DISCUSSION PAPER

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IMBALANCE, INEQUALITY, EQUITY, AND ADJUSTMENT IN PAY-AS-YOU-GO PENSION SYSTEMS IN DEVELOPING COUNTRIES: AN EMPIRICAL ANALYSIS WITH DATA FROM BRAZIL

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ABSTRACT

Using an empirical age-period-cohort model we analyze the effects of different Pay-As-You-Go (PAYG) public pension systems on financial imbalance, on inequality in permanent income, and on the rate of return of social security assets of heterogeneous agents in terms of gender and education. Apart from this heterogeneity, we include some aspects that are particularly relevant for developing countries such as the existence of labor informality (seen as non-contribution to the pension system) and of an old-age, social benefit granted to those that are not eligible for regular retirement. We consider several types of PAYG structures, including the defined benefit (DB) system, the notional defined contribution (DC) system, and a new mixed system that combines components of the two previous ones. Results are obtained both for a context in which there is no financial equilibrium and for a context with financial balance. In this last case, we use distinct schemes of partition of the imbalance between active and inactive generations, with one of the schemes inspired in the present German system where the contribution of the latter group is based on changes in the dependency ratio and in the wages net of contributions. The analysis for the inequality measure and the rate of return is conducted both for intra and intergenerational perspectives. The results of our simulations show that the mixed system herein proposed represents a compromise between the DB and DC systems. Within the context with financial balance, the German scheme tends to exhibit the highest rates of return and the highest horizontal equity in returns.

Keywords: public pension systems; financial imbalance; inequality; simulations.

SINOPSE

Usando um modelo empírico de idade-período-coorte, analisamos os efeitos de diferentes sistemas de previdência de repartição (SPR) no desequilíbrio financeiro, na desigualdade de renda permanente e na taxa de retorno dos ativos previdenciários de agentes heterogêneos em termos de gênero e educação. Para além desta heterogeneidade, incluímos alguns aspectos que são particularmente relevantes para os países em desenvolvimento, como a existência de informalidade no mercado de trabalho (vista como não contribuição para o sistema de previdência) e de um benefício social para idosos que é concedido aos não elegíveis para aposentadoria regular. Consideramos vários tipos de estruturas de SPRs, incluindo o sistema de benefício definido (BD), o sistema de contribuição definida nocional (CD) e um novo sistema misto que combina componentes dos dois anteriores. Os resultados são obtidos tanto para um contexto em que não há equilíbrio financeiro quanto para um contexto com equilíbrio financeiro. Neste último caso, utilizamos sistemas distintos para a repartição do desequilíbrio entre gerações ativas e inativas, sendo um dos sistemas inspirado no atual sistema alemão no qual a contribuição dos inativos se baseia nas mudanças da razão de dependência e dos salários líquidos de contribuições. A análise dos efeitos sobre a desigualdade e a taxa de retorno é realizada tanto para a perspectiva intrageracional quanto para a intergeracional. Os resultados de nossas simulações mostram que o sistema misto aqui proposto representa um compromisso entre os sistemas BD e CD. Dentro do contexto com equilíbrio financeiro, o esquema alemão tende a apresentar as maiores taxas de retorno e maior equidade horizontal nos retornos.

Palavras-chave: sistemas públicos de previdência; desequilíbrio financeiro; desigualdade; simulações.

1 INTRODUCTION

In PAYG pension systems the financial balance depends on the proportions of active and inactive people in the population. According to the projections of the United Nations (UN, 2019), the median dependency ratio of people over 65 years old per 100 people between 20 and 64 years of age will double in the next thirty years and more than triple at the end of the century in middle income countries. This process of demographic transformation places increasing pressures on the sustainability of PAYG systems in these countries, making the introduction of extensive reforms inevitable.[1]

In general, structural reforms of these systems impact a wide range of economic dimensions. In addition to affecting labor supply, aggregate savings, and economic growth, reforms in PAYG systems change the income distribution across generations, impacting differently permanent income and rates of return for agents born in different cohorts and, in developing countries, where heterogeneity among workers is high, impacting also among different groups of agents in the same cohort. Quantifying changes in permanent income inequality and equity in rates of return is thus an important exercise when there is concern as to the effects of different PAYG systems on vertical (between groups of agents) and horizontal (between generations) inequality.

