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IRREVERSIBLE INVESTMENT WITH EMBODIED TECHNOLOGICAL PROGRESS

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DISCUSSION PAPER

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SUMÁRIO

RESUMO

Implicitamente, nas teorias de investimento assume-se que as firmas devam utilizar toda capacidade instalada para investir. Entretanto, tal predição é inconsistente com a observação empírica. O objetivo deste trabalho é o de estender a literatura de investimento irreversível sob incerteza, tornando-a consistente com tal fato, além de duas outras constatações empíricas: O progresso tecnológico incorporado em novas máquinas, a infrequência e a os picos de investimento em nível micro. Além de ser consistente com tais evidências empíricas, a reposição de novas máquinas é adiada pelo aumento da incerteza. Mostra-se ainda que se o progresso tecnológico incorporado e incerteza são consideradas no modelo, a concessão de créditos tributários é ineficaz para estimular o investimento.

ABSTRACT

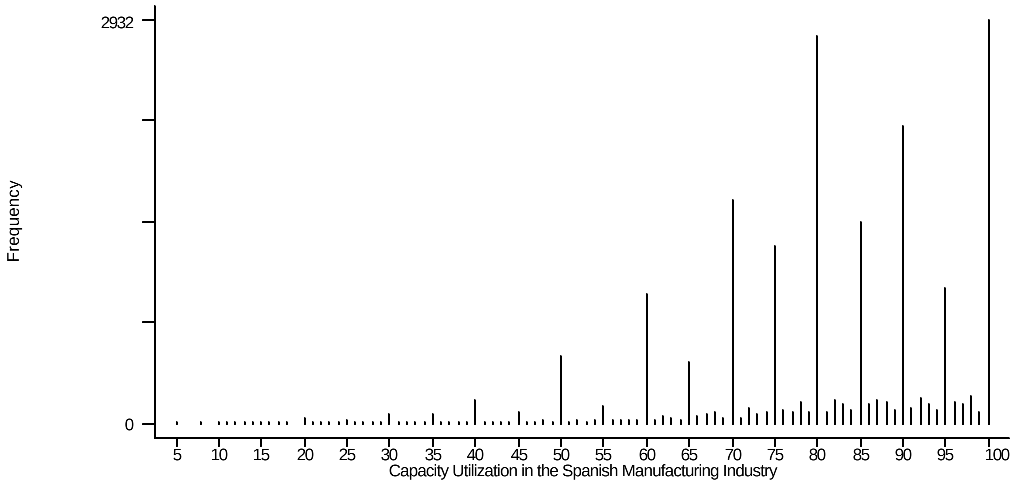
In this paper, we propose to explain capital accumulation in a stochastic framework by taking into account the two main motives for investment. Specifically, firms invest to expand capacity and to replace old machines. The model considers irreversible investment under uncertainty and embodied technological progress. It is shown to be consistent with the following empirical observations: Investment is lumpy and infrequent at the firm level; firms can invest even if they have not reached full capacity and technological progress is largely investment specific. We extend the paper of Pindyck (1988), by introducing embodied technological progress. To produce firms use irreversible capital, perfectly flexible labor, and energy whose price is stochastic. Capital and energy are complementary. We show that uncertainty makes firms to postpone investment, increasing the age of the oldest machine and reducing the proportion of new machines in the total stock of capital. We provide an exercise with tax credit to acquire new machines; it is shown that under the hypothesis of embodiment and uncertainty, the tax credit is not effective.

1 INTRODUCTON

Recent research on the investment behavior of firms in the US has shown that at the plant level, investment occurs infrequently and in burst. For example, using a 17 years sample, Doms and Dunne(1998) found that the five years biggest gross investment episodes for each firm account for more than 50% of aggregate US investment. Similar evidence has been documented for France by Jamet (2000): on a 13 years sample the three years of largest investments for each firm account for 75% of total investment in the French economy. Firms also stay for long periods inactive since each year, almost 20% of the firms do not invest. Such observations contrast with the result of the standard neoclassical model of investment with convex adjustment costs. Pindyck (1988) develops a model of capacity expansion, showing how uncertainty and irreversibility can affect the decision to invest. This model is able to reproduce the infrequency of investment at firm-level but not its lumpiness. Adjustment costs (see Caballero and Engle (1999)) or regime shifts (see Guo et alii (2002)) are then needed to generate lumpy investment. Nevertheless, in these models investment remains highly procyclical and only occurs to expand capacity. In addition, with homogenous units, all the capacity in place must be used before the firm starts investing. This contradicts the empirical observation. For instance, Figure 1 presents the distribution of capacity utilization of firms having an annual growth of capital higher than 20 percent in the Spanish manufacturing industry for the period 1991 to 2001. Even if the mode of the distribution is at 100 percent, there is still a large fraction of expanding firms using less than full capacity. Furthermore, the average growth rate of capital for firms with a capacity utilization less than 85% is not much lower than that for firms using full capacity (113.18% against 120.26%)[1]

FIGURE 1

Distribution of Capacity Utilization of firms presenting a peak of investment higher than 20% in the Spanish Manufacturing Industry - 1991 to 2001



Source: Fundacion SEPI - Encuestas sobre Estrategias Empresariales

1. Computations based on the Encuestas Sobre las Estragegias Empresiales a panel for the Spanish manufacturing firms, collected by Fundacion SEPI.

There is also evidence that technological progress is largely investment specific. For example, it has been documented that the probability that a peak of investment occurs is increasing with time (see among others Caballero, Engle and Haltiwanger, (1995), Cooper, Haltiwanger and Power (1999)). Moreover, as time passes, the relative price of capital goods is declining and the ratio equipment-GDP is raising. Therefore, investment decisions and technological progress seem to be interrelated. Indeed, technical advances are typically embodied in capital goods, implying that investment is the unique channel through which these innovations could be incorporated into the productive sectors. As a corollary, the old capital goods get less and less efficient over time, which might well induce the firms to scrap them (obsolescence). It seems then relevant to consider models with embodied technology which are able to generate endogenous scrapping (see for instance Cooley et alii (1997), Boucekkine, Germain and Licandro (1997)) that is, to explain replacement. Nevertheless, these models remain in a deterministic environment while it has been recognized that the stochastic nature of the environment matters a lot in explaining investment undertaken by firms.

In this paper, we propose to explain capital accumulation in a stochastic framework by taking into account the two main motives for investment. Specifically, firms invest to expand capacity and to replace old machines. The model considers irreversible investment under uncertainty and embodied technological progress. It is shown to be consistent with the following empirical observations:

- Investment is lumpy and infrequent at the firm level
- Firms can invest even if they have not reached full capacity.
- Technological progress is largely investment specific

Practically, we extend the paper of Pindyck (1988), by introducing embodied technological progress. To produce, firms use irreversible capital, perfectly flexible labor, and energy whose price is stochastic.[2] Capital and energy are complementary. Technological progress is energy saving ; since it is embodied only the new machines are more efficient in terms of energy requirements. Capital units are therefore not homogenous, and this induces firms to replace old energy-inefficient units by newer and less energy consuming units. Then, firms invest not only to expand capacity, as in Pindyck (1988), but also to replace old machines. Following the determination of the value-maximizing investment policy, we examine the implications of the model for capacity adjustment.

Our results differ from those obtained in a deterministic environment (see Boucekkine and Pommeret, 2004) : under uncertainty, the endogenous optimal age of the oldest machine evolves stochastically ; moreover, the optimal effective stock of capital (the one which is effectively used as opposed to the total stock of capital) is no longer constant as it is in the deterministic counterpart of the model. Due to an option value to invest in the future, the optimal effective capital stock is reduced by uncertainty. Moreover, the optimal age of the oldest machine increases and replacement is postponed as uncertainty increases. Therefore, by allowing for a

2. The framework proposed here can be generalized, to any complementar input to the stock of capital, like human capital, rather than only energy.

stochastic environment, this paper contributes to the literature on embodied technological progress.

Introducing embodied technology in a standard model of irreversible investment under uncertainty leads to some replacement. It is then no longer necessary for the firm to use all the old units before investing. As far as results are concerned; it allows generating lumpy investment and to get rid of the perfect "procyclical" behavior observed when capital units are assumed to be homogenous. Replacement may occur in unfavorable periods when firms need to replace old machines by new ones. This may generate a cleansing effect of recession as pointed out by Bresnahan and Raff (1991) and studied by Caballero and Hammour (1994, 1996) or Goolsbee (1998).[3] Moreover, we take into account the fact that the firm can react to shocks in two ways: through a variation in the rate of new units acquisition or through a variation in the rate of destruction. Even without adjustment costs or regime shifts, investment may then be lumpy, in order for the firm to replace a non-marginal amount of capital. Therefore, by allowing replacement investment, this paper also contributes to the literature on irreversible investment under uncertainty.

Finally, this paper is related to the real option literature focusing on technological progress. Abel and Eberly (2002) and (2004), Roche (2003) and Grenadier and Weiss (1997) consider models with stochastic technology. Abel and Eberly (2002) and (2004) study the optimal adoption of the stochastic available technology while in Roche (2003) it may be optimal for an upgrading firm to keep some distance with the frontier technology. Grenadier and Weiss (1997) also focuses on investment opportunities in stochastic technological innovations. But by adopting an innovation, the firm also receives an option value. This option reflects learning by doing generated by the use of the innovation and which makes it less costly to adopt the next innovation. In these models, technology adoption is costly and irreversible which prevents continuous upgrading ; as a result, investment in new technologies (and in reversible capital in Abel and Eberly (2002) and (2004)) occurs by gulps. Note that contrary to what is proposed in this paper, the firm can only operate one technology at one time and it cannot choose to only upgrade the oldest machines.

In the following section, we presented the model of investment under uncertainty with embodied technological progress. Section 4 provides expressions for the value of a marginal unit of capital. Boundary conditions are presented in section 5. Section 6 gives the optimal age of the oldest machine used. The optimal investment behavior is determined in section 7. Section 8 illustrates the dynamics of the model. In Section 9, we present an application to the efficiency of subsides to the acquisition of new machines. Section 10 concludes.

1.1 THE EMBODIED TECHNOLOGICAL MODEL

We consider a standard monopolistic competition economy under uncertainty in which the technical progress is embodied. It is a partial equilibrium model under

3. He finds that after the second oil shock, the probability of retirement of a Boeing 707 has more than doubled in the aircraft industry.

continuous time. We assume that the firm is risk neutral and discounts future cash-flows at a constant rate r.

2 TECHNOLOGY AND DEMAND

The infinitely-lived firm produces using capital, labor and energy. Specifically, capital and labor are inputs in a Cobb-Douglas production function with constant returns to scale. There exist operating costs whose size depends on the energy requirement of the capital,4 since we assume that to any capital use corresponds a given energy requirement. Labour and the energy use may be adjusted immediately and without any cost but following Pindyck (1988) and Abel and Eberly (1994), we consider that investment is irreversible: I(t)=dK(t)0 where K(t) represents the firm's total capital stock and investment is denoted by I(t). Nevertheless, any capital unit that has been installed may temporarily not be used for free. Therefore, we distinguish between the capital stock which is effectively used in the production which is denoted by Keff(t), and the capital the firm has installed which encompasses used units and unused units and which is denoted by K(t). We note to the acquisition date of the oldest machine currently used. The effective stock of capital is then:

$$\mathrm{K}_{eff}(t) = \int_{to}^{t} \mathrm{I}(z)dz \tag{1}$$

The firm faces an inverse demand function with a constant price elasticity: $P(t) = bQ(t)^{-q}$ with b>0 and q <1 where P(t) is the market price of the good produced by the firm and (-1/q) is the demand price elasticity. The firm's revenue, net of flexible factors, is given by:

$$P(t)AK_{eff}(t)^{b}\, L(t)^{1-b} - Pe(t)E(t) - vL(t) \tag{2}$$

where A is a scale factor, *L(t)* is labour *E(t)* stands for the energy use ; w is the constant wage rate, and *Pe(t)* is energy price. Since labor is completely reversible, it is straightforward to determine the optimal labor use at each point in time by maximizing the cash-flow in equation (2) with respect to L(t). The optimized value (with respect to labor) of the cash-flow is then

$$BK_{eff}(t)^{a} - Pe(t)E(t) \tag{3}$$

with $a = \dfrac{b(1-q)}{[1-(1-b)(1-q)]}$ and $B = \dfrac{[1-(1-b)(1-q)]}{\left[\dfrac{\left(A^{1}-q_{b}\right)}{\left[\left((1-b)(1-q)/w\right)^{-(1-b)(1-q)}\right]}\right]^{\frac{1}{(1-q)(1-q)-1}}}$

4. Such a complementarity is assumed in order to be consistent with the results of several studies showing that capital and energy are complements (see for instance Hudson and Jorgenson,1974 or Berndt and Wood, 1975).

2.1 EMBODIED TECHNOLOGICAL PROGRESS

Technological progress is assumed to make new machines becoming less energy-consuming over time. This means that as time passes, capital goods the firm can acquire are more efficient. But the stock of machine the firm already owns is not affected be this technological progress. Therefore, capital is heterogenous in terms of energy requirement and there is an incentive for the firm to replace old machines by the newest ones. This contrasts with the less realistic assumption of disembodied technological progress according to which all the stock of capital goods becomes more efficient over time whatever the age of the machines. Recall that to is the acquisition date of the oldest machine currently used. Energy use is then:

$$E(t) = \int_{to}^{t} I(z)e^{-gz}dz \tag{4}$$

$g > 0$ represents the rate of energy-saving technical progress. We assume[5] $y < r$.

