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THE MARKET VALUE OF PUBLIC EDUCATION

Sergei Soares

DISCUSSION PAPER



THE MARKET VALUE OF PUBLIC EDUCATION

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ABSTRACT

This article estimates the market value of public education by comparing standardized test scores of students in public and private schools. The idea is to assign to the education of each public school student a market value equivalent to the tuition paid by private school students with similar test score results. The implementation requires an expenditure survey, or other database to provide information on tuitions, and standardized test scores available for both private and public schools. This article uses Brazilian test score data, which are particularly good. The main results are not surprising. Pre-school, primary, and secondary education are all highly progressive government transfers. Furthermore, since their market value is superior to public expenditures in each of these educational levels, they are also welfare enhancing. The flip side is that public higher education is both highly regressive and welfare reducing.

Keywords: public education; value of education; educational proficiency.

1 INTRODUCTION

Public education is generalized fact of modern life. While there are still many children out of school on this planet, according to UNESCO worldwide school attendance is on the order of 91% for primary and 83% for secondary education. This is both an important public expenditure and a relevant in-kind transfer, often to the poorest households. It is important to value this public effort adequately.

By far the most common approach in recent times has been their valuation according to their cost to the public sector. An OECD (2008) report on income distribution leaves this clear:

“Imputation of public educational expenditures to individuals based on actual use requires, first, determining whether or not an individual is participating in different levels of the educational system; and second, increasing the income of the household where they live by the average public spending per student at the relevant educational level.”

The majority of the recent literature, such Atkinson (2005), follows this approach, which is to take how much it cost the state to provide the educational services and split it up evenly among the families with children in the public education system.

The popularity of this approach, however, is paradoxical. While it has to commend it the fact that it does not change the size of the welfare cake, it also has various shortcomings. First, it assumes that all students are receiving the same public education. This is clearly not the case even if the state spends the same on each student, which is usually not the case. Secondly, it flies in the face of the theory of provision of public goods (Samuelson, 1955), according to which the welfare value of a public good is the sum of the marginal utilities of all its users (as opposed to the simple marginal utility in the case of private goods). There are alternatives to this approach.

In this article, I will match educational proficiency data, assumed to represent well the quality of education received by a given student, with expenditure data on private education. This will yield the market value of private education at a given proficiency level. If each student in a public school can be matched to a student in a private school with the same proficiency score estimated in the same standardized test, then the market value of that student’s public education is the same as the tuition paid for by the private student with the same proficiency. For this I need proficiency and tuition data for the same sample of students.

With the generalization of national, regional, and global Standardized Tests, the estimation of the quality of instruction at some level of education has become possible for the majority of countries in the world. Apart from some very small, very poor, or failed states, almost all countries in the world have some kind of standardized test available with some kind of periodicity. Access to price data on private education can be found in either Household Expenditure Surveys or Living Standards Measurement Surveys. This means that the major hurdle to be overcome is one of matching price data from Household Surveys to school quality data from Standardized Tests.

Unfortunately, few, if any, Household Expenditure Surveys count on identifiers that can be mapped onto Standardized Tests. This means that in order to match the two, the smallest possible cell that will allow matching between Expenditure Surveys and Standardized Tests must be found. They will possibly be a combination of regional, urban/rural, and parents' education variables or whatever other set can be found in both databases. Household Expenditure Surveys or Living Standards Measurement Surveys almost always include these variables, as do many standardized tests in the socio-economic questionnaire (PISA, for example). Index these matching groups by k .

By matching test proficiency data from the k -th group in a given household survey with expenditure data from a Household Expenditure Survey, the two pieces of information needed to value education at market prices will be in the same database. If what people pay for when they pay for education is learning (as measured by test proficiency), the prices paid to private schools can be matched for a given number of proficiency points measured on whatever scale being used. Once the price in dollars (or other currency unit) per proficiency point is available, it is easy to impute this same value to free public education provided by the state, thus calculating its market value.

There are at least two approaches on how to do this. The first, which I call the naïve approach, is to use the imputation described above as is with no correction. I will calculate this in section 6 of this paper. The second is to consider that schools are only one of two crucial inputs producing learning. The other input is the families themselves. This means that a proficiency indicator net of family effects must be found. This is done in section 7 of this paper.

The remainder of this paper is divided up as follows. Section 2, immediately below, describes all the Brazilian data used in order to carry out the estimation described above. Section 3 describes the matching process, which is key to getting good results

and the fundamental step in the whole methodology. Section 4 and 5 succinctly describe the relation between tuition and income schools and between proficiency and tuition in private. Sections 6 and 7 finally estimate the market value of public education using what I call the naïve approach and the net of family inputs approach, respectively. Section 8 concludes.

2 DATA

In this effort to ascribe a monetary value to education, as close as possible to that provided by markets, two types of data are needed. These are data on educational expenditures and on educational proficiency as measured by standardized tests. What are the data sources for each in Brazil?

For educational proficiency as measured by standardized tests, three sources exist. The first is the *Sistema de Avaliação do Ensino Básico* (SAEB)/*Prova Brasil*, which tests a sample of fourth and eighth graders (it also tests secondary schoolers in their last year, but I will not be using this data here). The second is the *Exame Nacional do Ensino Médio* (ENEM), which tests almost all students finishing secondary school. The *Exame Nacional de Cursos* (ENC), or *Prova*, tests a sample of higher education students in different subject areas as they conclude their last year in universities and other higher education centers. All three are carried out by the *Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira* (INEP), the national educational statistical institute. While all are relatively high quality tests first implemented in the 1990s, they have also undergone major changes since then.

Brazil's current primary and secondary school evaluation system was created in 1995 under the name *Sistema de Avaliação do Ensino Básico* (SAEB) and has provided proficiency information for students every two years ever since. From 1995 to 2005, a sample of children in fourth grade was tested in Portuguese and Mathematics and a sample in eighth grade was tested in Portuguese, Mathematics, and Science. Testing time is towards the end of the school year, in November. The tests covered a sample of both private and public schools and only some very small rural schools were left out of the sampling scheme.

In 2007, however, INEP became much more ambitious and decided to test every public school in Brazil. This is when SAEB changed its name and became *Prova Brasil*. Needless to say, going from a sample of about four thousand classrooms to about a

hundred thousand classrooms provided considerable logistical difficulties and in 2007 and 2009 INEP concentrated on the public schools and although some private schools were tested, the results are not good and the data is not publicly available. In 2011 and 2013 private schools were back in the sample. Today we have all urban public schools and a sample of private and rural public schools tested. School proficiency is calculated using Item Response Theory and all tests are on the same scale, which means that learning can be compared across both years and grade levels. For 2003, the sample sizes were 8096 students for fourth grade and 5590 students for eighth grade. SEAB/*Prova Brasil* are relatively low stakes tests. While low test scores scare parents, and generate discussion in the school community, there is no money attached. For students, it is anonymous and really very low stakes.

The ENEM has a slightly more turbulent history. It was created in 1998 as a voluntary small scale test largely modelled upon the Programme for International Student Assessment (PISA) tests. Enrollment fees made it a largely elite test taken mostly by rich students and ENEM enrollment was on the order of 20% of each cohort. In 2001 the enrollment fees were dropped and ENEM became necessary to obtain student loans. Final year secondary students started participating in mass and today ENEM enrollment is close to universal, with some attrition, among final year secondary school students. In 2003 1,330,832 last year students enrolled in ENEM and 936,686 actually took the test. In addition, another 551,561 individuals who were either students in previous years taking the test for practice or graduates seeking student loans or admission to higher education also enrolled and 385,959 actually took it. While the tests are of excellent quality proficiency levels are not comparable across time since anchor questions are not used and only after 2010 has Item Response Theory been used to grade the ENEM tests. Testing is in December.

In 2009 the President of INEP reached an obvious conclusion – higher education admission would be much better served with a single national test than with the multiple institution-specific tests called *vestibulares*. The problem is that Brazil has a long tradition of content-heavy *vestibulares* which are more about memorization than reasoning and the university community resisted exchanging them for the competence-based ENEM. The result was a compromise in which the ENEM itself became much more content-heavy (and, in my opinion, a much worse test). ENEM today is very high stakes test. Students get to go to college or not depending largely on their ENEM score. Access to scholarships is based upon ENEM. The test has become the most important marking tool for private schools. This makes private schools and INEP run a cat and mouse game

over ENEM scores. Private schools select the best students from other grades and make them take the tests, they create fictitious schools within schools in which they “enroll” their best students, and use a variety of other schemes to increase their ENEM scores. ENEM questions have been stolen and sold for high prices on multiple occasions. INEP on its part today treats ENEM like a military operation shrouded in secrecy and applies various corrections in order to keep private schools from gaming the system, but private schools catch on to each correction and then find a way to game them. Today ENEM is far removed from the PISA-inspired evaluation focused on reasoning and application of knowledge and looks more like a 1950s test which measures rote memorization. The year I will be using, however, is 2003, for which ENEM data are great – near universal but still relatively low stakes.

