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DISCUSSION PAPER

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CORE INFLATION: ROBUST COMMON TREND MODEL FORECASTING

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DISCUSSION PAPER

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SUMMARY

RESUMO

As autoridades monetárias necessitam de uma previsão da tendência futura da inflação para agir preventivamente sobre a economia. Na literatura encontram-se muitas propostas para o núcleo da inflação que evitam algumas das deficiências do índice de preços usual como um previsor da inflação futura.

O índice de preços é definido como uma soma ponderada das taxas de variação de preços de uma lista de bens e serviços. A utilização desse índice como um indicador da inflação futura é criticada na literatura porque a variabilidade de preços dos produtos é heterogênea, e alguns dos preços apresentam componente sazonal relevante.

Este artigo propõe um modelo multivariado que descreve os movimentos dos preços dos produtos com uma componente comum, e componentes sazonais e irregulares definidas para cada elemento da lista de bens e serviços do índice de preços. É um modelo dinâmico que utiliza um filtro seqüencial robusto. As distribuições preditivas *a posteriori* das quantidades de interesse serão avaliadas utilizando a técnica estocástica do Monte Carlo Markov Chain (MCMC). Os diferentes modelos serão comparados utilizando como critério minimizar a variância preditiva.

Core In°ation: robust common trend model forecasting

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ABSTRACT

The monetary authorities need a future measure of in°ation trend to keep on tracking the in°ation on target. Many alternatives of the core in°ation measure have appeared in the recent literature pretending to avoid the de¯ciencies of the usual headline in°ation index as a predictor. This price index is de¯ned as some weighted average of the individual price change of a list of goods and services. To use it as the future in°ation indicator is criticized in the literature, as far as the products are heterogeneous in respect to the variability and some of the involved prices have relevant seasonal movements. A multivariate model including simultaneously the seasonal e®ects of each component of the price index and a common trend - the core in°ation - will be developed in this paper. The model will be phrased as a dynamic model and a robust sequential ¯lter will be introduced. The posterior and predictive distributions of the quantities of interest will be evaluated via stochastic simulation techniques, MCMC - Monte Carlo Markov Chain. Di®erent models will be compared using the minimum posterior predictive loss approach and many graphical illustrations will be presented.

Keywords: Core in°ation, Robust Kalman Filter, Common Trend, In°uence function.

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1 Introduction

Almost all the analysis of core in°ation assume that there is a well de¯ned concept of monetary in°ation that ought to be of concern to monetary policy makers. This kind of in°ation is not well captured by the standard price indexes as far as this concept is a bad predictor of future in°ation. The monetary authorities need a future measure of in°ation trend to keep on tracking the in°ation on target. Many alternatives of the core in°ation measure have appeared in the recent literature pretending to avoid the de¯ciencies of the usual headline in°ation index as a predictor. This price index is de¯ned as some weighted average of the individual price changes of goods and services with weights chosen on the basis of the expenditure shares. It is criticized by many authors to use it as the future in°ation prediction, given rise to two complementary alternatives to build up a core in°ation indicator.

The ¯rst group of arguments could be summarized as follows. If the weight average of the price changes is an in°ation predictor, the present index, although speci¯ed as a weighted mean of the price changes of individual components, is a ine±cient estimator of the mean variation, since each component has its own volatility. In this case, the price changes for each component of the index must be standardized by its volatility measure. It is clear that price variations of the same magnitude but associated to components with di®erent variability must have di®erent impact on the expectation of the future in°ation.

Certainly the products are not homogeneous with respect to their variability. There are products subject to periodic shortage { as vegetables or products with special harvest season { which present great variability and products with stable price for long time periods. In order to ¯x those drawbacks some authors, as for example Cecchetti (1997) introduced the use of trimmed mean of the cross section distribution of price changes to track trend in°ation.

A second approach emphasizes the predictive aspect of the problem and de¯nes the core in°ation as the common component involved in the description of the observed price changes. This approach, also introduced by Ceccheti, de¯nes the core in°ation as the common trend describing the joint distribution of the price changes of individual goods and services between two periods. Another class of criticism consider the fact that some prices have seasonal movement, as for example the school fare or the products subjected to harvest season, and therefore this regular movement must be considered in the building up of the core inflation.

The recent literature in core inflation includes Bakhshi and Yates (1999), Cecchetti and Groshen (2000), Bryan & Cecchetti (1999), Roger (1998), Wynne (1999) and it mainly goes around to argument about criterion to trim the price changes variation distribution. A stylized fact very well known in the literature is that the price change distribution has a heavy tail. This can be caused by the presence of outliers when the trimmed methods are justified or by distributions derived as mixtures. For instance, let $\mathbf{y}_i$ be a vector of random variables (the products price changes) normally distributed with mean μ and different and unknown variances σ_i^2, which will be assumed gamma distributed. The marginal distribution of y_i given μ will be a t-Student distribution. Although it is a heavy tail distribution it does not seem reasonable to use trimmed estimators.

Since the true data generation process is unknown it is an empirical question to decide the method to be used. The model proposed in this paper includes different processes allowing to choose empirically the best alternative. It is worth pointing out that the trimmed mean models are included in the above class.

The target inflation policy requires that the authorities can be able to advance the movements of the future inflation, so in this paper core inflation will be understood as the forecast of the inflation trend based on a broad class of models including the components: common factors, trend, seasonality and an idiosyncratic error term. The error term is assumed to have a symmetric location-scale multivariate distribution, unimodal and twice piecewise differentiable. This extension includes as particular case many recent attempts to improve upon existing core inflation measures like the trimmed mean, the moving average of the price index and the estimation of a common trend for the set of all price changes. One of the simplest member of the class of models introduced in this paper is obtained assuming that:

- the common component follows a first order autoregressive process;

- the seasonal component is deterministic; and
- the idiosyncratic error term do not have a dynamic structure.

From a methodological point of view a Bayesian approach was adopted. A robust common component model is presented and the posterior and predictive distributions are obtained via stochastic simulation methods – MCMC - Monte Carlo Markov chain. The robust sequential Bayesian estimation or, for simplicity, the robust Kalman filter involves some approximation in the sequential updating of the distribution of location parameters which could be easily avoided if the dimension of the vector of prices changes were not so huge. The approach adopted in this paper is mainly guided by the desire to keep the computational algorithm efficient. This model derives from the compromise of keeping in the model the price change of each product, avoiding the criticism of inefficient estimation, and considering the dynamic of the common price factor movement. The class of models we are introducing in this paper do not suffer from the criticism of independency and normality of the prices changes. Distributions with heavy tails can be used to describe the observed price changes and the observations are only assumed to be independent conditionally to the common factors. The use of the common factors impose a particular decomposition of the full variance and covariance structure of the prices chances.

The paper is organized as follows. In the next Section the proposed model, which includes the trimmed model of Cecchetti, is presented. In Section 3, a brief discussion of estimation in complex models is considered. The equations involved in the robust Kalman filter are derived and the MCMC procedure is discussed step by step. The main results obtained are presented in Section 4 and the conclusions and further remarks are discussed in the final Section.