2.2 DYNAMICS OF THE STOCHASTIC PROCESS

The energy price is uncertain and follows a geometric Brownian motion[6]

$$dPe(t) = mPe(t)dt + sPe(t)dz(t)$$

where Pe(t) is the energy price at time $t.m$ is the deterministic energy price trend which is disturbed by exogenous random shocks We assume[7] $m < r$. dz(t) is the increment of a standard Wiener process (E(dz)=0 and V(dz)=dt). s is the size of uncertainty, that is, it gives the strength with which this price reacts to the shocks.

2.3 MARGINAL VALUE OF CAPITAL

In this section we provide expressions for the value of a marginal unit of capital depending on whether it is used or not and owned by the firm or not. Since capital is not homogenous, expressions for these values differ from the standard ones obtained under disembodied or no technological progress (see Pindyck (1988)). In particular they depend on the acquisition date of the marginal unit we value. These values will be fully determined in the next section when we consider the boundary conditions.

Using equations (3) and (4) we obtain that the cash-flow generated between time t and (*t+dt*) by one used unit of capital acquired at time t is: $aBK_{eff}(t)^{a-1} - Pe(t)e^{-gt}$. Note that for bad realizations of the uncertain variable, this cash-flow may be negative . Since we assume that there is no cost to keep the machine unused, it is then optimal for the firm to stop using it as soon as the marginal cash-flow becomes negative. It can be deduced that *cf(t, t)*, the cash-flow generated between time t and (*t+dt*) by one unit of capital acquired at time t, depends on whether this unit is used or not:

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5. It is a standard assumption in the exogenous growth literature since it allows to have a bounded objective function.

6. See Epaulard and Pommeret (2003) for a justification.

7. If $m > r$, the firm would have an incentive to infinitely get into debt to buy an infinite amount of energy

$$cf(t,t) = \max[0,(aBK_{eff}(t)^{a-1} - Pe(t)e^{-gt})dt] \tag{5}$$

We then derive the value of a unit of capital depending on the date of acquisition of this unit and on whether this unit is currently used or not.

- The value V(Pe(t), t, t), of a unit of capital at time t acquired at time t and currently used has to satisfy the following Bellman equation:

$$rV(t,t) = (aBK_{eff}(t)^{a-1} - Pe(t)e^{-gt})dt - E_t(dV)$$

In the inaction region (that is, if it is optimal for the firm first not to reuse old units that were previously unused and second not to invest at time t), this differential equation leads to the following solution:[8]

$$V(t,t) = \frac{aB}{r}K_{eff}(t)^{a-1} - \frac{Pe(t)}{r-m}e^{-gt} - b_1(K_{eff}(t),t)Pe(t)^{b_1}$$

where $b_1 = \frac{1}{2} - \frac{(m-g)}{s^2} - + \sqrt{\left(\frac{(m-g)}{s^2} - \frac{1}{2}\right)^2 + \frac{2r}{s^2}} > 1$ is the positive root of the corresponding quadratic equation. Note that $\partial b_1 / \partial s^2 < 0.b_1(K_{eff}(t),t)Pe(t)^{b_1}$ gives the value of the option to stop using the unit ; it is of course an increasing function of the energy price.

The value W(t, t) at time t of a unit of capital acquired at time t and not currently used does not currently provide any cash-flow. It has to satisfy the following Bellman equation:

$$rW(t,t) = E_t(dW)$$

and the solution of this differential equation is[9]:

$$W(t,t) = b_2(k_{eff}(t),t)Pe(t)^{b_2}$$

where

$$b_2 = \frac{1}{2} - \frac{(m-g)}{s^2} - \sqrt{\left(\frac{(m-g)}{s^2} - \frac{1}{2}\right)^2 + \frac{2r}{s^2}} < 0$$

is the negative root of the corresponding quadratic equation. Note that $\partial b_2 / \partial s^2 > 0.b_2(K_{eff}(t),t)Pe(t)^{b_2}$ gives the value of the option to reuse the unit; it is a decreasing function of the energy price.

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8. An additional term including the negative root of the quadratic equation also enters the general solution of the differential equation. Such a term would imply that the value of the unit explodes as the energy price goes to zero. Therefore we eliminate this term from the solution.

9. An additional term including the positive root of the quadratic equation also enters the general solution of the differential equation. Such a term would imply that the value of the unit explodes as the energy price increases. Therefore we eliminate this term from the solution.

Value O(t) at time t of a unit that has not already been acquired has to satisfy

$$rO(t) = Et(dO)$$

that is[10]

$$O(t) = b_3(Keff(t))Pe(t)^{b_2}$$

where $b_2 < 0$ is the same as previously. $B_2(K_{eff}(t))Pe(t)^{b_2}$ gives the value of the option the firm has to give up if investing at time t. Such an option value comes from the fact that when acquiring a unit of capital at time t, the firm makes it more difficult (in the sense that it will require a better realization of the stochastic variable) to invest next period since the marginal productivity of the effective capital will be smaller. Note that this option is a function of the effectively used stock of capital and not of the capacity in place. It comes from the fact that it may be optimal for the firm to invest even if it is not using all the installed units of capital. This contrasts with what happens under disembodied or no technological progress (see Pindyck (1988)) since in these cases, investment only occurs once all the hold units are used.

3 CAPACITY CHOICE WITH EMBODIED TECHNOLOGICAL PROGRESS

In this section, by imposing boundary conditions on the expressions for the values of a marginal unit (depending on whether it is acquired or/and used), we provide full determination for these values as well as the rules for utilization and investment. The decision scheme is not the same as in the case of disembodied or no technological progress. In these latter cases, the firm has first to decide whether to invest or not depending on the relative values of the desired capital stock (given the observed value of the uncertain variable) and of the capacity in place. In the case in which it is not optimal to invest, the firm must then decide to use all the capacity in place or only part of it. Since any unit of capital has the same characteristics because technological progress benefits to all units, the firm first reuses old units before investing into new ones. In a way, the decisions of using installed units and of investing in new ones are taken independently since there is no incentive for the firm to replace old units by new ones.

It is no longer the case when technological progress is embodied because capital units differ according to their installation date. The intuition is the following: since a new capital unit may be a lot more energy saving than an old one, it may be interesting for the firm to stop using one old unit and to invest into a new one even if there is an acquisition cost for the new one while there is none if the firm keeps using the old unit. Therefore, the firm simultaneously has to decide to invest or not and to determine the age of the oldest machine to use. Indeed, these two decisions are now closely linked.

10. See previous footnote.

3.1 UTILIZATION RULE

As already stated (see equation (5)) a unit of capital will only be used if the cash-flow coming from its use is positive. For an observed energy price level, the acquisition date of the oldest machine it is optimal to use, to^*, has thus to satisfy

$$aBK_{eff}(t)^{a-1} = Pe(t)e^{-gro*} \tag{6}$$

This condition states that the marginal productivity, which is the same for any used machine, has to be equal to the marginal cost of using the oldest machine. Moreover, the firm uses an old machine acquired at time $Pe^*(t)$ until the realization of the energy is *Pe*(t)* such that it becomes indifferent between using it or keeping it unused: the value of the oldest machine used must be the same whether it is used or not. The transition between these two values of the unit has also to be smooth for the firm to be at the optimum. These two conditions are the usual value matching and smooth pasting conditions:

$$V(t,t) = W(t,t) \quad for\, Pe(t) = Pe^*(t) \tag{7}$$

$$\frac{\partial V(t,t)}{\partial Pe(t)} = \frac{\partial W(t,t)}{\partial Pe(t)} \quad for\, Pe(t) = Pe^*(t) \tag{8}$$

This leads to the expression of the marginal value of a unit acquired at time t and which is currently used:[11]

$$V(t,t) = \frac{aB}{r} K_{eff}(t)^{a-1} - \frac{Pe(t)e^{-gt}}{r-m} +$$

$$\underbrace{\frac{b_2}{b_1 - b_2}\left[aBK_{eff}(t)^{a-1}\right]^{1-b_1} e^{-rb_1 t}\left(\frac{1}{r} - \frac{(b_2 - 1}{b_2(r-m)}\right)}_{b_1(K_{ef}f(t),t)} Pe(t)^{b_1} \tag{9}$$

The value of the option to stop using the unit ($b^1(K_{eff}(t), t) Pe(t) b_1$) negatively depends on its acquisition date t : if a unit has been installed early, it is worth having the opportunity to stop using it.

We illustrate this value function using a numerical example. We assume $b = 0.3$, $q = 0.2$ (which correspond to a mark-up of 25%), $m = 0.02$ and B=100. For the technological parameter, we choose $g = 2\%$. Other parameters are those used in Pindyck (1988): r=0.05; k=10; s =0.2.

The value function is of course a decreasing function of the energy price (see figure 2 in Figures). We can also observe (on figure 2 in Figures) that for a given energy price, the higher the uncertainty, the higher the value of the marginal unit,

11. See appendix 1 for the derivation.

which is standard in the literature of investment under uncertainty. This comes from the value of the option to stop using the unit which rises with uncertainty. Figure 3 also shows that the value of the option to stop using a unit of generation t is increasing with $K_{eff}(t)$, since the higher the stock of the capital, the more it is valuable to have the opportunity to stop using old units (this is due to decreasing returns). Again, uncertainty increases the value of this option. The interesting result that can be seen on figure 4 is that, for a high enough t , the value of a marginal used unit is the same, no matter how large is the uncertainty parameter. Indeed, for sufficiently recent units, the technological progress will be high enough to reduce drastically the energy requirements. Having the opportunity not to use such units is of very low value, whatever the size of uncertainty.

4 INVESTMENT RULE

The firm invests for an energy price realization such that, for a given effective stock of capital, it is indifferent between acquiring an additional unit and doing nothing, that is, until the value of a newly used unit exactly compensates for the constant cost k to acquire it and for the value of the option to invest in the future the firm has to give up (it corresponds to the value matching condition). For the firm to be at the optimum of this stochastic program, the standard smooth pasting condition has to be satisfied as well. For given effective capital stock and technology levels, these optimality conditions provide the expression of the energy price level for which it is optimal for the firm to increase capacity. This expression may also be converted into that of the optimal effective stock of capital as a function of the observed energy price level and of the current level of technology.

$$V(t,t = t) = O(t) + k \tag{10}$$

$$\frac{\partial V(t,t)}{\partial Pe(t)} = \frac{\partial O(t)}{\partial Pe(t)} \tag{11}$$

Contrary to what has been obtained in the disembodied or no technological progress case, it is not the total amount of capital in place K(t) but the capital which is effectively used that appears in equations (10) and (11) (see the detailed expressions in appendix 1). To decide how much to invest, firms do not care about how much capital they have but about how much capital they use since it is the number of units currently in use that determines the marginal revenue product of capital. Due to the embodied technology, it may be interesting for the firm to acquire new units that are more energy saving even if all the old units are not used.

The resolution under disembodied or no technological progress left us with two conditions. One gives a requirement for the use of capital: if K(t) is greater than $K_{eff}(t)$, the stock $K(t)-K_{eff}(t)$ stays unused. The other gives a requirement for the investment in capital units: if K(t) is less than $K_d(t)$, the firm invests until its stock reaches the desired level whereas if K(t) is greater than $K_d(t)$, there is no investment. Since capital is homogenous when technological progress is disembodied, the desired capital does not coincides with the effective one for any realization of the uncertain variable. In fact, the expression for the desired stock of capital is only valid for the effective stock when the uncertain variable reaches its historically most favorable level (corrected to take account of the technological progress).

Under embodied technological progress, the stock of capital in place (which may as well be in excess) is no more determinant for investment. What is more interesting is the effectively used stock of capital, and since the firm can always decide the age of the oldest capital unit in use to adjust the used stock to its optimal level, the desired level of used capital always coincides with the effective level and the expression for the desired level of effectively used capital is valid whatever the realization of the uncertain variable. This will allow to get expressions for the optimal effective stock of capital and for the optimal acquisition date of the oldest machine.

The system (10)-(11) also provides the expression of the value of the option to invest (see appendix 1). Figure 5 shows that for a given t and K_{eff}, the value of this option decreases with the energy price, since the higher this price, the less valuable it is to have the opportunity to invest. Moreover, the higher the uncertainty, the higher this value. Note that even if uncertainty tends to zero, there will still exist an option to wait for newer units because of the existence of technological progress.[12]

Figure 6 in appendix simply reflects decreasing returns: the more one uses the capital, the less worth it is to hold the option to invest in the future. Figure 7 illustrates the fact that as time passes, it becomes more and more worth to have the opportunity to invest in the future. It comes from the fact that units the firm can buy become more and more efficient in terms of energy requirements due to technological progress.

Given the observed level of the energy price and the current state of the energy-saving technology, it is optimal for the firm to have an effective capital stock equal to K_{eff}^{*} (t) which is given by the following implicit expression:

$$\left[aBK_{eff}^{*}(t)^{a-1}\right]^{(1-b_1)}\left[\frac{(b_2-1}{b_2(r-m)}-\frac{1}{r}\right]Pe(t)^{b_1}e^{-b_1 gt}+ \qquad (12)$$

$$+\frac{aBK_{eff}^{*}(t)^{(a-1)}}{r}=\frac{Pe(t)e^{-gt}}{r-m}\left(\frac{b_2-1}{b_2}\right)+k$$

Note that thanks to potential decreases in the optimal age of the oldest machine used, it is possible for the optimal effective capital stock to be decreasing even if investment is irreversible (as we have already seen, under embodiment, the capacity in place does not really matter as far as the firm's decisions are concerned).

