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COORDENATES OF SPATIAL MODELS

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1. In a general way mathematical model has as purpose the collection of certain informations which are processed and then presented as a new set of informations. Some of the input informations are of a variable nature and they constitute the exogenous variables to the model. Others, of a permanent character, are the assumptions on which the model has been built. The resulting informations are the endogenous variables to the model.

Among the models used in Economics, those which try to describe the markets of certain goods, assume a particular importance. The description of the market is characterized by the nature of the supply and demand of the good and eventually the supply and the demand of the production factors used to supply the good which has been studied.

In the field of spatial economics, we have, among the fundamental characteristics of supply and demand, its location, which may be either endogenous or exogenous, its spatial distribution and so forth.

With these considerations in mind we have tried to establish a systematic way of characterizing the models of spatial economics through the attributes assumed by demand and supply of goods, demand and supply of the production factors and the linkage between supply and demand. For each of these elements for every model, many attributes are admitted. However, to fulfill our aims it will be possible to consider only some of the most important attributes; thus for the contact between supply and demand we have considered only the transportation problem, leaving aside the nature of the market.

Thus, we shall characterize a spatial model by the followings elements:

a) demand for the good;
b) supply of the good;
c) transportation of the good between the places of supply and demand;
d) demand for the production factors;
e) supply of the production factors.

Among the production factors we shall only consider the raw material for the manufacturing industries and the land for agriculture.

2. In order to a define spatial economic model it is necessary to define only some attributes of the elements formerly listed.

They are: location, spatial configuration and price elasticity.

The location of an element can be an exogenous or endogenous variable to the model.

The spatial configuration of supply and demand can assume the most diverse forms within reality, however in the models that we are going to consider it is assumed to be concentrated at points or distributed in areas. An intermediary solution of distribution along a line can also be considered.

Finally, we have the elasticity of both, supply as well as demand. The elasticity equal to zero excludes the price-variable, causing the quantity-variable to be exogenous to the model. An infinite elasticity excludes the quantity-variable and makes the price-variable exogenous. Intermediate levels cause both variables to be endogenous; only the function that relates them being exogenous.

These attributes related to each element will be called coordenates.

3. It is possible to define the several models of spatial economics by means of a very simple notation, as follows:

Let us call:

D - demand element of a good
S - supply element of this good
T - transportation between the places of supply and demand
F_1 - demand for production factors
F_2 - supply of the production factors

As we shall prove, the demand for production factors need not be characterized independently. Symbol F may be used for the supply of these factors.

Three coordinates may be associated with every one of these elements:

x - which will indicate the endogenous or exogenous character of the location in the model;
y - which will show the spatial configuration;
z - which will represent the price-elasticity.

The last coordinate is quantitative, and can assume any value in the field of real numbers. (1) The first two coordenates are qualitative and it becomes necessary to attribute numerical levels to them.

We shall represent by $x = 0$ an endogenous location and by $x = 1$ an exogenous location. By $y = 0$ a demand or supply concentrated at a point, by $y = 1$ a distribution according to a line and by $y = 2$ a distribution according to an area.

A model within this notation will be synthetically represented by:

$$D(x,y,z,) \quad S(x,y,z,) \quad T(x,y,z,) \quad F(x,y,z,)$$

where the variables x,y,z will be substituted by numbers within the notation formerly defined. If the value of one of these three last variables is irrelevant for the definition of the model it will be represented in the notation by the letter i. Finally, the notation m will indicate that it has been given more than one treatment to the variable in the model.

4. Using the above mentioned notation it will be possible to represent the different models of location theory.

In Weber's model the markets are admitted to be concentrated in points with location previously defined(therefore, exogenous to the model) with a constant demanded quantity(price elasticity equal to 0). In these conditions the coordinates of D will be $x = 1$, $y = 0$ and $z = 0$.

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(1) Positive sign will be attributed to elasticity of decreasing demand and increasing supply within the usual convention in Economics.

The demand for production factors in the present model as well as in all models that we shall analyze, will have the same characteristics of the supply of the good, namely, endogenous location, concentrated in points and price-elasticity equal to ZERO. In such conditions it will be necessary to add to the notation the characteristics of this element. However, this correspondence is not necessary. Thus, although the supply-elasticity is ZERO, the elasticity-demand of raw material can be assumed to be positive, there being possibilities of combining them in different proportions, according to price. This possibility has practical application, considering that by changing the supply-location the distance to the sources of raw material change too, and in consequence their relative prices.

