THE NAIRU, UNEMPLOYMENT AND THE RATE OF INFLATION IN BRAZIL

Elcyon Caiado Rocha Lima
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DISCUSSION PAPER

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RESUMO

Neste artigo estimamos a Non-Accelerating Inflation Rate of Unemployment (Nairu) do Brasil e investigamos diversas questões empíricas: o comportamento da Nairu ao longo do tempo, intervalos de confiança para a Nairu e sua utilidade na condução da política monetária no Brasil.


O artigo apresenta novas evidências empíricas que permitem responder a diversas indagações teóricas. Ele mostra que a Nairu tem crescido desde 1995 e conclui que existe uma correlação significativa e com sinal correto entre desvios da taxa de desemprego em relação à Nairu e à taxa de inflação. Conclui-se também que as estimativas da Nairu são muito pouco úteis na condução da política monetária já que os seus intervalos de confiança são demasiadamente amplos.
This paper estimates the Brazilian Nairu (Non-Accelerating Inflation Rate of Unemployment) and investigates several empirical questions: the behavior of Nairu along time, error bands for Nairu and the usefulness of Nairu to the conduct of monetary policy in Brazil.

There are many recent studies about the Nairu — Staiger, Stock and Watson (1997), Blanchard and Katz (1997) and Portugal, Madalozzo and Hillbrecht (1999). This article innovates with respect to previous ones because it adopts an econometric model that, in our judgment, is more adequate to deal with the still recent instability of Brazilian economy. We estimate two different state-space models: one with ARCH residuals and another with a Markov-switching regime.

The article presents some new evidence on several questions. It shows that the Nairu has been increasing since 1995. It concludes that there is a statistically significant relationship, with correct sign, between deviations of unemployment from the Nairu and inflation. It also shows that the usefulness of the Nairu to the conduct of monetary policy is very limited because its error bands are too wide.
1 - INTRODUCTION

In this article we estimate the Non- Accelerating Inflation Rate of Unemployment (Nairu) for Brazil. We investigate the relationship between the rate of inflation and the deviation, from Nairu, of the observed rate of unemployment. We intend to determine to what degree the observed rate of unemployment, which summarizes what happens in the labor market, can be used to forecast the rate of inflation and to what extent the Central Bank should observe the rate of unemployment before taking decisions about how to conduct monetary policy (i.e., fixing the short-run interest rate, controlling domestic credit, establishing targets for monetary aggregates, etc.).

There are many recent studies about the Nairu, such as Staiger, Stock and Watson (1997), Blanchard and Katz (1997) for the United States and Portugal, Madalozzo and Hillbrecht (1999) for Brazil. Our research innovates when compared to others by adopting an econometric procedure that, in our view, is more adequate to deal with structural breaks in the model's equation. We also estimate error bands for the Brazilian Nairu.

Estimating the relationship between nominal and real variables in Brazil is a very hard task due to the instability experienced by the Brazilian economy in its recent past. Macroeconomic models are estimated using information provided by time series data and the number of required observations outnumber what is available for the most recent period of stability, after the Real Plan. This is a challenge for anyone who works with Brazilian macroeconomic data. Fortunately, since the beginning of the 90's there has been a variety of theoretical developments in time series analysis that enable us to extract information from data even when instabilities, like the ones observed for Brazilian data, are present [Kim and Nelson (1999)]. These developments allow us to deal with a larger amount of data. It is also true that the greater stability shown by the Brazilian economy since the Real Plan at the middle of 1994, despite the recent change in the exchange rate policy, gives greater confidence that the investment in econometric models is going to payoff in the future.

Two different models are used to calculate the natural rate of unemployment and its confidence interval: a state-space model with ARCH residuals (TVP model) and a state-space model with a Markov-switching regime (MSR model). A detailed description of both models and of the estimation methods can be found in Kim and Nelson (1999). The models are estimated using quarterly data for the average rate of open unemployment and for the rate of inflation measured by the national consumer price index (INPC) both collected by IBGE, respectively, in six and eleven Metropolitan Regions of Brazil, for the period 1982:1/1999:3.

The article is organized as follows: in Section 2 we describe the data used, how the monthly data were transformed into quarterly data and the estimated models; in Section 3 we present the estimation procedures adopted; in Section 4 we show the estimation results and discuss some statistical tests; in Section 5 we conclude.
2 - DATA AND ESTIMATED MODELS

The basic data comprise the national consumer price index (INPC) of IBGE from 1981:12 to 1999:9 and the average rate of open unemployment of IBGE from 1982:1 to 1999:9. These monthly data were averaged into quarterly data and the model was estimated using only quarterly data, from the first quarter of 1982 to the third quarter of 1999. The monthly data were transformed into quarterly data as follows:

\[ \pi_t = \frac{1}{3} \log \left( \frac{P_{t,f}}{P_{t-1,f}} \right) = \text{monthly geometric average of the quarterly rate of inflation}; \]

\[ P_{t,f} = \text{centered INPC (national consumer price index) of IBGE for the last month of quarter } t; \]

\[ U_t = \text{quarterly average of the monthly average rate of open unemployment of IBGE}. \]

2.1 - The Basic Model

Our basic model, for the relationship between the change of the rate of inflation and the rate of unemployment, can be represented by the following equations:

\[ \Delta \pi_t = \mu_t + \sum_{s=1}^{3} \beta_s (U_{t-s} - \bar{u}) + Z_t \gamma + \varepsilon_t, \quad (1) \]

and

\[ \sum_{s=0}^{3} \mu_{t+s} = 0 \quad (\text{restriction that allows for the identification of } \bar{u}) \quad (2) \]

where:

\[ \mu_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} \quad (3) \]

\[ D_{it} = \text{seasonal dummy of quarter } i; \]

\[ \bar{u} = \text{Nairu}; \]

\[ Z_t = \text{is a row vector of control variables with the two first lags of } \Delta \pi_t; \]

\[ \varepsilon_t \sim N(0, \sigma^2) \]