2 The Proposed Model

The main concern of the core inflation methodology is to predict the inflation trend defined as the moving average of the future inflation (1), where the inflation π_t is defined as the weighted mean of the price changes ($\mathbf{y}_t$ an $m \times 1$

vector) for all the components of the price index.

$$\omega_t^h = \frac{\pi_{t+1} + \cdots + \pi_{t+h}}{h}; \quad \text{where} \quad \pi_t = \mathbf{g}'_t \mathbf{y}_t \tag{1}$$

and $\mathbf{g}_t = (g_{1;t}; \cdots; g_{m;t})'$ is the weight vector assumed known for each time.

The expected value of each one of the m components of the price changes vector, $y_{i;t}$, is modeled by a common factor μ_t, a seasonal components and an idiosyncratic shocks $e_{i;t}$. The common component dynamic evolution is described by a first order autoregressive stationary process.

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\phi} \mathbf{D}_t + \mathbf{F}\mu_t + \mathbf{e}_t \\ \mu_t &= a + b(\mu_{t-1} - a) + w_t \end{aligned} \tag{2}$$

where: $\mathbf{e}_t \sim p(\beta)$ and $\mathbf{F} = (1; \cdots; 1)'$, $w_t \sim N[0; W_t]$, $W_t = b^2(\frac{1}{f} - 1)V[\mu_{t-1|t-1}]$, $\mathbf{D}_t$ is the matrix of monthly dummy seasonal indicators of dimension $s - 1$, where s is the seasonal period; $\boldsymbol{\phi} = (\phi_1; \cdots; \phi_m)'$ the matrix, $(s-1) \times m$, of deterministic seasonal factors; $(a; b)$ defines the time evolution of the common component, with innovation variance, W_t, defined as proportional to the variance of the former time period[1]; and, finally, the distribution of the idiosyncratic shocks, generically denoted by p, is parameterized by the vector β, which dimension varies from model to model.

As will be seen in Section 3.1 the distribution p determines the effect of each observation on the estimation of the common trend. Four alternative distributions will be discussed in this application. In one extreme case all the information are used in equal foot and, in the other, the observations on the tail of the distribution are not taken in consideration, because they are supposed to be outliers. The intermediate cases permit information to have influence declining to zero as they go far away from the center of the distribution. The following table presents the alternative models, where ν denotes the degree of freedom of the t-Student distribution, $V = \mathrm{diag}(v_i)$ is the variance matrix of the idiosyncratic shocks. The assumptions made about p and the content of the parametric vector β, for each model, are also presented. It is worth noting that in TRIM-α model, $e_{100\%(1-\alpha)}$ denotes the $100\%(1-\alpha)$ percentile defining the cutting point in the trimmed mean procedure.

Table 1: Alternative forms for the p distribution

Case	Name	Model	β
1	Multivariate normal	$\mathbf{e}_t \sim MN[0; \mathbf{V}]$	$\mathbf{V}$
2	Jointly t-Student for products	$\mathbf{e}_t \sim MtSt[0; \mathbf{V}; \nu]$	$\mathbf{V}$, ν
3	t-Student for each product	$e_{i;t} \sim tSt[0; v_i; \nu]$	$\mathbf{V}; \nu$
4	Trimmed	$e_{i;t} \sim D(\beta)$	$e_{100\%(1-\alpha)}$

In the case in where p is a multivariate normal, the equation 2 describes a multivariate model, otherwise the generalized model will be called robust multivariate model. Given the hyperparameters $\theta = (a; b; f; \phi; \beta)'$, the expected value of the predictive distribution of π_t and ω_t^h can easily be obtained since they are function of the common component plus the seasonal factor. These quantities will be the best prediction assuming the square error loss function.

$$
\begin{aligned}
E[\pi_t | t-1; \theta] &= \mathbf{g}_t' E[\mathbf{y}_t | t-1; \boldsymbol{\theta}] = \mathbf{g}_t' E[(\phi D_t + F\mu_t + \mathbf{e}_t) | t-1; \boldsymbol{\theta}] \\
&= \mathbf{g}_t' \phi D_t + E[\mu_t | t-1; \theta]; \quad \text{since } \mathbf{g}_t'\mathbf{F} = 1 \\
E[\omega_t^h | t; \theta] &= \frac{E[\pi_{t+1} | t; \theta] + \cdots + E[\pi_{t+h} | t; \theta]}{h} \\
&= \frac{\sum_{i=1}^{k} (\mathbf{g}_{t+i}' \phi D_{t+i} + E[\mu_{t+i} | t; \theta])}{h} \qquad (3)
\end{aligned}
$$

In the multivariate normal case, ω_t^h is normally distributed since it is a linear combination of normal distributed random variables and otherwise it will be approximately normal, since it is the sum of a large number [2]of identically distributed random variables. For the parametric models the influence function is the first derivative of the log-density and $V[\omega_t^h | t; \theta]$ can be evaluated in a close form. The robust log-likelihood function that approximates the likelihood of θ can also be obtained as (see Appendix):

[1]see West e Harrison (1997)

[2]Since ($m = 512$ e $h = 4$) we have more than 2000 parcels involved in the sum, corresponding to mh

$$l(\Psi|\omega) = \log(p(\omega|\Psi)) \approx -\sum_t \log(V[\omega_t^h|_t;\Psi]/2 - \rho(\omega_t^h - E[\omega_t^h|_t;\Psi])/(2V[\omega_t^h|_t;\Psi]) \quad (4)$$

where: $\omega = (\omega_1^h; \cdot\cdot; \omega_n^h)'$ and $\rho(\omega) = \omega^2$ in the normal case.

In the non-parametric cases - the trimmed mean - the variance of ω_t^h is not analytically available. Nevertheless, if we assume that the variance is time invariant the above expression for the log-likelihood function can be used as an approximated criterion.

Many alternative models for the core inflation are nested to the one we are proposing in this paper. If a unit root is assumed, $b = 1$ in equation (2), the common component describes permanent movements of the inflation and a similar model to that one proposed by Fiorêncio and Moreira (2000) is obtained. If, by the other hand, the second part of equation (3) is eliminated from the model specification, the common component looses its intertemporal restriction, giving a simple measurement of the current inflation taking in consideration that index components have different precision, making the model similar to that one proposed by Cecchetti (1997).

It is worth remembering that the quantity ω_t^h is a forecasting of the mean inflation in the next h time periods, given the available information until time t. Therefore this quantity is only available till h periods of time before the end of the sample and the densities specified before could only be evaluated till this period time. In the results presented in this paper the last four values of this quantity are forecasting.

3 Inference for Robust Common Trend Models

In this sort of complex models closed form expressions for the point estimates of the quantities of interest are not often available. Adopting a Bayesian approach the posterior and predictive distribution for all the quantities of interest can be calculated from the prior distribution via Bayes theorem. The Bayesian computation of those distributions can be done using MCMC

- Monte Carlo Markov Chain techniques. This is a stochastic iterative algorithm which decompose the computation of the joint posteriori distribution of the quantities of interest in more simple sub-problems. One of those sub-problems is just the evaluation of the trajectory of the common component given all the other parameters and the available information, $p(\mu_1, \cdots, \mu_T | \psi, \Omega_T)$, where Ω_T represents the global information available.

