A PLANNING MODEL FOR A MIXED ECONOMY

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My motive in adding yet another paper to the mounting file of EPEA documents on planning is a general disatisfaction with the lack of distinction between the treatment of the public and private sectors in the usual planning models. The heart of the planning problem in Brazil is to adapt the planning mechanism to the operations of a market economy. Where the price mechanism is allocating resources efficiently, there is little reason for government intervention, and one of the principle goals of the planning agency is to discover where the market economy should be overridden by direct action. The planner should also recognize that in a free enterprise economy, investors and consumers are free to make their own decisions. This means that the rate of investment necessary to fulfill a governmental output goal may differ from the amount which businessmen plan to invest. Similarly the level of demand may not be equal to the output planned by the government. Planners must have some method of estimating the difference so that steps can be taken to ensure equality. Technically we are suggesting that a planning model for a mixed or free market economy must contain behavioral equations for both consumption and investment.

Another distinction which is useful is that between the capacity to produce and money demand. Equality between the two is an equilibrium condition and should not be assumed in advance. Capacity is a function of labor and capital and is increased by investment and additions to the labor force. The incremental capital output ratio (ICOR) refers to the technical relationship between investment and capacity. The relationship between investment and demand is the subject of Keynesian multiplier analysis, which is distinct from the proceeding relationship. The general planning mechanism is to set a desired capacity goal. If one knows the output production function and the rate of growth of the labor force, this clearly determines a rate of investment. Where investors are free, the government may be required to generate special incentives such as tax credits or low interst loans. It may also happen that the output goal set by the government does not equal private demand thus requiring government action. The point is that one cannot simply assume equality between desired an actual investment or between capacity and demand in an economy where investors and consumers are free to make their own decisions.

A simple one sector model which expresses this general planning approach is the following:

1) $Y_t^d = C_t + I_t + G_t$

2) $C_t = a + c\,(Y_t^d - T)$

3) $Y_t^s = \emptyset\,(L_t K_t)$

3a) $Y_{t+1}^s = \emptyset \left[L_t(1+r),\ K_t + I_t \right]$

4) $pY_t^s = Y_t^d$

5) $I_t = b\,(Y_t - Y_{t-1}) + b'\,(Y_{t-1} - Y_{t-2}) + e$

6) $\dfrac{Y_{t+1}^s}{Y_t^s} = W$

7) $K_t = K_{t-1} + I_{t-1}$

8) $L_t = L_{t-1}(1 + r)$

9) $T = T^o$

Exogeneous Variables

W - the preset desired rate of growth of output
T - taxes
p - rate of inflation

Exogeneous Parameters

(derived from regression analysis)