In the supply of production factors, we shall consider only the raw-materials. They are of two natures: the ubiquities of endogenous location and the specifically located ones of exogenous location. In any one of them a configuration concentrated in points is assumed with quantities defined "a priori", therefore, with price elasticity equal to ZERO. Thus we shall have the following coordinates for F $x = m$, $y = 0$ and $z = 0$.

Thus, Weber's model will be defined by:

D(1,0,0,) S(0,0,0,) T(0,1,0,) F(m,0,0)

5. Lösch's model characterization, does not present difficulties. Demand has its location defined "a priori", being distributed in an area presenting a positive price-elasticity.

Supply is concentrated at points whose locations are defined by the model itself. Elasticity is negative due to economies of scale. Transportation will be defined by the model itself which explains its location. As the exchanged quantities result from the model itself and as the transportation price is admitted unaltered with this quantity, it results that the price-elasticity will be infinite.

The supply of production factors, in the case of raw materials will have endogenous location, since only ubiquities are considered. As in the model it is not taken into account the cost increase of the raw material transported related to the increase of the production scale, this supply is assumed to be concentrated at one point and presenting an infinite elasticity.

In these conditions the model will be given by:

D (1,2,$>$ 0) S (0,0,$<$ 0) T (0,1, ∞) F (0,0, ∞)

6. It will also be easy to characterize Thünen's model. Demand represented by the demand of an urban center has exogenous location and it is assumed to be concentrated in a point with positive price-elasticity.

Supply has its location explained by the model itself which determines in what position will each culture be produced and is considered distributed in an area. Supply may present elasticity equal to ZERO in a model in which coefficients are constant and output per unit of area is admitted to be constant, or it may be positive in a model in which coefficients are variable and the possibility of an increase in output per unit of area with FOB price increase of the finished good is considered.

The same considerations made for Lösch's model are valid for transportation. The supply of production factors, in this case the land, will be of elasticity ZERO besides being distributed according to a given area and will have an endogenous location, since the model will indicate the land that will be used for the culture.

In these conditions, the notation for Thünen's model will be:

D(1,0,$>$ 0) S(o,2,0) T(0,1,∞) F(0,2,0)

for constant coefficients and

D(1,0, 0) S(0,2$>$ 0) T(0,1,∞) F(0,2,0)

for variable coefficients.

7. The comparison of the notations of the 3 models shows that they bear many aspects in common defined by the identity of the coordinates of the same element. Thus the three models present coordinate $x = 0$ for the element S. This is one characteristic of a category of spatial models which we shall call supply location model.

The class of spatial models is however of a larger range and includes other categories. Thus we would have the demand location models in the case of intermediate goods. As examples of models belonging to this last category we have Lösch's and Thünen's ones, both in a reverse form. The first one (see for

instance Silva Leme 1964-3.312) (1) has the notation:

$$D(0,0,1) \quad S(1,2,0) \quad T(0,1,\infty)$$

The second one (see for instance Silva Leme 1964 - 4.15) has the notation:

$$D(0,2,0) \quad S(1,0,0) \quad T(0,1,\infty)$$

Demand may have an endogenous location only in the case of production goods, since in case of consumers goods it will be defined by the location of the population of consumers.

Other categories of spatial models are:

a) transportation models in which demand as well as supply has exogenous location;
b) models of spatial price variation of which Thünen's model in the aspect of explaining income and prices of lands is an especial case;
c) the distribution of markets among several producers, etc.

8. The advantage of the synthetic representation of the models that we have presented is that they provide a panoramic view of the points which have already been considered in the theory of location and of those which have not yet been considered too. Causing to vary one or more indexes in the notation of an existing model we will have a completely new condition which may either have a practical interest or not, to be studied.

Many of these new models have already been considered in publications, yet the amount of possibilities is so large that much has to be done to exhaust it. Thus, for instance, whenever coordinate $z = 0$ it will be interesting to generalize the model, including the influence of price.

Many of these generalizations are being made as the adequate mathematical devices are developed. The development of integer linear programing for instance allows to solve an analogous problem solved by Lösch's model with much more realistic assumptions (demand concentrated at points, transportation constrained to lines) with the following notation:

$$D(1,0,0) \quad S(0,0,0) \quad T(1,1,\infty) \quad F(1,1,\infty)$$

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(1) Silva Leme (1964) Contribuições à Teoria de Localização Publicação da Faculdade de Ciencias Economicas e Administrativas da Universidade de São Paulo - São Paulo.