Figure 8 in appendix shows that the higher is the price of the energy, the less firms use machines. Moreover, uncertainty reduces the optimal effective stock of

12. It can be easily checked that if $s^2 \rightarrow 0$ and $g=0,$ then O(t)=0.

capital. This comes from the option value to invest in the future (the opportunity cost of investment) which increases with uncertainty. Finally, figure 9 shows that for a given energy price, the effective stock of capital increases with time because more recent machines consume less energy.

5 OPTIMAL AGE OF THE OLDEST MACHINE USED

Since in this model there exists no cost to temporally not use a machine13, there is no incentive for the firm to definitively scrap any machine. Thus we only derive an optimal age for the oldest machine used but not really an optimal scrapping age. This is a significative departure from what is obtained in a deterministic environment (see Boucekkine and Pommeret, 2004). Using equations (6) and (12) provides an implicit expression for the optimal acquisition date of the oldest machine as a function of the observed price:

$$e^{-g\,to*(t)} = \frac{rk}{Pe(t)} + \frac{(b_2 - 1)}{b_2(r - m)} e^{-gt} - \left(\frac{(b_2 - 1)r}{b_2(r - m)} - 1 \right) e^{-gb_1 t} e^{g\,to*(t)(b_1 - 1)}$$

We also can express the optimal age of the oldest machine used T* (t)=t- t o*(t):

$$e^{-gb_1 T*(t)} = \frac{b_2(m-r)}{b_2 m - r} \left[1 - e^{-gT*(t)} \left(\frac{b_2 - 1)r}{b_2(r-m)} + \frac{rk}{Pe(t)} e^{gt} \right) \right]$$

Given the observed level of the energy price and the current state of the energy-saving technology, the firm desires to use only capital units that have been acquired at time $to^*(t)$ or more recently. Due to the option value to invest in the future, uncertainty urges firms to reduce their optimal effective stock of capital, therefore increasing the marginal productivity of capital and allowing for the use of older machines. As shown on figure 10, firms dealing with uncertainty are more reluctant to renew the machines; replacement is in some sense postponed. It is also possible to see on figure 10 that a higher energy price reduces the age of the oldest machine ; one could claim that the model can reproduce the "cleansing effect of recessions" : firms would tend to use newer machines in periods of higher energy prices. For a given energy price, as time passes, new technology becomes available and we have seen that the optimal effective stock of capital increases. Nevertheless, figure 11 shows that rising $K_{eff}^*(t)$ is not achieved through the use of older machines. Indeed, the optimal age of the oldest machine used is a decreasing function of time. This means that not only capacity is expanded through investment but that there is also replacement of old machines by new ones which takes place.

13. It is also the case in Pindyck (1988). Introducing a cost to keep the machine unused would generate an option to scrap the machine ; this would complicate the model a lot without significantly alter the results.

5.1 OPTIMAL INVESTMENT

Equation (1) implies

$$dK_{eff}(t) = d\left[\int_{to*}^{t} I(z)dz\right] \Leftrightarrow I*(t) = \frac{dto^*}{dt} + \frac{dK_{eff}^*}{dt}$$

Current investment is therefore the sum of the "destruction term" $I(to^*)(dto^*/dt)$ that can be positive or negative, and of the variation over time of the desired effective capital stock ; that is, past history of investment matters for contemporaneous investment. This is why lumpy investment may appear: for instance, if at the beginning of the program the firm immediately adjusts to its desired level of effective capital, it will invest in a lumpy way when replacement of that equipment is needed. Moreover, replacement makes it possible that the firm invests in unfavorable times and investment is therefore no longer highly procyclical. As already shown in the previous section, the model may give rise to the so-called "cleansing effect of recession" since in bad times, old machines may be replaced by new ones.

Nevertheless it cannot be stated that investment exhibits echoes since variations in both the optimal date of acquisition of the oldest machine and the optimal desired capital highly depend on the realization of the uncertain variable.

Let us consider the special case for which we observe that during a time period dt the energy price evolves exactly to compensate for the gains in technology : dPe(t)/Pe(t)=ydt. The optimal stock of effective capital becomes then constant as it is also the case in a deterministic framework (see Boucekkine and Pommeret, 2004): $dK_{eff}{}^*(t)=0$. This does not mean that no investment is undertaken, since the optimal acquisition date of the oldest machine is not constant:

$$dto^* = \frac{1}{g}\frac{dPe(t)}{Pe(t)} - \frac{aB}{gPe(t)} e^{gto*} K_{eff}^*(t)^{(a-1)} \frac{dK_{eff}^*(t)}{K_{eff}^*(t)} = dt$$

Therefore, in this special case, the optimal acquisition date of the oldest machine increases exactly with time and old machines are replaced by new ones:

$$I^*(t) = I(to^*)$$

5.2 ILLUSTRATION OF THE DYNAMICS

This section provides an illustration of the dynamics of the key variables of the model proposed in this paper (effective stock of capital, age of the oldest machine and investment). Results are compared with those of the deterministic and/or disembodied counterparts of this model.

In this dynamic example, we use the same parameters as previously. Simulations are driven over 100 periods. In order to get the dynamics of Pe(t), a geometric Brownian motion is simulated using parameters m =0.02, s^2=0.04 and Pe(0)=10 as a starting value. Figure 12 gives the sample path for Pe(t). The firm observes the energy price and then decides about utilization and investment.

5.3 REPLACEMENT

Investment occurs infrequently in all models considered here (in fact, investment irreversibility is sufficient to generate such a characteristics). However it is only under the assumption of embodied technological progress that replacement is possible which can generate lumpiness. Moreover, in the case of homogenous capital, the firm should reach full capacity before investing. In such a case, the firm will present a very strong "procyclical" behavior, since the energy price should reach its historically lowest level (corrected to take account of the rate of technological progress if relevant) to induce the firm to use all its units and finally invest. In figure 14, it can be seen that, under disembodied technological progress, firms barely invest : they increase their total stock of capital only twice in this example, at the beginning of the program and at the very end of the period considered, when there is a significant decrease in the energy price. In the embodied case, investment is driven part by the willingness to increase the effective stock of capital and part by the possibility to acquire a more efficient machine in terms of energy requirements, that is, replacement.

5.4 UNCERTAINTY

Comparing the two cases of embodied technology shows (see figure 14) that the echoes effects is no longer identifiable when firms operate in a stochastic environment. Moreover, the total stock of capital become smaller (see figure 13) and firms are more reluctant to renew the machines, leading to a higher optimal age for the oldest machine in use[14] (see figure 15). For these two reasons, firms under uncertainty will tend to invest less in new capital (see figure 14).

5.5 TECHNOLOGICAL PROGRESS

Not surprisingly, a higher rate of embodied technological progress induces more capital accumulation (see figure 16 in appendix); investment peaks are higher and more frequent (see figure 17 in appendix). Replacement occurs more intensively since the age of the oldest machine used is smaller (see figure 18 in appendix).

Summary of the results:

- Investment occurs in spurts, and the so-called lumpiness of the investment appears
- Since firms can invest even if they are not using all the units they have(which is a much more realistic assumption), they may invest for very unfavorable realizations of the uncertain variable in order to replace old machines. The higher the rate of technological progress, the more active the replacement. To some extent, this model support the cleansing effect of recessions argument.

14. Since the firm may not own old enough machines, its maximal age of the oldest machine may be smaller than the optimal one. This would not affect the effective capital stock which can still be optimal, but it would of course affect investment.

- Uncertainty reduces both the total stock of capital and the proportion of new machines in this stock. Both capacity expansion and replacement are postponed.

Next section presents one application of this to the case of subsidies and tax credit aiming at the diffusion on new machines more efficient in terms of energy consumption.

6 SUBSIDIZING ENERGY SAVING CAPITAL ACCUMULATION: A REAL OPTION APPROACH

Global climate change has resulted in policy makers becoming more and more concerned about energy conversation investments. Indeed, public policies, like tax-incentives, have been developed, which aim at the adoption of energy saving machines and equipment. Nevertheless, these policies face the so-called "energy paradox": very attractive investment opportunities in energy efficient capital are ignored by investors, even if these opportunities have very high ex-ante rates of return. The diffusion of apparently cost-effective energy-efficient technologies is very low. Empirically, it has been shown by Walsh (1987,1989) that tax incentives decrease investment and by Dubin and Henson (1988) that the relationship between investment and tax incentives is statistically insignificant. Such a lack of effectiveness has substantially reduced policy makers' enthusiasm.

From a theoretical point of view, Hasset and Metcalf (1992) explains the discrepancy between tax incentives and investment using the combination of irreversibility of investment and uncertainty. They construct a model in which residential energy conservation investment is irreversible, and the price of energy as well as the cost of energy conservation capital are stochastic. In this framework, the authors study the decision of households to invest in one project. There is an option value to invest that leads to postponing the decision to invest. Households require a better environment to acquire new energy-efficient machines than they would if there were no irreversibility or no uncertainty. This is why a tax incentive may not be sufficient to trigger investment.

Nevertheless, such modelling only focuses on the household decision to invest, while energy conservation investment is also an issue for firms. Moreover, it only considers one project, which implies that it ignores the main grounds for investment. Investment is driven by two motives: capacity expansion and replacement. Pindyck (1988) proposes the first stochastic model of capacity expansion, in which investment is irreversible. His model deals with homogenous units, that is, all the machines are similar. Therefore, the firm has to reach full capacity before undertaking investment, which seems to be an unrealistic assumption. Due to the assumption of homogenous units, the replacement of machines is ignored in his model. However, replacement is an important feature of investment decisions, and technological progress is highly embodied in new machines, i.e. it is investment-specific. Greenwood et. al. (1997) argues that almost 60% of the US post-war growth can be accounted for by investment-specific technological progress. Therefore, when examining possible explanations for the energy paradox, irreversibility and uncertainty are certainly part of the story, but the fact that the energy-saving technological progress is largely investment-specific should be considered as well.

In this part of this chapter, we use the model developed in this chapter, which exhibits these three characteristics (uncertainty, irreversibility and embodiment) to assess the efficiency of a tax credit. We compare the effects of the tax credit we obtain in such a framework with those which result in a model of disembodied technological progress under uncertainty and those that emerge from the deterministic counterpart of the embodied case developed in Boucekkine and Pommeret (2004).

Our results show that uncertainty, irreversibility and the embodied technological progress assumption are crucial to explain the low degree of tax credit effectiveness. In a dynamic example, we compare the stochastic and deterministic cases of embodied/disembodied technological progress. In our example, firms reduce the optimal scrapping age with the tax credit; however, due to uncertainty, irreversibility and embodiment, the option to postpone the machines renders this impact almost negligible compared with those which exist in the deterministic case or in the disembodied case under uncertainty. Even if the returns on new equipment and machines are high, the acquisition is postponed because of uncertainty, irreversibility and embodiment. This leads to a slowdown in the replacement and the diffusion of more energy-efficient machines. Our model, therefore, can provide some theoretical explanations for the empirical observation about the ineffectiveness of tax credit, and the so-called energy paradox.

The deterministic model of embodied technological progress and the impact of a tax credit are presented in the next sub-section.

6.1 OPTIMAL CAPITAL STOCK UNDER EMBODIED TECHNOLOGICAL PROGRESS

Presentation of the model

We consider that the energy saving technical progress is embodied in the new capital goods acquired by the firm. The firm's problem is:

$$\max \int_0^\infty [P(t)Q(t) - Pe(t)\,E(t) - w(t)L(t) - kI(t)]e^{-rt}dt \tag{13}$$

subject to constraints taking embodiment into account :

$$P(t) = bQ(t)^{-q} \qquad with \;\; q < 1 \tag{14}$$

$$Q(t) = AK(t)^b\, L(t)^{1-b} \tag{15}$$

$$K(t) = \int_{t-T(t)}^{t} I(z)dz \tag{16}$$

$$Pe(t) = \overline{P}e\; e^{mt} \;\; with\; m < r \tag{17}$$

$$E(t) = \int_{t-T(t)}^{t} I(z)e^{-gz}\ dz\ with g < r \tag{18}$$

$$w(t) = w \tag{19}$$

$$I(t) = dK(t) \geq 0 \tag{20}$$

$$K(t) \geq K_{eff} \tag{21}$$

P(t) is the market price of the good produced by the firm, Q(t) is the production, the demand price elasticity is ($-1/q$), K(t) is capital, L(t) is labor, E(t) stands for the energy use and I(t) is investment ; Pe(t) is energy price ; the wage rate w and the purchase cost of capital k are supposed to be constant for simplicity; r is the discount rate, m is the energy price rate of growth, and $g > 0$ represents the rate of energy saving technical progress. We assume that there is no physical depreciation. Moreover, we assume that $m < r$ *and* $g < r$. *If* $m > r$, the firm would have an incentive to infinitely get into debt to buy an infinite amount of energy. $g < r$ is a standard assumption in the exogenous growth literature since it permits to have a bounded objective function.

The Cobb-Douglas production function exhibits constant returns to scale but there exists operating costs whose size depends on the energy requirement of the capital: to any capital use K(t) corresponds a given energy requirement $K(t)e^{-gt}$. Such a complementarity is assumed in order to be consistent with the results of several studies showing that capital and energy are complements (see for instance Hudson and Jorgenson, 1974, or Berndt and Wood, 1975).

T(t) denotes the age of the oldest machine still in use at t or scrapping age. Also the capital variable is effective capital, since only active machines are taken into account in the definition of the capital stock. Note that only the new machines incorporate the latest technological advances, i.e. are more energy-saving than the machines acquired in the past. Such an assumption is consistent with Terborgh (1961) and Smith (1949) set-ups in which it is hypothesized that the operation cost of a machine is a decreasing function of its vintage.[15] However, the rate of technical progress g enters linearly into their operation costs functions whereas it is exponential in our model.