In this version the Nairu is an unknown parameter. The model, with this representation, is not linear in its parameters. Nevertheless, the above equation can be presented in a different form that makes it linear in its parameters:

\[ \Delta \pi_t = \beta_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \sum_{s=1}^{3} \beta_s u_{t-s} + Z_t \gamma + \varepsilon_t, \quad (4) \]

where:
\( \beta_0 = \alpha_0 - \sum_{s=1}^{3} \beta_s u \) \hspace{1cm} (5)

Estimating the above model and considering the hypothesis that \( \sum_{s=0}^{3} \mu_{t+s} = 0 \), we obtain an estimate of \( u^- \) and \( \alpha_0 \). We explain below how to compute this estimate:

\[
\sum_{s=0}^{3} \mu_{t+s} = 0 \quad \Rightarrow \quad \sum_{s=0}^{3} \left( \beta_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} \right) = -4 \cdot \sum_{s=1}^{3} \beta_s u
\]

therefore,

\[
\bar{u} = - \left( \beta_0 + \alpha_1/4 + \alpha_2/4 + \alpha_3/4 \right) / \left( \sum_{s=1}^{3} \beta_s \right) \hspace{1cm} (6)
\]

The inflation stabilization plans adopted by Brazil [Cruzado (1986:1 and 1986:2), Bresser (1987:3), Verão (1989:1), Collor (1990:2) and Real (1994:3)] have produced, in the quarters of their implementation, an abrupt reduction of the rate of inflation. To deal with these shocks, interventions were made in the model at each quarter of implementation of each stabilization plan. The basic model with interventions at each stabilization plan is described by the following equation:

\[ \Delta \pi_t = \beta_0 (1 + \theta_{t}) + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \sum_{s=1}^{3} \beta_s u_{t-s} + Z_t \gamma + \varepsilon_t, \] 

\( \theta_{t} = 0, \) if there was no stabilization plan at quarter \( t; \)

\( \theta_{t} = \theta_{t} = \) nonlinear intercept intervention parameters when stabilization plan "\( \tau \)" happens at period \( t \) [\( \tau = 1 \) (Cruzado Plan), \( \tau = 2 \) (Bresser and Verão Plans), \( \tau = 3 \) (Collor and Real Plans)]

To simplify the description of the model, while changing notation to allow for time varying parameters, we summarize the representation of the model as follows:

Let,

\[
y_t = \Delta \pi_t ; \quad x_t = [(1 + \theta_{t}) D_{1t} D_{2t} D_{3t} U_{t-1} U_{t-2} U_{t-3} Z_t] , \quad \beta^* = [\beta_0 \alpha_1, \alpha_2, \alpha_3, \beta_{1t}, \beta_{2t}, \beta_{3t}, \gamma_t],
\]

therefore the model can be represented, in a compact form, by

\[
y_t = x_t \beta^* + \varepsilon_t \] 

\( \beta^* \)
The Brazilian economy, in its recent past, has experienced a period of great economic instability and the above equation may show structural breaks in its parameters. To deal with possible structural breaks we estimate two different versions of the above model: \( a \) the TVP model (that allows for parameter change over time and ARCH residuals); and \( b \) the MSR model (that allows for parameter change over time and Markov-switching regimes).

### 2.2 - The TVP and the MSR Models

#### The TVP Model

The TVP model is a state-space model with ARCH residuals and is a simplified version of the model proposed by Harvey, Ruiz and Sentana (1992):

\[
y_t = x_t \beta_t^* + \varepsilon_t^* + \varepsilon_t, \quad \text{measurement equation,} \tag{9}
\]

\[
\beta_t^* = \beta_{t-1}^* + \omega_t, \quad \text{transition equation,} \tag{10}
\]

\[
\varepsilon_t \sim N(0, \sigma^2), \quad \omega_t \sim N(0, Q) \quad \text{and} \quad \varepsilon_t^{*}/\psi_t \sim N(0, h_{1t}).
\]

Where: \( \Lambda, \varepsilon_t^{*} \) and \( \sigma \) are scalars and \( Q \) is, by hypothesis, diagonal and \( 9 \times 9 \). The ARCH effect is introduced through the scalar residual \( \varepsilon_t^{*} \). The variance of \( \varepsilon_t^{*} \) is given by:

\[
h_{1t} = 1 + \gamma_0 \varepsilon_{t-1}^{*2} \tag{11}
\]

#### The MSR Model

The MSR model is a state-space model with a Markov-switching regime. It is a simplified version of the model suggested by Kim and Nelson (1999):

\[
y_t = x_t \beta_t^* + \varepsilon_t, \quad \text{measurement equation;} \tag{12}
\]

\[
\beta_t^* = \beta_{t-1}^* + \omega_t, \quad \text{transition equation;} \tag{13}
\]

\[
\varepsilon_t \sim N(0, \sigma_{St}^2), \quad \omega_t \sim N(0, Q_{St})
\]

The subscript \( St \) denotes that \( \sigma^2 \) and the parameters at the diagonal of matrix \( Q \) take values that depend on a discrete, non-observed variable, that follows a Markov-switching process with two different states (regimes). The transition probabilities are given by:

\[
P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}
\]

\[
(14)
\]
where, $p_{ij}$ = probability of state $J$, at period $t$, given state $i$ at period $t-1$ and

$$\sum_{j=1}^{2} p_{ij} = 1, \quad i = 1, 2$$ \hfill (15)

