THE TRADE-OFF BETWEEN INFLATION AND ECONOMIC ACTIVITY IN BRAZIL

Alexis Maka¹
Fernando de Holanda Barbosa²

¹ Senior Economist at Diretoria de Estudos e Políticas Macroeconômicas (Dimac)/Ipea.
² Professor at Fundação Getulio Vargas (FGV)/EPGE.
DISCUSSION PAPER

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JEL: E12, E31, E32.
ABSTRACT

This paper contrasts empirically four leading models of inflation dynamics – the accelerationist Phillips curve (APC), the new Keynesian Phillips curve (NKPC), the hybrid Phillips curve (HPC), and the sticky information Phillips curve (SIPC). We employ an encompassing Phillips curve specification that allows us to derive tests for these models within a single framework. According to the generalized method of moments (GMM) estimator, the evidence suggests that the restrictions implied by the NKPC, HPC and SIPC are rejected for the period after the Real Plan in Brazil. Only the restrictions implied by the APC are not rejected. However, when we construct confidence regions that are robust to weak instruments, it is not possible to reject any of the Phillips curve specifications, including the NKPC.

Keywords: Phillips curves; weak instruments; fully robust confidence regions.
1 INTRODUCTION

The Phillips curve has been playing a central role in policymakers’ understanding of the macroeconomy and the formulation of monetary policy. It is not surprising then that empirical challenges in estimating a Phillips curve relationship have been closely intertwined with challenges in conducting monetary policy. Much work has been done, both theoretically and empirically, since Phillips’s seminal 1958 paper. Yet economists have not converged to a widely agreed specification that is satisfactory both from the theoretical and empirical standpoints.

This paper contrasts empirically four leading models of inflation dynamics – the accelerationist Phillips curve (APC), the new Keynesian Phillips curve (NKPC), the hybrid Phillips curve (HPC), and the sticky information Phillips curve (SIPC). Our method of testing the Phillips curves is different from the approaches taken by previous studies of the inflation dynamics in Brazil because it is based on an alternative specification of this curve that encompasses the APC, NKPC, HPC and SIPC.¹ This encompassing specification has the advantage of reducing part of the huge specification uncertainty surrounding the Phillips curve by making it possible to test each of these alternative specifications within a single framework. Furthermore, to draw inferences about the parameters we use methods that are robust to weak instruments.

To the best of our knowledge, none of the studies that have employed the generalized method of moments (GMM) to estimate the Phillips curve for Brazil have used identification-robust methods, making their results unreliable according to the weak instruments literature (e.g., Stock, Wright and Yogo, 2002; Andrews and Stock, 2005).

The evidence from using the GMM estimator suggests that the NKPC, HPC and SIPC are not consistent with data for Brazil after the Real Plan. Only the APC is consistent with these data. However, when we construct confidence regions that are robust to weak instruments, it is not possible to reject any of the Phillips curve specifications, including the NKPC. This happens because the GMM confidence regions understate the sampling uncertainty, compared to regions that are robust to weak instruments.

¹ For a survey on the studies about the Phillips curve in the Brazilian economy, see Sachsida (2013).
The paper is organized as follows. Section 2 presents the encompassing Phillips curve for the open economy (EPCOE) and shows how different Phillips curve specifications considered in the literature can be seen as special cases of the EPCOE. Section 3 tests for Brazil the restrictions implied by different Phillips curve specifications using the GMM approach on a quarterly sample from 1996 to 2015. Section 4 discusses the issue of weak instruments. Finally, section 5 brings the concluding remarks.

2 AN ENCOMPASSING PHILLIPS CURVE FOR THE OPEN ECONOMY

The exchange rate is important in the study of inflation dynamics in open economies because it allows additional channels for the transmission of monetary policy. In an open economy, the real exchange rate will affect the relative price between domestic and foreign goods, which, in turn, will affect both domestic and foreign demand for domestic goods, and hence contribute to the aggregate-demand channel for the transmission of monetary policy. There is also a direct exchange rate channel for the transmission of monetary policy to inflation, in that the exchange rate affects domestic currency prices of imported final goods, which enter the consumer price index (CPI) and hence CPI inflation. Finally, there is an additional exchange rate channel to inflation: the exchange rate will affect the domestic currency prices of imported intermediate inputs, affecting the cost of domestically produced goods and hence domestic inflation (inflation in the prices of domestically produced goods).