We assume that labor may be adjusted immediately and without any cost and this standard problem reduces to the following conditions for optimal inputs use:

$$L^*(t) = \left[\frac{A^{1-q} b(1-b)(1-q)}{w}\right]^{\frac{1}{1-(1-b)(1-q)}} K(t)^{\frac{b(1-q)}{1-(1-b)(1-q)}} \tag{22}$$

$$with\, B = (A^{(1-q)} b)^{\frac{1}{1-(1-b)(1-q)}} [1-(1-b)(1-q)][(1-b)(1-q)]^{\frac{(1-b)(1-q)}{1-(1-b)(1-q)}} w^{\frac{(1-b)(1-q)}{(1-b)(1-q)-1}}$$

15. On the contrary, Brems (1967) assumes a constant operation cost.

The vintage structure does matter in capital accumulation decisions, investment and scrapping. Noting that J(t)=T(t+J(t)) is the lifetime of a machine of vintage t, the problem may be transformed, following Malcomson (1975), into a more tractable one (see the Boucekkine and Pommeret, 2004) and the following first order conditions then result:

$$\int_t^{J(t)+t} \left\{ Ba[K(t)]^{a-1} - \overline{P}e\ e^{mt-gt} \right\} e^{-r(t-t)}\, dt = k(t) \tag{23}$$

$$aB[K(t)]^{a-1} = \overline{P}e\, e^{-g[t-T(t)]+mt} \tag{24}$$

$with\ a = [b(1-q)]/[1-(1-b)(1-q)].\ Note\ that\ 0 < a < 1.$

Equation (23) gives the optimal investment rule according to which the firm should invest at time t until the discounted marginal productivity during the whole lifetime of the capital acquired in t exactly compensates for both its discounted operation cost and its marginal purchase cost in t. Equation (24) is the scrapping condition: it states that a machine should be scrapped as soon as its marginal productivity (which is the same for any machine whatever its age) no longer covers its operating cost (which rises with its age).

It is shown in Boucekkine and Pommeret (2004) that if $g = m$, the Terborgh-Smith result $T^*(t)=J^*(t)=T$ is also reproduced in our case with T given by:

$$e^{-rT} = \frac{g-r}{g}\left[1-e^{-gT}\left(\frac{r}{r-g}+\frac{rk}{\overline{P}e}\right)\right] \tag{37}$$

Moreover, the optimal capital stock is:

$$K^{E*}(t) = \left[\frac{aB}{\overline{P}e}e^{-gT}\right]^{\frac{1}{1-a}} = K^{E*} \tag{38}$$

6.2 TAX CREDIT AND OPTIMAL STOCK OF CAPITAL

The tax credit s is modeled as a cut in the investment acquisition cost. The optimal scrapping time becomes then:

$$e^{-rT} = \frac{g-r}{g}\left[1-e^{-gT}\left(\frac{r}{r-g}+\frac{rk(1-s}{\overline{P}e}\right)\right]$$

and the optimal stock of capital can be deduced from equation (26). The behavior of the optimal scrapping time and of the optimal capital stock with respect to a tax credit are as follows :

$$\frac{\partial T}{\partial s} = \frac{1}{-g(1-s) - \frac{g}{r-g}\frac{\overline{P}e}{k}(1-e^{(g-r)T})} < 0 \tag{27}$$

$$\frac{\partial k^{E*}}{\partial s}=\underbrace{\frac{\partial K^{E*}(T,\overline{P}e)}{\partial T}}_{<0}\frac{\partial T}{\partial s}$$

$$=\frac{K^{E*}}{\underbrace{(1-a)(1-s)+(1-a)\frac{\overline{P}e}{rk}}_{A}+\underbrace{(1-a)\frac{\overline{P}e}{rk}\left[\frac{r(1-e^{(g-r)T})}{r-g}-1\right]}_{B}} \tag{28}$$

First note that there is a negative relationship between K^{E*}, a given scrapping age T. The underlying mechanism is the following. The greater the age of the operated machines, the greater the operation cost associated with those machines, and thus the higher the marginal productivity required for all machines whatever their age due to the optimality condition (24) Since the production function exhibits decreasing returns with respect to capital, a higher marginal productivity can only be achieved by lowering the stock of capital.

The tax credit affects the optimal capital stock through the optimal scrapping age: the higher the credit tax, the more interesting it is to scrap old machines to replace them with new ones whose cost is subsidized. This negative relationship between the tax credit and the scrapping age is expressed in equation (27). Moreover, the total effect of the credit tax on the effective stock of capital can be broken down into 2 parts :

- A>0 results from the effect of the tax credit in the absence of embodied technological progress
- B<0 results from the effect of the tax credit due to the embodied technological progress.

On the one hand, embodiment makes it even more attractive to acquire new machines. On the other hand, it also encourages scrapping. Combining both effects leads to an unambiguous positive impact. In the simulations exercise proposed in the last section, it can be seen that the positive effect prevails.

7 EMBODIED TECHNOLOGICAL PROGRESS UNDER UNCERTAINTY

In this sub-section, we analize the impact of subsidies when it is considered the introduction of uncertainty into the Embodied Technological progress.

7.1 TAX CREDIT AND OPTIMAL STOCK OF CAPITAL

Introducing a tax credit s, the optimal effective stock of capital becomes:

$$\frac{\partial T^{*}(t)}{\partial s}=\left[\underbrace{-g\,(1-s)-\frac{g}{r-g}\frac{\bar{P}e}{k}(1-e^{(g-r)T})}_{D}+\underbrace{\frac{gPe(t)e^{-gt}}{k}\left(\frac{1-e^{(g-r)T}}{r-g}+\frac{b_2-1}{b_2}\right)}_{E}\right.$$

$$\left.+\underbrace{(1-a)\,b_1\left[aBK^{*}_{eff}(t)^{a-1}\right]^{1-b_1}\left[\left(\frac{b_2-1}{b_2}\frac{1}{(r-m)}\right)-\frac{1}{r}\right]\frac{(Pe(t)e^{-gt})^{b_1}}{k}}_{C}\right]^{-1} \quad (31)$$

$$\frac{\partial K^{*}_{eff}(t)}{\partial s}=\left[\underbrace{(1-a)(1-s)+\frac{(1-a)Pe(t)e^{-yt}}{rk}}_{D}+\underbrace{\frac{(1-a)Pe(t)e^{-yt}}{rk}\left[\left(\frac{r(b_2-1)}{b_2(r-m)}\right)-1\right]}_{B}\right.$$

$$\left.-\underbrace{\frac{(1-a)b_1[aBK^{*}_{eff}(t)^{a-1}]^{1-b_1}(Pe(t)e^{-yt})^{b_1}}{k}\left[\left(\frac{b_2-1}{b_2(r-m)}\right)-\frac{1}{r}\right]}_{C}\right]^{-1}K^{*}_{eff} \quad (32)$$

and the optimal age of the oldest machine will be:

$$e^{-yb_1T^*(t)}=\frac{b_2(m-r)}{b_2m-r}\left[1-e^{-yT^*(t)}\left(\frac{b_2-1)r}{b_2(r-m)}+\frac{rk(1-s)}{Pe(t)}e^{yt}\right)\right]$$

The behavior of the optimal age of the oldest machine and of the optimal effective capital stock with respect to a tax credit are as follows :

$$\frac{\partial T^{*}(t)}{\partial s}=\left[\underbrace{-g\,(1-s)-\frac{g}{r-g}\frac{\bar{P}e}{k}(1-e^{(g-r)T})}_{D}+\underbrace{\frac{gPe(t)e^{-gt}}{k}\left(\frac{1-e^{(g-r)T}}{r-g}+\frac{b_2-1}{b_2}\right)}_{E}\right.$$

$$\left.+\underbrace{(1-a)\,b_1\left[aBK^{*}_{eff}(t)^{a-1}\right]^{1-b_1}\left[\left(\frac{b_2-1}{b_2}\frac{1}{(r-m)}\right)-\frac{1}{r}\right]\frac{(Pe(t)e^{-gt})^{b_1}}{k}}_{C}\right]^{-1} \quad (31)$$

$$\frac{\partial K^*_{eff}(t)}{\partial s} = \left[\underbrace{(1-a)(1-s) + \frac{(1-a)Pe(t)e^{-yt}}{rk}}_{A} + \underbrace{\frac{(1-a)Pe(t)e^{-yt}}{rk}\left[\left(\frac{r(b_2-1)}{b_2(r-m)}\right)-1\right]}_{B} \right.$$

$$\left. \underbrace{-(1-a)b_1\left[aBK^*_{eff}(t)^{a-1}\right]^{1-b_1}(Pe(t)e^{-yt})^{b_1}\left[\left(\frac{b_2-1}{b_2(r-m)}\right)-\frac{1}{r}\right]}_{C} \right]^{-1} K^*_{eff} \quad (32)$$

Now, in this stochastic case, the total effect on the effective stock of capital can be broken down into 3 parts:

- A>0 comes from the effect of the tax credit if technological progress is disembodied in deterministic environment.
- B>0 comes from the effect the tax credit due to the embodied technological progress affecting the option to invest in the future. Embodied technological progress increases the option value to invest in the future, therefore discouraging even more investment and reducing the effect of the tax credit. We will see in the simulations exercise that this effects is very strong a reduces a lot the effect of the tax credit.
- C<0 comes from the effect the tax credit due to the option to reuse old units under embodied technological. Unambiguously, the tax credit reduces the option to invest, therefore favoring the effective stock of capital.

The impact of the tax credit on the optimal age can be also divided in 3 parts:

- D<0 gives the effect of the tax credit in a deterministic framework: lowering the price of new machines by tax credit, creates an incentive to scrap older machines, and therefore reduces the age of the oldest machine.
- E results from the option to invest in new machines. The tax credit increases this option therefore increasing the age of the oldest machine.

In the dynamic example proposed in the next section,

- C <0 comes from the option to invest machines. The tax credit reduces this option therefore increasing the age of the oldest machine.
- In the dynamic example proposed in the next section, E>0, and the total effect of the tax credit is to lower the age of the oldest machine.

8 DYNAMICS OF THE EFFECTIVE STOCK OF CAPITAL, AGE OF THE OLDEST MACHINE AND TAX CREDIT

In this dynamic example, simulations are driven over 100 periods. In order to get the dynamics of Pe(t), a geometric brownian motion is simulated using parameters

$g = m = 0.02,\ s^{2} = 0.04 and\ Pe(0) = 10$ as a starting value. Figure 1 gives the sample path for Pe(t). The firm observes the energy price and derives how much effective capital to use. The firm observes the energy price and derives how much effective capital to use. It then has to decide whether to use more or fewer old units and, at the same time, decide whether it should invest in new units or not.

Figure 21 shows the dynamics of the total stock of capital in the deterministic case. It exhibits the usual echoes effects. Since the optimal effective stock of capital is constant, the positive effect of the tax credit accumulates over time. Considering the total capital stock in a stochastic framework with disembodied technological progress, Figure 22 leads to a dynamics consistent with investment occurring infrequently and in bursts. Due to tax credits, the initial investment is higher. Since in this case technological progress reduces the energy requirements of all installed machines, the tax credit unambiguously results in higher initial investment and higher capital over the whole period. However, the positive effect of the credit tax is less striking when one takes into account the fact that technological progress is embodied in new machines. In fact, the effective total stock of capital is barely increased in the model of embodied technological progress under uncertainty (Figure 23); also note that firms leave their scrapping policy almost unchanged (Figures 25 and 27).

Figure 5 compares the percentage increase in the total stock of capital over time, depending on whether we consider a stochastic environment or a deterministic one and whether technological progress is disembodied or embodied. First note that under disembodied enviroment, the tax credit is more efficient under uncertainty than in a deterministic framework. This is clearly due to the fact that we take capacity expansion into account since it contradicts the result of Hasset and Metcalf (1992) based on a single investment project. Second, introducing embodiment into the stochastic environment drastically reduces the effect of the tax credit under uncertainty while the reverse takes place in a deterministic environment. This means that taking embodiment into account is crucial when assessing the effectiveness of a tax credit.

9 CONCLUSION

The literature on investment under uncertainty has focused separately on investment in technology and on irreversible investment and capacity; however, replacement is an important motive for investment, as suggested by the large literature on vintage capital. This paper has proposed a model of irreversible investment under uncertainty with embodied technological progress, in which firms invest not only to expand the capacity but also to replace old machines. By introducing heterogenous capital, replacement occurs and the model no longer requires all old units to be used before the firm invests. The discussion on the firms behavior with respect to capital accumulation and on the effect of shocks on the economy is indeed significantly enriched. Investment may be lumpy and the so-called cleansing effect of recessions appears since replacement can occur for bad realizations of the stochastic process. The scrapping decision or the age of the oldest machine is endogenous; it is no longer constant as in the literature of vintages, and evolves stochastically. As shown by a dynamic example, uncertainty increases the optimal age of the machines in use, and due to uncertainty, not only capacity expansion but replacement as well, are postponed.

One clear extension of this paper is to introduce heterogenous firms to study the dynamics of the aggregate capital stock, and to eventually test it empirically. Note moreover that the discussion of energy utilization as well as some recent crisis in this sector have strengthened debates on how society should deal with macroeconomic impacts of energy price shocks ; an extension of the model proposed in this paper could compute the social benefits of energy policies, for instance by predicting the impact of an energy tax on the opportunity of replacement and more broadly on the economy.

