383	1.06478	0.97296	0.99856	0.99984
Intervention	Type 1	0.03073	0.96705	-	0.842	0.92456	1.02738	1.00067	1.15689	1.09045
	Type 2	0.03080	0.96875	-	0.989	0.92651	1.00303	1.00104	1.14386	1.08516
		MS-AR(0) RMSE	MSE of the model relative to the MSE of the MS-AR(0) model							
No intervention		0.03022	0.99409	1.04119	-	1.02436	1.10864	1.01303	1.03969	1.04102
Intervention	Type 1	0.02819	1.14916	1.18832	-	1.09867	1.22085	1.18911	1.37475	1.29580
	Type 2	0.03062	0.97983	1.01144	-	0.93712	1.01451	1.01249	1.15695	1.09757

5 CONCLUSIONS

This paper estimates Hamilton's model and Lam's model, with Brazilian quarterly GDP data from 1975:1 to 2000:2 (see Table 13), allowing for breaks at the Collor Plans. Based on the likelihood ratio test, relative mean squared forecast error and the filtered probabilities, we selected a MS-AR(2) (Hamilton's model) and a MSG-AR(2) (Lam's model) as presenting the best fit to the Brazilian business cycle under two different types of interventions. The estimated parameters from both models are very similar.

The sample identifies two significant states for the Brazilian economy. The MS-AR(2) model estimates that in state 1 the economy grows at a negative rate of around 1.4% per quarter (–5.6% a year) while in state 0 the Brazilian economy grows at a rate of 1.6% per quarter (6.4% a year). The MSG-AR(2) model estimates that in state 1 the economy grows at a negative rate of around 1.5% per quarter (–6% a year) while in state 0 the Brazilian economy grows at a rate of 1.7% per quarter (6.8% a year). Recessions in Brazil last a short time, averaging between two and three quarters for both models. Expansions last twice as long. The MS model estimates that periods of positive growth last on average between six and seven quarters, while for the MSG model the duration of expansions is around four and five quarters.

We compared the out-of-sample performance of several Markov switching models to a MS-AR(0), ARMA(1,1) and an autoregressive model [AR(3)]. The models were estimated from 1976:2 up to 1992:1, and then recursively re-estimated for each subsequent quarter, from 1992:2 until the last quarter of the sample, 2000:2, to generate the out-of-sample forecasts. Overall, the MS-AR(2) model display the best forecasting performance, with the smallest relative MSE for two to seven quarters ahead. This finding corroborate the evidence, obtained by several authors, that modeling nonlinearities, underlying changes in GDP growth, improves forecasting performance. This is particularly true for the case of Markov switching models that take into account asymmetries of business cycle phases.

We also checked the out-of-sample performance of several Markov switching models, estimated under both types of intervention with their counterparts without intervention. The results indicate that the interventions have improved forecast ability. The MSG models and the MS-AR(2) model have, overall, the smallest relative MSE. Nevertheless, because the standard errors are relatively high, for most models the relative MSE is not significantly smaller than one. However, the greatest advantage of introducing interventions is that they characterize the Brazilian business cycle without loss of forecasting ability.

As an illustration of the recent performance in forecasting GDP growth, the models were estimated from 1976:2 up to 2000:2, and then were used to predict the annual rate of growth of GDP from 2000:3 to 2001:4. The best overall model, under intervention type 2, was the MS-AR(2) model.

TABLE 13
Brazilian Quarterly Real GDP: 1975 — 2001

	GDP index				Seasonally adjusted GDP index			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1975	56.787	64.705	63.008	62.329	59.723	61.493	62.260	63.435
1976	63.574	70.248	69.003	67.759	66.839	66.802	68.148	68.940
1977	66.930	75.828	72.170	71.190	70.222	72.245	71.233	72.395
1978	68.966	77.563	77.487	76.394	72.325	74.066	76.422	77.645
1979	74.245	82.880	81.598	80.956	77.795	79.344	80.313	82.257
1980	81.082	87.657	86.827	84.192	84.919	84.062	85.313	85.595
1981	80.977	85.348	81.539	77.232	84.912	81.993	79.911	78.518
1982	77.256	85.975	84.637	79.832	80.876	82.743	82.838	81.054
1983	74.828	82.507	81.690	78.957	78.486	79.599	79.812	79.992
1984	77.935	86.282	85.909	84.694	81.965	83.419	83.798	85.558
1985	83.251	91.076	93.835	93.089	87.677	88.270	91.355	93.874
1986	89.245	98.020	101.486	99.861	94.335	95.141	98.565	100.594
1987	96.359	104.822	102.110	99.812	102.075	101.805	98.743	100.712
1988	96.474	104.507	104.557	97.619	102.467	101.335	100.708	98.815
1989	93.737	108.126	110.049	104.183	99.904	104.594	105.687	105.719
1990	96.181	98.081	105.870	98.044	102.900	94.492	101.729	99.736
1991	89.375	105.629	106.607	98.389	95.395	101.528	102.723	100.355
1992	94.101	103.706	101.491	97.624	99.973	99.339	98.416	99.781
1993	96.864	109.072	105.726	102.014	102.171	104.298	103.169	104.225
1994	101.786	113.110	112.373	112.159	106.962	108.063	110.218	114.430
1995	112.430	118.861	113.268	112.252	117.945	113.684	111.211	114.309
1996	109.976	121.541	121.607	118.251	115.639	116.293	119.304	120.263
1997	112.863	126.953	125.191	121.022	118.980	121.436	122.597	123.010
1998	115.162	128.082	125.734	118.945	121.640	122.347	122.937	120.950
1999	114.752	128.668	125.366	123.049	121.445	122.780	122.443	125.269
2000	121.153	134.096	134.669	132.747	128.569	127.856	131.141	134.869
2001	134.612	141.834	137.395	131.292	141.461	135.226	133.835	133.325