In this paper, we extend the encompassing Phillips curve (EPC) proposed by Maka and Barbosa (2017) by including the level of the real exchange rate gap \( q_t \) in order to capture the importance of the exchange rate for inflation dynamics in an open economy:

\[
\Delta \pi_t = \alpha_1 \Delta \pi_{t-1} + \alpha_2 \Delta \pi_{t-2} + \beta_0 \Delta x_t + \gamma_1 x_{t-1} + \delta_1 \pi_{t-1} + \xi_0 q_t + \varepsilon_t,
\]

where \( \pi \) is the inflation rate, \( x \) is a measure of inflation pressure (output gap or marginal cost), \( \Delta z_t \equiv z_t - z_{t-1} \) is the rate of change of variable \( z \) and \( \varepsilon_t \) is an error term which can be correlated with the explanatory variables. The encompassing Phillips

2. The level term of the real exchange rate gap can be interpreted as capturing the presence of intermediate imported goods as in McCallum and Nelson (2000). They provide a detailed development of an open economy model that is based explicitly on the decisions of the agents in the model. Alternative ways of introducing the real exchange rate gap, like using its change (\( \Delta q_t \)) instead of its level (\( q_t \)), do not change our conclusions.
curve for the open economy (EPCOE) – equation 1 – is a model of inflation dynamics general enough to encompass the APC, NKPC, HPC and the SIPC as special cases.

Table 1 contains the implications of each Phillips curve to the signs of the coefficients of the EPCOE. The EPCOE provides a simple set-up to test the role of expectations in the inflation process.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\beta_0$</th>
<th>$\gamma_{1,1}$</th>
<th>$\delta_1$</th>
<th>$\zeta_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>APC</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>-,-,0 or +</td>
</tr>
<tr>
<td>NKPC</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+</td>
<td>-,-,0 or +</td>
</tr>
<tr>
<td>HPC</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>+ or 0</td>
<td>-,-,0 or +</td>
</tr>
<tr>
<td>SIPC</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>-,-,0 or +</td>
</tr>
</tbody>
</table>

### 3 EMPIRICAL EVIDENCE: BRAZIL (1996Q1-2015Q3)

The sample goes from 1996Q1 to 2015Q3, the period following the Real Plan. Graphs 1-3 plot the inflation rate, output gap and real exchange rate gap.

Our empirical analysis starts with the EPCOE [equation (1)]. When the EPCOE has forward-looking elements as in the case of the NKPC and HPC, the error term becomes a function of $\varepsilon_{t-1}$, which makes the error term correlated with $\pi_{t-1}$ and $\Delta\pi_t$, by construction. A solution to the endogeneity problem lies in the use of generalized method of moments (GMM) estimators. In this article we use the continuously updated (CU) GMM estimator whose estimates are independent of any normalization applied to the data.

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3. See Maka and Barbosa (2017) to see how one can derive the different Phillips curve models by restricting the coefficients of equation (1).
4. The Data Appendix gives details on the definitions of the variables employed in the estimations.
Table 2 reports CU-GMM estimates of the open-economy EPCOE on the 1996Q1-2015Q3 sample. According to table 2 the coefficients of both lagged inflation acceleration terms are negative within the 95% confidence interval, while the coefficients of the change in output gap and the output gap lag are positive. The coefficient of the inflation lag is not significantly different from zero. Using equation (1) and comparing the signs of tables 1 and 2 we observe that none of the restrictions implied by the NKPC are verified. Only two restrictions implied by the HPC are valid ($\alpha_1 > 0$ and $\delta_1 = 0$). Three of the restrictions implied by the SIPC are accepted ($\beta_0, \gamma_1 > 0$ and $\delta_1 = 0$) and two are not ($\alpha_1, \alpha_2 = 0$). All restrictions implied by the APC are accepted with the exception of the sign of the coefficient of $\Delta \pi_{t-1}$, which according to table 1 should be negative ($\alpha_1 < 0$). However, it is possible to show that the APC model is consistent with $\alpha_1 > 0$ and $\alpha_2 < 0$. Consider the APC given by

$$\pi_t = (1 + \omega_1) \pi_{t-1} + \omega_2 \pi_{t-2} + \omega_3 \pi_{t-3} + \kappa_0 x_t + \kappa_1 x_{t-1} + \varepsilon_t,$$

where $(1 + \omega_1) + \omega_2 + \omega_3 = 1$ and $\omega_1, \omega_3 > 0$.

In this model the coefficient of $\pi_{t-1}$ is an overshooting mechanism that allows cyclical fluctuation of inflation in its dynamic adjustment toward the equilibrium, in the spirit of Friedman (1971). This equation can be rewritten as

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where $(1 + \omega_1) + \omega_2 + \omega_3 = 1$ and $\omega_1, \omega_3 > 0$.