Policies focusing on the implementation of energy-efficient machines face the so-called energy paradox: efficient machines in terms of energy requirements, even if profitable for the firms, have a very low diffusion rate. Furthermore, some of these policies, such as tax credits, have been shown to be inefficient. We have proposed a stochastic model that accounts for heterogeneous units and technological progress being investment-specific. We show that the assumption of embodied technological progress is very important when assessing the effectiveness of a tax credit. The existence of an option value (due to uncertainty and irreversibility) is not sufficient to explain the relative inefficiency of the tax credit in a capacity expansion framework. Indeed, in the disembodied case, the total impact on the capital stock is even higher than in the deterministic embodied case. Due to the combination of the option value and of embodiment, firms postpone replacement, and following a tax credit, they do not significantly increase their capital stock and reduce the scrapping age of the oldest machines. When devising energy conservation policies therefore, policy makers should not only take the option value into account as a feature of the investment decision. They should consider embodiment as well.

APPENDIX 1

he system:

$$V(t,t) = W(t,t) \qquad for \;\; Pe(t) = Pe^{*}(t) \tag{33}$$

$$\frac{\partial V(t,t)}{\partial Pe(t)} = \frac{\partial W(t,t)}{\partial Pe(t)} \qquad for \;\; Pe(t) = Pe^{*}(t) \tag{34}$$

$$\Leftrightarrow \; \frac{aB}{r} K_{eff}(t)^{a-1} - \frac{Pe^{*}(t)e^{-gt}}{r-m} + b_1(K_{eff}(t),t)Pe^{*}(t)^{b_1} = b_2(K_{eff}(t),t)Pe^{*}(t)^{b_2} \quad (35$$

$$\frac{e^{-gt}}{r-m} + b_1 b_1(K_{eff}(t),t)Pe^{*}(t)^{b_1-1} = b_2 b_2(K_{eff}(t),t)Pe^{*}(t)^{b_2-1} \tag{36}$$

Taking into account the fact that $a\,\mathrm{BK_{eff}(t)}^{a-1} = Pe^{*}(t)e^{-yt}$, this leads to the expression of the marginal value of a unit acquired at time t and which is currently used (see equation (9)).

The system:

$$V(t,t=t) = \mathrm{O}(t) + k$$

$$\frac{\partial V(t,t)}{\partial Pe(t)} = \frac{\partial \mathrm{O}(t)}{\partial Pe(t)} \quad (37$$

may be rewritten:

$$\frac{aB}{r} K_{eff}(t)^{a-1} - \frac{Pe(t)}{r-m} e^{-yt} + b_1(K_{eff}(t,t))Pe(t)^{b_1} = b_3(K_{eff}(t),t)^{b_2} + k \tag{38}$$

$$-\frac{1}{r-m} e^{-gt} + b_1 b_1(K_{eff}(t),t)Pe(t)^{b_1-1} = b_2 b_3(K_{eff}(t),t)Pe(t)^{b_2-1} \tag{39}$$

The system (38) e (39) provides the expression of the value of the option to invest:

$$O(t) = \left[\frac{b_1}{b_1 - b_2}[aBK_{eff}(t)^{a-1}]^{1-b_1} e^{-gb_1t} Pe^{**}(t)^{b_1-b_2}\left(\frac{1}{r} - \frac{(b_2-1}{b_2(r-m}\right)\right]Pe(t)^{b_2} \tag{40}$$

$$-\frac{e^{-yt}Pe^{**}(t)^{1-b_2}\,Pe(t)^{b_2}}{b_2(r-m)}$$

with Pe**(t) satisfying:

$$\left[aBK_{eff}(t)^{(a-1)}\right]^{(1-b_1)}\left[\frac{b_2-1}{b_2(r-m)} - \frac{1}{r}\right]Pe^{**}(t)^{b_1} e^{-b_{1yt}} \tag{41}$$

$$+\frac{aBK_{eff}(t)^{(a-1)}}{r} = \frac{Pe^{**}(t)e^{-gt}}{r-m}\left(\frac{b_2-1}{b_2}\right) + k$$

FIGURES

FIGURE 2

Value at time t of a marginal unit acquired at time t and currently used as a function of the energy price (Keff(t)=1 and to *(t)=1)

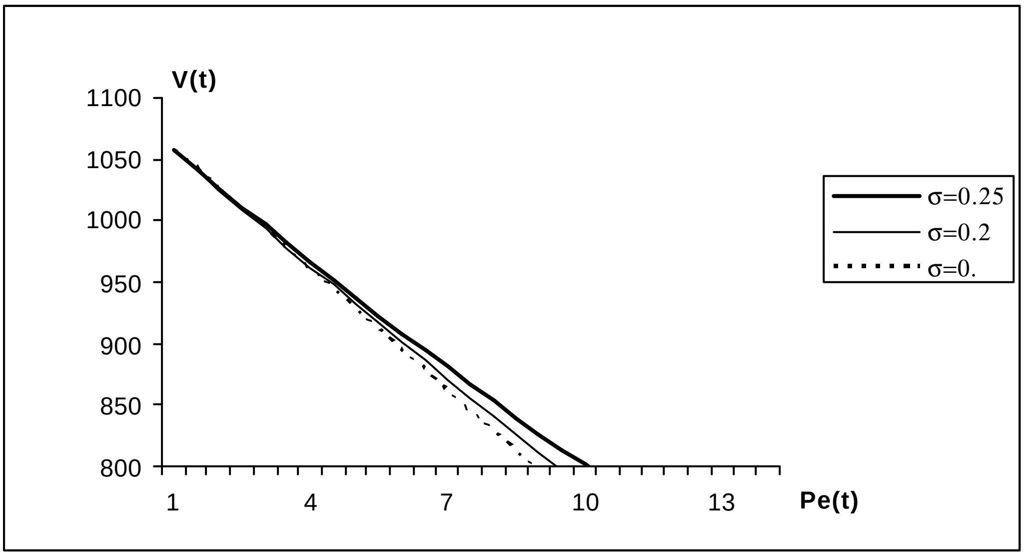


FIGURE 3

Value at time t of the option to stop using a unit of generation t as a function of Keff(t) (Pe(t)=0.5)

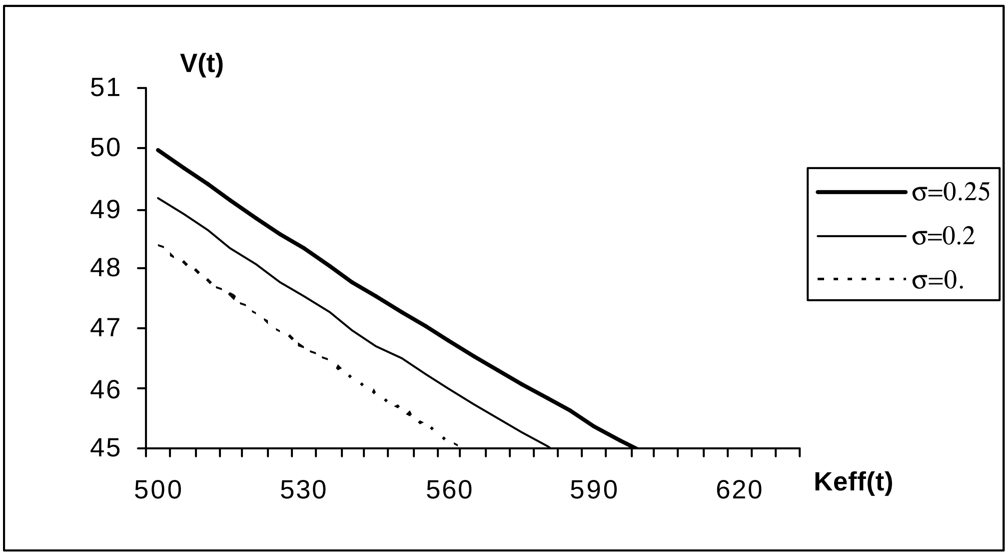


FIGURE 4

Value of a marginal used unit as a function of its acquisition date *t* (Keff(t)=1 and Pe(t)=10)

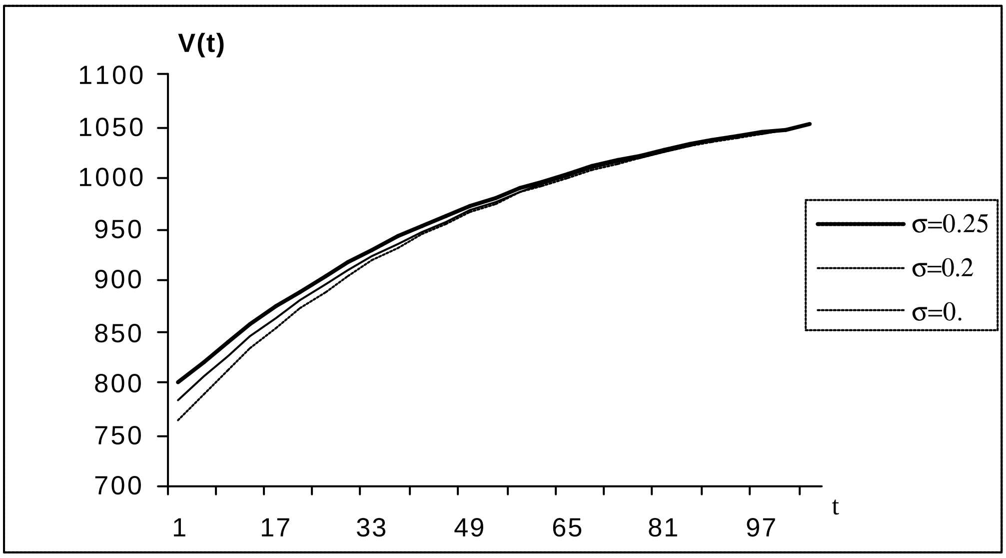


FIGURE 5

Option to postpone investment as a function of the energy price (t=10 and Keff(t)=1)

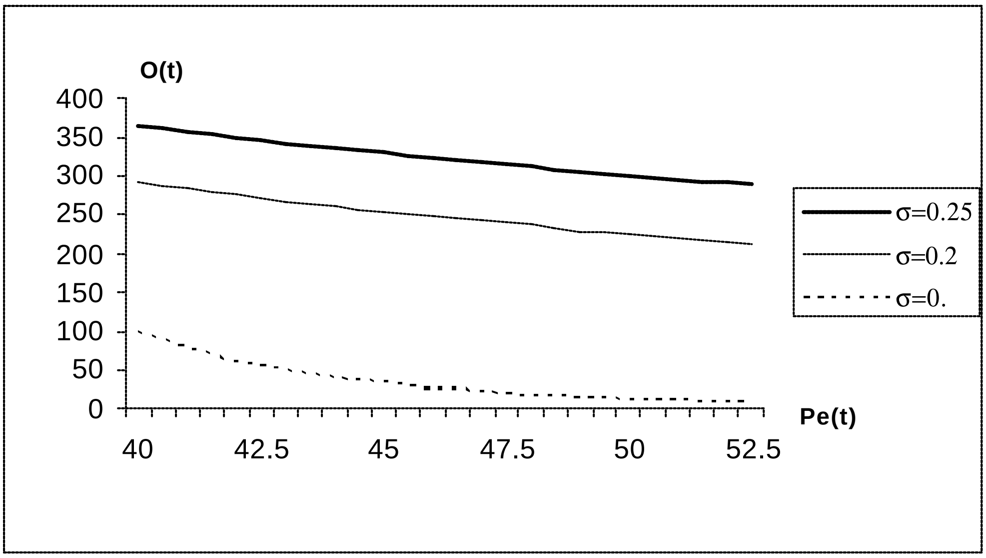


FIGURE 6

Option to postpone investment as a function of the effective stock of capital (t=10 and Pe(t)=50)

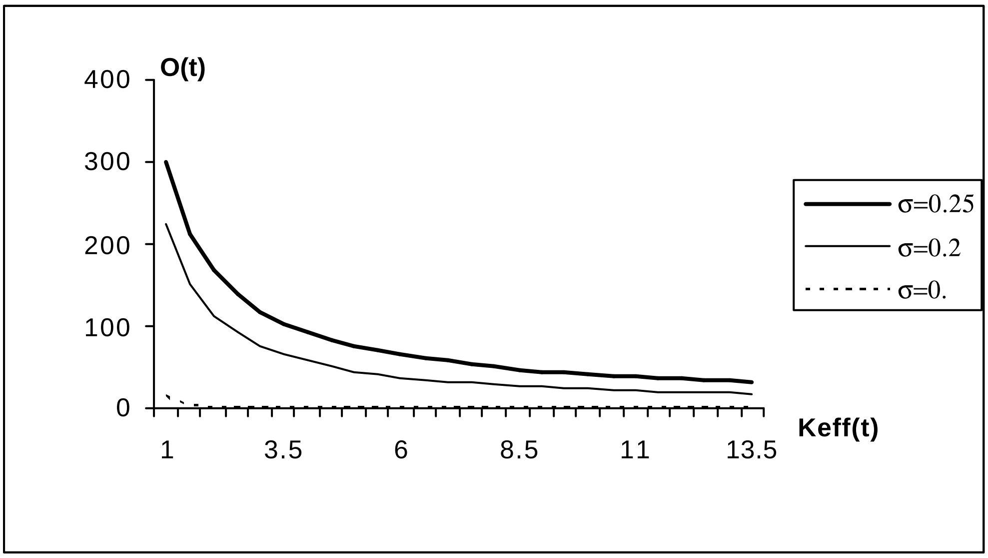


FIGURE 7

Option to postpone investment as a function of time (Keff(t)=1 and Pe(t)=80)