In the normal case, the multivariate dynamic model formulation of West and Harisson (1997) can be used to calculate the mean and variance of all the distribution involved via the recurrence equations sometimes called Kalman filter. In the case where p do not represent a normal distribution there are not analytical expressions to describe the trajectory of those parameters. Nevertheless, assuming that p is unimodal, symmetrical and twice differentiable, an approximate procedure, due to West (1981) and closely related to Marseliez (1975) and Raftery and Martin (1996), is available.

3.1 Robust Sequential Filter

When p is the multivariate normal, conditional on the hyperparameters ψ, the model described by (1-2) corresponds to the usual multivariate dynamic model, that is:

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\phi}\mathbf{D}_t + \mathbf{F}\mu_t + \mathbf{e}_t; \quad \mathbf{e}_t \sim N[0; \mathbf{V}] \\ \mu_t &= a + b(\mu_{t-1} - a) + w_t; \quad w_t \sim N[0; b^2 W_t]] \end{aligned}$$

where: $W_t = (\frac{1}{f} - 1)V[\mu_{t-1}|_{t-1}]$.

Assuming that $E[\mu_{t-1|t-1}]$ and $V[\mu_{t-1}|_{t-1}]$ are known for each time $t-1$ we can easily obtain the mean and the variance of all the distributions involved, as showed in the Appendix (West and Harisson (1997)). A simplifying assumption that does allow calculation of the posterior mean and variance even when the observations are not normally distributed was introduced by Masreliez (1975) and involves the score function for the predictive density - $p(y_t|y_{t-1})$ - and its first derivative. Those densities are in general intractable in the presence of outliers and so the score function and its first derivative must be approximated by appropriately chosen bounded continuous functions, as for example the Hampel's two part redescending function. Nevertheless, when p is a heavy-tailed distribution the approach of West (1981) provides approximate Bayesian methods for time series analysis which extend considerably the works of Masreliez (1975) and Masreliez and Martin (1977). An alternative approximation for the recurrence equation of Masreliez is obtained after some Taylor series expansion for the log-likelihood function.

The equations for the posterior mean and variance are replaced by:

$$\begin{aligned} E[\mu_t|t] &\simeq E[\mu_t|t-1] + V[\mu_t|t-1]\mathbf{F}'g(\hat{e}_t) \\ V[\mu_t|t] &\simeq V[\mu_t|t-1](1 - V[\mu_t|t-1])\mathbf{F}G(\hat{e}_t)\mathbf{F}' \end{aligned} \tag{5}$$

where: $\hat{\mathbf{e}}_t = \mathbf{y}_t - E[\mathbf{y}_t|t-1]$, $g(\hat{\mathbf{e}}_t) = -\partial \log(p(\hat{\mathbf{e}}_t|t-1)/\partial\hat{\mathbf{e}}_t$ and $G(\hat{\mathbf{e}}_t) = \partial g(\hat{\mathbf{e}}_t)/\partial\hat{\mathbf{e}}_t$

For the normal case it is easy to show that $g(\hat{\mathbf{e}}_t) = \mathbf{Q}_t^{-1}\hat{\mathbf{e}}_t$ and $G(\hat{\mathbf{e}}_t) = \mathbf{Q}_t^{-1}$. Then under the normality hypothesis the robust Kalman filter coincides with the classical solution. If, when updating beliefs about location surprisingly large observations must be ignored, then $g(\hat{e}_t)$ and $G(\hat{e}_t)$ must tend to zero when $\hat{e}_t \to \infty$. This ensures that prior and posterior mean and variance are not impacted from the current observation, leading to the concept of robust likelihood. The equation (5) shows that the influence function g determines the impact of the deviation $\hat{\mathbf{e}}_t$ in the estimation of the common component. The hypothesis behind each alternative specification of p could help in clarifying the understanding of the influence function.

- Multivariate Normal (Mn): All the observations are supposed to come from the same normal distributions and therefore the magnitude of the deviations are not relevant to discriminate the observed values of the components;

- Multivariate t-Student (MtSt): It assumes that the observations associated to large deviations, evaluated in the m-dimensional space of all the products, have less chance to belong to the sample and so the size of those deviations are useful to discriminate the observations. Large deviations imply less effect in the index formation.

- Univariate t-Student (by product) (tSt): It admits that the observation of each product associated to the larger deviations has less chance to belong to the sample and therefore the magnitude of the deviations are useful to discriminate the relevance of the observations. The larger deviations must have less impact in the inflation index evaluation. An example will be useful to make some distinction between this alternative and the former. Let us consider a situation where only few products have large deviations. In this case it is possible that alternative (2) do not penalize an observation relatively to all products.

- Trimmed mean (Trim): It assumes that the deviation after some threshold are spurious and then must be eliminated from the analysis. The cutting value is well chosen percentile of the distribution of the deviations in a certain time period.

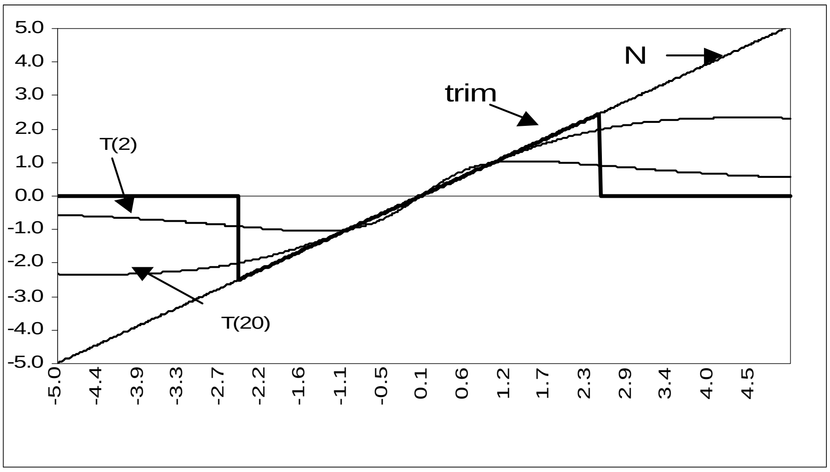
Table 2: Influence Function

p	$g(\hat{e}_t)$	$G(\hat{e}_t)$
Mn	$\Omega_t^{-1}\hat{e}_t$	Ω_t^{-1}
MtSt	$(\nu + m)\frac{\Omega^{-1}\hat{e}_t}{\nu+\hat{e}_t'\Omega^{-1}\hat{e}_t}$	$(\nu + m)\frac{\Omega^{-1}(\nu+\hat{e}_t'\Omega^{-1}\hat{e}_t)-2(\Omega^{-1}\hat{e}_t)(\Omega^{-1}\hat{e}_t)'}{\nu+\hat{e}_t'\Omega^{-1}\hat{e}_t}$
tSt	$(\nu + 1)\frac{\Omega^{-1}_{(i;i);t}\hat{e}_{i;t}}{\nu+\hat{e}^2_{i;t}\Omega^{-1}_{(i;i);t}}$	$(\nu + 1)\frac{(\nu-\hat{e}^2_{i;t}\Omega^{-1}_{(i;i);t})\Omega^{-1}_{(i;i);t}}{(\nu-\hat{e}^2_{i;t}\Omega^{-1}_{(i;i);t})^2}$
Trim	$\hat{e}_{i;t} I_{[0;e_{100\%(1-\alpha)}]}(\lvert\hat{e}_{i;t}\rvert)$	n–

where $I_A(x) = 1$ if $x \in A$; 0 c.c. Other alternatives for the influence function include the Huber family, the logistic distribution as described in West (1981).