**3 - ESTIMATION PROCEDURES**

**3.1 - The TVP Model Estimation Procedure**

Harvey, Ruiz and Sentana (1992) substitute the $\varepsilon_{t-1}^{*,2}$ variable in equation (11), which is non-observed, by its conditional expectation, $h_{1t} = 1 + \gamma E [\varepsilon_{t-1}^{*,2}/\psi_{t-1}]$. Therefore, the algorithm is an approximation. To get $E [\varepsilon_{t-1}^{*,2}/\psi_{t-1}]$, Harvey, Ruiz and Sentana augmented the original state vector, in the transition equation (10), in the following way:

$$\begin{bmatrix} \beta_t^* \\ \varepsilon_t^* \end{bmatrix} = \begin{bmatrix} I_9 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta_{t-1}^* \\ \varepsilon_{t-1}^* \end{bmatrix} + \begin{bmatrix} I_9 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_t \\ \varepsilon_t^* \end{bmatrix}$$

where $I_9$ is the identity matrix with 9x9 dimension. Therefore, the measurement equation (9) is replaced by:

$$y_t = [x_t \Lambda] \begin{bmatrix} \beta_t^* \\ \varepsilon_t^* \end{bmatrix} + \varepsilon_t$$

It is not hard to show that,

$$E [\varepsilon_{t-1}^{*,2}/\psi_{t-1}] = E [\varepsilon_{t-1}^{*,2}/\psi_{t-1}]^2 + E [(\varepsilon_{t-1}^{*,2} - E(\varepsilon_{t-1}^{*,2}/\psi_{t-1}))^2]$$

The two expectations at the right hand side of the last equation can be computed, recursively, using the Kalman filter. The first expectation is equal to the recursive estimation of the second element of the state vector ($\varepsilon_{t-1}^{*,2}$) squared and the second expectation is equal to its recursively estimated covariance matrix.

Given the intercept intervention parameters $\theta_\tau$ [\tau = 1, 2 and 3], $\sigma$, $\Lambda$, $\gamma_0$ and $Q$ it is possible to obtain the model's likelihood value and $\beta_t^*$ through the Kalman Filter recursions. That is, we can concentrate the likelihood with respect to $\beta_t^*$, using the Kalman filter and, with the help of a numerical optimization routine, estimate the value of the other parameters [ $\theta_\tau$ (\tau = 1, 2 and 3), $\sigma$, $\Lambda$, $\gamma_0$ and the elements at the diagonal of matrix $Q$] that maximize the likelihood.

To arrive at a more parsimonious model the following restrictions were imposed a priori:
Q(5,5) = Q(6,6) = Q(7,7), equality between parameters, at the diagonal of matrix Q, that controls for the time change of lag unemployment coefficients.

Q(8,8) = Q(9,9), equality between parameters, at the diagonal of matrix Q, that controls for the time change of lag Δπ, coefficients.

With these restrictions matrix Q has only four unknown parameters. Therefore the TVP model has, if we exclude the parameters of β* from the counting, a total of 10 parameters to be estimated. In the next section we describe how, departing from this more general model and using a few statistical tests, we are able to reduce, from 10 to 5, the number of parameters to be estimated.

3.2 - The MSR Model Estimation Procedure

In order to estimate the MSR model it is necessary to make inferences about both the unobserved states and the latent Markov state. The regime shifts of the MSR model imposes a nonlinearity that precluded the estimation of this type of model until the estimation methods by Albert and Chib(1993), Shephard (1994) and Kim (1994) were developed. I use Kim's method to estimate the MSR model. The Kim filter is an optimal estimator in the sense that no other estimator, based on a linear function of the information set, yields a smaller mean squared error [Smith and Makov (1980)].

Let,

\[ I_{t-1} = [y_{t-1}, y_{t-2}, ..., y_1], \] information available up to time \( t-1 \);

\[ \beta_{y^{(i,j)}_{t-1}} = \mathbb{E}(\beta_t^* / I_{t-1}, S_t = j, S_{t-1} = i); \]

\[ \sum^{(i,j)}_{y_{t-1}} = \mathbb{E}[(\beta_t^* - \beta_{t-1}^*)(\beta_t^* - \beta_{t-1}^*)' / I_{t-1}, S_t = j, S_{t-1} = i]; \]

\[ \beta_{y^{(i)}_{t-1}} = \mathbb{E}(\beta_t^* / I_{t-1}, S_{t-1} = i); \] and

\[ \sum^{(i)}_{y_{t-1}} = \mathbb{E}[(\beta_{t-1}^* - \beta_{t-1}^*)^2 / I_{t-1}, S_{t-1} = i]. \]

\[ Pr[S_t = j, S_{t-1} = i] = p_{ij} \]

\[ P = [p_{ij}] = \text{matrix of transition probabilities}; \]

\[ Pr[S_t = i \mid I_t] = \text{probability of state } i, \text{ at period } t, \text{ given information up to time } t. \]

The objective of Kim’s nonlinear filter is to obtain estimates of the unobserved state vector (β∗) and its associated mean squared error matrix (Σ), and of the marginal probability of the latent Markov state variable St, at each date t. The
estimates are based on information available up to time \( t \). That is, at each date \( t \), given \( \beta^{(i)}_{t-1} \), \( \Sigma^{(i)}_{t-1} \), \( Pr[S_{t-1} = i \mid I_{t-1}] \), \( y_t \), \( Q_i \), \( \sigma_j \) (for \( i = 1, 2 \) and \( j = 1, 2 \)), \( \theta \) (\( \tau = 1, 2 \) and 3) and the matrix of transition probabilities (\( P \)) Kim's filter allows us to obtain approximate estimations of \( \beta^{(i)}_t \), \( \Sigma^{(i)}_t \), and \( Pr[S_t = j \mid I_{t}] \) \( (j = 1, 2) \). To start the filter we need the initial values of \( \beta^{(i)}_0 \), \( \Sigma^{(i)}_0 \) and \( Pr[S_0 = i \mid I_0] \) \( (i = 1, 2) \). We set \( \beta^{(i)}_0 = 0 \), \( \Sigma^{(i)}_0 = I \times 100.000 \) and \( Pr[S_0 = I / I_0] \) \( (i = 1, 2) \) equal to the ergodic distribution of the Markov chain (the steady-state marginal distribution of the states).