a - the intercept in the consumption function

c - the marginal propensity to consume

b, b' - investment reaction coefficients

r - rate of growth of labor force

Endogeneous Variables

Y_t^d - money demand

Y_t^s - capacity

C_t - consumption

I_t - net investment

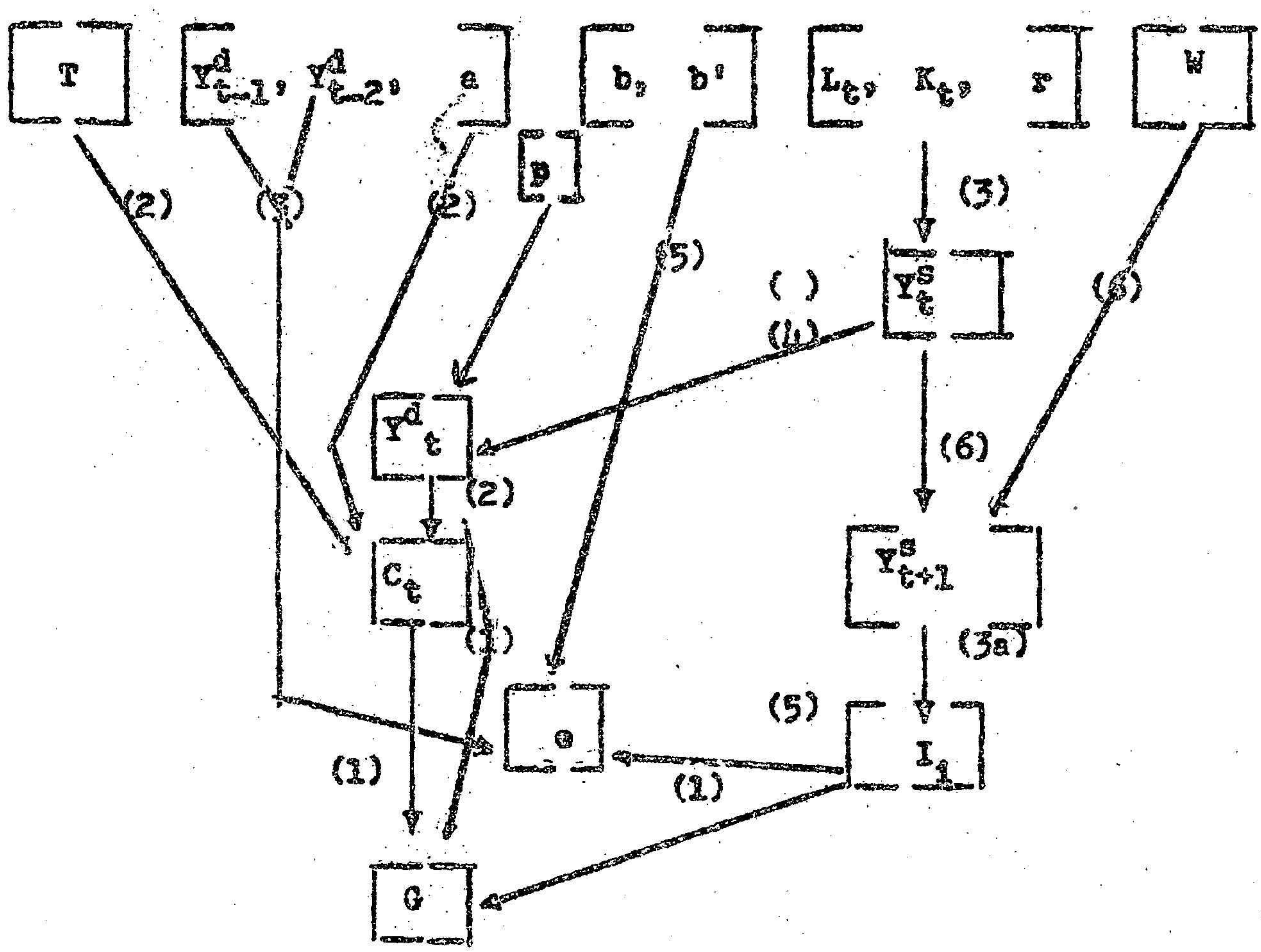
G - Government expenditure

K_t - capital stock at the beginning of time t

e - an environmental parameter expressing credit availability and government policy.

Equation (1) is an accounting identity; (2) expresses consumption as a function of disposable income; (3) gives capacity as a function of the supply of labor and capital; (4) is the equilibrium condition. It can be modified to express inflationary pressure by allowing money demand to exceed capacity by some amount; (5) is a behavioral equation for investment which has been applied with good results in the U.S. The dependence of investment on lagged changes in output comes from the assumption that investors build capacity on the basis of today's output and that investment takes time; (6) contains the planning element of the model. Given today's capacity, tomorrow's capacity is determined by the growth goal W. The system can be interpreted as saying that if consumers and investors behave in the future as they did in the past, and if the production function remains constant, setting the growth goal determines the amount of income, consumption, investment and government expenditure and further guarantees that the system is consistent.

Given this background one can lay out the sequential solution to the model in diagrammatic terms. The arrows indicate the variables entering into the solution of any variable while the numbers refer to equations.



The solution of the model is sequential. Given the present stock of capital and labor, today's capacity is determined. This now determines tomorrow's capacity and investment given W and the production function (3a). With the capacity limit set by (3), (4) determines the total money demand which can be produced in a non-inflationary manner. This amount less taxes determines consumption. At this point the difference between desired and investment enters the model. Investment depends on present and past changes in output. However it also depends on environmental factors. In effect equation (5) determines the value of the constant e, given the desired value of investment and present money income. Finally, G is determined as a residual. It is amount of government spending which is consistent with capacity and total private demand.