The story of the *Exame Nacional de Cursos* (ENC) is almost as turbulent as ENEM’s. The ENC, or *Prova*, was born more or less at the same time as SAEB and ENEM in the effort to create a national evaluation system for education. The first ENC was in 1997. Due to poor relations between the government and the academic community at the time, however, the ENC always faced strong resistance. There were boycotts, articles in the press denouncing the ENC, and marches against the test. This resistance came mostly from the public universities and continued until the government changed in 2003. One of the first steps of the new government was exchange ENC for a new test, called *Exame Nacional de Desempenho de Estudantes* (ENADE). The ENADE was an improvement upon the ENC but most of all it was negotiated with the academic community. Today the ENADE is an accepted method for evaluating higher education. The last ENC is exactly that of 2003, which will be the one used here. One of the difficulties in using the ENC is that it is not composed of a single exam given to all students, but of separate exams, each given to those studying a different course. The test taken by students of engineering, for example, has no questions in common with the test taken by future lawyers.

For school characteristics, I use the *Censo Escolar*, or School Census. Brazil has been collecting data on schools, teachers, and students for almost a century. The first centralized data on school enrollment refers to 1932, and was published in 1939. From the forties to the eighties, the *Censo Escolar* underwent improvements but no disaggregated data existed. The data were aggregated at the state level and sent in municipal-level tables to the Ministry of Education. The legal basis for the school-level *Censo Escolar* came in 1996 with the *Lei de Diretrizes e Bases da Educação Nacional*, which organizes education in Brazil, established the need for disaggregated data collection on public and private education. The first *Censo Escolar* microdata slightly precede the law and

are from 1995, when INEP began to collect disaggregated data on schools, classrooms, teachers, and students. The first *Censo da Educação Superior*, which collects analogous information for higher education, also begins in 1995.

Both the *Censo Escolar* and the *Censo da Educação Superior* are collected annually. The collection dates are usually in May, when all the enrollments are more or less final. From 1995 to 2007, the smallest unit of the *Censo Escolar* is the school. From 2008 onwards, INEP began to collect information by student. This should allow for student trajectories to be analyzed once all the considerable matching problems have been ironed out. In 2003, the *Censo Escolar* was collected by school and the microdata are composed of 1853 variables which allow a wealth of information about schools, their teachers, and their students.

For educational expenditures, the obvious choice is an expenditure survey. The Brazilian *Pesquisa de Orçamentos Familiares* (POF) is collected every five years or so by the *Instituto Brasileiro de Geografia e Estatística* (IBGE). The collection period is about 18 months so the POFs are usually referred by IBGE using two years. There has been a very detailed expenditure survey called *Estudo Nacional da Despesa Familiar*, whose data was collected from 1974 to 1975, and there have been POFs covering only the metropolitan areas with data collected in 1987-1988 and 1995-1996. There have been two POFs with national coverage, the 2002-2003 POF and the 2008-2009 POF. As of writing, the 2015-2016 POF is finishing its data collection phase, but the microdata will not be available until, at the earliest, mid-2017. The 2003 POF sample is composed of 48,470 households in which 182,333 people live. The survey is carried out from 2002 to 2003.

Since no private school data for 2009 exists due to the implementation of the universal coverage of the *Prova Brasil* and the 2016 POF data are not yet ready, there is no choice but to use 2003 as the year for matching data.

Finally, a word on incomes. Only POF has reliable income questions. As an expenditure survey, POF allows researchers to calculate different incomes. The income we will use here is the most convenient: gross income, which is income from all sources including government transfers, but not taking into account direct taxes on income. In order to make sure the results are robust with regards to the type of income used, we will also calculate them using market income (with no government transfers but social security) and disposable income, where taxes on income (bar social security taxes) are excluded. The results do not change much.

3 MATCHING

Whatever approach is used to relate public and private student proficiency data, the first step is to match Standardized tests with Expenditure Surveys. In the case of Brazil, educational expenditures can be found in the POF and learning assessments in SEAB (for primary education), ENEM (for end of secondary), and ENC. Unfortunately, only the ENEM is identified. This means individuals cannot be exactly matched between the various surveys and thus cell or group matching is the only solution. By cell matching I mean building k groups, which are as narrowly defined as possible, and ascribing to each group in the POF the average standardized grade for the same group in SEAB, ENEM, and ENC.

The smallest merge groups which can be created for POF and proficiency data are defined by the following variables: UF (the state in which people live); sex, grade, and age of the student; the type of household (headed by a male, a female, or both); and the type of school the child studies in (private or public), called in Brazil *Rede*. Socio-Economic Background can be ascertained by the presence of various durables such as television, freezer, car, and computer. Ideally, the education of parents should be used but since the SAEB socioeconomic data is self-reported by 10 and 14 year olds, more than a third do not know the highest level of education completed by their parents (ENEM test-takers do better but a single methodology is necessary). The variables that make up SAEB identification information (these are measured with no error) are state (UF), type of school (public or private), and grade level. All other information is reported by the children themselves and thus susceptible to some kind of reporting error. The matching variables chosen are those for which the reporting error, proxied by the number of children who report “do not know” for each question, is lowest.

Other than reporting error, which can be minimized using variables with low “do not know”, there are two dangers in matching using observed variables. The first is to aggregate too much by using few matching variables and have little variation. The second is to have matching variables that are too detailed and too many and thus have many observations that do not match. Table 1 shows both perils. If only the administrative variable measured with no error are used – UF (state) and type of (school public or private) all 5,264 fourth grade children in the POF can be matched but this will yield only 53 cells and the average number of observations per cell is a gigantic 849. At the other extreme, if using the full matching code – including also sex, years born, computer,

automobile and television – yields an average cell size of eight but loses 244 observations. Why do we lose them? Because as we increase the detail of the match code, there will be more and more cells that exist in POF but have no counterpart in SAEB, or vice-versa.

TABLE 1
Matching information for fourth grade

Variables making up match code	Number of categories	Average cell size	Largest cell	Nº of merges
Only UF, Rede	53	849	1,878	5,264
UF, Rede, Sex	106	425	949	5,264
UF, Rede, Sex, YOB	696	65	587	5,213
UF, Rede, Sex, YOB, Computer	1,249	36	421	5,196
UF, Rede, Sex, YOB, Computer, Car	3,086	15	250	5,138
UF, Rede, Sex, YOB, Computer, Car, TV	5,744	8	178	5,020

Source: POF; Prova Brasil (2003).

An easy way to circumnavigate the problem is to use the full code and for those observations that do not match fall back upon the average cell proficiency of the less detailed code. For example, for the 118 observations that lose their match when we introduce the variable *TV* into the match code (represented by the last line in table 1), I use the proficiency values matched without television (but with UF, type of school, sex, year born, computer, and automobile). Likewise, the introduction of *Car* loses 58 matches relative to the previous line. By going backwards until the full 5,264 observations are matched and no child in POF is left without a proficiency.

Table 2 shows the same data for eighth grade. While there are fewer students, the story is more or less the same.

TABLE 2
Matching information for eighth grade

Variables making up match code	Number of categories	Average cell size	Largest cell	Nº of merges
Only UF, Rede	54	683	1,555	3,871
UF, Rede, Sex	108	342	825	3,871
UF, Rede, Sex, DOB	831	44	452	3,839
UF, Rede, Sex, DOB, Computer	1,475	25	314	3,805
UF, Rede, Sex, DOB, Computer, Car	3,707	10	183	3,711
UF, Rede, Sex, DOB, Computer, Car, TV	6,133	6	176	3,364

Source: POF; Prova Brasil (2003).

Finally, table 3 shows the same process using ENEM for the end of secondary school.

TABLE 3
Matching information for the end of secondary

Variables making up match code	Number of categories	Average cell size	Largest cell	Nº of merges
Only UF, Rede	54	16,205	198,119	2,945
UF, Rede, Sex	108	8,102	118,162	2,945
UF, Rede, Sex, YOB	3,393	258	56,262	2,832
UF, Rede, Sex, YOB, Computer	5,586	157	34,025	2,796
UF, Rede, Sex, YOB, Computer, Car	13,483	65	16,854	2,732
UF, Rede, Sex, YOB, Computer, Car, TV	25,305	35	10,017	2,654

Source: POF, ENEM (2003).

Since ENEM is almost a Census of individuals finishing secondary education and not a sample, the cell sizes should be larger, and this is effectively the case. There are more cells (since there are very few empty cells) and the average number of students in each cell is much larger. The number of successful matches with the POF is more or less the same, being limited mostly by POFs sample size.