What we have called Kim's filter is actually a nonlinear version of the Kalman filter combined with Hamilton's nonlinear filter. We describe both filters below.\(^1\)

### The Nonlinear Kalman Filter – Part I

Given \( \beta^{(i)}_{t-1} \), \( \Sigma^{(i)}_{t-1} \), \( y_t \), \( Q_i \), \( \sigma_j \) (for \( i = 1, 2 \) and \( \theta \tau \) \( \tau = 1, 2, 3 \)), the nonlinear Kalman Filter, in its first part, obtains \( \beta^{(i,j)}_t \), \( \Sigma^{(i,j)}_t \), the conditional one step prediction error (\( \eta^{(i,j)}_t \)), and the conditional variance of the one step prediction error, \( f^{(i,j)}_t \) \( (i = 1, 2 \) and \( j = 1, 2) \). The algorithm estimates two (number of different possible regimes) state vectors for each possible value of \( S_{t-1} \). Therefore, at each date \( t \), the algorithm estimates four state vectors. The first part of Kalman filter recursions are presented below:

#### Prediction equations:

\[
\beta^{(i)}_{t|t-1} = \beta^{(i)}_{t-1},
\]

\[
\Sigma^{(i)}_{t|t-1} = \Sigma^{(i)}_{t-1} + Q_i
\]

#### Updating equations:

\[
\eta^{(i,j)}_{t|t-1} = y_t - \beta^{(i,j)}_{t|t-1}
\]

\[
f^{(i,j)}_{t|t-1} = x_t \Sigma^{(i,j)}_{t|t-1} x_t^\prime + \sigma_j^2
\]

\[
\beta^{(i,j)}_{t|t} = \beta^{(i,j)}_{t|t-1} + \Sigma^{(i,j)}_{t|t-1} x_t^\prime \left[ f^{(i,j)}_{t|t-1} \right]^{-1} \eta^{(i,j)}_{t|t-1}
\]

\[
\Sigma^{(i,j)}_{t|t} = (I - \Sigma^{(i,j)}_{t|t-1} x_t^\prime \left[ f^{(i,j)}_{t|t-1} \right]^{-1} x_t) \Sigma^{(i,j)}_{t|t-1}
\]

\(^{1}\) A detailed discussion of the Kim’s filter can be found in Kim (1994) and Kim and Nelson (1999).
The Hamilton Filter

Given $Pr[S_{t-1} = i \mid I_{t-1}]$, $P$, $\eta_{t|t-1}^{(i,j)}$, $f_{t|t-1}^{(i,j)}$, the Hamilton Filter obtains $Pr[S_{t} = i \mid I_{t}]$ and $Pr[S_{t} = j \mid I_{t}]$. The Hamilton Filter is presented below:

$$Pr[S_{t} = j, S_{t-1} = i \mid I_{t-1}] = Pr[S_{t} = j \mid S_{t-1} = i]Pr[S_{t-1} = i \mid I_{t-1}], \quad (i, j = 1, 2)$$

$$Pr[S_{t-1} = i, S_{t} = j \mid I_{t}] = \frac{f(y_{t} \mid S_{t-1} = i, S_{t} = j, I_{t-1}) Pr[S_{t} = j, S_{t-1} = i \mid I_{t-1}]}{f(y_{t} \mid I_{t-1})}$$

$$Pr[S_{t} = j \mid I_{t}] = \sum_{i=1}^{2} Pr[S_{t-1} = i, S_{t} = j \mid I_{t}]$$

Where,

$$f(y_{t} \mid S_{t-1} = i, S_{t} = j, I_{t-1}) = (2\pi)^{-\frac{1}{2}} f_{t|t-1}^{(i,j)} \left| \frac{1}{2} \exp\left(-\frac{1}{2} \eta_{t|t-1}^{(i,j)} f_{t|t-1}^{(i,j)-1} \eta_{t|t-1}^{(i,j)} \right) \right| (i, j = 1, 2)$$

and

$$f(y_{t} \mid I_{t-1}) = \sum_{i=1}^{2} \sum_{j=1}^{2} f(y_{t} \mid S_{t} = j, S_{t-1} = i, I_{t-1}) Pr[S_{t} = j, S_{t-1} = i \mid I_{t-1}]$$

The Nonlinear Kalman Filter – Part II

Given $\beta_{t|t}^{(i,j)}$, $\Sigma_{t|t}^{(i,j)}$ (both estimated by the first part of the nonlinear Kalman filter) and $Pr[S_{t} = i, S_{t} = j \mid I_{t}]$, $Pr[S_{t} = i \mid I_{t}]$ (both estimated by the Hamilton filter) the nonlinear Kalman filter, in its second part, collapses $\beta_{t|t}^{(i,j)}$ into $\beta_{t|t}^{(i,j)}$ and $\Sigma_{t|t}^{(i,j)}$ into $\Sigma_{t|t}^{(j)}$. The collapsing is of crucial importance because without it the number of filter evaluations is multiplied by 2, at each date $t$, and it is computationally unfeasible to estimate the model. The collapsing is an approximation based on the work of Harrison and Stevens (1976). The approximation consists of a weighted average of the updating procedures by the probabilities of the Markov state, in which the mixture of four Gaussian densities is collapsed, after each observation, into a mixture of two densities. It is done as follows:

Collapsing equations:

$$\beta_{t|t}^{(j)} = \sum_{i=1}^{2} \frac{Pr[S_{t-1} = i, S_{t} = j \mid I_{t}] \beta_{t|t}^{(i,j)}}{Pr[S_{t} = j \mid I_{t}]}$$
The Likelihood

The likelihood is given by: \( L = \prod_{t=1}^{T} f(y_t | I_{t-1}) \)