The influence function for the four alternative models previously described can be appreciated in Figure 1, where the t-Student with 2 and 20 degrees of freedom are shown. For the normal multivariate case (Mn) the effect is the same independently on the deviation size. The influence function corresponding to the trimmed case abruptly decreases to zero and the influence function corresponding to a t-Student with 2 (T(2)) and 20 (T(20)) degrees of freedom present a intermediate behavior.

Figure 1: The Influence Function for the standard deviation and for the four alternatives



3.2 Estimation

Let $\boldsymbol{\Psi}$ denote the vector of hyperparameters and $\boldsymbol{\Psi}_{(k)}$ the former excluding the k^{th} element, let Ω_T denote the available information at time t and consider the model defined by 1-4. The posterior distribution of the vector of hyperparameters $\boldsymbol{\Psi} = (\Psi_1; \ldots; \Psi_k)'$ is obtained sampling from the conditional distribution when they are available for sampling. Then the joint distribution of $p(\boldsymbol{\Psi}|\Omega_T)$ is obtained sampling sequentially from $\Psi_k^r \sim p(\Psi_k|\boldsymbol{\Psi}_{(k)}^{r-1}; \Omega_T)$; $k = 1; \cdots; n$; $r = 1\cdots$. If same of those conditional posterior distribution were not available for sampling some acceptance/rejection method can be used to approximate them [3]. The following algorithm permits to obtain the posterior and predictive distribution for the multivariate normal case. Denote the initial conditions by $\mu_t = \omega_t^h$; $\Psi = \Psi_0$; $r = 1$.

Algorithm:

1. Sample $\mu_t^r \sim p(\mu_t|\Psi; \Omega_T)$

[3] Metropolis-Hastings, for details see Gamerman (1997).

2. Sample $\phi \sim p(\phi | a; b; f; \mu_t; \Omega_T)$

3. Sample $\beta \sim p(\beta | a; b; f; \phi; \mu_t; \Omega_T)$

4. For $k = 1; 2; 3$

   - Sample $\Psi_k^{r} \sim N(\Psi_k^{r-1}; v_k)$
   - Obtain $l(\boldsymbol{\Psi}^{r})$ using the robust Kalman filter using the desired influence function
   - Sample $u \sim U(0; 1)$, if $l(\boldsymbol{\Psi}^{r}) - l(\boldsymbol{\Psi}^{r-1}) > \ln(u)$, accept Ψ_k^{r}, otherwise let $\Psi_k^{r} = \Psi_k^{r-1}$

5. check for the convergence of the chain, go back to (1) up to the convergence can be accepted.

Problem 1:

Given $\boldsymbol{\Psi}$, $p(\mu_1; \cdots; \mu_T | \Psi; \Omega_T)$ can be obtained via the Kalman filter or even its robust version. Alternatively the FFBS (forward filtering, backward sampling) developed by Fruhwirth-Schnatter (1994) can be used to get efficiently the joint distribution given Ψ, $p(\mu_1; \cdots; \mu_T | \boldsymbol{\Psi}; \Omega_T)$ as follows:

- sample μ_T from $(\mu_T | \boldsymbol{\Psi}; \Omega_T)$
- for each $t = T - 1; T - 2; \cdots; 1; 0$ sample μ_t from $(\mu_t | \mu_{t+1}; \boldsymbol{\Psi}; \Omega_T)$

The marginal distribution of $p^*(\mu_t | \boldsymbol{\Psi}; \Omega_T)$ is then easily obtained.

Problem 2:

The parameter ϕ is conditionally independent of $(a; b; f; \beta)$ given μ_t, ie.: $p(\phi | a; b; f; \beta; \mu_t; \Omega_T) = p(\phi | \mu_t; \Omega_T)$. Since the seasonal components are idiosyncratic given μ_t, their distributions are independent for each product i. Therefore $p(\boldsymbol{\phi} | \mu_t; \Omega_T) = \prod_i p(\phi_i | \mu_t; \Omega_T)$ and the posterior distribution of the seasonal components for each product

$$(\phi_i | \mu_t; \Omega_T) \sim N[(D'D)^{-1} D'(yi - \mu); V_i]$$

where[4] $D = (D_1, \ldots, D_T)'$.

Problem 3:

When p is the multivariate normal the parameter β corresponds to the idiosyncratic variance $V = diag(v_i)$. Its posterior distribution do not depend on (a, b, f) given μ_t, that is $p(V|a, b, f, \phi, \mu_t, -) = p(V|\mu_t, \phi, -)$. Since the socks are independent then $p(V|\mu_t, \phi, \omega) = \prod_i p(V_i|\mu_t, \phi_i, -)$ with inverted gamma distribution given by [5]:

$$v_i^r \sim GaI(T + n_0, s_0 + \sum_i \hat{e}_{i,t}); \text{ where } \hat{e}_{i,t} = y_{i,t} - \phi_i^r D_t$$

Problem 4:

The posterior distribution of the parameters $(a, b, f) = (\psi_1, \psi_2, \psi_3)$ involved in the dynamic of μ_t will be accessed via a rejection algorithm. One value of Ψ_k is obtained sampling from the proposal distribution $\Psi_k^r \sim N[\Psi_k^{r-1}, v_k]$ and the Kalman filter used to get $l(\Psi^r)$. Comparing $l(\Psi^r)$ with $l(\Psi^{r-1})$ it will be decided if the draw value is accepted or not.

3.3 Estimation for the other cases

The main modification involved in the estimation of the other model are:

- Alternatives 2 (MtSt) and 3 (tSt): the number of degrees of freedom must be included in the step 4 of the former algorithm;

- Alternative 4 (Trim): the former algorithm must be used excluding the step 2 and including the cutting factor in step 4. The likelihood in step 4, $l(\Psi)$, supouse that the variance of ω_t^h is constant.

[4] The hypothesis that $\phi_i = 0$ tested at the 1% significance level. When not rejected the coefficient was set at the value zero. About 20 products, mainly agriculture products, have ϕ significantly different from zero. seasonal component were calculated only for productcs that are present in the two samples, until 1999 and after

[5] The list of components of the inflation index - IPCA - changes in 08/1999, from 350 to 512 items. The variance is estimated summing the squares deviation for the first sample - until 08/1999 - and for the second one. For the new items we can not calculate the first part. This component were approximated by the mean sum of squares of the products of the same type

In the cases where p is not a multivariate normal distribution the results obtained depend on the accuracy of the robust Kalman filter as an approximation for the true evaluation of the distribution of μ_t. In the non-parametric case - trimmed function - the approximation depends also on the hypothesis of constant variance. Certainly the approximation is more crucial when we are far way from the multivariate normal assumption.