In the case of higher education, things become slightly more complicated. The ENC has a separate test for each course so that the tests taken by engineering and social service students have no questions in common and cannot be compared. This would not be a problem if all courses were equally represented in private and public higher education. However, table 4 shows that this is not the case. While high academic prestige and high socio-economic status courses such as engineering, medicine, and the hard sciences (physics and chemistry) are under-represented in the private sector, low prestige and low SES courses such as journalism, administration, and law are over-represented.¹

TABLE 4
Odds ratio of being in a private university vs a public one

Course	Odds ratio	Course	Odds ratio
Physics	0.29	Pedagogy	0.93
Agronomy	0.34	Nursing	0.93
Geography	0.49	Odontology	1.00
Chemistry	0.59	Architecture	1.05
History	0.64	Science	1.13
Engineering	0.69	Journalism	1.20
Mathematics	0.70	Psychology	1.24
Medicine	0.70	Law	1.26
Literature	0.87	Administration	1.27
Pharmacy	0.88	Phono audiology	1.32
Economics	0.88		

Source: ENC (2003).

1. The alignment is far from perfect. Pedagogy is low prestige and slightly under-represented in the private sector, as are history or geography.

In order to resolve this issue, the influence of the course must be removed from the grade. This is not difficult: simply regress proficiency score on course dummies and then use the residuals.

Once the proficiency is free of course effects, the merge file with the matching groups must be created. In higher education, the matching group is created by the variables: UF, *Rede* (private or public school), year of birth, sex, family size, and income groups. These are not the same variables as for the basic education but variables such as computer, car and TV do not exist in the ENC data. Furthermore, the role of private education in higher education is different from its role in basic education. In higher education, private schools are in general lower quality schools for either the hard-working poor or the lazy rich.

TABLE 5
Matching information for the end of college

Variables making up match code	Number of categories	Average cell size	Largest cell	Nº of merges
Only UF, Rede, Sex	106	3,216	57,731	3,668
UF, Rede, Sex, YOB	1,156	295	18,892	3,666
UF, Rede, Sex, YOB, Household size	5,271	65	8,340	3,599
UF, Rede, Sex, YOB, HH size, Income	18,760	18	4,464	3,460

Source: POF; ENC (2003).

In any case, table 5 shows the matching process using the ENC data. It shows the same process as the previous matching processes. Number of categories is closer to that of ENEM but cell size is closer to that of SEAB. Once again all students found in POF are matched.

The next section will analyze the relations between income, tuition, and proficiency scores using the database created by the merges just described.

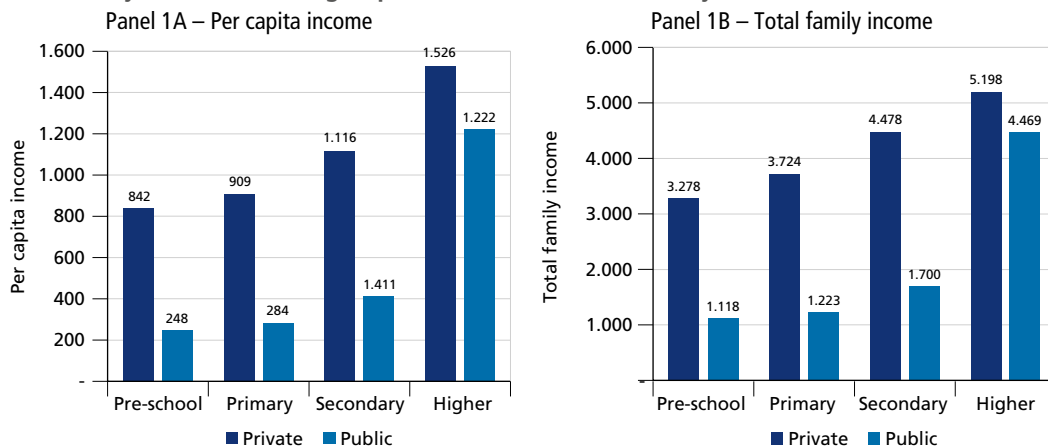
4 INCOMES AND TUITION

Once the perils of merging have been navigated, the next step is to calculate per capita incomes and per student educational expenditures, which is readily done using the POF. Figure 1 shows total family and per capita incomes for families with students in various education levels. Note that the categories are not exclusive. A family can, for

example, have children studying both in pre-school and in primary school. A family could even have some children in public school and others in private school. All values are in 2003 Reais.

Figure 1 results show, as expected, that families with children in private schools are richer than those whose children go to public school. The income difference for families with children in primary school is a factor of three. This large difference is because, with notable exceptions, Brazilian public primary schools are low quality schools and those upper class families who can afford it send their children to private school. The supply of high quality public schools increases somewhat in secondary education with the entry of high quality public technical schools and military schools into educational supply. In addition, many of the poorest students do not go beyond the end of primary, thereby increasing the average socio-economic status of the public secondary schools. The income difference between families with children in private and public school falls somewhat, but is still considerable at 2.6.

FIGURE 1
Family income according to presence of students in family



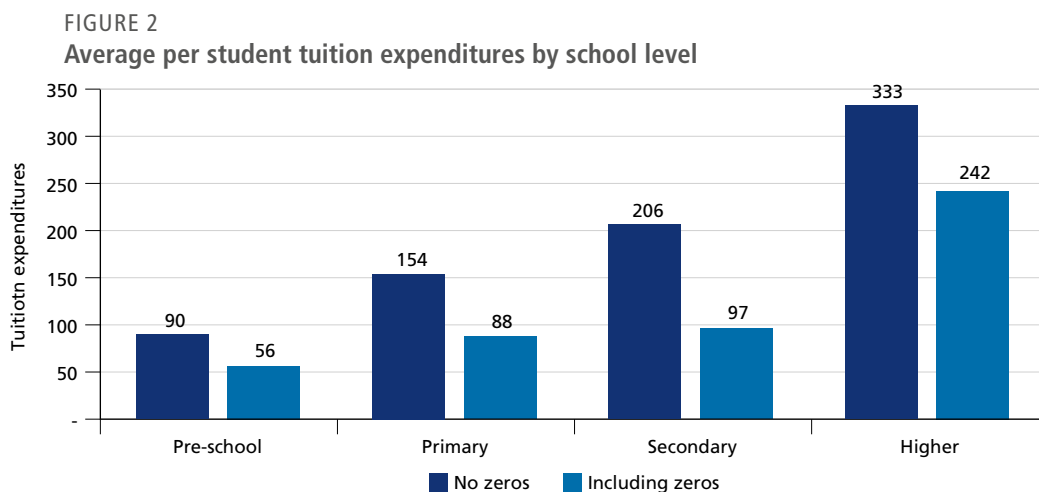
Source: POF.

For higher education the tables are turned. Private college families have incomes that are only 16% higher than those of public college families. This is both because of student selection that leaves children of poor families out of higher education altogether and because the supply of high quality public education increases dramatically and, again with notable exceptions, public universities are in general better than private ones.

What about school expenditures?

Figure 2 shows two numbers for monthly tuition expenditures. Those in black include zeros and those in blue do not. Zero tuition expenditures means that a family declared to the IBGE that they had children in private school but when asked how much they paid in tuition they declared they pay nothing. Some of these may be families with children on scholarships or whose tuition was paid by grandparents or other relatives, but undoubtedly the number also includes those who paid tuition but forgot how much and thus did not declare a value. I believe tuition with no zeros is the better indicator. In any market, zero prices for things that are not free are complicated to understand.

The increase in values is striking, but again, expected. Per student tuition for pre-primary school is almost four times smaller than for higher education.



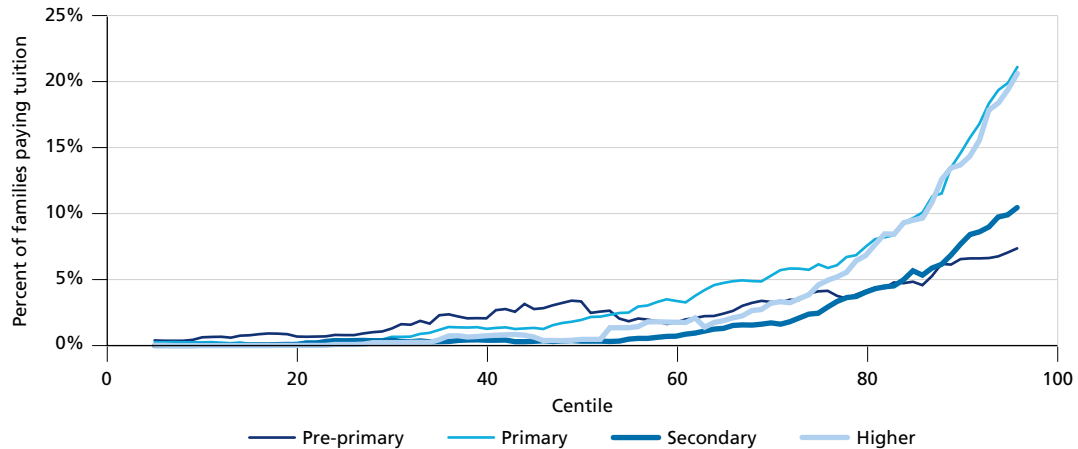
Source: POF.

The results also show that the average cost of sending a child to school varies from 10% of per capita family expenditures for pre-school to about 20% for higher education. The main result here is that we should use expenditure considering zeros as missing data and not zeros.