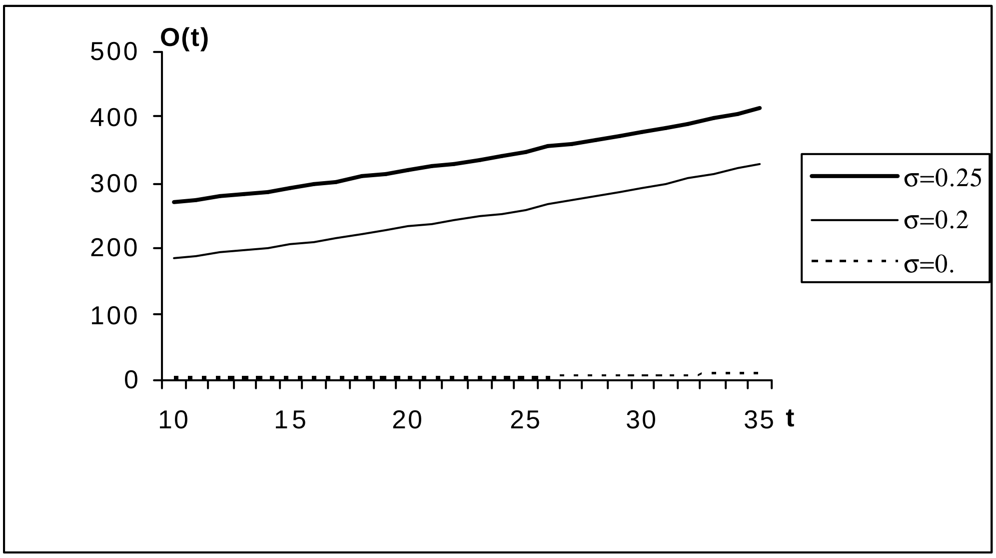


FIGURE 8

Optimal effective stock of capital as a function of the energy price (t=1)

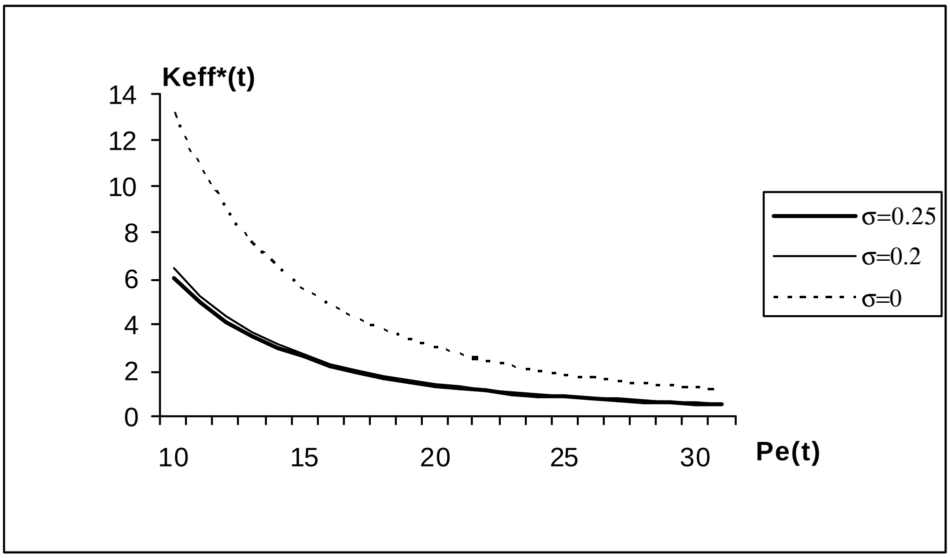


FIGURE 9

Optimal effective stock of capital as a function of time (Pe(t)=10)

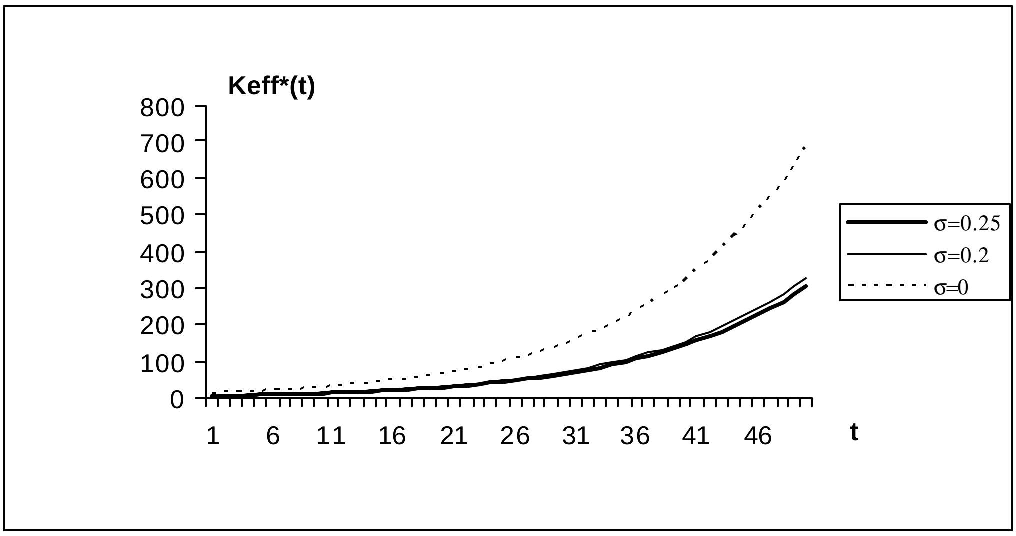


FIGURE 10
Optimal age of the oldest machine as a function of the energy price (t=1)

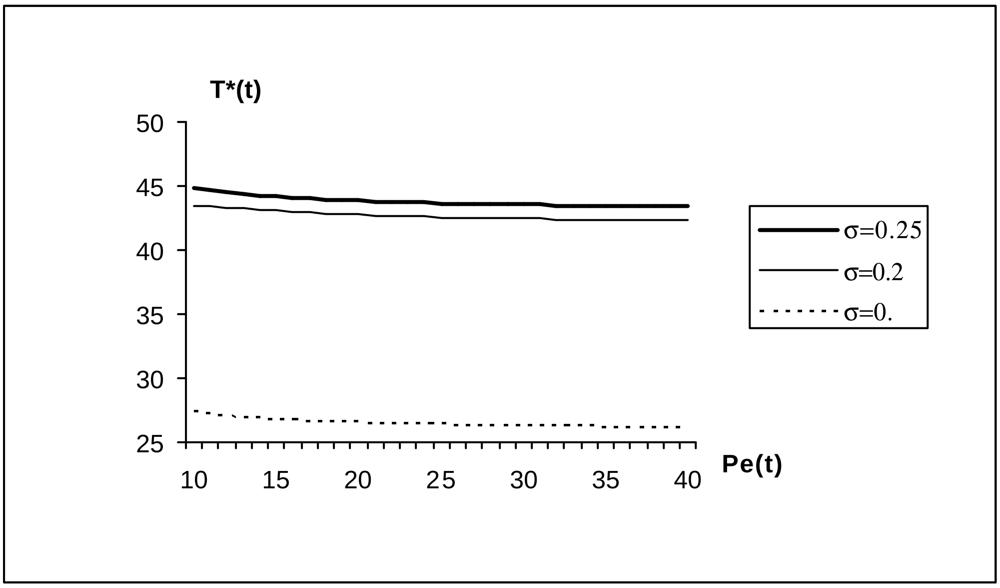


FIGURE 11
Optimal age of the oldest machine as a function of time (Pe(t)=10)