For the MSR model we adopt the same restrictions on the parameters of the diagonal of matrix \( Q \) that were imposed on the TVP model in Subsection 3.1. Furthermore equation (15) shows that we can estimate only \( p_{11} \) and \( p_{21} \) to obtain the entire matrix \( P \). Therefore, given \( p_{11}, p_{22}, \theta \tau (\tau = 1, 2, 3), \sigma_1, \sigma_2 \) and the parameters at the diagonal of matrix \( Q \) at states 1 and 2, the fixed initial values of the state vector and it's covariance matrix we can use Kim's filter to estimate \( \beta_1 \) and calculate the value of model's likelihood. That is, the likelihood can be concentrated with respect to \( \beta_1 \) making it easier to estimate \( p_{11}, p_{22}, \theta \tau (\tau = 1, 2, 3), \sigma_1, \sigma_2 \) and the parameters at the diagonal of matrix \( Q \), at each state, with the help of a numerical optimization routine. The Kim's filter obtains the mapping from the hyperparameters — \( \sigma_i \) and \( Q_i \) — the parameters in \( P \), and \( \theta \tau (\tau = 1, 2, 3) \) to the likelihood value. We use this mapping to estimate the likelihood maximizing values of these hyperparameters and parameters using a numerical optimization routine. The results of this estimation are presented in Section 4.

If we exclude the parameters of \( \beta_1 \) the MSR model has 15 parameters to be estimated. We show, in the next section, how departing from this more general model and using a few statistical tests we end up with a model with only eight parameters to be estimated through the numerical optimization routine.

4 - THE MODEL'S ESTIMATION RESULTS

The two models were estimated using computer programs developed with the help of the Matlab software.

4.1 - The TVP Model Results

The TVP model was initially estimated with 10 parameters if we exclude, from the counting, the parameters that belong to \( \beta_1 \) and that were concentrated out of the likelihood using the Kalman filter. We could not reject — at a significance level higher than 10\% and using the likelihood ratio test — that \( \Lambda, \gamma_0 \) and the parameters of the diagonal of matrix \( Q \) which controls for the time variation of coefficients of the dummy variables, of unemployment rate and of the lags of the
inflation rate, are all equal to zero. The log likelihood values with and without the restrictions are presented in Table 4.

Therefore, we could not reject that the TVP model, without ARCH residual in the measurement equation, with time variation only in the intercept and with intercept intervention at the stabilization Plans fits the data well. With the above mentioned restrictions the TVP model shows only five parameters to be estimated through the numerical optimization routine. The estimated values of these five parameters can be found in Table 1. The one-step ahead forecast and fitted values of the TVP model are presented at the set of Graphics 2. At the set of Graphics 3 we show the smoothed estimates of the Nairu, as defined in Section 2, and its confidence interval computed from the smoothed estimates of $\beta^*$ and of $\text{Var}(\beta^*)$, at each period $t$. The confidence intervals for the Nairu were computed from 10,000 extractions of the distribution of the estimator of $\beta^*$ and, in each extraction, the Nairu was computed as explained in Section 2.

<table>
<thead>
<tr>
<th>Hyperparameters</th>
<th>Parameter</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q(1,1)$</td>
<td>0.0088</td>
<td>0.0053</td>
</tr>
<tr>
<td>$\sigma (\varepsilon, \text{Standard Deviation})$</td>
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<td>0.0036</td>
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<td>Intercept Intervention Parameters</td>
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<tr>
<td>Cruzado Plan ($\theta_1$)</td>
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<td>Bresser and Verão Plans ($\theta_2$)</td>
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<td>0.2549</td>
</tr>
<tr>
<td>Collor I and Real Plans ($\theta_3$)</td>
<td>-3.7243</td>
<td>0.7364</td>
</tr>
</tbody>
</table>

### 4.2 - The MSR Model Results

The MSR model was initially estimated in its more general version, with 15 parameters if we exclude, from the counting, the parameters that belong to $\beta^*$. At a significance level higher than 10% and using the likelihood ratio test it could not be rejected that all the parameters at the diagonal of matrix $Q$ are equal to zero at state 2 and that these parameters – with the exception of the one that controls the time variability of the intercept – are also equal to zero at state 1. At Table 4 we present the log likelihood for the MSR model with and without these restrictions.

The MSR model, with the above restrictions, has eight parameters to be estimated through the numerical optimization routine. At Table 2 we present the estimated values of these eight parameters. The estimated transition probability matrix ($P$) can be found in Table 3. The state 1 probability, at each period, can be viewed in Graphic 1. The one-step-ahead forecast and fitted values of the MSR model are presented at the set of Graphics 2. At the set of Graphics 3 we show the smoothed
estimates of the Nairu, and its confidence interval computed using the same procedure adopted for the TVP model.

Table 2

**MSR Model**

<table>
<thead>
<tr>
<th>Hyperparameters</th>
<th>Parameter</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q(1,1)$ at State 1</td>
<td>0.0059</td>
<td>0.0024</td>
</tr>
<tr>
<td>$\sigma_1$ ((\varepsilon), Standard Deviation at State 1)</td>
<td>0.0087</td>
<td>0.0043</td>
</tr>
<tr>
<td>$\sigma_2$ ((\varepsilon), Standard Deviation at State 2)</td>
<td>0.0389</td>
<td>0.0044</td>
</tr>
<tr>
<td>Transition Probabilities</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{11}$ [(Pr(St=1</td>
<td>St-1=1))]</td>
<td>0.9607</td>
</tr>
<tr>
<td>$p_{21}$ [(Pr(St=1</td>
<td>St-1=2))]</td>
<td>0.1325</td>
</tr>
<tr>
<td>Intercept Intervention Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cruzado Plan ((\theta_1))</td>
<td>-1.1472</td>
<td>0.3207</td>
</tr>
<tr>
<td>Bresser and Verão Plan ((\theta_2))</td>
<td>-1.9991</td>
<td>0.4604</td>
</tr>
<tr>
<td>Collor and Real Plan ((\theta_3))</td>
<td>-5.3728</td>
<td>1.3375</td>
</tr>
</tbody>
</table>