Figure 3 shows the percentage of families (weighted by number of family members) that pay nonzero tuition. All families, including those who have no children in school, are included in the denominator.

Below the 60th percentile, less than 5% of families pay any tuition at all. This is both because many families do not have school age children but mainly because of public education which accounts for about 90% of primary and 80% of secondary school enrollments.

FIGURE 3
Percent of families with tuition expenditures (weighted by family size)



Source: POF (2003).

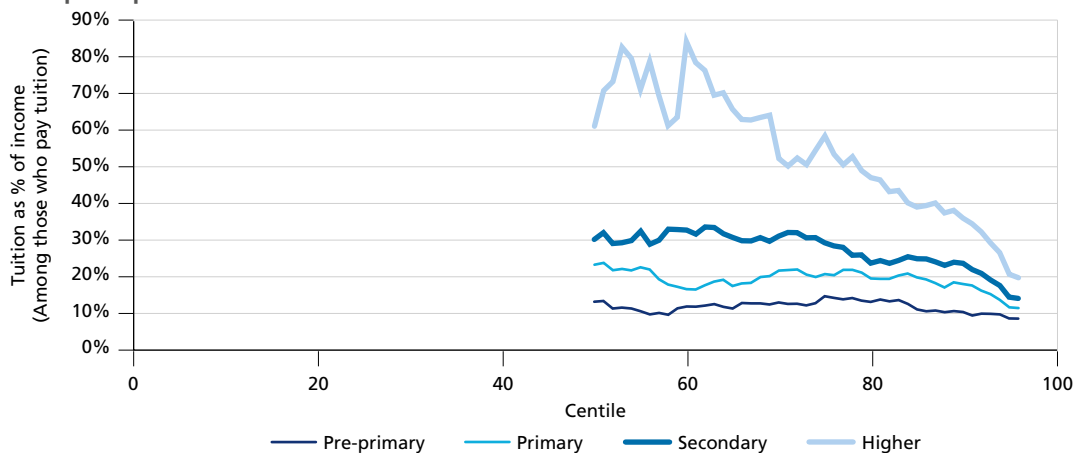
At the upper end of the income distribution about 20% of families pay for some kind of private education and about 20% pay for some kind of higher education. About 10% pay for private secondary education and about 7% for private pre-school.

Many factors explain the differences. One of them is the duration of each level of education. Primary education lasts eight years (today it is nine) as opposed to three years for secondary, so naturally a higher percentage of families will be paying for private primary than private secondary education. In the case of higher education, however, the number also reflects the prevalence of private supply: about 80% of enrollments in higher education are in private institutions.

Figure 4 shows per student tuition expenditures as a percentage of per capita income for those families who pay nonzero tuition. The results show a surprisingly large education burden for families paying for education, particularly higher education. For families that are not in the top decile, per capita tuition expenditures are slightly more than 10% of per capita income for pre-primary school, about 20% for primary, a little less than 30% for secondary and between 40% and 80% for higher education.

The very large numbers for higher education may be due to student loans and/or a large effort on the part of families in the 90 poorest centiles to send at least one member to college. While impressive, the numbers should be read with figure 3 in mind: less than 5% of families actually pay any higher education tuition at all.

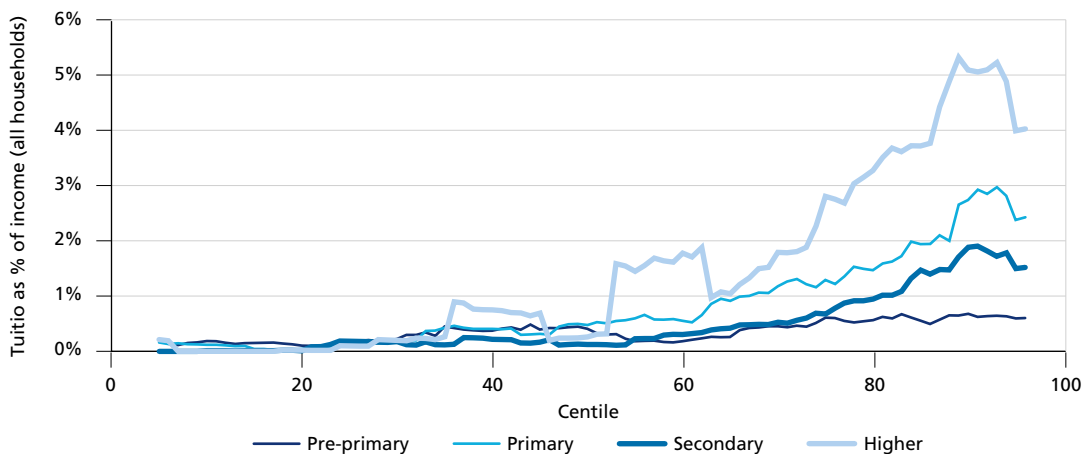
FIGURE 4
Average tuition expenditures (of families with positive expenditures) as a percentage of per capita income



Source: POF (2003).

Finally, figure 5 puts the two previous figures together. It shows the percent of total family expenditures allocated to tuition, including both families that pay it and those that do not.

FIGURE 5
Percent of budget allocated to tuition (weighted by family size)



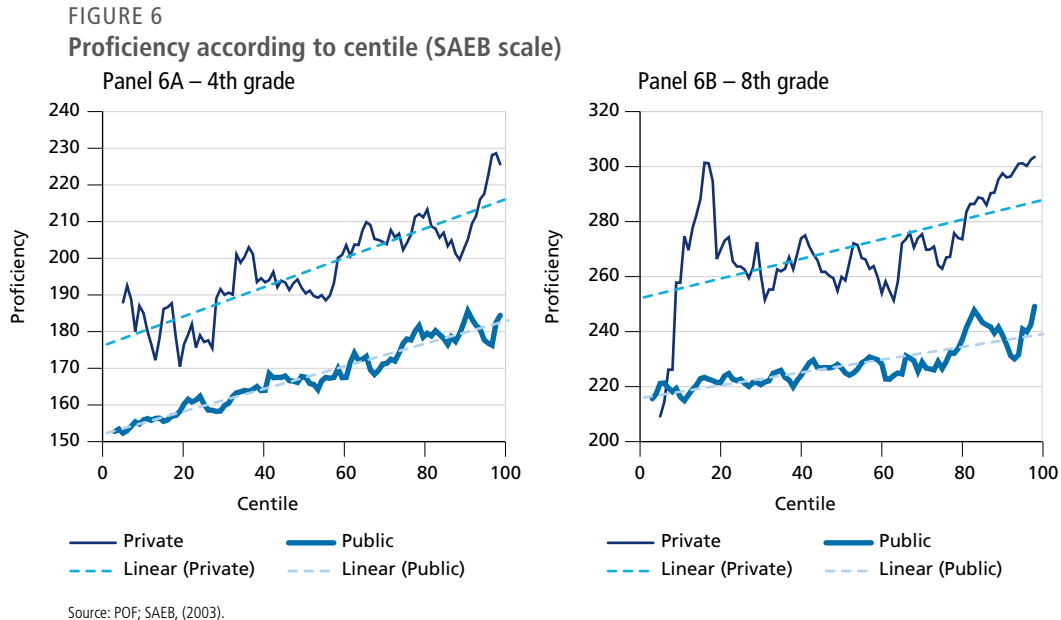
Source: POF (2003).

Almost all tuition expenditures are in the top two deciles, which is to be expected given figure 3. The figures also show that the heaviest investment is in higher education, followed by primary (due to the long duration of the course), then secondary and pre-primary.

The most important conclusion from this section is that education is a significant financial burden only to those in the upper quintile of the income distribution. Since almost everyone in Brazil attends primary school and a good 80% of the age cohort attend at least some secondary, this means that public education is a very important contribution of the state to families in the lower four deciles of the income distribution.