The most adopted type of PAYG system is that of a DB structure. Typically, the DB system offers high benefits relative to the taxation levels and operating rules of this system, which generates growing deficits over time, especially in a context of accelerated demographic transformation. Furthermore, with the aim of compensating agents with lower contributory densities (e.g., women and informal workers), DB replacement rates commonly include a component that is independent of the number of contributions. Despite playing a redistributive role, this component tends to increase fiscal imbalances and generates heterogeneity in rates of return between groups of agents in the same cohort and, due to economic trends and shocks, also between different cohorts.

To deal with the financial unsustainability of the DB system, some countries (e.g., Sweden, Italy, and Poland) reformed their PAYG systems by adopting a DC structure. Basically, in this

1. In the last decades, various developed countries have implemented substantial reforms in their PAYG schemes, making the systems' parameters more consistent with long term financial sustainability (see, e.g., Fouejieu et al., 2021, for an account of the reforms in European countries). In Latin America, there was a first wave of reforms in the 1980s and the 1990s which introduced funded defined contribution schemes in some countries. More recently, a second wave of reforms have taken place in those countries aiming at recovering some components of the PAYG systems, in particular the universal pillar (see, e.g., Mesa-Lago and Valero, 2020).

system, agents contribute to an individual non-financial (or notional) account that receives remuneration based on some economic indicator (e.g., the average wage growth rate, as in Sweden) and receives a lifetime annuity whose value depends on the total amount accumulated in the notional individual fund at the time of retirement and the life expectancy of the agent's birth cohort.[2] By construction, a DC system of this type does not affect permanent income inequality and inequity of returns between agents. In this sense, the DC system is neutral from both an intra- and inter-generational perspective. However, unlike the DB system, the DC system tends to generate low-value benefits for agents who have low contributory densities or low wages (or both), which makes it an unsuitable system for developing countries. In this article, we propose a new mixed system that combines features of DB and DC systems.[3] More specifically, we defined a formula for calculating retirement benefits that combines the component that does not depend on the number of contributions of the DB system with the individual fund component of the notional DC system. This combination of components makes it possible to mix the redistributive role of the DB system with the neutrality of the DC system.

Many countries incorporate a non-contributory pillar in their social security systems that offers a minimum benefit to those who do not comply with regular retirement rules. Often linked to the minimum wage – for example, Brazil, Colombia, and other Latin American countries (Gill, Packard and Yermo, 2005) –, this floor aims to guarantee a minimum income that avoids poverty in the elderly phase of life. Typically financed with general taxes, this pillar benefits the most vulnerable workers and in developing countries accounts for a high proportion of social spending. In addition, it tends to represent a disincentive to labor supply and contributions to social security, which has repercussions on revenues of PAYG schemes.

The theoretical and empirical literature that assesses the economic effects of public pension systems is quite extensive.[4] More related to our study, Auerbach and Lee (2011) analyze how various stylized PAYG programs distribute the risks of demographic and economic shocks across generations. The analysis is conducted in a stochastic environment for financially-balanced systems, and considers the steady state situation. Auerbach et al. (2018) use a model of overlapping generations with balanced government budget to compare the performance of several stylized PAYG systems in the

2. For details on the functioning and implementation of notional DC schemes, see Holzmann and Palmer (2006).

3. Using an overlapping generation model, Matsen and Thøgersen (2004) study the (optimal) mix between PAYG and funded DC systems. As our focus is on pure PAYG schemes, funded DC structures are not included in our analysis.