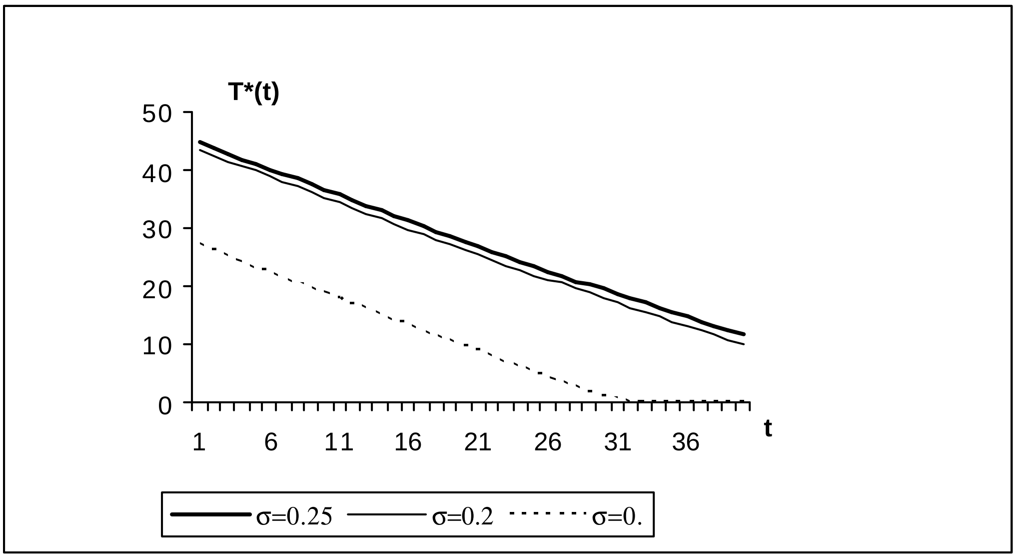


FIGURE 12

Energy Price as a Geometric Brownian Motion, m =0.02 and s^2=0.04

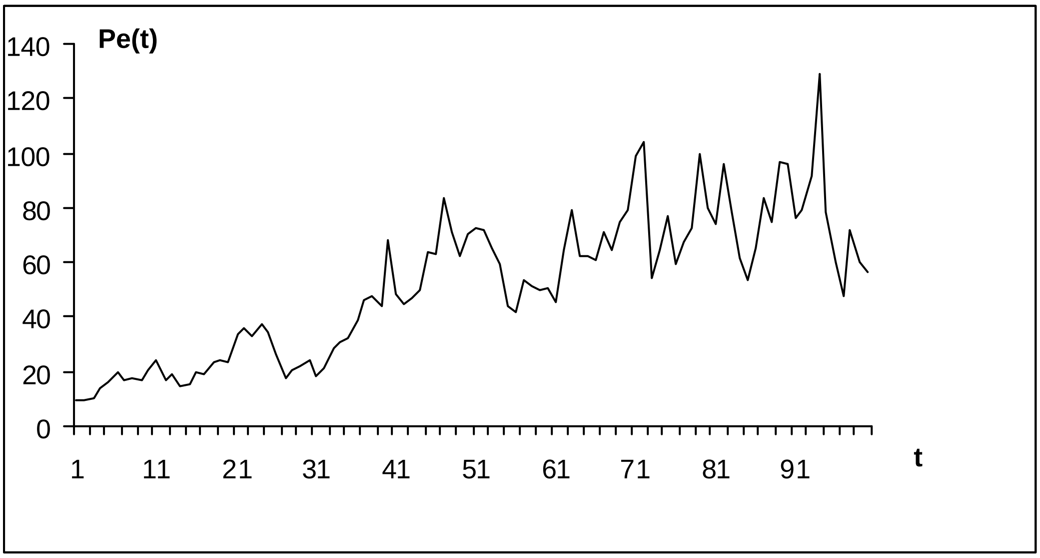


FIGURE 13

Total Stock of Capital

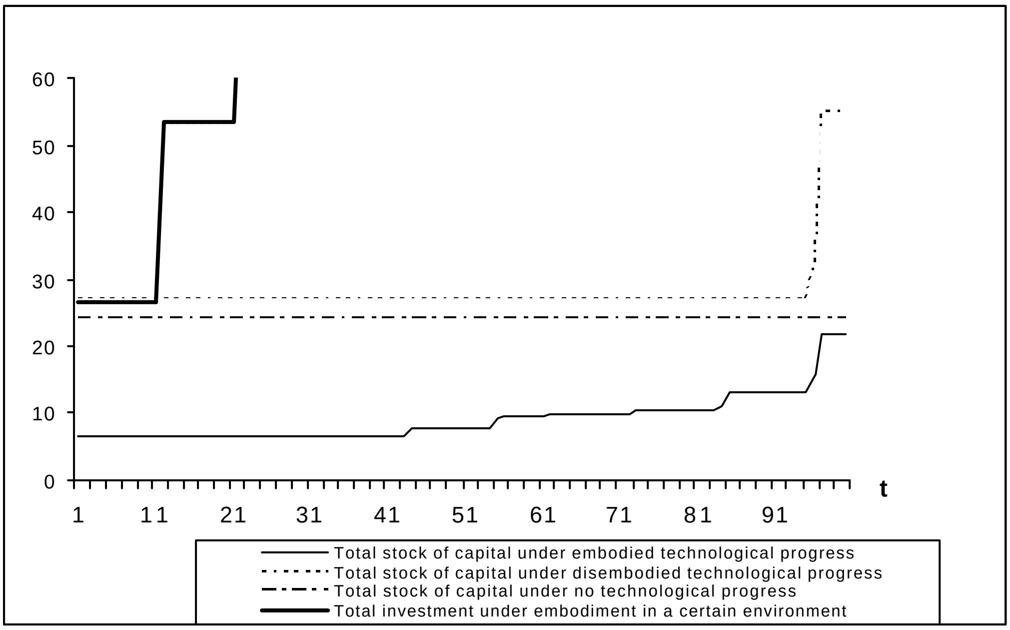


FIGURE 14
Investment

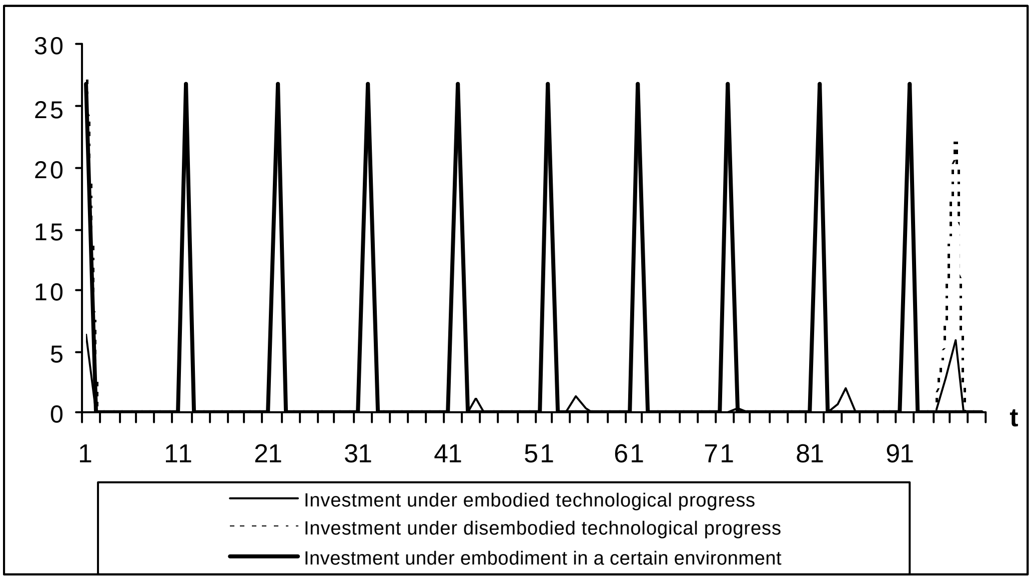


FIGURE 15
Age of the Oldest Machine

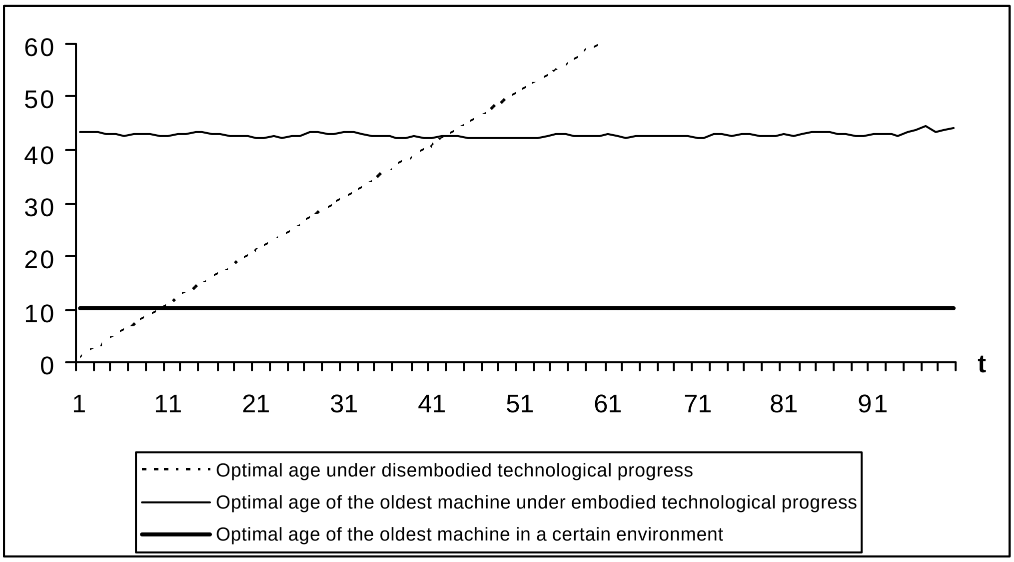


FIGURE 16

The Ratio Investement over Total Stock of Capital and the Capacity utilization in the embodied model with uncertainty

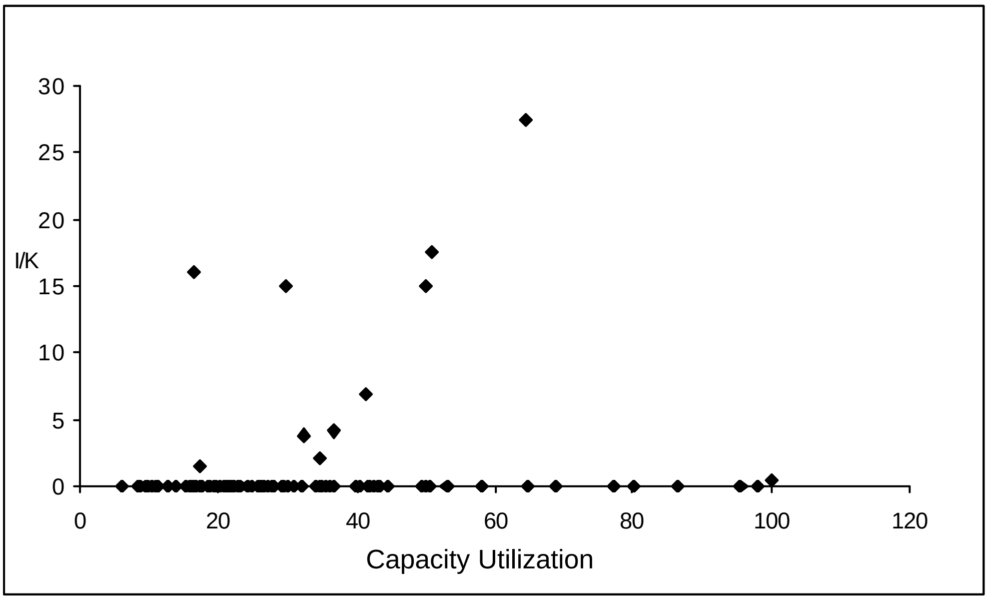


FIGURE 17

Total Stock of capital under different rates of embodied technological progress

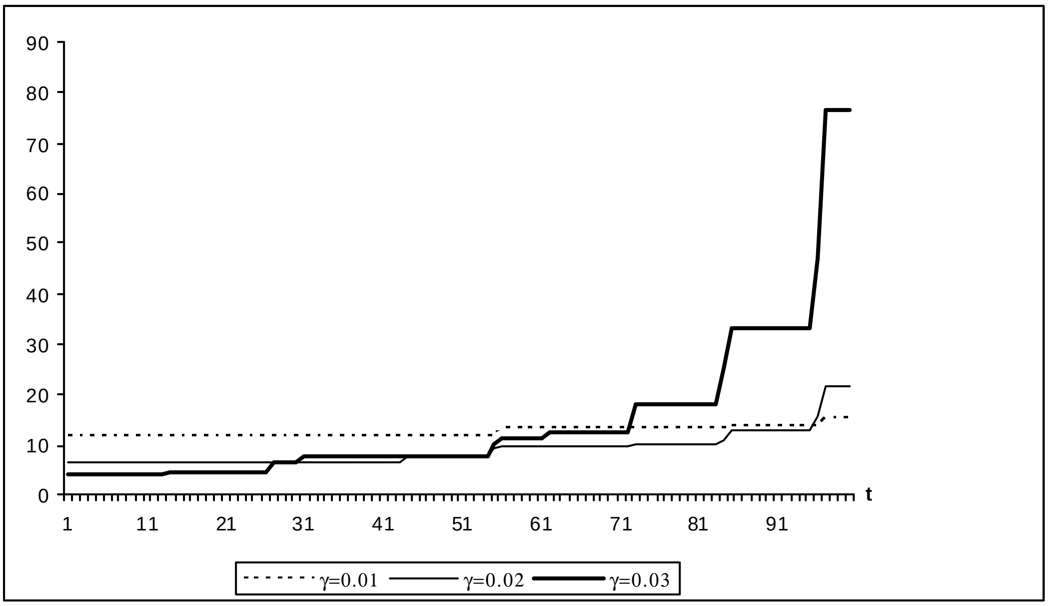


FIGURE 18

Investment under different rates of embodied technological progress

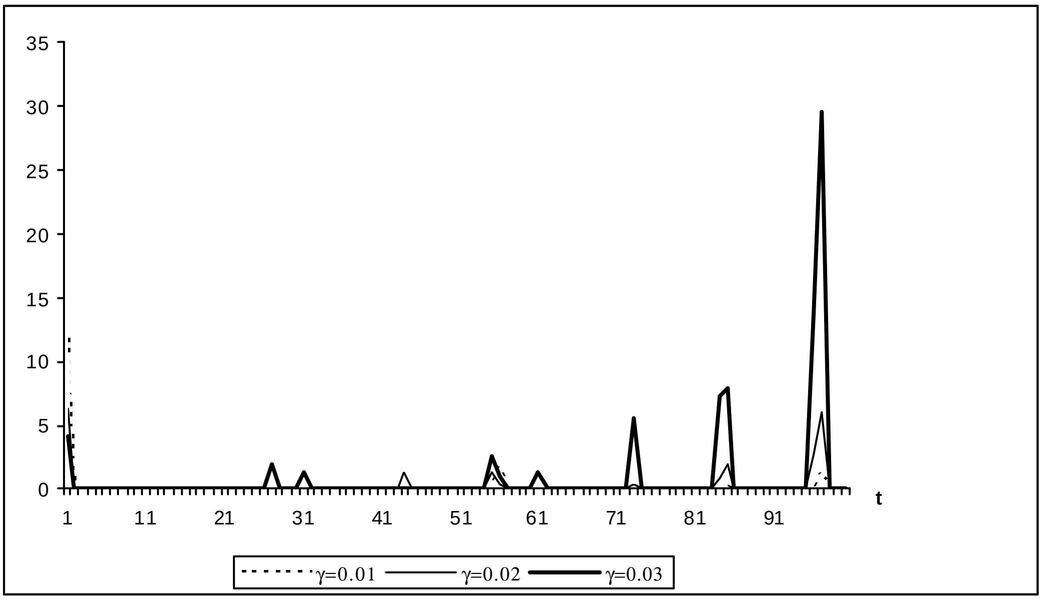


FIGURE 19

The Spikes of investment: Investment over Total Capital Stock (I/K) under different rates of embodied technological progress

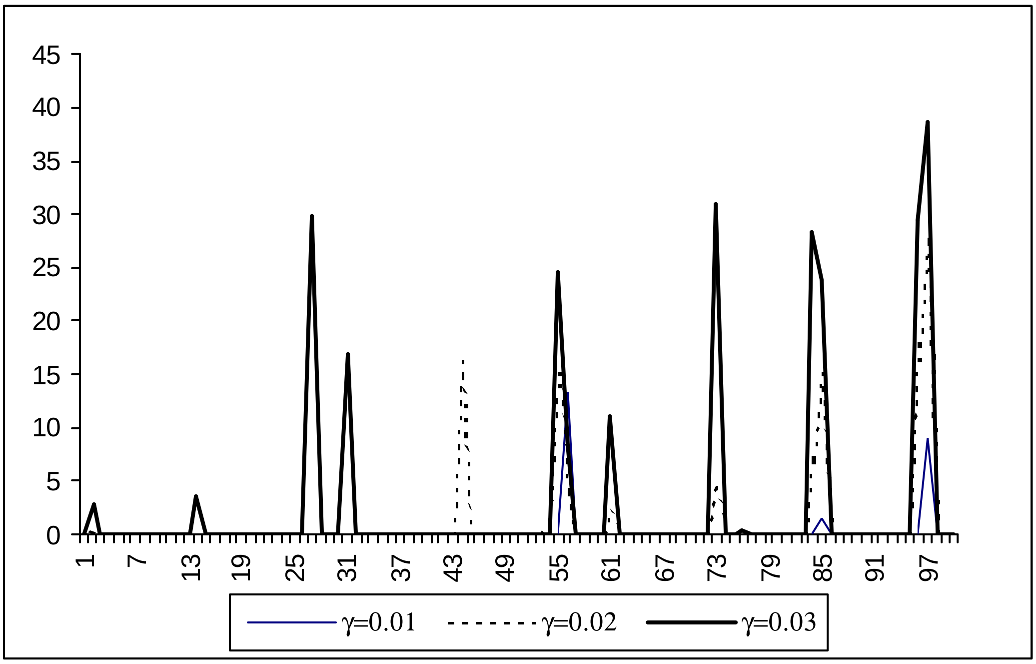


FIGURE 20

Age of the oldest machine under different rates of embodied technological progress

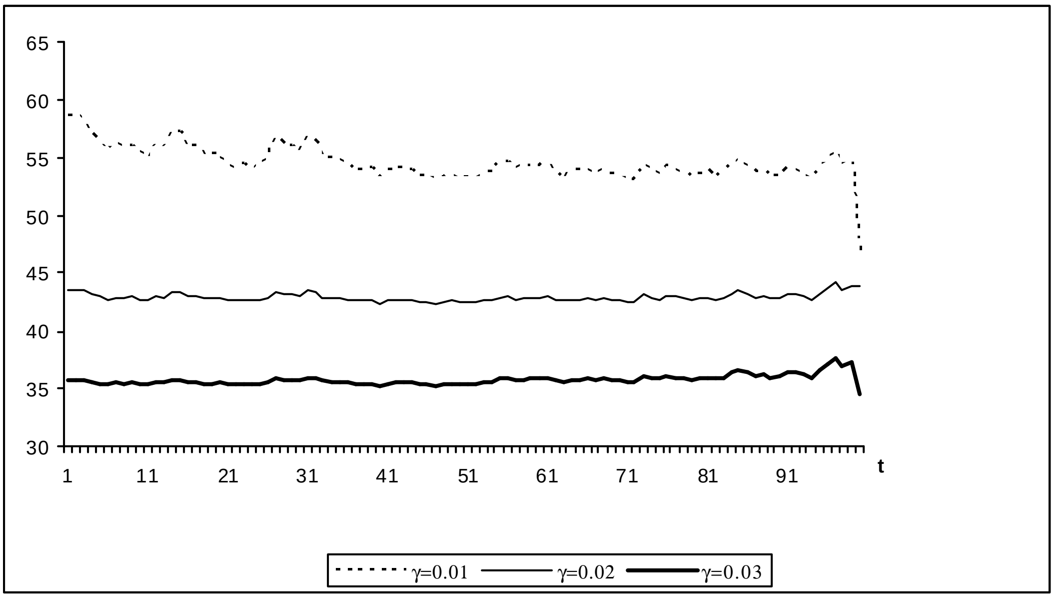


FIGURE 21

Total capital dynamics with and without tax credit; deterministic case under embodiment