5 DISTRIBUTIVE ANALYSIS OF PROFICIENCY SCORES

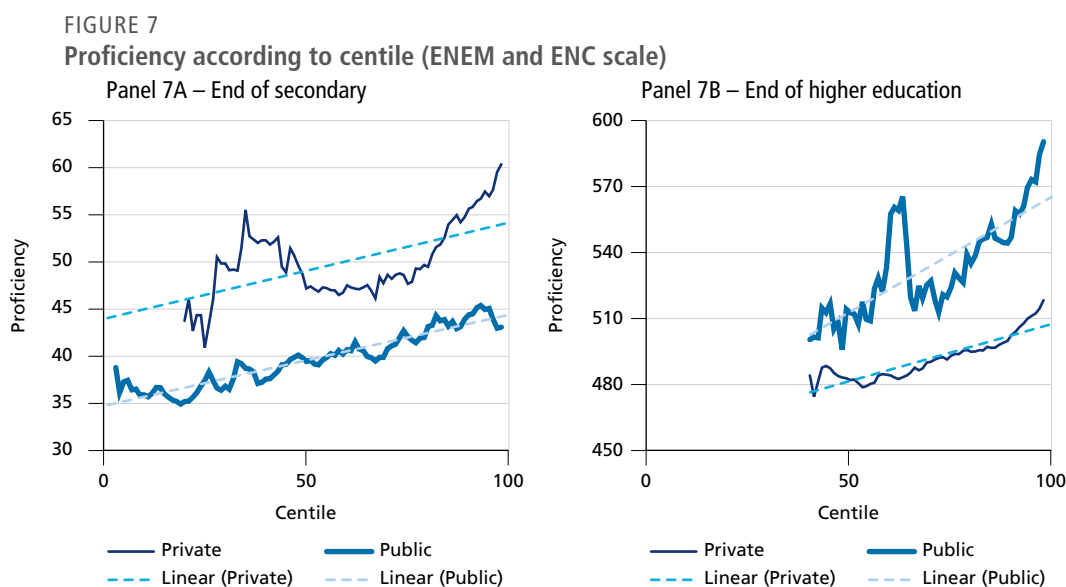
What about proficiency scores by centile? The two panels of figure 6 and figure 7 show proficiency as measured by standardized tests for grades 4, 8, and 11 according to centile of per capita income distribution.



The two panels of figure 6 show some expected results. The fact that the blue curves lie higher than the red ones shows that private schools have higher standardized

test results than public ones. The fact that the slope of the linear trend line for private schools is more inclined than that for public schools shows that money (or other variables correlated to income) can buy quality in both but that the relation is slightly stronger for private schools.

Panel 7A of figure 7 shows more or less the same results for the 11th grade. The ENEM proficiency scale is different, so the grades themselves are not comparable, but it does appear that the centile dependency of proficiency is less than in 4th and 8th grades.



Panel 7A of the figure above looks a lot like the two panels of figure 6: for secondary schooling private school students also score higher on standardized tests and the relation between income and proficiency is stronger in private schools. In panel 7B of figure 7, however, the roles are reversed. Public education is both better and more closely linked to income. This suggests that much of the positive relation between income and proficiency may be peer effects and family background rather than more or better educational inputs.

To see the relation between proficiency and type of schooling in more detail, table 4 shows the results of a regression in which centile explains test proficiency. As was already suggested in the Figures above, the coefficient of centile on income is always greater for

private schooling than for public schooling, but is always positive and significant for public schooling as well. Likewise, the constant shows that private schools have better test results even for the same income level.

TABLE 6
Relation between income centile and proficiency

Type of education	Centile coefficient	Constant
4th grade private (SAEB scale)	0.436	171.7
4th grade public (SAEB scale)	0.230	152.5
8th grade private (SAEB scale)	0.581	230.1
8th grade public (SAEB scale)	0.185	216.7
11th grade private (ENEM scale)	0.187	37.3
11th grade public (ENEM scale)	0.077	34.4
College private (ENC scale)	0.792	431.7
College public (ENC scale)	0.938	459.2

Source: POF; SAEB; ENEM; ENC (2003).

How do high income parents get better test results even in public schooling? One channel is that they are able to move to neighborhoods with better schools, as well as “capture” high quality public elementary education such as military schools and federal technical schools, for which access is often through competitive examination. The other, of course is that high income parents are usually high Socio Economic Status parents who directly impart proficiency to their children as well as provide better study environments at home. Peer effects are also potentially important and serve as a reinforcing factor. For private schools, in addition to the previous two channels, there is of course the direct channel that allows more expensive private schools to provide more and better educational inputs, particularly better teachers.

For higher education the tables are turned. Public schools exam results are better (as seen by the constant) but they are also more highly correlated with income, suggesting that selection is more important than inputs in defining quality of education.

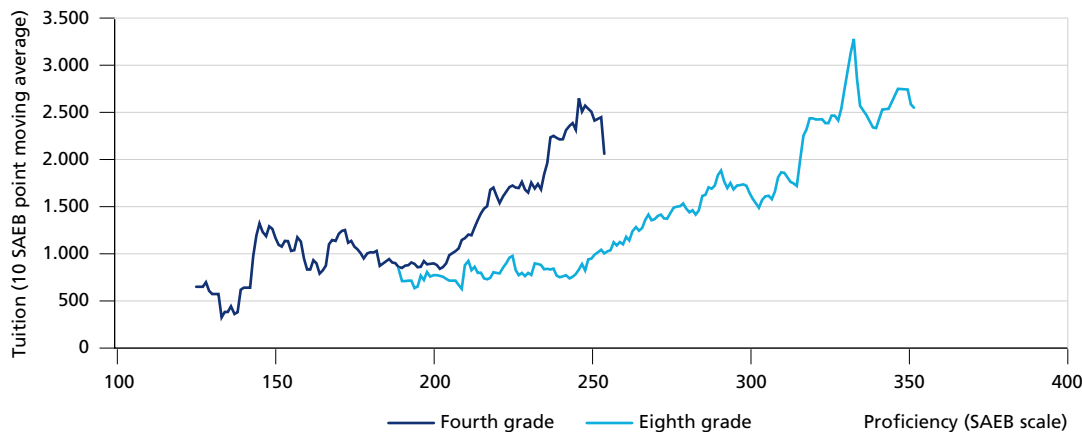
6 PRIVATE EDUCATION AT MARKET VALUE: THE NAÏVE APPROACH

Almost all that is needed to estimate the market value of public education is ready. Matching has produced a database containing both tuition (and income), and proficiency scores and the relations between income, tuition, and proficiency show nothing unexpected.

Subsection 4 showed that families of students in higher income centiles pay more tuition for private education than the families in lower income centiles. The previous subsection (subsection 5) shows that there is positive relation between income and education quality as measured by standardized test proficiency.

Given these two observations, it is to be expected that schools whose students score higher on standardized tests also charge higher tuition. Figures 8 and 9 show that there is, for the private sector, also a positive relation between how much schools charge and their students' proficiency scores.

FIGURE 8
Tuition according to proficiency for 4th and 8th grades (SAEB scale)



Source: SAEB (2003).

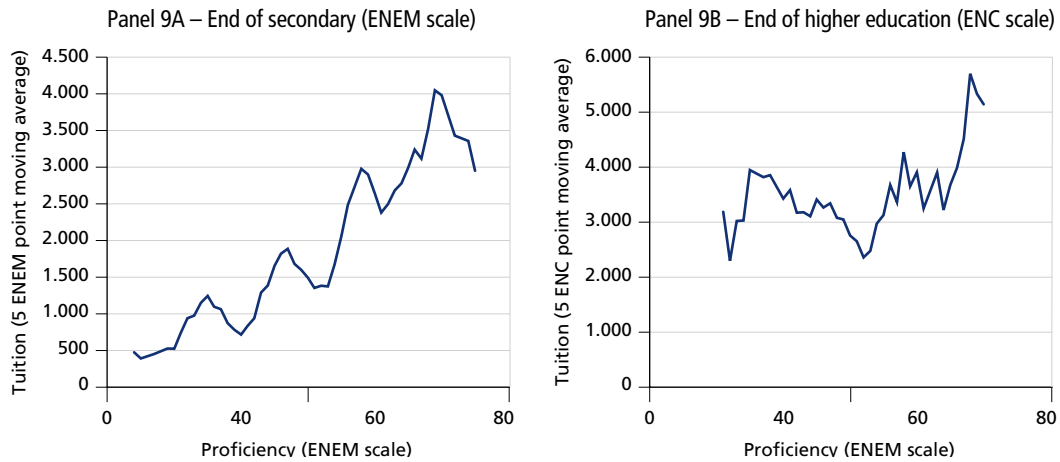
Luckily, figure 8 shows nothing strange: tuition increases with scores. Interpretation, of course, is not straightforward. This relation may mean that better schools charge higher tuition or it may mean that better (and wealthier) students are selected into better schools, or both. Nevertheless, better schools as measured by SAEB charging less tuition would be an awkward result to explain.

The two panels of figure 9 show the same relation for secondary and higher education.

Once again, there is nothing strange: upward sloping curves show that better schools (or schools somehow with better students) charge more tuition. The price of education also increases with grade level. While top 4th and 8th grade schools are charging around R\$ 2500 (of 2003) per year, the same figures for secondary and higher education are closer to R\$ 4000/year and R\$ 5000/year, respectively.

FIGURE 9

Tuition according to proficiency for end of secondary and end of higher education



Source: ENEM; ENC (2003).

All is now ready to attribute a market value to public education. This involves taking the tuition paid by students or their parents at private schools and assigning the same value to public school students with the same proficiency, as measured by standardized tests. The previous sections have shown the relations between proficiency, income, and expenditures and they all more or less conform to a priori expectations. This means that the discussion now becomes one of functional form and how to carry out the matching. A limiting factor will be the number of observations we have to work with. Since all students were matched, the number of observations is given by the POF survey sample size.

Table 7 shows unweighted totals of observations by schooling level. Higher education includes both undergraduate and graduate education.