When the influence function is multidimensional the matrix Q_t^{-1} has rank equal to the number of components involved. Since in the algorithm presented before this matrix must be inverted as many times as the Monte Carlo sample size are and the periods of time the computational cost is almost infeasible. Nevertheless, this matrix has some properties that can ease the computational burden. An alternative analytical expression is obtained in the appendix. Expressions for the efficient calculation of $\mathbf{F}'g(\hat{e}_t)$ and $\mathbf{F}G(\hat{e}_t)\mathbf{F}'$ are presented in the following table. It is worth mentioning the difference in the influence function when the components are jointly or individually considered. In one case the expression depends on the ratio of the means and, in the other case on the mean of the ratios.

Table 3: Efficient Expressions for Evaluation of Influence Function

Models para p	$\mathbf{F}'g(\hat{e}_t)$	$\mathbf{F}G(\hat{e}_t)\mathbf{F}'$
Mn	$X_t(1-\delta_t\gamma)$	$\gamma(1-\delta_t\gamma)$
MtSt	$\frac{\nu+n}{\nu+Z_t}X_t(1-\delta_t\gamma)$	$(1-\gamma\delta_t)\frac{v-m}{v+Z_t}\gamma - 2X_t^2(\frac{1-\gamma\delta_t}{v+Z_t}$
tSt	$(\nu+1)\frac{Q_t[i;i]^{-1}\hat{e}_{it}^2}{\nu+\hat{e}_{it}^2Q_t[i;i]^{-1}}$	$(\nu+1)\frac{(\nu-\hat{e}_{it}^2Q_t[i;i]^{-1})Q_t[i;i]^{-1}}{(\nu+\hat{e}_{it}^2Q_t[i;i]^{-1})^2}$

where: $\gamma = \sum_i v_i^{-1}$; $\delta_t = (\gamma + r_t^{-1})^{-1}$; $X_t = \sum_i \hat{e}_{i;t}$ and $Z_t = \sum_i \hat{e}_{i;t}v_i^{-1}$.

In this paper we introduce a broad class of models including or not a common trend component and its dynamics, the seasonal factors and different data generation descriptions. The number of parameters varies from model to model so the model selection criterion must take into account this fact. Gelfand and Ghosh (1998) developed a criterion with a solid decision theoretical basis. Model complexity is penalized and a parsimonious choice stimulated, in the spirit of penalized likelihood approaches, e.g. the now popular BIC criterion due to Schwarz. This criterion, defined on the prediction space, includes two components: one is a measure of the goodness of

¯tting and the other is the variance of the predictive distribution and could be interpreted as the punishment component. The use of the MCMC samples permit to take in account the uncertainties derived on the parameters estimation and will be used to access the components mentioned above.

For the non-parametric models it is not possible to get the predictive distribution expression since the matrix involved is not full rank. Then, to make the comparisons possible we introduced the hypothesis of constant predictive variance $V(!_t^h) = V_!$.

4 The Main Results

In this paper we deal with IPCA monthly observations in the period of 09/1994 to 05/2001. Clearly the same approach could be applied to any in°ation index. Since di®erent assumption about the forecast horizon do not impact too much the main results obtained, we decided to ¯x it in four months.

The in°uence function and also the speci¯cation of the transition equation of the common trend are empirically accessed. The normal model do not depend upon approximations in the evaluation of the common trend but involves a large number of idiosyncratic variances. The t-Student model by its turn has an in°uence function more reasonable given less weight for the more extreme observations but its performance strongly depends on the approximation involved in the robust Kalman ¯lter and also on a large number of idiosyncratic variances. The other speci¯cations of the in°uence function correspond to procedure already presented in the core in°ation literature and are not free of the approximations of the robust ¯lter.

The proposed model is °exible and can be estimated with four alternative speci¯cations for the error term and three di®erent speci¯cation for the transition in the common component: i) the unrestricted case, corresponding to the transitory component of the in°ation (T), where the common trend follows a mean reversion process; ii) the restricted case where the common trend follows a random walk (P), that is $b = 1$ and $a = 0$, measuring the permanent component of the in°ation and, ¯nally, iii) the case where the common component evolves unrestricted throughout time (C), which means the current in°ation. The model can also be speci¯ed including (PS) or not the seasonal factor.

In table 4, the expected likelihood function (L ¡ Lik) and the total variance (Tv) derived from the Gelfand and Ghosh criterion under square loss function. The total variance is decomposed in a goodness of ¯tting measure (Gv) and the predictive variance (Pv), Tv = Gv + Pv.

Table 4: Exact Performance Measures

		Tv$^{1=2}$				L ¡ Lik		
In°uence	P	T	C	TS	P	T	C	TS
Mn	0.524	0.519	0.591	0.522	45.3	46.3	-46.1	46.0
t-St	0.537	0.524	0.596	0.524	42.6	45.2	8.2	45.3
Mt-St	0.535	0.522	0.579	0.523	42.7	45.7	-38.3	45.6

In table 5 approximate results assuming constant predictive variance are presented for all the models. In order to compare the di®erent core in°ation measure proposed in the literature we also calculated the asymmetric trimmed mean model.[6] It is worth pointing out that the model C estimated under the asymmetric trim corresponds exactly to the Cecchetti proposal.

Table 5: Approximate Performance Measures

		Tv$^{1=2}$				L ¡ Lik		
In°uence	P	T	C	TS	P	T	C	TS
Mn	0.398	0.387	0.740	0.389	95.2	99.5	48.6	99.6
t-St	0.431	0.396	0.585	0.398	89.1	99.1	66.3	98.5
Mt-St	0.428	0.391	0.720	0.392	89.6	98.6	50.7	98.1
Trim	0.402	0.384	0.480	0.388	94.5	98.7	81.5	97.8
TrimA	0.408	0.480	0.525	0.431	94.9	98.0	76.7	98.2

The main conclusions that can be drawn from the above results are:

[6] Although in this case the use of the robust Kalman ¯lter is not recommenced, since it corresponds to an asymmetric in°uence function, we can interpret the result as a smoothed asymmetric trim, see Fiorêncio and Moreira.

² Assuming multivariate normality, the model T (transitory component) presents the smallest total variance and the highest expected log likelihood. It is clearly the best model for those data set.

² The inclusion of seasonal e®ects is supported by both performance criterion.

² Although the asymmetric trim presents reasonable results (expected log-likelihood slightly smaller than the normal case) it is the worst when the total variance is taken into account, probably due to the uncertainty on the cutting point estimation.

² The model of current in°ation when estimated with the trim method presents reasonable goodness of ¯tting variance, showing how strongly it can smooth the data, as can be seen in table 6. Nevertheless, the best performance from this point of view was obtained by the normal transitory model (T).

Table 6: Goodness of Fitting Variance (Gv) (100 £)

In°uence	P	T	C	TS
Mn	0.58	0.31	7.67	0.30
t-St	0.81	0.29	2.83	0.32
Mt-St	0.80	0.37	6.84	0.39
Trim	0.55	0.40	1.03	0.42
TrimA	0.45	0.20	0.98	0.27

All the above numerical estimates are based on the MCMC output. A few hundred iterations seemed enough for the estimates to reach reasonable stability. The convergence was access graphically and also via the Geweke criterion. Actually, after convergence the remaining 1000 iterations are used in the estimations. In table 7, the 95% posterior probability intervals were presented for the hyperparameters. The full empirical posterior distribution for the discount factor (f) and for the other parameters of Normal Model are presented in ¯gure 4.