4. Seminal contributions are Samuelson (1958), Feldstein (1974), Rothschild and Stiglitz (1976), Diamond (1977), Auerbach and Kotlikoff (1987), Stock and Wise (1990), and Tabellini (2000). For surveys of the literature, see Feldstein and Liebman (2002a; 2002b).

distribution of intergenerational risks arising from specific shocks in productivity, fertility, mortality, and migration. Ludwig and Reiter (2010) also use a model of overlapping generations to investigate how different PAYG designs affect intergenerational well-being from specific fertility shocks. The analysis of these studies is carried out for the representative agent of each generation – that is, there is no intragenerational heterogeneity –, which makes their frameworks and results more appropriate for developed countries. Unlike these studies, our analysis evaluates the performance of different types and designs of stylized PAYG systems for developing countries, where agents are more heterogeneous, inserted in a labor market with informal employment (i.e., where agents do not always contribute to social security), and in which large portions of the population reach the elderly stage in poverty. To take these characteristics into account, our framework incorporates several relevant dimensions. First, recognizing the differences in labor market insertion between men and women, the analysis is carried out separately for the genders. Second, we model both formal and informal labor income, as well as pension contribution patterns and the educational distribution of agents. Third, we control for the effects of the business cycle on these variables and allow them to evolve differently by age and birth cohort of agents. Fourth, we incorporated the existence of a floor for retirement benefits and a pillar that guarantees a minimum income for the elderly poor. Finally, we considered different designs of the DB and the notional DC schemes, in addition to the aforementioned system that combines elements of these two structures. Naturally, the inclusion of all these dimensions makes our framework more complex but allows us to obtain a wealth of results for different types of agents, both within the same cohort and for different cohorts.

Methodologically, we use an empirical age-period-cohort model whose estimates are used, together with some hypotheses about the age profile and trends by cohort, to project the future trajectories of labor income, the probability of contributing, and the educational distribution of observed and unborn cohorts for each type of agent. These trajectories make it possible to simulate the outcome variables of interest for different types and designs of PAYG systems, namely: measures of the financial imbalance over time, permanent income inequality, and rates of return of social security assets between types of agents and between cohorts. This analysis is carried out for two different contexts in relation to the financial stability of the systems. In the first one, we analyze PAYG structures in which system rules are pre-defined, and the financial imbalance is endogenous. In the second, contributions of active and inactive generations are endogenously adjusted over time so that the systems do not present imbalances. In this last context, we consider two adjustment rules: i) the one proposed by Auerbach and Lee (2011) and Auerbach et al. (2018) in which there is a proportional sharing mechanism between active and inactive groups to cancel the financial imbalance; and ii) an adjustment inspired in the present German pension system, in which inactive groups share with the active ones the risks of imbalance and productivity gains after retirement.

We used data from Brazil, a country that, like other middle-income countries – especially in Latin America –, is experiencing an accelerated population aging process, has significant skill heterogeneity in the workforce, and a large informal sector. The database is the Pesquisa Nacional por Amostra de Domicílios – PNAD (National Household Sample Survey) available between 1992 and 2015. PNAD is an annual cross-section sample representative of the country's population and collects demographic and socioeconomic information, including labor income and formal-informal insertion in the labor market. Our simulations start in 2016 and cover the time horizon up to 2140. Unlike other studies (e.g., Auerbach and Lee, 2011), we did not impose that the trajectory of the demographic profile was stationary and, therefore, we used the available projections – IBGE (2019) until the year 2030 and UN (2019) from there until 2100 – making it possible to assess long-term effects of trends on the simulated systems.

In addition to this *Introduction*, the article contains three other sections. In the second, we present the age-period-cohort model, the basic rules of contribution and benefits (including those of the assistance pillar for the elderly poor), the measures for our outcome variables of interest (for the context without financial balance), and the way we project the trajectories of labor income, contributory density, and educational distribution of different types of agents across the cohorts. The third section is dedicated to the presentation of the results. Given the complexity of our framework and the different contexts regarding the imposition or not of financial balance, the presentation is carried out in subsections. In the first one, we allow for financial imbalances and exclude the benefit floor and the assistance pillar to analyze the performance of various PAYG schemes without the influence of discontinuities generated by these components. In the second subsection, we maintain the possibility of imbalances, but now include these components. In the third subsection, which maintains these components, we impose adjustments to balance the systems and introduce new measures to gauge their burden on active and inactive generations. The fourth section summarizes the results, points out some conclusions and indicates possibilities of using our framework for analysis of reforms in PAYG systems.