An example

Suppose that we have found the following behavioral relationship in time series analysis.

$$C = .10 + .67Y$$

$$Y_t^s = \frac{1}{3} \left[K_t \right] \quad \text{(labor is surplus and the COR = 3)}$$

$$I_t = 1.5 \left[Y_t^d - Y_{t-1}^d \right] + 1.5 \left[Y_{t-1}^d - Y_{t-2}^d \right]$$

$$W = 1.05$$

$$Y_{t-1}^d = 100$$

$$Y_{t-2}^d = 90$$

$$K_t = 309$$

$$T = 10$$

The system generates the following path for the various endogeneous variables in the model.

time	Y^s	Y^d	I	e	C	G
t	103	103	15.45	- 4.05	69.10	18.45
t + 1	103.15	103.15	16.23	4.03	72.56	19.36
t + 2	113.56	113.56	17.04	1.20	76.18	20.31
t + 3	119.24	119.24	17.89	1.26	79.99	21.36
t + 4	125.20	125.20	18.78	1.32	83.98	22.41
t + 5	131.46	131.46	19.72	1.39	88.18	23.56

In this economy full employment is maintained only by an increasingly large government deficit. This is due to the particular form of the consumption function. If there had been no constant, all the endogeneous variables would have grown at the preset rate of 5%. The important point in the example is to demonstrate the time pattern of response of a free market economy to a growth goal set by the government.

Somewhat more realism can be introduced into this aggregate model by including an international sector. Imports rather than government expenditures become the residual. This requires the following adjustments to the first model.

1a) $Y_t^d = C_t + I_t + G_t - M_t + E_t$

10) $G_t = gY_t$

11) $E_t = j\left(\frac{P^B}{W}, Y_{t-1}^W\right)$

P^B = price of Brazilian Exports

P^W = world price for the good Brazil exports

Y_{t-1}^W = world income, previous period

(11) expresses Brazilian exports as a function of the relative price of Brazilian products and world income, ignoring supply considerations. The solution is similar to the one just shown except that imports become the balancing residual, derived from equation (1a). A side condition such as requiring imports to be no greater than exports sets an upper bound on the growth goal W by curtailing the amount of investment.

The next step in the analysis is to introduce additional sectors into the model. Using the same notation as the first model and indicating the i^{th} sector by subscript i we have:

1. $Y_t^d = \sum C_i + \sum I_i + \sum E_i - M_i - M_a - M_c$

2. $C_i = a_i + c_i(Y_{t-1}^d - T)$ $(i = 1 \ldots\ldots n)$

3. $Y_{i_t}^s = \sum p_{i_t} \phi (L_i , K_i) = \sum p_{i_t} Q_{i_t}$

4. $Y_t^s = Y_t^d$

5. $I_i = k_i (p_i Q_{i_t} - p_i Q_{i_{t-1}}) + e_i$ $(i = 1 \ldots\ldots n)$

6. $\frac{p_{t+1} Q_{i_{t+1}}}{p_t Q_{i_t}} = W_i$ (i = 1.....n)

7. $E_i = j_i \left(\frac{p_i^B}{p_i^W}, Y_t^W\right)$ (i = 1.....n)

8. $p_i Q_i = p_i \phi (L_i K_i)$ (i = 1.....n)

9. $M_a = 1 \left[Y^d_{t-1} - T \right]$

10. $M_i = m \sum I_i$

11. $I_i = K_{i_{t+1}} - K_{i_t}$ (i = 1.....n)

M_a - imports of agricultural products
M_i - imports of investment goods
M_c - imports of consumption goods
$p_i Q_i$ - <u>value added</u> in sector i.

The exogeneous variables are n W's, T and a vector of prices for each period. We measure demand by agregating across goods which are sold in final form. We avoid double counting by eliminating demand functions for intermediate products. This means that there is no connection between the demand and supply functions, since only an input output table permits one to translate final output into demands for intermediate products. The resulting system is one of 5n + j + 5 equations in 5n + j + 5 endogeneous variables, namely:

jC_i 's M_a
$np_i Q_i$'s

nI_i 's	M_i
npQ_i 's	M_c
nE_i 's	Y_t^s
ne_i 's	Y_t^d

This model can be solved along the same lines as the one sector model with imports of consumption goods as a residual. A five year plan for a four sector economy is worked out in appendix A to demonstrate the method. We have set no limit on imports in the model which may allow for the attainment of unrealistic investment rates. A side condition limiting imports focuses the attention of the planner on the choice of the W_i's. The problem can be stated as that of choosing a vector W such that the discounted value of the future income stream is maximized within the import restriction. This would be a simple linear programming problem if one had the import coefficient and demand equation for each sector for which a W_i is to be chosen. At the present state of our knowledge one acceptable alternative to the above procedure is to approximate the optimum by making alternative choices of W_i. It can be seen that it there is an import constraint, no information on sectoral import coefficients and if imports of investment goods are a simple function of the total amount of investment, then the optimal strategy is to grow as fast as possible in those sectors with the lowest capital output ratio, or highest return on investment. This choice can be improved upon only by more knowledge about sectoral import requirements.