Table 7: Posterior Density Interval for transitory Normal Model

Influence Parameter	P_{05}	Mn Mean	P_{95}	Corr f	a	b
Discount factor (f)	0.60	0.75	0.88	1.00	-0.48	0.44
Transient Cons (a)	0.33	0.68	0.96	-0.48	1.00	-.22
AR(1) coeff. (b)	0.91	0.95	0.99	0.44	-0.22	1.00

Table 8: Posterior Density Interval for transitory Trim Models

Influence Parameter	P_{05}	Mn Mean	P_{95}	Corr f	a	b	p
Discount factor (f)	0.51	0.60	0.72	1.00	-.35	0.41	0.27
Transient Cons (a)	0.58	0.74	0.93	-.35	1.00	0.07	0.10
AR(1) coeff. (b)	0.90	0.92	0.95	0.41	0.07	1.00	0.15
Trim percentile(p)	0.08	0.10	0.14	0.27	0.10	0.15	1.00

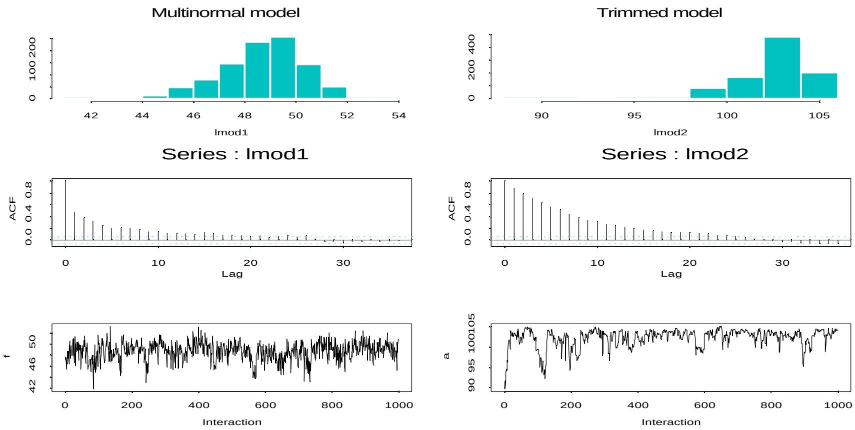
The conditional distribution of the common trend $(\mu_t|\theta;\Omega_t)$ and of the future inflation trend $(\pi_t^h|\theta,\Omega_t)$ can be obtained from the robust sequential filter. Since our main interest is in the marginal distribution for $(\mu_t|\Omega_t)$ and $(\pi_t^h|\Omega_t)$ the hyperparameters θ must be eliminated. The integral involved can be solved numerically using the empirical distribution of the hyperparameters got from the MCMC iterations after the elimination of some initial values. The mean and variance of the above marginal distributions are given by:

$$E[\mu_t|\Omega_t] = \sum_r \{E[\mu_t|\theta^r;\Omega_t]\}/R \quad \text{and} \quad E[\pi_t^h|\Omega_t] = \sum_r \{E(\pi_t^h|\theta^r;\Omega_T)\}/R$$

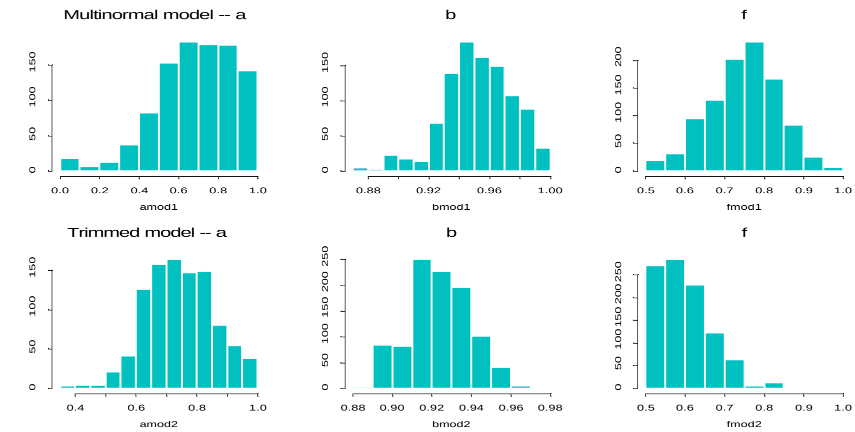
$$\begin{aligned} V[(\mu_t|\Omega_T] &= \sum \{V[\mu_t|\theta^r;\Omega_T) + (E[\mu_t|\theta^r;\Omega_T] - E[\mu_t|\Omega_T])^2\}R \\ V[(\pi_t^h|\Omega_T] &= \sum_r \{V[\pi_t^h|\theta^r;\Omega_T) + (E[\pi_t^h|\theta^r;\Omega_T] - E[\pi_t^h|\Omega_T])^2\}R \end{aligned}$$