Table 3

**Transition Probability Matrix (P)**

<table>
<thead>
<tr>
<th>(St=1)</th>
<th>(St=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(St-1=1)</td>
<td>0.9610</td>
</tr>
<tr>
<td>(St-1=2)</td>
<td>0.1330</td>
</tr>
<tr>
<td>Steady –State Probabilities</td>
<td>0.7713</td>
</tr>
</tbody>
</table>

Graphic 1

State 1 Probabilities (smoothed)
Graphics 2

TVP Model
One-Step Ahead Forecast of
$\Delta \pi$
Theil-U = 0.29

MSR Model
One-Step Ahead Forecast of
$\Delta \pi$
Theil-U = 0.30
4.3 - Comparison between the PVT and MSR Models and some Additional Statistical Tests

The maximizing value of log likelihood for the MSR model is 88.58 and the same value for the TVP model is 80.91. The log likelihood for the two models and for a variety of alternative hypothesis is presented at Table 4. The TVP model is more parsimonious than the MSR model since it has three parameters less. It is also true that the TVP model (without ARCH residuals) is nested in the MSR model. Nevertheless, as it has been pointed out by Engel and Hamilton (1990) in the region of the parameter space where the MSR model becomes the TVP model the information matrix is singular and the standard regularity conditions, needed to establish asymptotically valid hypothesis tests, are not satisfied. Therefore we cannot test, using the usual tests, which of the models better fits the data and the results for both models are going to be presented.

The MSR model, as can be seen in Table 5, passes satisfactorily two tests: one suggested by Engel and Hamilton (1990) designed to test if the measurement equation residuals are homoscedastic without regime switching \( (p_{11} = 1-p_{22}) \) and with a time-varying intercept in the measurement equation and another to test if
there is no time-varying intercept at state 1 \((Q_{1}(1,1)=0)\) when the hypothesis of Markov-switching, for the standard deviation of the residual of the measurement equation, is in place. When the hypothesis \(p_{11} = 1-p_{22}\) is imposed to the MSR model there is no longer a Markov-switching process and the rate of inflation is an i.i.d. sequence with individual densities given by a mixture of two normals. We are able to reject the first hypothesis, at a significance level smaller than 1%, using the likelihood ratio and Wald tests. The second hypothesis is rejected at a significance level smaller than 1% if the likelihood ratio test is used or at a significance level smaller than 5% if the Wald test is used.

Table 4

<table>
<thead>
<tr>
<th>Log Likelihood and Alternative Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TVP Model</strong></td>
</tr>
<tr>
<td>Initial Model (10 parameters)</td>
</tr>
<tr>
<td>Simplified Models ((Q_{j,j} = 0, \ for \ j \neq 1, \Lambda = 0 \ and \ \gamma_0 = 0))</td>
</tr>
<tr>
<td>(Q_{1}(1,1) \neq 0) (5 Parameters)</td>
</tr>
<tr>
<td>(Q_{1}(1,1) = 0) (4 Parameters)</td>
</tr>
<tr>
<td><strong>MSR Model</strong></td>
</tr>
<tr>
<td>Initial Model (15 Parameters)</td>
</tr>
<tr>
<td>Simplified Models ((Q_{i,j} = 0, \ for \ j \neq 1 \ and \ i=1,2))</td>
</tr>
<tr>
<td>(Q_{1}(1,1) \neq Q_{2}(1,1)) (Regime Dependent) (9 par.)</td>
</tr>
<tr>
<td>(Q_{1}(1,1) = Q_{2}(1,1)) (Regime Independent) (8 par.)</td>
</tr>
<tr>
<td>(Q_{1}(1,1) \neq 0) and (Q_{2}(1,1) \neq 0) (Regime Dependent) (8 par.)</td>
</tr>
<tr>
<td>(Q_{1}(1,1) = 0), Both Regimes (7 par.)</td>
</tr>
</tbody>
</table>

**Note:** The parameter count, in this table, does not include the parameters of \(\hat{\beta}^*_{1}\) (\(\hat{\beta}^*_{1}\) has nine other parameters).

Table 5

<table>
<thead>
<tr>
<th>Additional Hypothesis Tests – MSR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ho</strong>: (p_{11} = 1 - p_{22})</td>
</tr>
<tr>
<td>Wald Test</td>
</tr>
<tr>
<td>Likelihood Ratio Test</td>
</tr>
<tr>
<td>Log Likelihood</td>
</tr>
<tr>
<td>Under Ho*</td>
</tr>
<tr>
<td><strong>Ho</strong>:(Q_{1}(1,1) = 0)</td>
</tr>
<tr>
<td>Wald Test</td>
</tr>
<tr>
<td>Likelihood Ratio Test</td>
</tr>
<tr>
<td>Log Likelihood</td>
</tr>
</tbody>
</table>

\* Suggested by Engel and Hamilton (1990).

We also tested both models (TVP and MSR) for serial correlation in the standardized forecast errors and in the squares of the standardized forecast errors. Tables 6 and 7 present these tests. Using the \(Q\) statistic, for the TVP (MSR) model, we do not reject at the 4.7% (4.4%) significance level the null of no serial correlation in the standardized forecast errors for lags smaller than 21 (19). For the MSR model, at all investigated lags and at the 5% significance level, we do not reject the hypothesis of no serial correlation in the squares of the standardized forecast errors. For the TVP model the last statement is true only at the 1% significance level.
Table 6
Tests for Serial Correlation of Standardized Forecast Errors