Ideally to operate the model the planner needs a production consumption and investment function plus a growth goal for each sector plus a growth goal. Exports are determined more or less outside the model. But the advantage of this model is in its flexibility in the face of lack of information or determination of output by the free market. One can divide economic activity exactly on the basis of the need for governmental intervention and reduce information requirements drastically.

PRIVATE SECTOR:

The private sector can be defined as that area in which the government can accept the decisions of the market as final and which the control of production is in private hands. Here the government allows demand to determine the growth rate.

W. No estimate of investment is needed since the assumption is that the rate of growth of output will be sufficient to satisfy demand. While a production function is desirable it is not essential because capacity is being assumed to be greater than or equal to demand.

One of the chief problems of the planner is to decide which sectors are to be regarded as private under the definition just given. This is both a realistic acceptance of government's inability to plan all areas of economic activity at the present stage of knowledge and attempt to economize on the use of planning resources. One interesting criterion which could be used to divide industries into those needing and those not needing governmental intervention is the historic shape of the industrial supply schedule. When it is relatively flat, entrepreneurs have been able to meet past demands in a non-inflationary manner. Provided that the projected output level is acceptable to government, such industries could be declared private. A simple way of estimating the supply schedule is to regress output on prices or changes of output on changes in prices in time series analysis.

PUBLIC SECTORS:

The public sector is defined to be the collection of those industries and economic activities in which the means of production are owned by the state. In this sector the behavioral investment equation, (5), drops out. Setting the rate of growth of capacity automatically sets the level of per period investment given a production function. The planning agency should devote a good deal of its attention to this sector since here its ability to plan is accompanied by meaningful direct control of production. A demand forecast must be made and a growth goal chosen. Given an ICOR, this determines per period investment. If a discrepancy exists between capacity and demand, the price system or the consumption constant can be relied upon to ensure equality.

MIXED SECTOR:

A mixed sector is one in which the control of production is private but the government wishes to regulate the amount of production forthcoming. The natural rate of investment of entrepreneurs in the sector is to be changed to conform with some preset growth goal by setting the environmental parameter e_1. In order to do this successfully the planners must be able to forecast investment. This requires an attempt to estimate an investment equation. Clearly there should be a strong reason for shortcircuiting the market mechanism as one does in altering the normal investment rate of entrepreneurs. The best such reason may be the historic inability of the sector to meet demand at constant prices, as evidenced by a high rate of price increases in the past.

Recapitulating, we can now lay out the information needed to complete the plan.

Private Sector

1. Consumption function, or some alternative forecast of demand.
2. Production function - since we are assuming that capacity is greater than or equal to demand in any period, this information is not essential for the plan. Capacity can be set equal to forecast demand, less imports.

Public Sectors

1. Consumption function or some alternative forecast of demand.
2. Production function or at least an estimate of the ICOR.
3. Growth goal. If the growth goal implies a capacity level smaller than projected demand, then steps must be taken to reduce demand.

Mixed Sector

1. Consumption function or some alternative forecast of demand.
2. Production function
3. Investment function
4. Growth goal - The growth rate may be that which ensures that capacity in every period is greater than or equal to forecast demand. Here the government is accepting the market demand decision but intervening because projected supply is insufficient. In practice the information necessary to estimate functions two and three may not be available. One way around this difficulty is to estimate the ICOR. One can then get a rough idea of the difference between capacity and demand by the amount of price change between period. With the ICOR one then

shortcircuiting the market mechanism as one does in altering the normal investment rate of entrepreneurs. The best such reason may be the historic inability of the sector to meet demand at constant prices, as evidenced by a high rate of price increases in the past.

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knows how much investment is needed to equate capacity and demand. If it is impossible to fore cast private investment, one must be content to measure investment as the plan unfolds. If discrepancies are occurring between targets and actual amounts, the government can then create tax incentives or extend low interest loans.

In conclusion, this paper has attempted to lay out a model which would be useful to planners in a mixed economy. We have shown the minimum information and planning requirements with which one can make a complete and consistent forecast of the economy, sector by sector. The main difference between our approach and that of Simonsen is in our explicit recognition of the private sector. Only in equilibrium is the desired rate of capital formation derived from the growth goal W equal to the rate set independently by investors. Similarly if consumers are free to consume what they please, one cannot simply assume that demand will be equal to capacity. The challenge of planning in a private economy is to maintain resource allocation by market forces where this is efficient, adding governmental direction only where the market fails.