Sources: IBGE and IPEA.
Note: Fix base (1980) GDP. The GDP was seasonally adjusted using the X12-ARIMA software.

APPENDIX A

Hamilton's Filter

Hamilton's nonlinear filter uses as input the ergodic and transition probabilities:

$$\text{Prob}(S_{t-1} = i,\, S_t = j\,|I_{t-1}) = p_{ij} \sum\nolimits_{h=0}^{1} \ \text{Prob}\,(S_{t-2} = h,\, \text{S}_{t-1} = i \mid I_{t-1}) \tag{10}$$

From these joint conditional probabilities, the density of Δy_t conditional on S_{t-1}, S_t, and I_{t-1} is:

$$f(\Delta y_t\,|S_{t-1} = i,\, S_t = j,\, I_{t-1}) = \ [(2\pi)^{-k/2}\,|\,Q_t^{(i,j)}\,|^{-1/2}\ \exp(-\frac{1}{2}N_{t|t-1}^{(i,j)'}Q_t^{(i,j)^{-1}}N_{t|t-1}^{(i,j)}) \tag{11}$$

The joint probability density of states and observations is then calculated by multiplying each element of (10) by the corresponding element of (11):

$$F(\Delta y_t,\, S_{t-1} = i,\, S_t = j|I_{t-1}) = f(\Delta y_t \mid S_{t-1} = i,\, S_t\ j,\, I_{t-1})\ \text{Prob}(S_{t-1} = i, S_t = j \mid I_{t-1}) \tag{12}$$

The probability density of Δy_t given I_{t-1} is:

$$F\,(\Delta y_t \mid I_{t-1}) = \sum\nolimits_{j=0}^{1}\sum\nolimits_{i=0}^{1}\ f(\Delta y_t,\, S_{t-1} = i,\, S_t = j\,|I_{t-1}) \tag{13}$$

The joint probability density of states is calculated by dividing each element of (12) by the corresponding element of (13):

$$\text{Prob}\,(S_{t-1} = i,\, S_t = j \mid I_t) = f(\Delta y_t,\, S_{t-1} = i,\, S_t = j \mid I_{t-1})\,/\,f(\Delta y_t \mid I_{t-1}) \tag{14}$$

Finally, summing over the states in (14), we obtain the filtered probabilities of recessions and expansions:

$$\text{Prob}\,(S_t = j \mid I_t) = \sum\nolimits_{i=0}^{1}\ \text{Prob}\,(S_{t-1} = i,\, S_t = j \mid I_t) \tag{15}$$

The first-order assumption of the Markov chain implies that all relevant information for predicting future states is included in the current state. Thus, Δy_t depends only on the current and r most recent values of s_t, on r lags of Δy_t, and on a vector of parameters θ:

$$p(\Delta y_t \mid s_t, s_{t-1}, \ldots, \Delta y_{t-1}, \Delta y_{t-2}, \ldots; \theta) = p(\Delta y_t \mid s_t, s_{t-1}, \ldots, s_{t-r}, \Delta y_{t-1}, \Delta y_{t-2}, \ldots, \Delta y_{t-r}; \theta)$$

Lam's Filter

The first step of the algorithm is initialized with the distribution of the states in this period conditional on information in the previous periods. From this, the distribution of the states is generated, for the following period, using the Markov process. Thus, the first step calculates:

First Step

$$P\,[S_t{=}1,\ S_{t-1}{=}s_{t-1},\ldots,\ S_{t-r+1}{=}s_{t-r+1},\ \sum_{i=1}^{t} S_i\ = x|\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots]=$$

$$P\,[S_t{=}1|\ S_{t-1}{=}s_{t-1}]\ x\ \sum_{S_{t-r}=0}^{1} P\,[\ S_{t-1}{=}s_{t-1},\ldots,\ S_{t-r+1}{=}s_{t-r+1},\ \sum_{i=1}^{t-1} S_i\ = x{-}1|\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots] \quad (16)$$

and:

$$P\,[S_t{=}0,\ S_{t-1}{=}s_{t-1},\ldots,\ S_{t-r+1} = s_{t-r+1},\ \sum_{i=1}^{t} S_i\ = x\ |\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots]=$$

$$P\,[S_t{=}0|\ S_{t-1}{=}s_{t-1}]\ x\ \sum_{S_{t-r}=0}^{1} P\,[\ S_{t-1} = s_{t-1},\ldots,\ S_{t-r+1} = s_{t-r+1},\ \sum_{i=1}^{t-1} S_i\ = x{-}1|\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots] \quad (17)$$

where $\sum_{i=1}^{t} S_i = x$ is the sum of the past states up to period t.

Second Step

The second step, which uses the result from the first step as input, computes the joint distribution of the current observation and of the states:

$$f(\Delta y_t,\ S_t,\ S_{t-1},\ldots,\ S_{t-r+1},\ \sum_{i=1}^{t} S_i\ |\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots) =$$

$$f(\Delta y_t|\ S_t,\ S_{t-1},\ldots,\ S_{t-r+1},\ \sum_{i=1}^{t} S_i\ ,\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots)\ P\,[S_t =$$

$$= s_t,\ldots,\ S_{t-r+1} = s_{t-r+1},\ \sum_{i=1}^{t-1} S_i = \ x|\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots] \quad (18)$$

and:

$$f(\Delta y_t,\ S_t,\ S_{t-1},\ldots,\ S_{t-r+1},\ \sum_{i=1}^{t} S_i\ |\ \Delta y_{t-1},\ \Delta y_{t-2},\ldots) =$$

$$\left(1/\sqrt{2\pi\sigma}\right)\cdot \exp\left\{(1-(2\sigma^2))\cdot(1-\phi_1 L-\phi_2 L^2-\ldots-\phi_r L^r)x\left[\Delta y_t+\sum_{i=1}^{t-1}\Delta y_i-\alpha_0 t\right]+\right.$$

$$\left.+(1-\phi_1-\phi_2-\ldots-\phi_r)z_0-\alpha_1(1-\phi_1-\phi_2-\ldots-\phi_r)\sum_{i=1}^{r}S_i+\alpha_1\sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right)S_{t-j+1}\right\}^2 \quad (19)$$

Third Step

In the third step, the joint distribution obtained above is used to compute the likelihood of the observation conditional to its past:

$$f(\Delta y_t, S_t, S_{t-1},..., S_{t-r+1}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},...) =$$

$$= \sum_{s_t=0}^{1} \cdots \sum_{s_{t-r+1}=0}^{1} \sum_{x=0}^{t} f\ (y_t, S_t = s_t,..., S_{t-r} = s_{t-r}, \sum_{i=1}^{t} S_i = x \mid \Delta y_{t-1}, \Delta y_{t-2},...) \tag{20}$$

Fourth Step

In the fourth step, the algorithm uses the result from the second and third steps to calculate the distribution of the states conditional on the current information:

$$P\ [S_t = s_t,..., S_{t-r} = s_{t-r}, \sum_{i=1}^{t-1} S_i = x \mid \Delta y_t, \Delta y_{t-1},...] =$$

$$= f(y_t, S_t, S_{t-1},..., S_{t-r}, \sum_{i=1}^{t} S_i \mid \Delta y_{t-1}, \Delta y_{t-2},...)\ /\ f(\Delta y_t \mid \Delta y_{t-1}, \Delta y_{t-2},\ldots) \tag{21}$$

Through these four steps the algorithm generates the conditional likelihood value to each observation (third step) and the distribution of the states (from the fourth step), which is then used to initialize again the algorithm for the following observation. The algorithm is repeated for all observations, and the conditional likelihood function is obtained from the sum of its value for each observation:

$$L\ [\Delta y_T, \Delta y_{T-1}, \Delta y_{T-2},..., \Delta y_1] = \sum_{t=1}^{T} \log f(\Delta y_t \mid \Delta y_{t-1}, \Delta y_{t-2}..., \Delta y_1) \tag{22}$$