2 METHODOLOGY

The profile of the agent of birth cohort c, educational group e and gender g – denoted by the triple (c, e, g) – is characterized by the wage (s), the contributory pattern to social security (p), and the probability of being in the group of education e ($z(e)$). The age-period-cohort model (hereinafter, APC), described in section 2.1, allows decomposing the trajectory of $(s, p, z(e))$ according to variables indicating age, period (year), and cohort, which measure respectively the effects of: age-indexed life cycle (i); the macroeconomic conditions of period (t); and long-term trends indexed to the cohort (c). As is known, it is impossible to simultaneously identify the age, cohort, and period parameters without imposing restrictions on the model. Deaton and Paxson (1994) make the model identifiable by admitting that the effect of the business cycle is transitory and imposing the restriction that the coefficients associated with the period component add up to zero.

2.1 APC model for the agent profile

The heterogeneity of the agents' profile is represented by the segmentation of the population into 6 education groups (e) by gender (g) in a total of 12 groups by cohort (c).[5] Then, for agent (c, e, g) at age i, let $p_{c,e,g,i}$ be the probability of contributing to social security, $s_{c,e,g,i}$ the logarithm of the wage of those who contribute, and $z_{c,g,e,i}$ the probability of being in educational group e.[6] The APC model applied to the quantities that characterize the agent's profile, $(s, p, z(e))$, can be written for the individual who belongs to cohort c, has schooling e, is of gender g and is age i as:

$$s_{c,e,g,i} = \alpha 1_{i,g} + \beta 1_{t,g} + \gamma 1_{c,g} + \delta 1_{e,g} + \varepsilon 1_{c,e,g,i} \; g = 1,2 \, , \tag{1}$$

$$p_{c,e,g,i} = \alpha 2_{i,g} + \beta 2_{t,g} + \gamma 2_{c,g} + \delta 2_{e,g} + \varepsilon 2_{c,e,g,i} \; g = 1,2 \, , \tag{2}$$

$$z_{c,e,g,i} = \alpha 3_{i,e,g} + \beta 3_{t,e,g} + \gamma 3_{c,e,g} + \varepsilon 3_{c,e,g,i} \; g = 1,2; \; e = 2,\ldots,6, \tag{3}$$

where εk, $k = 1,2,3$, represent the error terms. The equations for s and p are estimated separately by gender and those for z by gender and education group. The parameter vectors $\alpha(i) = (\alpha 1_{i,g}, \alpha 2_{i,g}, \alpha 3_{i,e,g})$, $\beta(t) = (\beta 1_{t,g}, \beta 2_{t,g}, \beta 3_{t,e,g})$, $\gamma(c) = (\gamma 1_{c,g}, \gamma 2_{c,g}, \gamma 3_{c,e,g})$, and $\delta(e) = (\delta 1_{e,g}, \delta 2_{e,g})$ represent respectively the life cycle, the economic cycle, and the long-term trends associated with birth cohorts and schooling.

The collinearity between the period, cohort, and age components makes the APC model unidentifiable. To circumvent this problem, Deaton and Paxson (1994), interpreting the period component as a pure effect of the business cycle, proposed the restriction: $\sum_t \beta k_{t,g} = 0$, $\forall k = 1,2,3$ and $g = 1,2$, which makes the model identifiable. The equations are estimated using PNAD for

5. The education groups are illiterate (0 years of schooling), incomplete primary first cycle (1 to 4), complete primary first cycle and incomplete primary second cycle (5 to 8), complete primary and incomplete high school (9 to 11), complete high school and incomplete tertiary (12 to 15), and at least complete tertiary (16 and over).

6. Throughout the paper we suppose that the wage as well as other monetary values are deflated. The variables p and z are dummy variables that indicate respectively whether the individual contributed to social security and whether he/she belongs to educational group e.

the period 1992-2015 and, therefore, provide estimates only for the periods and cohorts existing in the sample. Long-term projections require extrapolation of the cohort and period components out of the sample and will be presented in section 2.4.