Figure 2: Model Assessment and Hyperparameters Posterior Distribution

(a) Marginal Predictive Likelihood, Autocorrelation Function and Trace

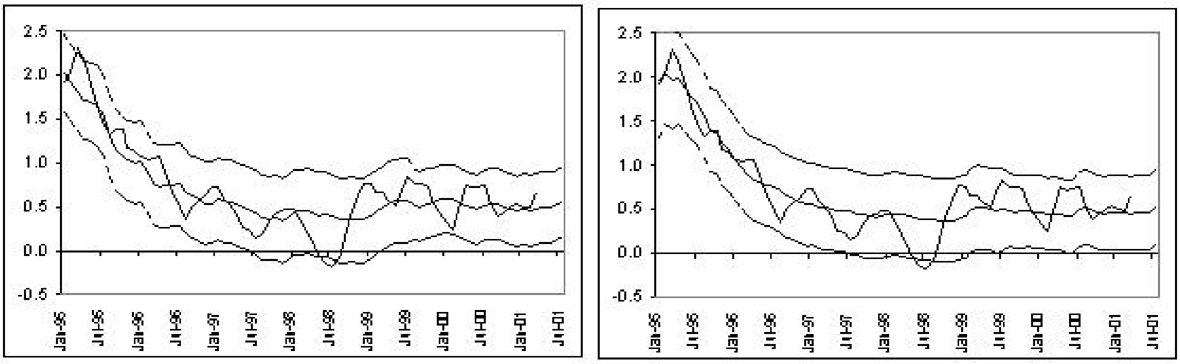


(b) Posterior Distribution for the AR Coe±cients and the Discount Factor



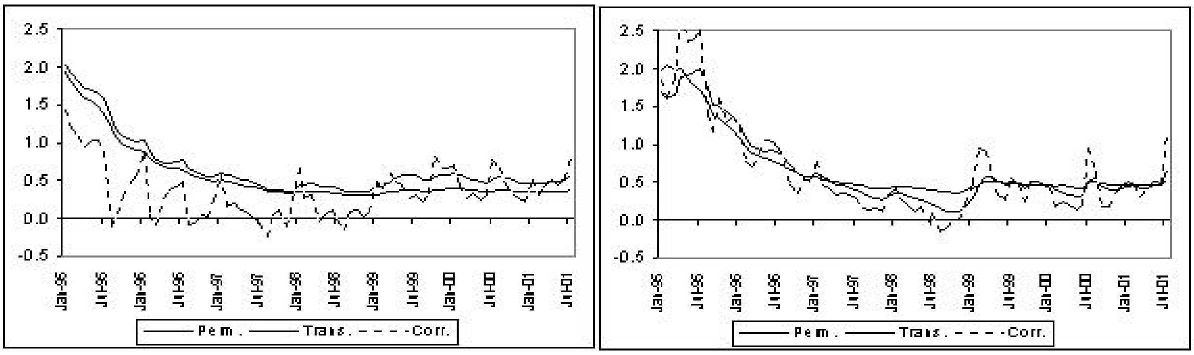
In the following graphics we can appreciate the performance of the models developed. In ¯gure 2, the observed values of $!_t^h$, the IPCA trend, and its h = 4 months ahead forecast - $E[!_t^h j -_t]$ - can be observed for the the multivariate normal model and also for the asymmetric trimmed model. Both ¯gures include the Bayesian 95% probability intervals. One point to stress is that the probability interval width do not increase around October 1999, a well known period of high volatility in the economy. The con¯dence intervals obtained with the trimmed models besides to be very narrow have an almost constant width. The ¯rst comment must be due to the spurious uncertainty elimination involved in the trimmed process and the second is related to the hypothesis of constant variance associated with this class of model.

Figure 3: IPCA trend and $E(!_t^h)$ for Models Mn and Trim



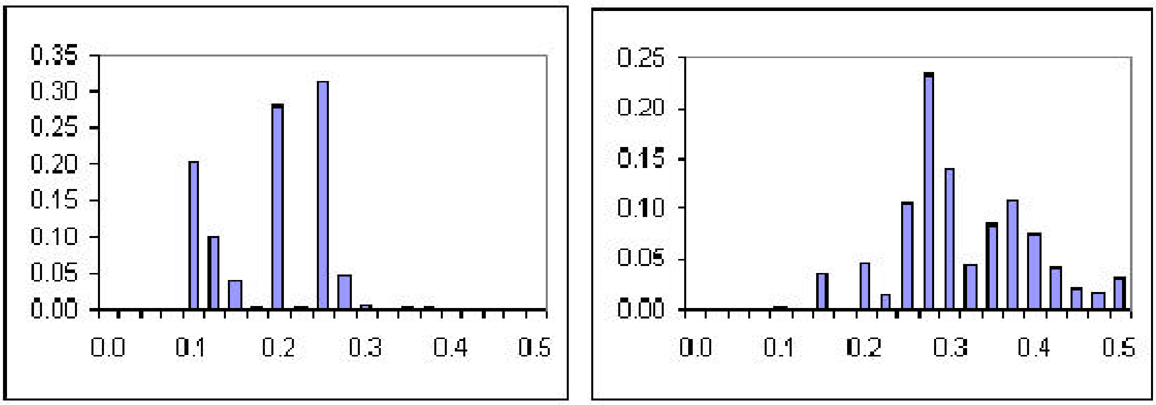
In ¯gure 3, only the four months ahead point forecast are shown for the multivariate normal and the asymmetric trimmed mean models. The three lines represents, respectively, the point forecast obtained with the transitory in°ation component model (T), the permanent component model (P) and the current in°ation model (C). The models P and T have a similar behavior, but the current in°ation is not useful for forecasting as far as it is not smooth enough.

Figure 4: $E(\omega_t^h)$ for Normal Models T, P, C and Trim Models



The posterior marginal distribution of the parameter involved in the symmetric trimmed mean model [7] - the cutting percentile - is presented in figure 5 and shows how difficult it is to estimate this quantity. In the model C - current inflation - this distribution has multiple modes. Therefore, to cut at the 10% percentile is as good as to cut at 20% percentile or even at 30%.

Figure 5: Distribution of the Cutting Point - Symmetric Case (C) e (P)



[7] Estimated via a grid search over the sample space in Fiorêncio and Moreira (2000)

5 Concluding Remarks and Extentions

There is a huge literature discussing alternative approaches to the measurement of core in°ation, including various trimmed mean models (Cecchetti and others) and smoothing techniques introduced by Cogley (1998). In this paper we have introduced a large class of models which contemplate as special cases the former measurement approaches as well as the dynamic factor index model proposed by Bryan and Cecchetti (1993) and Cecchetti (1997).

The trend in°ation rate is de¯ned as the moving average of the future headline in°ation rate, a slight variation on the Cecchetti's de¯nition. The model proposed to forecast this quantity is composed by a common trend component, a deterministic seasonal factor and idiosyncratic shocks. The common trend dynamics is described as an autoregressive not excluding the possibility of the mean reversion. A more fundamental advantage of the proposed model is that it allows the idiosyncratic socks to be modeled by a general class of multivariate probability distribution. The components of the price index are taken as endogenous variables and their uncertain jointly modeled. It is worth pointing out that in this model the common trend in°ation and the mean of the future in°ation play di®erent games. The former is, in same sense, a measurement of the current trend in°ation while the latter is predictive in nature.

After state a so broad class of models it is natural to ask what were the advantages obtained. Then, some ¯nal words are in order:

- The seasonal factor: the introduction of the seasonal factors do not improve the forecasting capability of the model.

- Non-parametric model: the cutting point is a central quantity to apply the trimmed means models. Nevertheless it estimation is, often, unstable. In the asymmetric trimmed means the posterior obtained from the MCMC output is multi modal. All those comments suggest that the parametric in°uence function models have a good chance of succeeding.

- The form of the idiosyncratic socks distribution: the multivariate normal models are better in many aspects and do not depend on approximations like that involved in the robust Kalman ¯lter. The comparative study developed in this paper permits to conclude that for the IPCA, in the period from 09/94 up to 05/01, there is no space for models with heavy tails or even for trimmed means models. It is worth paying attention to the fact that the components of the index have di®erent volatility factors estimated from the data. The ¯tted t-Student models are very close to Normal models since the estimated degrees of freedom are so big.

² Common component dynamic: the dynamic evolution of the common component does not include an unit root. Although the transitory model has the best expected log-likelihood its total variation is bigger than the permanent model, showing the limitation of this speci¯cation.

Finally, we pretend to validate the core in°ation measures obtained considering the di®erent criterion available in the literature.

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Appendix:

In this Section a brief summary of some methodological aspects will be present. The mean an variance describing all the distributions involved in the Multivariate Normal Dynamic Linear (DLM) models, the efficient MCMC in normal DLM proposed by Frühwirth-Schnatter (1994) and some simplifications associated with the common component model.

A1 - Multivariate Common Component Dynamic Model

$$\begin{aligned} \mathbf{y}_t &= \psi\mathbf{D}_t + \mathbf{F}\mu_t + \mathbf{e}_t \\ \mu_t &= a + b(\mu_{t-1} - a) + w_t \end{aligned}$$

where: $\mathbf{e}_t \sim p(\beta)$ and $w_t \sim N[0;\ b^2(\frac{1}{f} - 1)V[\mu_{t-1}|t-1)]$.