Since the second step requires data from r previous periods, the algorithm is initialized in the observation $r + 1$. For the first step, the probabilities below are required, which are obtained from their non-conditional counterparts.

$$P\ [S_r = s_r,..., S_1 = s_1, \sum_{i=1}^{t-1} S_i = x \mid \Delta y_r,...] \tag{23}$$

The filter used to estimate Lam's model involves substantial more computation than Hamilton's algorithm for two reasons. First, in the calculation of the error, the states for each observation include all the history of the Markov process, which is treated as an additional variable. Second, the initial value of the autoregressive component is treated as an additional free parameter to be estimated. These two components are represented in the third and second terms of equation (24), respectively. When α_0 and α_1 are independent from t, the computation of the error E is:

$$E = (1-\phi_1 L-\phi_2 L^2-\ldots-\phi_r L^r)\left[\sum_{i=1}^{t}\Delta y_i-\alpha_0 t\right]+(1-\phi_1-\phi_2-\ldots-\phi_r)z_0$$

$$-\alpha_1(1-\phi_1-\phi_2-\ldots-\phi_r)\sum_{i=1}^{t}S_i \;-\alpha_1\sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right)S_{t-j+1} \tag{24}$$

When dummies are introduced in Lam's model, the parameters α_0 e α_1 depend on t and the error is then calculated as:

$$E = (1-\phi_1 L-\phi_2 L^2-\ldots-\phi_r L^r)\left[\sum_{i=1}^{t}\Delta y_i-\alpha_0^t t\right]+(1-\phi_1-\phi_2-\ldots-\phi_r)z_0$$

$$-(1-\phi_1-\phi_2-\ldots-\phi_r)\sum_{i=1}^{t}S_i\alpha_1^i \;-\sum_{j=1}^{r}\left(\sum_{k=j}^{r}\phi_k\right)S_{t-j+1}\alpha_1^{t-j+1} \tag{25}$$

APPENDIX B

One-step-ahead Predictions

As an illustration of the procedure, the predicted one-step ahead mean for the MS AR(2) at the first forecast date T+1 = 1992:2 is given by:

$$\Delta\hat{y}_{t+1}\,|\,I_t = \hat{\mu}_{t+1}+\phi_1(\Delta y_t-\mu_t)+\phi_2(\Delta y_{t-1}-\mu_{t-1})$$

where $\hat{\mu}_{t+i}=\alpha_0\hat{P}(S_{t+i}=0)+\alpha_1\hat{P}(S_{t+i}=1)$ are the estimated drifts for each state. The estimated probabilities are obtained from the filtered probabilities and from the transition matrix. For example, the one-step-ahead predicted probability of a recession is given by:

$$\hat{P}(S_{t+1}=0)=P(S_t=0)\,p_{00}+P(S_t=1)\,p_{10}$$

where $P(S_t = i)$ for i = 0,1 are the ergodic probabilities. At time T + 2 = 1992:3, a new observation of Δy_t is considered, and the models are re-estimated to obtain the parameters and filtered probability. This procedure is repeated for each subsequent observation up to T = 2000:3 in order to obtain the recursive one-step-ahead forecasts of the filtered probability and the forecasts the Brazilian GDP growth.

Two-step-ahead Predictions

A similar procedure is used to obtain two-step-ahead prediction of the mean and filtered probabilities of a recession at the first forecast date, which are now given by:

$$\Delta\hat{y}_{t+2} \mid I_t = \hat{\mu}_{t+2} + \phi_1(\Delta\hat{y}_{t+1} - \hat{\mu}_{t+1}) + \phi_2(\Delta y_t - \mu_t)$$

$$\hat{P}(S_{t+1} = 0) = P(S_t = 0)(p_{00}p_{00} + p_{01}p_{10}) + P(S_t = 1)(p_{10}p_{00} + p_{11}p_{10})$$

Three- steps- ahead and on Predictions

$$\Delta\hat{y}_{t+h} \mid I_t = \hat{\mu}_{t+h} + \phi_1(\Delta\hat{y}_{t+h-1} - \hat{\mu}_{t+h-1}) + \phi_2(\Delta\hat{y}_{t+h-2} - \hat{\mu}_{t+h-2}) \quad \forall h > 2$$

$$\hat{P}(S_{t+h} = 0 \mid I_t) = P^h \hat{P}(S_t = 0 \mid I_t)$$

where P is the transition probability matrix with elements $p_{ij} = pr[s_t = j \mid s_{t-1} = i]$, where i denotes the i^{th} column and j the j^{th} row. Each column of P sums to one, so that 1_2' $P = 1_2$', where 1_2 is a column vector of ones. For h-step ahead there are 2^h possible cases for the probabilities, which are computed directly from Hamilton's filter.

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