TABLE 7
Unweighted sample sizes after the various matching processes

Type of information	Pre-school	Primary (Lower)	Primary (Upper)	Secondary	Higher
Private education in POF	1,860		4,692	2,093	2,743
Public education in POF	4,658		36,550	8,270	2,388
All education in POF	6,518		41,242	10,363	5,131
Private with tuition	1,159		2,599	962	1,600
Public with proficiency	0	4,740	3,370	2,352	1,423
Private with proficiency	0	524	501	593	2,245
Private with proficiency and tuition	0	317	290	287	1,597

Source: POF; SAEB; ENEM; ENC (2003).

While POF provides large samples of children and youth in school, there are far fewer observations of children and youth in private school paying tuition even fewer numbers paying tuition with valid test scores. For example, of the 41 thousand children in primary education, only 6.3% are in private school with nonzero tuition and only 1.4% are in fourth or eighth grade private education with nonzero tuition and valid test scores. The reason for the drop in observations with valid tests scores is not a limited matching process but rather the fact that tests only occur in 4th, 8th, and 11th grades, as well as the end of undergraduate education.

The numbers we are left with – 607 observations for primary school, 287 for secondary, and 1597 for higher education – are not enough for non-parametric matching of scores and tuition values to children in public education.

This means a parametric model will have to be used. In principle, this presents no difficulty: a regression in which a quadratic on proficiency explains positive tuition is all that is needed. State dummies are used to control for the fact that educational markets are local. I then predict what public schools would cost were they private, supposing that parents only care about proficiency. This will yield market values for the schooling of all children in 4th, 8th, and 11th public schools. The regression results are on table 8 (full results including state dummies are in the appendix).

TABLE 8
Regression coefficients of proficiency and proficiency squared and their joint statistical significance (dependent variable is annual tuition) – Naïve approach

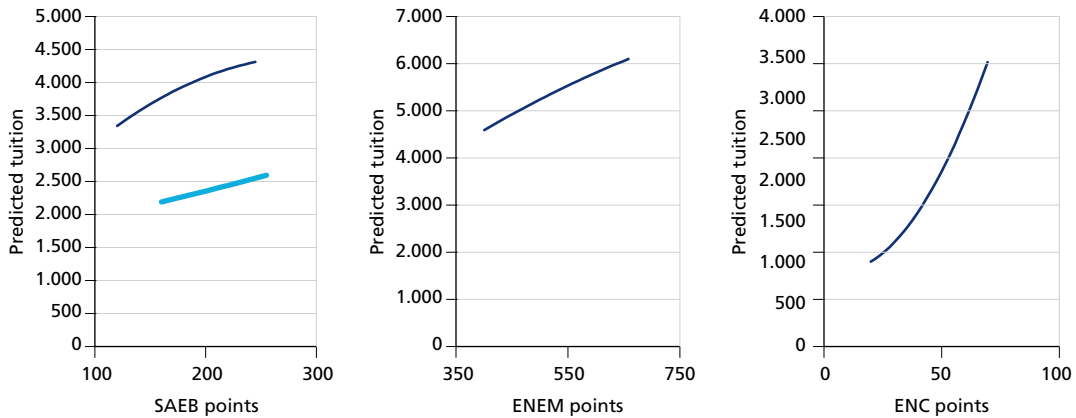
	4th	8th	11th	15th
Proficiency	20.42	3.29	-8.15	10.07
Proficiency squared	-0.035	0.002	0.631	-0.004
Joint significance	1.2%	4.8%	0.0%	1.4%
R ²	68.3%	72.2%	71.6%	59.3%

Source: POF; SAEB; ENEM; ENC (2003).

While the functions linking Proficiency and Tuition are sometimes concave and sometimes convex, they are always increasing over the relevant proficiency intervals. This can be seen on figure 10, which shows the predicted tuitions as a function of proficiency for the state of São Paulo. The functions for other states will be shifted up or down but will have the same slope.

The functions are, *grosso modo*, coherent with the non-parametric curves on figures 8 and 9. All that remains for imputation of the market value of public education in the grades covered by the standardized tests is to calculate predicted values.

FIGURE 10
Predicted annual tuition according to proficiency – Naïve approach



Source: Table 8.

What about students in other grades? The results can be extended to other grades within the same level of education using a Hot Deck imputation procedure. This is done by sorting students by the same variables used in matching excluding type of school

(since only public schools are relevant here) and year of birth (which makes no sense when talking about different grade levels). This leaves UF, Sex, Computer, Car, and TV as sorting variables to which we can add income. Then the closest valid observation is then used as data donor for those children who have none. As shown by Andridge; Little (2010), Hot Deck imputation has a series of desirable properties, the most important of which is that it does not change the shape of the underlying distribution.

And the result is the Market Value of Public Education for all students in all levels of public education. Table 9 shows the main results.

TABLE 9
Incidence and concentration analysis for the market value of public education (Naïve)

Source	Population (millions)	Average values				Incidence coefficient	Concent. coefficient
		Ex-ante	% of income	Ex-post	% of income		
Per cap income	176	499.93	100.0%	535.06	100.0%	0.5919	0.5532
Public education	176	35.13	7.0%	35.13	6.6%	-0.0521	0.0533
Pre-school	176	3.31	0.7%	3.31	0.6%	-0.0968	0.0126
Lower primary	176	11.51	2.3%	11.51	2.2%	-0.1930	-0.0752
Upper primary	176	11.46	2.3%	11.46	2.1%	-0.1276	-0.0156
Secondary	176	6.86	1.4%	6.86	1.3%	0.1329	0.2195
Higher	176	1.99	0.4%	1.99	0.4%	0.6333	0.6886

Source: POF; SAEB; ENEM; ENC (2003).

The difference between the last two columns, Incidence and Concentration coefficients, requires explanation. By Incidence coefficient, I mean the area between the Line of Perfect Equality and the Incidence curve. By Incidence curve, I mean the curve defined by Cumulative Population on the horizontal axis and a single Cumulative Income source on the vertical axis, with individuals ordered by per capita income *without* imputed income from public education. The Concentration coefficient and Concentration curve are analogously defined, but with individuals ordered by per capita income *with* imputed income from public education. This means that while the Incidence coefficient measures the ex-ante concentration, in which the imputed value of education has not been yet been added to per capita income, the Concentration coefficient measures the ex-post concentration, once the imputed value of education has been added to per capita income.

Once again income is per capita household gross income, which includes all public transfers but does not take into account direct taxes on income.

The first interesting result refers to the total market value of all public education. According to our estimates, it was worth R\$ 35.13 per person in Brazil. Annualized and multiplied by the 176 million people who lived in the country in 2003, we have R\$ 74 billion total market value. This is 2.8% higher than the R\$ 72 billion that the government spent producing it, indicating that Brazilian public expenditures in education are welfare producing and not welfare reducing. These are only notional values, but interesting nevertheless.

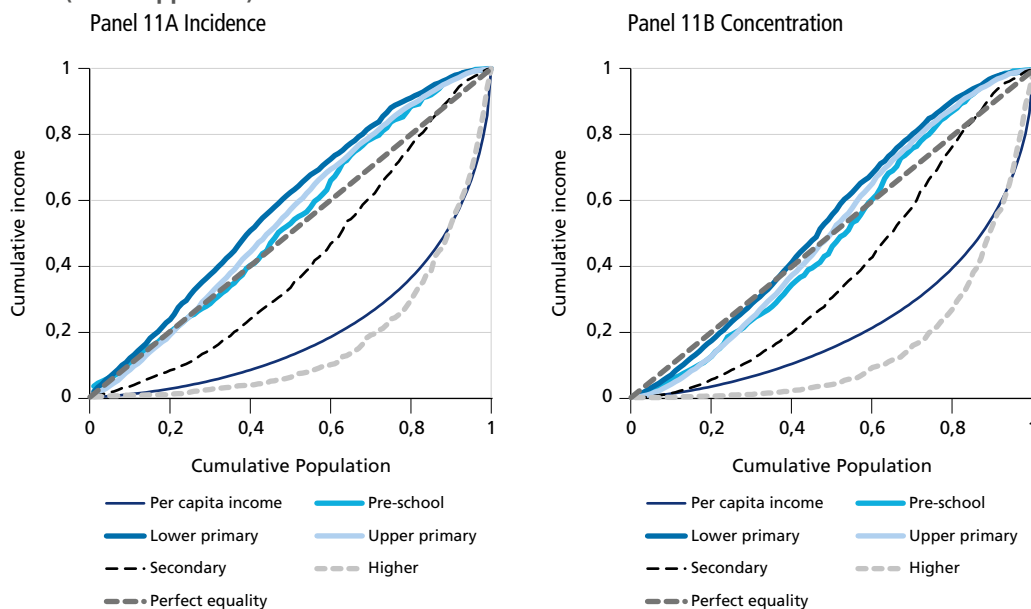
The second result is that the imputation of public education at market value reduces the Gini coefficient by a hefty 3.9 Gini points (from 0.592 to 0.553). The direction of the reduction is hardly news, since it has long been known that children are concentrated in the lower deciles of the population. The magnitude, however, is relevant, particularly considering that public education is a universal, non-targeted service.