Within the private sector it should be a goal of the planner to separate these industries whose performance has been satisfactory from those needing detailed supervision. A simple criterion for this categorization on the basis of past rates of price increases was suggested.. Industries found able in the past to expand output at a reasonable rate can then be labelled as the private sector while those industries whose supply curve is steeper than some given figure should be put into the mixed sector for which the government will attempt to stimulate investment to achieve a rate of growth high enough to meet projected demand. Within this sector there is obviously no necessary equality between the desired rate of capital growth determined jointly be the production function and the growth goal, W, and the rate private investors would arrive at independently. The government must recognize this and take steps to make the two equal. Within the government sector the model makes clear the need for some production function information, be it only an estimate of the ICOR.

APPENDIX A

In the following example a four sector economy is hypothesized. The agricultural sector (subscript a) is mixed in the sense that the government is imposing a growth rate. The industrial sector (subscript i) is private and investment in the sector is determined under the assumption that it is just sufficient to meet the charge in consumer demand in the sector. The last two sectors an the public sector and a basic industry sector regarded here as public. No import constraints were imposed. The necessary information to operate the model was supposedly obtained through regression analysis and observation of present and past magnitudes. For convenience the defining equations for each endogeneous variable are also shown.

DEFINING EQUATIONS

$$Y_s = Y_d = \Sigma P_i Q_i$$

$$I_a = .12 \ (PQ_{a_t})$$

$$I_i = (Y_t - Y_{t-1})$$

$$I_p = .24 \ (pQ_{p_t})$$

$$I_B = .18 \ (pQ_{B_t})$$

$$M_i = .30 \ \Sigma_i I_i$$

$$M_a = .03 \ [Y_{t-1} - T]$$

$$C_a = 40 + .2 \ [Y_{t-1} - T]$$

$$C_i = .5 \ [Y_{t-1} - T]$$

$$C_p = 10 + .2 \ [Y_{t-1} - T]$$

$$D = 26$$

$$pQ_{i_{t+1}} = pQ_{i_t} + I_t/ICOR \text{ for all sectors.}$$

	Y_s	Y_d	pQ_a	pQ_i	pQ_p	pQ_B	I_a	I_i	I_p	I_B	M_a	M_i	M_c	C_a	C_i	C_p	E_x	e	Aggregate growth rate
t	250	250	70	100	30	50	8.4	20	7.2	9	5.4	17.8	10.4	76	90	46	26	2.4	7.1%
t+1	267.7	267.7	72.8	110	31.8	53	8.7	17.7	7.6	9.5	6.0	17.4	8.4	80	100	50	26	1.5	6.4
t+2	284.5	284.5	75.7	118.9	33.7	56.2	9.1	16.8	8.1	10.1	6.5	17.6	7.4	3.5	108.9	53.5	26	.6	5.9
t+3	301.3	301.3	78.7	127.3	35.7	59.6	9.4	16.8	8.6	10.7	7.0	18.2	6.1	86.9	117.3	56.9	26	.6	6.0
t+4	319.5	319.5	81.8	136.7	37.8	63.2	9.8	18.2	9.1	11.4	7.5	19.0	3.8	90.3	125.7	60.3	26	.7	

EXOGENOUS INFORMATION

$C_a = 40 + .2 [Y^d_{t-1} - T]$

$C_i = .5 [Y^d_{t-1} - T]$

$C_p = 10 + .2 [Y^d_{t-1} - T]$

$ICOR_a = 3$

$ICOR_p = 4$

$ICOR_i = 2$

$ICOR_B = 3$

$PQ_{a_t} = 70$

$PQ_i = 100$

$PQ_p = 30$

$PQ_B = 50$

$W_a = 1.04$

$W_p = 1.06$

$W_b = 1.06$

$Y^d_{t-1} = 230$

$T = 50$

$E_x = 20$

$M_a = .03 [Y^d_{t-1} - T]$

$M_i = .4 [I_t]$

$I_a = 1.5 [pQ_{a_t} - pQ_{a_{t-1}}] + 1.5 [pQ_{a_{t-1}} - pQ_{a_{t-2}}] + e$

$pQ_{a_{t-1}} = 68$

$pQ_{a_{t-2}} = 66$