Denoting by

- $m_0;\ C_0$, respectively the mean and variance of the posterior distribution at time $t = 0$,
- $y_t;\quad n \times l$: the price changes for the products involved in the headline inflation,
- $\mathbf{D}$: $n \times 12$ matrix of seasonal components, μ_t : 1×1 common trend component,
- e_t : $n \times 1$: idiosyncratic error term, $F = (1; 1; \ldots 1)'$: a vector of unitary constants.

Prior Distribution

$$\begin{aligned} E[\mu_t|_{t-1}] &= a + b[E[\mu_{t-1}|_{t-1}] - a] \\ V[\mu_t|_{t-1}] &= b^2 V[\mu_{t-1}|_{t-1}]/f \end{aligned}$$

Predictive Distribution

$$\begin{aligned} E[y_t|_{t-1}] &= \mathbf{F}E[\mu_t|_{t-1}] + \psi D_t \\ V[y_t|_{t-1}] &= V[\mu_t|_{t-1}] + \mathbf{V} = Q_t \end{aligned}$$

As soon as the data vector $\mathbf{y}_t$ is observed, the posterior distribution can be evaluated, with mean and variance given by: Posterior Distribution

$$m_t = E[\mu_t|_t] = E[\mu_t|_{t-1}] + V[\mu_t|_{t-1}]\mathbf{F}'\mathbf{Q}^{-1}\hat{e}_t$$

$$c_t = V[\mu_t|_t] \;=\; V[\mu_t|_{t-1}](1 - V[\mu_t|_{t-1}])\mathbf{F}'\mathbf{Q}^{-1}\mathbf{F} \tag{6}$$

where: $\hat{\mathbf{e}}_t = \mathbf{y}_t - \mathbf{F}'E[\mu_t|_{t-1}] - \boldsymbol{\phi}\mathbf{D}_t$ and $\mathbf{Q}_t = V + \mathbf{F}V[\mu_t|_{t-1}]\mathbf{F}'$.

A2 - The efficient MCMC in normal DLM :

The filter proposed by Frühwirth-Schnatter (1994) is given by:

i) Let $h_{t+1} = m_T$ and $H_{t+1} = C_T$

ii) Sample $x_{t+1} \sim N[h_{t+1}; H_{t+1}]$, where x_{t+1} denote the state at time $t+1$.

iii) Obtain:

$$\begin{aligned} h_t &= \{I - B_{t+1}G\}m_t + B_{t+1}x_{t+1} \\ H_t &= \{I - B_{t+1}G\}C_t \end{aligned} \tag{7}$$

where: $B_{t+1} = C_tG'\{GC_tG' + W)^{-1}\}$

iv) Let $t = t - 1$ and reapit (i) to (iii) till $t = 0$

Since $B_{t+1} = C_t b(b^2C_t/\delta)^{-1} = \delta/b$ the above equations simplify, for the model in this paper, to:

$$h_t = (1-\delta)m_t + (\delta/b)x_{t+1} \quad \text{and} \quad H_t = (1-\delta)C_t$$

A3 – Predictive Variance

Let $Z_t^h = \sum_{i=1}^h g'_{t+i}z_{t+i}$, where $z_t = y_t - \phi D_t = \mathbf{F}'\mu_t + e_t$. Then the predictive variance will be:

$$\begin{aligned} V[Z_t^h|_t;\Theta] &= \sum_{i=1}^{h} V[g'_{t+i}z_{t+i}|_t;\Theta] + 2\sum_{i<j} COV[(g'_t z_{t+i}; g'_t z_{t+j}|_t;\Theta] \\ &= \sum_{i=1}^{h} V[g'_{t+i}z_{t+i}|_t;\Theta] + 2\sum_{i<j} b^{j-i}V[g'_{t+i}z_{t+i}|_t;\Theta] \end{aligned}$$

Assuming conditional independence between the common component (μ_t) and the seasonal factor (ϕ), given the observed data and Θ, it follows $V[g'_t\phi|\psi] = g'_tV[\phi|\psi]g_t = g'_t(D'D)^{-1}V g_t = (D'D)^{-1}\sum_i v_i g_{it}^2$. Then we obtain

$$V[\omega_t^h|\psi) = V[Z_t^h|\psi] + D'_tV[g'_t\phi|\psi]D_t \tag{8}$$

A4 – Efficient Calculation of the Posterior Mean and Variance

The posterior mean and variance (6) depend upon the factors $\mathbf{F}\mathbf{Q}^{-1}\hat{e}_t$ and $\mathbf{F}\mathbf{Q}^{-1}\mathbf{F}'$. From the definition of $\mathbf{F}$ and remembering that $\mathbf{Q}_t = \mathbf{F}'R_t\mathbf{F} + \mathbf{V}$, then using a well known result in matrix theory it follows:

$$\mathbf{Q}_t^{-1} = V^{-1} - V^{-1}\mathbf{F}[\mathbf{F}'V^{-1}\mathbf{F} + R_t^{-1}]^{-1}\mathbf{F}'V^{-1}$$

Since: $\mathbf{F}'V^{-1}\mathbf{F} = \sum_{i=1}^{n} v_i^{-1}$ and $\mathbf{q} = V^{-1}\mathbf{F}\mathbf{F}'V^{-1} = (v_i^{-1}v_j^{-1})$, then it follows

$$\mathbf{Q}_t^{-1} = V^{-1} - \mathbf{q}\delta_t; \quad \text{where } \delta_t = \left(\sum v_i^{-1} + r_t^{-1}\right)^{-1}; \ \text{with } r_t = V[\mu_t|_{t-1}]$$

Denoting $\hat{e}_t = y_t - \phi D - \mathbf{F}'m_{t-1}$ and remembering that in the normal case $g(\hat{e}_t) = \mathbf{Q}_t^{-1}\hat{e}_t$ and $G(\hat{e}_t) = \mathbf{Q}_t^{-1}$, it follows:

$$\begin{aligned} \mathbf{F}'g(\hat{e}_t) = \mathbf{F}'\mathbf{Q}_t^{-1}\hat{e}_t &= \sum \hat{e}_{i;t}[1 - \delta_t \sum v_i^{-1}] \\ \mathbf{F}'G(\hat{e}_t)\mathbf{F} = \mathbf{F}'\mathbf{Q}_t^{-1}\mathbf{F} &= \sum v_i^{-1}[1 - \delta_t \sum v_i^{-1}] \end{aligned} \qquad (9)$$

In the t-Student case similar calculation provides:

$$\begin{aligned} \mathbf{F}'g(\hat{e}_t) &= K \sum_i \hat{e}_{i;t}[1 - \delta_t \sum_i v_i^{-1}] \\ \mathbf{F}'G(\hat{e}_t)\mathbf{F} &= K \left[\sum_i v_i^{-1} - 2\left(\sum_i \hat{e}_{i;t}\right)^2 \left(\frac{1 - \delta_t \sum_i v_i^{-1}}{\nu + \sum_i \hat{e}_{i;t}v_i^{-1}}\right)\right] \end{aligned} \qquad (10)$$

where: $K = \frac{\nu + n}{\nu + [\sum_i \hat{e}_{i;t}v_i]^{-1}}$.

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