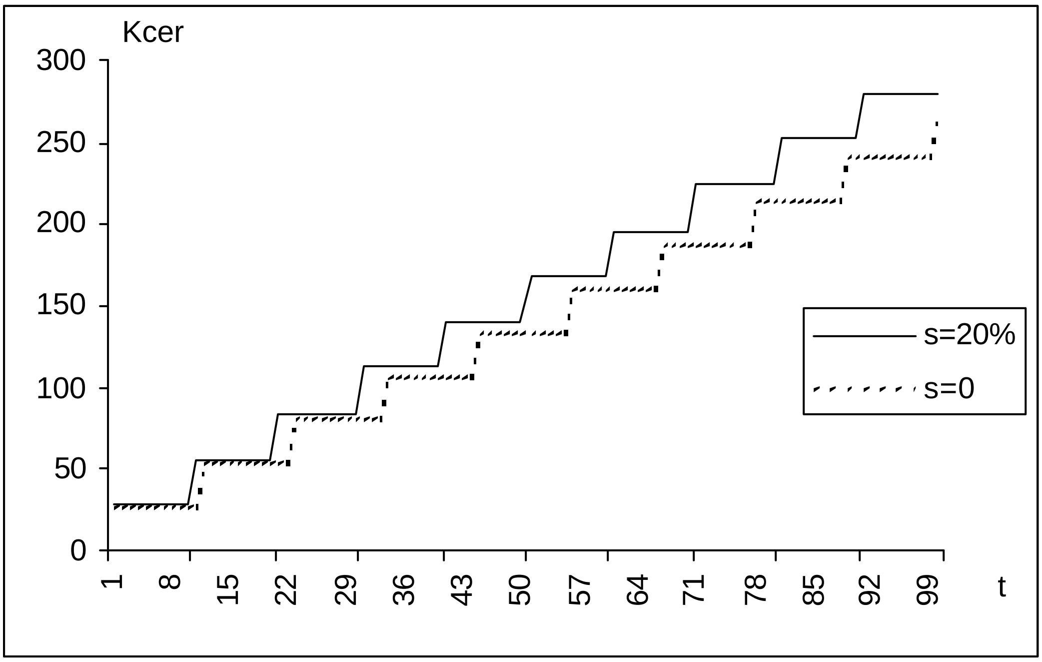


FIGURE 22

Total capital dynamics with and without tax credit; stochastic case under disembodiment

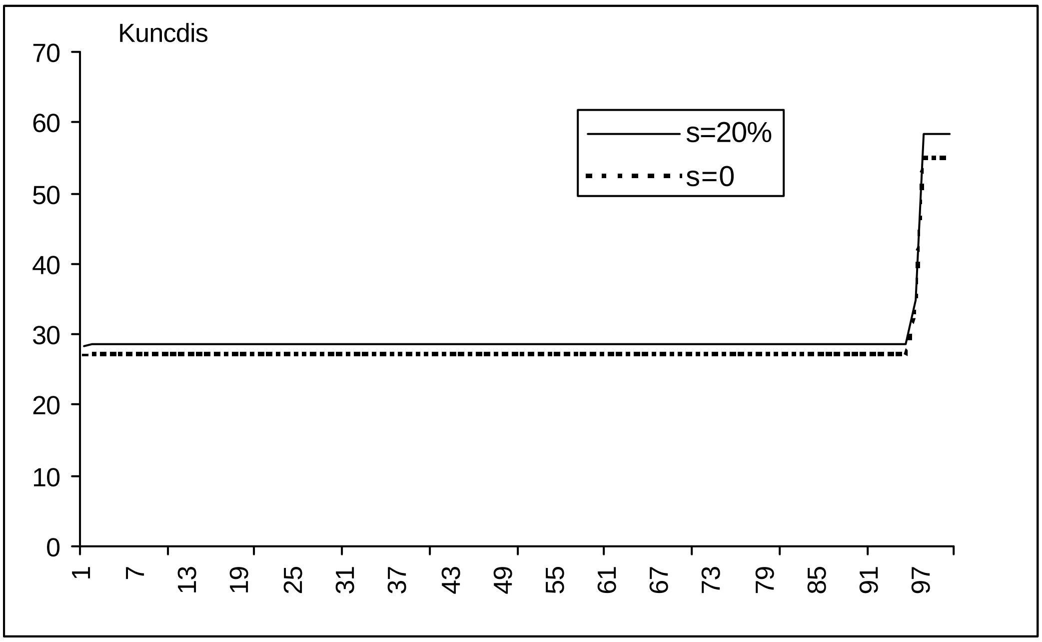


FIGURE 23

Total capital dynamics with and without tax credit; stochastic case under embodiment

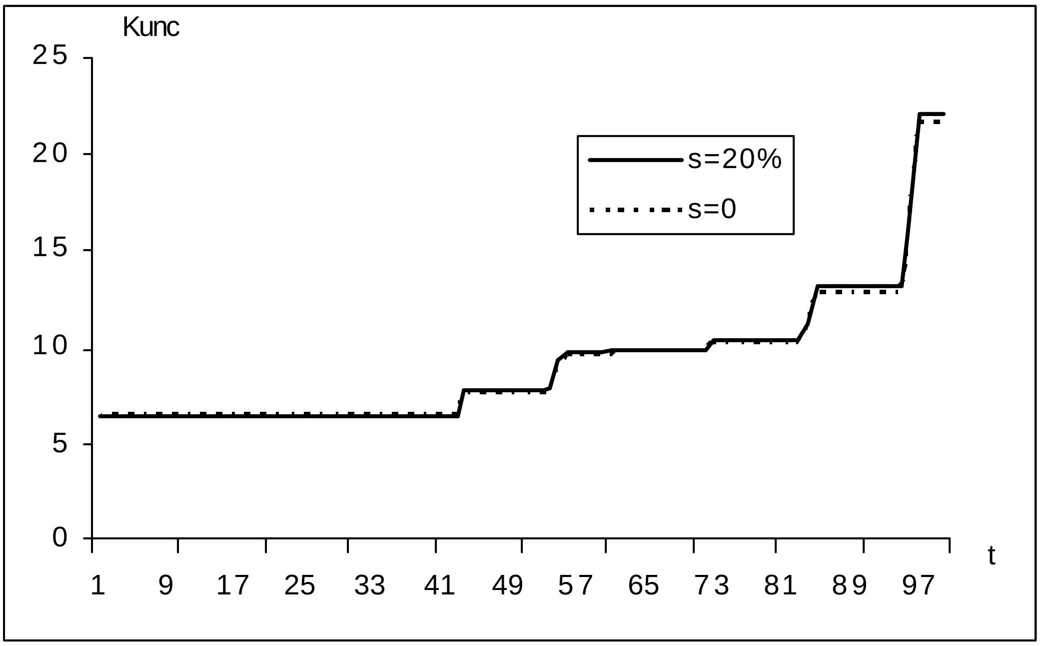


FIGURE 24
Percentage increase in capital due to tax credit

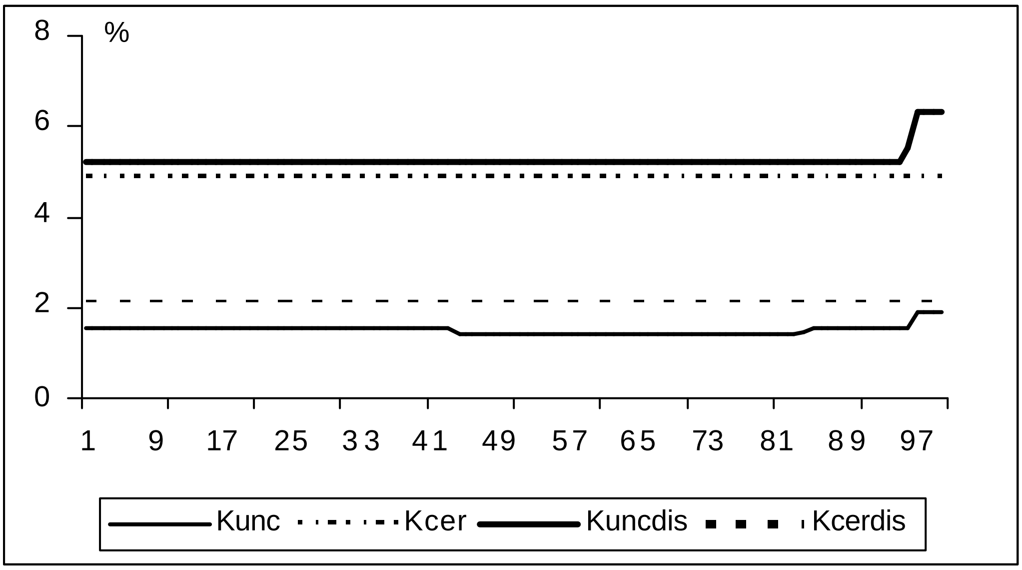


FIGURE 25
Percentage increase in capital due to tax credit

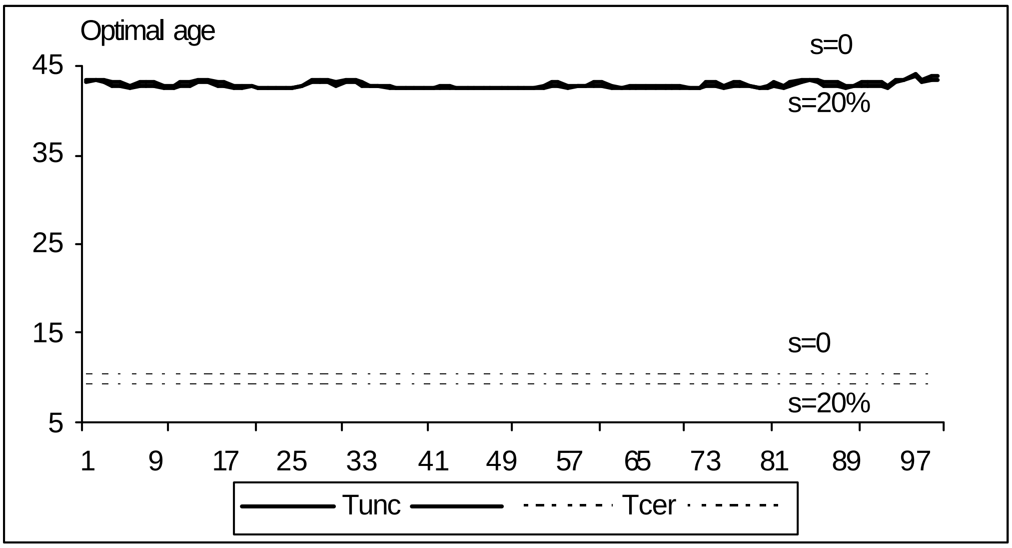
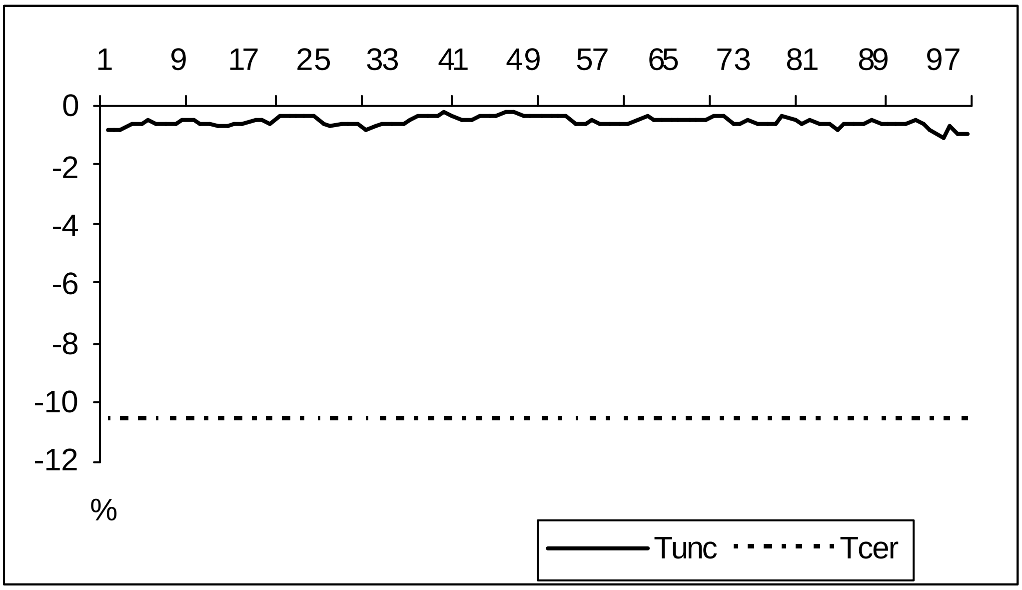


FIGURE 26

Percentage decrease in the optimal scraping age due to tax credit



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