<table>
<thead>
<tr>
<th></th>
<th>MRS MODEL</th>
<th></th>
<th></th>
<th>TVP MODEL</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Autocorrelation</td>
<td>Partial</td>
<td>Q-Statistic</td>
<td>Probability</td>
<td>Autocorrelation</td>
</tr>
<tr>
<td>1</td>
<td>0.256</td>
<td>0.256</td>
<td>4.054</td>
<td>0.044</td>
<td>0.170</td>
</tr>
<tr>
<td>2</td>
<td>-0.110</td>
<td>-0.188</td>
<td>4.824</td>
<td>0.090</td>
<td>-0.262</td>
</tr>
<tr>
<td>3</td>
<td>-0.050</td>
<td>0.036</td>
<td>4.982</td>
<td>0.173</td>
<td>-0.091</td>
</tr>
<tr>
<td>4</td>
<td>-0.055</td>
<td>-0.079</td>
<td>5.181</td>
<td>0.269</td>
<td>0.044</td>
</tr>
<tr>
<td>5</td>
<td>-0.087</td>
<td>-0.060</td>
<td>5.687</td>
<td>0.338</td>
<td>0.017</td>
</tr>
<tr>
<td>6</td>
<td>-0.256</td>
<td>-0.258</td>
<td>10.151</td>
<td>0.118</td>
<td>-0.210</td>
</tr>
<tr>
<td>7</td>
<td>-0.155</td>
<td>-0.041</td>
<td>11.812</td>
<td>0.107</td>
<td>-0.145</td>
</tr>
<tr>
<td>8</td>
<td>-0.045</td>
<td>-0.092</td>
<td>11.955</td>
<td>0.153</td>
<td>0.060</td>
</tr>
<tr>
<td>9</td>
<td>-0.083</td>
<td>-0.116</td>
<td>12.447</td>
<td>0.189</td>
<td>-0.006</td>
</tr>
<tr>
<td>10</td>
<td>0.015</td>
<td>0.007</td>
<td>12.464</td>
<td>0.255</td>
<td>0.030</td>
</tr>
<tr>
<td>11</td>
<td>0.065</td>
<td>-0.024</td>
<td>12.777</td>
<td>0.308</td>
<td>0.047</td>
</tr>
<tr>
<td>12</td>
<td>-0.131</td>
<td>-0.273</td>
<td>14.100</td>
<td>0.294</td>
<td>-0.108</td>
</tr>
<tr>
<td>13</td>
<td>-0.124</td>
<td>-0.121</td>
<td>15.306</td>
<td>0.289</td>
<td>-0.152</td>
</tr>
<tr>
<td>14</td>
<td>-0.017</td>
<td>-0.110</td>
<td>15.329</td>
<td>0.356</td>
<td>-0.002</td>
</tr>
<tr>
<td>15</td>
<td>0.231</td>
<td>0.165</td>
<td>19.699</td>
<td>0.184</td>
<td>0.164</td>
</tr>
<tr>
<td>16</td>
<td>0.048</td>
<td>-0.187</td>
<td>19.895</td>
<td>0.225</td>
<td>-0.047</td>
</tr>
<tr>
<td>17</td>
<td>-0.048</td>
<td>0.010</td>
<td>20.090</td>
<td>0.270</td>
<td>-0.062</td>
</tr>
<tr>
<td>18</td>
<td>0.236</td>
<td>0.152</td>
<td>24.973</td>
<td>0.126</td>
<td>0.235</td>
</tr>
<tr>
<td>19</td>
<td>0.255</td>
<td>0.112</td>
<td>30.845</td>
<td>0.042</td>
<td>0.286</td>
</tr>
<tr>
<td>20</td>
<td>-0.030</td>
<td>-0.150</td>
<td>30.926</td>
<td>0.056</td>
<td>-0.126</td>
</tr>
<tr>
<td>21</td>
<td>-0.199</td>
<td>-0.071</td>
<td>34.682</td>
<td>0.031</td>
<td>-0.188</td>
</tr>
<tr>
<td>22</td>
<td>-0.086</td>
<td>0.011</td>
<td>35.406</td>
<td>0.035</td>
<td>-0.019</td>
</tr>
<tr>
<td>23</td>
<td>-0.065</td>
<td>-0.063</td>
<td>35.824</td>
<td>0.043</td>
<td>0.057</td>
</tr>
<tr>
<td>24</td>
<td>-0.119</td>
<td>-0.040</td>
<td>37.279</td>
<td>0.041</td>
<td>-0.053</td>
</tr>
</tbody>
</table>
Table 7