In this model the coefficient of $\pi_{t-1}$ is an overshooting mechanism that allows cyclical fluctuation of inflation in its dynamic adjustment toward the equilibrium, in the spirit of Friedman (1971). This equation can be rewritten as
\[ \Delta \pi_t = \omega_1 \Delta \pi_{t-1} - \omega_3 \Delta \pi_{t-2} + \kappa_0 \Delta x_t + \kappa_1 x_{t-1} + \varepsilon_t, \]

since \((1 + \omega_1) + \omega_2 + \omega_3 = 1, \omega_1, \omega_3 > 0 \implies \omega_2 < 0.\]

Therefore, all restrictions implied by the APC are accepted \((\alpha_1 > 0, \alpha_2 < 0, \beta_0, \gamma_1 > 0 \text{ and } \delta_1 = 0).\) We conclude that only the APC model appears to be consistent with inflation dynamics in Brazil from 1996Q1 to 2015Q3.

### TABLE 2
**Open – economy encompassing Phillips curve: GMM estimates**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>P-value</th>
<th>5% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \pi_{t-1})</td>
<td>0.278</td>
<td>0.101</td>
<td>0.006</td>
<td>[0.079, 0.476]</td>
</tr>
<tr>
<td>(\Delta \pi_{t-2})</td>
<td>-0.508</td>
<td>0.087</td>
<td>0.000</td>
<td>[-0.680, -0.336]</td>
</tr>
<tr>
<td>(\Delta x_t)</td>
<td>0.204</td>
<td>0.095</td>
<td>0.033</td>
<td>[0.016, 0.391]</td>
</tr>
<tr>
<td>(x_{t-1})</td>
<td>0.135</td>
<td>0.055</td>
<td>0.014</td>
<td>[0.027, 0.243]</td>
</tr>
<tr>
<td>(\pi_{t-1})</td>
<td>-0.027</td>
<td>0.045</td>
<td>0.548</td>
<td>[-0.116, 0.061]</td>
</tr>
<tr>
<td>(q_t)</td>
<td>0.029</td>
<td>0.008</td>
<td>0.000</td>
<td>[0.013, 0.045]</td>
</tr>
</tbody>
</table>

Hansen J statistic (overidentification test of all instruments): 2.533 Chi-sq(2) P-value = 0.281.
Instrumented variables: \(\pi_{t-1}, \Delta \pi_{t-1}, \Delta x_t\)
Included instruments: \(\Delta \pi_{t-2}, x_{t-1}\)
Excluded instruments: \(\pi_{t-2}, \Delta \pi_{t-3}, \Delta x_{t-1}, \Delta x_{t-2}, x_{t-3}\)

### 4 WEAK INSTRUMENTS

However, in order to be valid, the set of instruments chosen must satisfy two statistical conditions. First, each instrument must be uncorrelated with the error term (instrument exogeneity). Second, an instrument must be highly correlated with that portion of the endogenous regressors that cannot be explained by the other instruments (instrument relevance). Despite the fact that both criteria are necessary for an instrument to be valid, the studies that have employed the generalized method of moments (GMM) to estimate the Phillips curve for Brazil – mostly them concerned only with the NKPC – have ignored the issue of instrument relevance and instead focused solely on instrument exogeneity. When the instruments are only weakly correlated with the endogenous regressors, we have what is known as weak instruments or weak identification. Weak instruments pose considerable challenges to inference with GMM.
methods. If instruments are weak, then the sampling distributions of GMM statistics are in general non-normal, and standard GMM point estimates, hypothesis tests, and confidence intervals are unreliable.

4.1 Approaches to inference with weak instruments: detecting weak instruments

One approach to deal with weak instruments is to conduct tests of underidentification and weak identification.\(^6\) The first diagnostic tool for assessing the strength of identification is based on a LaGrange-Multiplier (LM) test for underidentification using the Kleibergen and Paap (2006) \(rk\) statistic (see table 3). We cannot reject the hypothesis that the model is underidentified. The second set of diagnostics is based on the Stock and Yogo (2005) characterization of weak instruments using the Kleibergen-Paap Wald statistic (see table 4).\(^7\) As the test statistic is less than the critical value tabulated by Stock and Yogo, we do not reject the hypothesis that the instruments used are weak.

<table>
<thead>
<tr>
<th>TABLE 3</th>
<th>Underidentification test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kleibergen-Paap (rk) LM statistic: 4.851 Chi-sq(3) P-value = 0.183 (underidentified)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE 4</th>
<th>Weak identification test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kleibergen-Paap (rk) Wald statistic: 5.018 (equation is not weakly identified)</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Approaches to inference with weak instruments: fully robust confidence regions

The weak instruments literature (e.g., Andrews and Stock, 2005; Kleiberger and Mavroeidis, 2009) has shown that using conventional inference methods after pretesting for identification is both unreliable and unnecessary. A better approach is to construct confidence regions that are fully robust to weak instruments.