Then we have the Incidence coefficients. The Incidence coefficients (calculated using per capita incomes which do not take into consideration the value of public education) are negative up to secondary education. Higher education is highly regressive with an Incidence coefficient of 0.633. Luckily, its value is only about 6% the value of all public education. For public education as a whole, the Incidence coefficient is negative at -0.0521, which is very progressive.

The Concentration coefficients (calculated sorting per capita income which has incorporated the value of public education) are somewhat less progressive. This is to be expected since very poor households with children in school move up the income distribution when the value of public education is included in their incomes. The Concentration coefficients range from -0.075 for lower primary public education to 0.689 for higher education. Public education as a whole comes in at 0.053, very close to a universal per capita income transfer.

Finally, figure 11 shows the Incidence and Concentration curves from which their respective coefficients are calculated.

FIGURE 11
**Incidence and concentration curves for the market value of public education
 (Naïve approach)**



Source: POF; SAEB; ENEM; ENC (2003).

But this approach ignores the fact that learning occurs both in schools and at home and thus would overstate how much better the schools of the high Socio-Economic Status (SES) children are relative to those of the low SES children. Correcting for this requires a slightly different approach.

7 CONSIDERING SCHOOL SOCIO-ECONOMIC BACKGROUND – NET PROFICIENCY APPROACH

To correct for household inputs in the educational production function, the data available in different socioeconomic questionnaires of the different standardized tests (and other socio-economic data) can be used to set up a model such as:

(1) $p_i = \beta_{SES} X_i^{SES} + \beta_{School} X_m^{School} + e_i$ where p_i represents the measured proficiency of student i , X_i^{SES} represents his or her socioeconomic characteristics, and X_m^{School} represents the characteristics of school m in which student i studies.

Once a model such as this has been set up, what is the proficiency level of students, net of their own socio-economic characteristics? It is simply:

$\bar{p}_i = \beta_{SES} \bar{X}^{SES} + \beta_{School} X_m^{School} + e_{ik}$ where \bar{X}^{SES} is some fixed SES variable vector applied to all students. It can be the average of all the SES variables, or any other value which does not vary. Thus, \bar{p}_i represents the proficiency explained by school and idiosyncratic individual effects, but net of family effects. The fact that peer effects explain most of the school effects is irrelevant, since peer effects are outside the family and thus part of what parents are paying for when they pay for schooling. This was done and the regression coefficients can be found in the appendix.

Once the proficiency net of family effects is found, we merge the new values with the expenditure surveys in exactly the same way as the observed proficiency was merged. The matches proceeded in exactly the same way so there is no need to reproduce tables 1 through 3.

Once merged, we can estimate means, standard deviations, and the coefficient of variation, using the POF survey weights. The results are on table 10.

TABLE 10
Comparison of measured proficiency and proficiency net of family effects

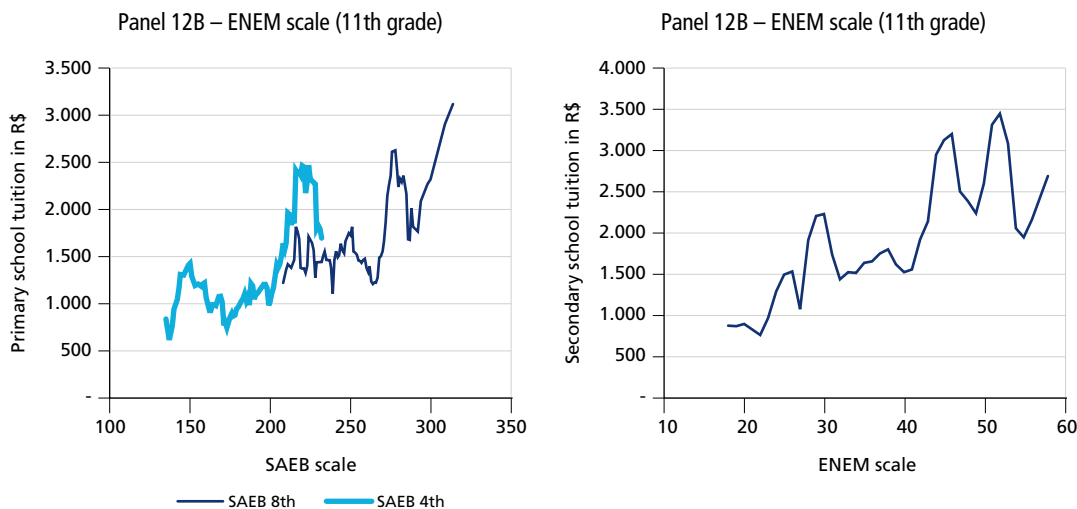
Grade	Measured (Naïve approach)			Net of family effects		
	Mean	Standard deviation	Coefficient of variation	Mean	Standard deviation	Coefficient of variation
4th	170.14	24.94	0.15	175.41	19.36	0.11
8th	235.88	35.10	0.15	226.75	26.61	0.12
11th	43.49	9.17	0.21	37.04	6.53	0.18

Source: POF; SAEB; ENEM (2003).

As expected, the variances and coefficients of variation are smaller for the proficiencies net of family effects, but not much smaller. They are about ¼ smaller when family effects are netted out for the SAEB tests and about 15% smaller for the ENEM test. Less variance in the proficiency scores will lead of course to more egalitarian scores, but not necessarily to a more egalitarian valuation of education. This will depend on the relation between proficiency and tuition.

The relation between net proficiency and tuition in private schools is shown on figure 12 (analogous to figure 8 for the naïve model). As for the naïve model, the panels of figure 12 show a 10 SAEB point moving average and a five ENEM point moving average. As in the naïve model, the curves slope upwards. This shows that better schools continue to be more valued by parents, as expected.

FIGURE 12
Relation between proficiency and tuition (raw data using moving average)



Source: SEAB; ENEM; POF microdata.

Unfortunately, as with the naïve model, there are not enough observations to assign market value to public education without appealing to a parametric relation between proficiency and tuition. The regression specification used in the net proficiency model is similar to that used in the Naïve model: a quadratic on proficiency explaining tuition. Since the net proficiency data have somewhat less variance (as shown on table 10), only region and not state dummies were used to control for different markets. A regression was done for each educational level: 4th, 8th, and 11th. For higher education, I kept the naïve model due to lack of socioeconomic questions in the ENC questionnaire.

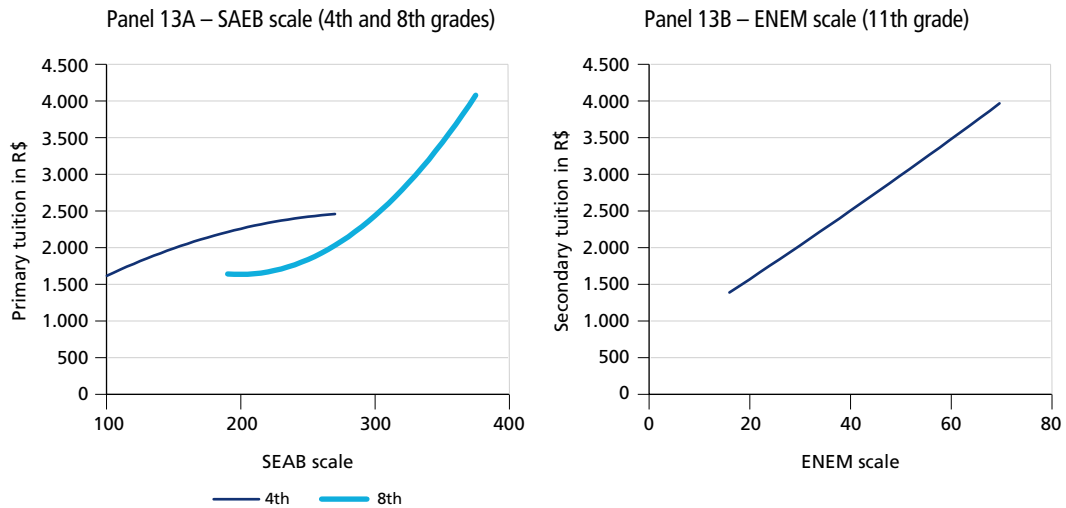
TABLE 11
Imputation regression results

	4th	8th	11th
Proficiency	12.79	-31.63	43.29
Proficiency squared	-0.0212	0.0793	0.0547
Joint significance	16.51%	1.45%	0.07%
R ²	64.15%	66.38%	63.46%

Source: SEAB; ENEM; POF microdata.

Using the regression coefficients shown above, it is easy to predict tuition as a function of proficiency for 4th, 8th, and 11th grades. The results are shown on figure 13 below and are roughly (as in move in the same direction with the same average slope) comparable to the results in figure 12. The relation between proficiency and tuition is stronger for 8th than for 4th grade and even stronger for 11th grade. Since there are no studies on this issue in Brazil, it is impossible to know whether this is in line with the literature, but the results are slightly different from those found in the naïve model, in which 8th grade, in particular, had a flatter proficiency-tuition profile than 4th grade.