Tests for Serial Correlation of Squared Standardized Forecast Errors

<table>
<thead>
<tr>
<th></th>
<th>MRS MODEL</th>
<th></th>
<th></th>
<th>TVP MODEL</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Autocorrelation</td>
<td>Partial Correlation</td>
<td>Q-Statistic</td>
<td>Probability</td>
<td>Autocorrelation</td>
<td>Partial Correlation</td>
</tr>
<tr>
<td>1</td>
<td>0.244</td>
<td>0.244</td>
<td>3.707</td>
<td>0.054</td>
<td>0.285</td>
<td>0.285</td>
</tr>
<tr>
<td>2</td>
<td>-0.052</td>
<td>-0.119</td>
<td>3.879</td>
<td>0.144</td>
<td>-0.222</td>
<td>0.153</td>
</tr>
<tr>
<td>3</td>
<td>-0.112</td>
<td>-0.074</td>
<td>4.685</td>
<td>0.196</td>
<td>-0.031</td>
<td>-0.143</td>
</tr>
<tr>
<td>4</td>
<td>-0.117</td>
<td>-0.081</td>
<td>5.585</td>
<td>0.232</td>
<td>-0.137</td>
<td>-0.147</td>
</tr>
<tr>
<td>5</td>
<td>-0.061</td>
<td>-0.028</td>
<td>5.831</td>
<td>0.323</td>
<td>-0.083</td>
<td>0.026</td>
</tr>
<tr>
<td>6</td>
<td>0.172</td>
<td>0.187</td>
<td>7.833</td>
<td>0.251</td>
<td>0.151</td>
<td>0.261</td>
</tr>
<tr>
<td>7</td>
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<td>-0.001</td>
<td>8.685</td>
<td>0.276</td>
<td>0.064</td>
<td>-0.043</td>
</tr>
<tr>
<td>8</td>
<td>-0.112</td>
<td>-0.150</td>
<td>9.575</td>
<td>0.296</td>
<td>0.173</td>
<td>0.041</td>
</tr>
<tr>
<td>9</td>
<td>-0.141</td>
<td>-0.058</td>
<td>11.014</td>
<td>0.275</td>
<td>-0.105</td>
<td>-0.204</td>
</tr>
<tr>
<td>10</td>
<td>0.033</td>
<td>0.121</td>
<td>11.095</td>
<td>0.350</td>
<td>-0.123</td>
<td>-0.048</td>
</tr>
<tr>
<td>11</td>
<td>0.018</td>
<td>-0.016</td>
<td>11.119</td>
<td>0.433</td>
<td>-0.084</td>
<td>0.104</td>
</tr>
<tr>
<td>12</td>
<td>0.056</td>
<td>0.002</td>
<td>11.360</td>
<td>0.498</td>
<td>-0.050</td>
<td>-0.002</td>
</tr>
<tr>
<td>13</td>
<td>-0.032</td>
<td>-0.113</td>
<td>11.438</td>
<td>0.574</td>
<td>-0.115</td>
<td>-0.204</td>
</tr>
<tr>
<td>14</td>
<td>-0.072</td>
<td>0.003</td>
<td>11.855</td>
<td>0.618</td>
<td>-0.051</td>
<td>-0.125</td>
</tr>
<tr>
<td>15</td>
<td>-0.120</td>
<td>-0.036</td>
<td>13.030</td>
<td>0.600</td>
<td>-0.072</td>
<td>0.100</td>
</tr>
<tr>
<td>16</td>
<td>-0.069</td>
<td>-0.072</td>
<td>13.423</td>
<td>0.642</td>
<td>0.024</td>
<td>0.157</td>
</tr>
<tr>
<td>17</td>
<td>0.089</td>
<td>0.072</td>
<td>14.106</td>
<td>0.660</td>
<td>-0.040</td>
<td>-0.088</td>
</tr>
<tr>
<td>18</td>
<td>0.140</td>
<td>0.079</td>
<td>15.817</td>
<td>0.605</td>
<td>0.062</td>
<td>-0.044</td>
</tr>
<tr>
<td>19</td>
<td>0.055</td>
<td>0.020</td>
<td>16.087</td>
<td>0.651</td>
<td>-0.054</td>
<td>-0.102</td>
</tr>
<tr>
<td>20</td>
<td>0.059</td>
<td>0.070</td>
<td>16.406</td>
<td>0.691</td>
<td>-0.022</td>
<td>0.089</td>
</tr>
<tr>
<td>21</td>
<td>-0.023</td>
<td>-0.023</td>
<td>16.456</td>
<td>0.743</td>
<td>-0.077</td>
<td>0.066</td>
</tr>
<tr>
<td>22</td>
<td>-0.068</td>
<td>-0.025</td>
<td>16.912</td>
<td>0.768</td>
<td>-0.129</td>
<td>-0.228</td>
</tr>
<tr>
<td>23</td>
<td>-0.012</td>
<td>0.017</td>
<td>16.927</td>
<td>0.813</td>
<td>-0.082</td>
<td>-0.155</td>
</tr>
<tr>
<td>24</td>
<td>0.090</td>
<td>0.040</td>
<td>17.762</td>
<td>0.814</td>
<td>-0.086</td>
<td>-0.060</td>
</tr>
</tbody>
</table>

5 - CONCLUSIONS

In our analysis of the results we will emphasize two main aspects: is there a statistically significant relationship between an upward (downward) deviation of the rate of unemployment from our estimated value of the Nairu and a reduction (increase) in the rate of inflation? To what degree our measure of the Nairu can be used as a guide to the conduct of monetary policy in Brazil?

The first question can be easily answered by testing if the sum of coefficients of lag unemployment is negative and significantly different from zero in the two models. This sum belongs, with a 95% degree of confidence, to the (-1.9, -1.2) interval for the TVP model and to the (-1.2, -0.7) interval for the MSR model. Neither of the models allows for time variation in the coefficients of lag unemployment. Therefore, it cannot be rejected that the deviations of the unemployment rate from the Nairu can have a significant effect, with correct sign, on the inflation rate. Nevertheless, it is also true that the above estimates of the effect are not very precise. If we consider the uncertainty with respect to what is the right model then no value between (-1.9, -0.7) can be rejected.
The answer to the second question depends on the degree of precision with which the Nairu is estimated. Observing the set of Graphics 3 we can conclude that the estimates of the Nairu, considering both models, are very imprecise and that from the second quarter of 1995 the estimated value of the Nairu is contained within its error bands and therefore it cannot be rejected that the observed rate of unemployment was equal to the Nairu. That is, from the second half of 1995 the error bands estimated for the Nairu do not give a good guidance as to what should be the monetary policy followed by the Central Bank.

Despite what we have said one cannot conclude that our estimates of the Nairu are useless. It should be pointed out that, during a good part of the period of high inflation faced by Brazil, the rate of unemployment was, systematically, below the error bands estimated for the Nairu. Therefore there was a continuous up trend in the rate of inflation whose upward trajectory was only broken by the stabilization plans. This is true for the MSR model from the beginning of 1985 until the second half of 1995. For the TVP model this is true from the beginning of 1986 to the beginning of 1991. It is also interesting to note that both models indicate that there was a decrease in the Nairu from 1994 until the middle of 1995 when it started increasing again until it reached values next to its previous 1994 level by the third quarter of 1999.

Unfortunately there is not a large number of articles with different models and with estimates of the Nairu for Brazil. The degree of confidence in the results depends critically on the increase of research in this area.

**BIBLIOGRAPHY**


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