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6. See Baum et al. (2007).
7. The Kleibergen-Paap Wald statistic correspond to the heteroscedasticity-robust multivariate analogue to the first-stage F statistic.
In order to conduct inference on the parameters of the EPCOE we use methods that are robust to weak instruments in the sense that identification of the coefficients is not assumed. This is in contrast to the conventional IV/GMM method, where the validity of tests of estimated coefficients requires the assumption that they are identified. We construct fully robust confidence regions by inverting the conditional likelihood ratio (CLR) test of Moreira (2003). Moreira’s test overcomes the distortions of standard tests by adjusting the critical values for hypothesis tests from sample to sample so that, for given data, the critical values used yield a correct significance level. Thus, his critical values are “conditioned” on the data at hand, not constant. The projection-based confidence regions are obtained by grid search over the parameter space and are centered around the point estimates from the continuously updated GMM estimator, with width set as a multiple of the Wald confidence interval.\(^8\)

Table 5 shows the projection-based CLR confidence sets for the baseline EPCOE model where there are three endogenous regressors \((\pi_{t-1}, \Delta \pi_{t-1} \text{ and } \Delta x_t)\) and two exogenous regressors \(\Delta \pi_{t-2}\) and \(x_{t-1}\). Figures 4–6 display the scatter plots for the 2-dimensional confidence regions.\(^9\) The results are consistent both with the view that price setting is purely backward-looking, as well as with the view that forward-looking expectations are very important in price setting. Furthermore, we do not reject the NKPC, in contrast with the findings of the previous approach, which is not robust to weak instruments.

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8. To construct the fully robust confidence regions we employ the Stata routine weakiv that can estimate models with any number of endogenous regressors. See Finlay, Magnusson and Schaffer (2013).
9. The confidence regions are estimated over \(8^{8}=32768\) grid points.


**GRAPH 4**
CLR robust confidence region for $\Delta \pi_{t-1}$ and $\Delta \pi_{t-2}$.

**TABLE 5**
Projection-based inference

<table>
<thead>
<tr>
<th></th>
<th>95% CLR Confidence set</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \pi_{t-1}$</td>
<td>[-2.446, 2.887]</td>
</tr>
<tr>
<td>$\Delta \pi_{t-2}$</td>
<td>[-2.378, 1.377]</td>
</tr>
<tr>
<td>$\Delta x_t$</td>
<td>[-2.444, 2.946]</td>
</tr>
<tr>
<td>$x_{t-1}$</td>
<td>[-0.976, 1.385]</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>[-1.070, 1.011]</td>
</tr>
</tbody>
</table>
5 CONCLUDING REMARKS

This paper contrasted empirically four leading models of inflation dynamics using an encompassing specification that allowed us to derive tests for each model within a single framework. When we used the GMM estimator, only the restrictions implied by the
APC were not rejected. However, according to the weak instruments literature the usual GMM confidence intervals are unreliable due to the problem of weak instruments. When we used methods that are robust to the issue of weak instruments in GMM, the confidence regions became so wide that it was not possible to reject any models’ restrictions, meaning that the evidence is consistent with all four models of inflation dynamics. These confidence regions were constructed using projection-based methods, which are very conservative, especially when many dimensions of the structural parameter vector are projected out.\textsuperscript{10} To the best of our knowledge, there is currently no alternative way to making fully robust inference with weak instruments with more than one endogenous variable, as in our case. As pointed out by Mikusheva (2010), this seems extremely difficult to do. Nonetheless, we hope this will become a topic of research for those working at the frontier of inference with weak instruments.

At this point, micro-founded versions of the Phillips curve can be viewed as complementary to standard backward-looking specifications. So far, there is little evidence suggesting that forward-looking Phillips curve specifications provide more accurate inflation forecasts than a standard backward-looking specification.

As a result, the traditional backward-looking specification, possibly augmented to account for supply shocks, continues to play a role in shaping the inflation outlook and the conduct of monetary policy. Still, the importance of expected future inflation for determining current inflation is finding its way into the policy discourse.

REFERENCES


\textsuperscript{10} It is conservative in the sense that it has asymptotic size less than or equal to nominal size. Intuitively, whereas a standard correctly-sized test will commit a Type I error 5\% of the time, a conservative projection-based test will commit a Type I error at most 5\% of the time.


APPENDIX

The inflation rate ($\pi_t$) is measured as the quarter-to-quarter change in the Consumer Price Index (IPCA), where the quarterly inflation rates are calculated by arithmetic averaging of the monthly series.

The output gap ($x_t$) is given by 100 times the log of the quarterly real GDP seasonally adjusted, detrended by the HP filter.

The real exchange rate gap ($q_t$) is calculated as 100 times the log of the arithmetic average of monthly indexes of real effective exchange rates, detrended by the HP filter.
The manuscripts in languages other than Portuguese published herein have not been proofread.
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Enhance public policies that are essential to Brazilian development by producing and disseminating knowledge and by advising the state in its strategic decisions.