FIGURE 13
Relation between proficiency and tuition (using regression analysis)



Source: SEAB; ENEM; POF microdata.

In any case, once in possession of the imputation regression coefficients, it becomes a simple task to predict market value of public education for 4th, 8th and 11th grades with the net proficiency model. For the end of higher education, I use the same data as in the naïve model.

To generalize this market value to other grade levels, once again a hot deck imputation procedure is the solution. The numbers for the distributional analysis of public education are shown on table 12.

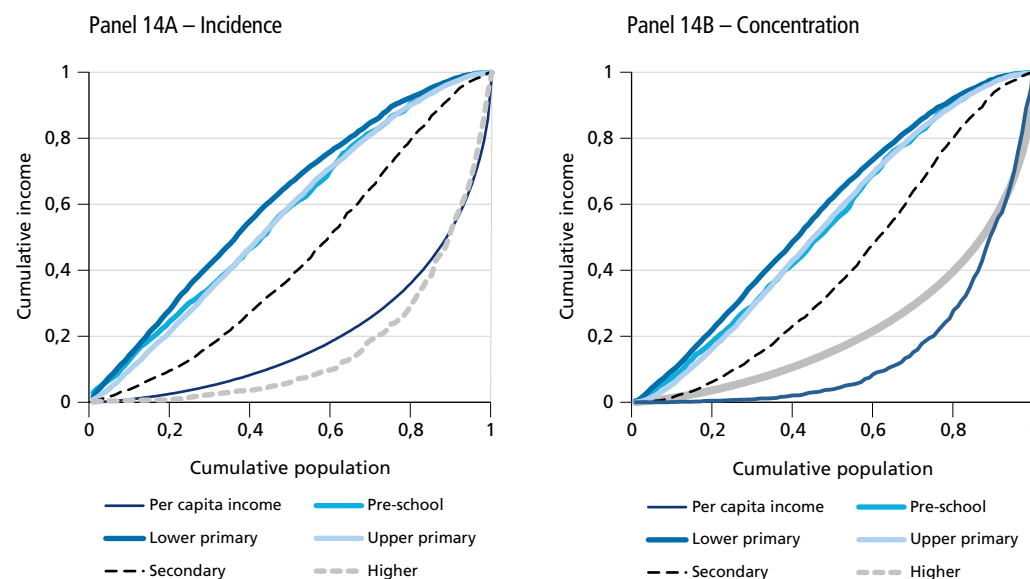
TABLE 12
Incidence and concentration analysis for the market value of public education

Source	Population (millions)	Average values				Incidence coefficient	Concent. coefficient
		Ex-ante	% of income	Ex-post	% of income		
Per cap income	176	499.93		535.73		0.592	0.549
Public education	176	35.80	7.2%	35.80	6.7%	-0.093	-0.004
Pre-school	176	3.30	0.7%	3.30	0.6%	-0.167	-0.081
Lower primary	176	12.26	2.5%	12.26	2.3%	-0.245	-0.151
Upper primary	176	10.42	2.1%	10.42	1.9%	-0.161	-0.075
Secondary	176	7.84	1.6%	7.84	1.5%	0.083	0.174
Higher	176	1.99	0.4%	1.99	0.4%	0.633	0.692

Source: SEAB; ENEM; ENC; POF microdata.

The total market value of public education increases slightly from the Naïve model (R\$74) to the net proficiency model (R\$76 billion) and continues to be welfare enhancing (but perhaps not too much should be made of this result since the values are notional). What is much more solid is how this value is distributed. Table 12 and figure 14 show a very progressive distribution of educational value. In particular, it is more progressive than that observed in the Naïve model. This is only half a surprise.

FIGURE 14
Incidence and concentration curves for the market value of public education (net proficiency approach)



Source: SEAB; ENEM; ENC; POF microdata.

It has long been known that the distribution of enrollments is concentrated among the lower half of the income distribution. The Incidence coefficient of primary education enrollment is -0.28 and even secondary education enrollment boasts of an Incidence coefficient of 0.04. What is a surprise is that the quality of public education in the lower half of the income distribution is not so much worse as to negate the effects of the distribution of enrollments. If primary education were to be monetized, it would be more progressive than any income source but the means tested and highly targeted *Bolsa Familia* and BPC.

When family effects are netted out, the distribution of educational value becomes even more progressive. Once again, this should be no surprise: part of the reason why the children of richer families have higher test scores is purely socioeconomic and has nothing to do with the school. Once this effect is removed, the distribution of educational value becomes more progressive.

Finally, the perverse distributive consequences of public higher education are highly visible in figure 14. The Incidence coefficient of public higher enrollment is higher than the Gini coefficient of per capita income: 0.59 vs 0.55. When the quality, assessed at market value, of higher education is taken into consideration the Incidence coefficient soars to 0.63 and the Concentration coefficient to nearly 0.70! Public higher education in Brazil is a massive benefit for the upper half of the income distribution and one of the most regressive income sources in family income.

Table 13 shows public expenditures in education for 2003 as well as the market value of public education calculated in this paper. Expenditures in pre-school, primary, and especially secondary education are welfare enhancing since the market value of what they provide is greater than what the state spends providing it. This is especially so for secondary education, although this may be partially due to its being underfunded until 2007.

TABLE 13
Public expenditure vs market value of public education

Level	Public expenditure	Market value
Pre-school	5.6	7.0
Primary	46.2	47.9
Secondary	8.0	16.6
Higher	12.9	4.2
Total	72.7	75.6

Source: Table 12; (Castro, 2011).

Once again, higher education is different. Since the state spends almost three times its market value in its production, it is welfare-reducing. Some care should be taken in interpreting this result since higher education expenditures also fund research, whose value to society is not being calculated here. The total effect of education remains welfare enhancing due to the three lower levels of education.

What about other income definitions? For comparison, table 14 shows education incidence (ex-ante) and concentration (ex-post) coefficients for three different income definitions. Market Income is Gross Income but without public transfers.

TABLE 14
Incidence and concentration coefficients according to different definitions of income

Source	Market income		Gross income		Disposable income	
	Incidence coefficient	Concent. coefficient	Incidence coefficient	Concent. coefficient	Incidence coefficient	Concent. coefficient
Per cap income (Gini)	0,593	0,550	0,592	0,549	0,583	0,538
Public education	-0,094	-0,005	-0,093	-0,004	-0,083	0,009
Pre-school	-0,167	-0,080	-0,167	-0,081	-0,157	-0,067
Lower primary	-0,247	-0,153	-0,245	-0,151	-0,227	-0,129
Upper primary	-0,162	-0,076	-0,161	-0,075	-0,145	-0,055
Secondary	0,084	0,174	0,083	0,174	0,087	0,183
Higher	0,629	0,690	0,633	0,692	0,571	0,645

Source: SEAB; ENEM; ENC; POF microdata.

The year we use is 2003, which is before the *Bolsa Família* rollout, so there are few public transfers and the Gini coefficient barely changes. Direct taxes on income have a slightly stronger effect, reducing the Gini coefficient by one Gini point. By comparison, public education reduces the Gini coefficient by almost five points, independently of the income definition used.

8 NEXT STEPS

I believe this article contributes to the literature in two ways.

First, it suggests a methodology to estimate how much public education is worth.

The idea is to compare students in public schools with students in private schools whose proficiency scores in standardized tests are the same and assign to the public school student the tuition paid by the equivalent private school student. The implementation requires an expenditure survey, or other database to provide information on tuitions, and standardized test scores available for both private and public schools. Brazil counts on a particularly good set of educational evaluations based upon standardized tests – SEAB, ENEM, and ENC – but this approach can also be taken using tests such as PISA or LLECE.

If good socio-economic data are available, such as they are for PISA, proficiencies can be calculated net of family effects. In general, this will make public education more valuable and more progressive.

The second contribution is to restate the importance of public education as one of the most important policies for both income and wealth distribution. The Concentration coefficients are highly progressive and the values quite relevant. Higher education is the glaring exception. Nevertheless, we must not jump to conclusions based upon these results, since public universities fulfill other roles other than just providing schooling. I believe that some type of cost recovery based upon student payment must be thought of for higher education.

This is, of course, a first exercise in this direction. Many improvements might be thought of. For example, the netting out of family effects was undertaken ignoring the hierarchical structure of schooling. Mostly this was done because the school variables were there as controls since it was the family variables I was interested in. But a hierarchical model might be worth pursuing nevertheless.

Very soon, new Brazilian data from the 2016 POF will be available. It will be possible to use better data than what was used here. *Prova Brasil* can be used instead of SAEB for 4th and 8th grades and ENADE provide a much more complete socioeconomic questionnaire than ENC. I doubt the results will change much, but nevertheless it will be good to count on better data.

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