2.2 PAYG rules

The starting point in our analysis is a stylized pension system inspired in the PAYG scheme in force in Brazil after 2019.[7] We restrict the analysis to social security contributions and benefits of urban, private sector workers, including the self-employed. The option for this trimming is to focus on the central component of a PAYG system and, therefore, we disregard other related expenditures such as survivor pensions and other types of allowances.[8] The analysis incorporates an assistance component of social security systems – often called zero pillar (World Bank, 1994; Gill, Packard and Yermo, 2005) – for low-income, old-age workers who do not meet the criteria for regular retirement. We consider a broad set of rules typical of PAYG schemes that in Brazil directly affect a large portion of the population.[9]

In this stylized system, given the profile $(s_{c,e,g,i}, p_{c,e,g,i}, z_{c,e,g,i})$ of the agent (c, e, g) throughout his age i, the PAYG rules determine the contribution rate during the active period and age $A(c, e, g)$ from which the agent starts to receive an annuity $b_{c,e,g}$. In the sequel, we define the main variables and rules of the system. Table 1, at the end of this subsection, presents the values of the parameters on which these rules are based.

7. In that year, a comprehensive reform of the public pension system was approved in Brazil. Amongst the main changes it is worth citing the end of the possibility of retirement based only on the contributory density of the individual, the rise in the minimum retirement age, and the change in the replacement rate formula for pension benefits. A detailed description of the new system can be found in Peixoto (2019). Souza, Monteiro and Paiva (2021) offers an account of the potential effects of some components of the reform on poverty and inequality.

8. In 2015, expenditures solely with the payment of pensions represented 62% of all expenditures of the Brazilian social security system (Brasil, 2015). See Caetano et al. (2016), Giambiagi and Sidone (2018), and Brasil (2017) for medium-term forecasts of the Brazilian system's imbalances and detailed results for expenditures with survivor pensions and other types of allowances.

9. As in other countries, in Brazil there is a set of specific regulations that depends on the type and activity of the worker (for instance, public servants, military personnel, teachers, and police officers). In order not to further complexify our model, we apply the same general rules of the systems we analyze to these groups of workers.

The population in period t can be expressed as the sum of population segments defined for agents at each age present in t:

$$pop_t = \sum_{c,g} \sum_{i \in I(c,t)} pop_{c,g,i},$$

where $I(c,t) = \{i;\ c + i = t\}$.

Based on the agents' wages, the payroll is obtained by:

$$fs_t = \sum_{c,e,g} \sum_{i \in I(c,t)} \exp(s_{c,e,g,i}) * z_{c,e,g,i} * pop_{c,g,i}, \qquad (4)$$

an aggregate variable that, together with that for population, define the productivity gain in period t:

$$y_t = \frac{(fs_t/pop_t)}{(fs_{t-1}/pop_{t-1})}. \qquad (5)$$

The minimum wage plays an important role in PAYG schemes because, in addition to serving as the wage floor in the formal labor market, it is also linked to the pension floor and the social assistance benefits in several countries (e.g., Brazil, Colombia, and Mexico). Although the way to readjust its value varies between countries, it is reasonable to assume that its evolution can be related to variations in productivity, albeit in a modulated way. Adopting this assumption, the minimum wage in period t is expressed by:

$$smin_t = smin_{t-1} * (1 + \alpha min * (y_t - 1)), \qquad (6)$$

where αmin represents the parameter that modulates how productivity growth is translated to the minimum wage.

In Brazil, as in several countries, there is a wage cap that limits the worker's social security contribution. Incorporating this constraint, the contribution wage is then expressed by:

$$sc_{c,e,g,i} = min\{\exp(s_{c,e,g,i}), fM * smin(c + i)\}, \qquad (7)$$

where fM corresponds to the factor, expressed as a proportion of $smin_t$, which controls the maximum value of the contribution.[10] The total amount of social security contributions is the sum of the worker's contribution, which in Brazil is calculated in cascade with a progressive rate, with the proportional contribution of the employer.[11] Using (7), the value of the total contribution per agent is then given by:

$$vc_{c,e,g,i} = [fc(sc_{c,e,g,i}) + \tau\, sc_{c,e,g,i})], \tag{8}$$

where τ is the employer's rate and the $fc(sc)$ is a non-linear function that determines the value of the worker's contribution.[12]

Generally, the calculation of the retirement benefit in a DB system considers a time window of the l last contribution wages of the worker. The average wage, which corresponds to these l contribution wages, can then be expressed as:

$$sm_{c,e,g} = (\textstyle\sum_{i-l<j<i} sc_{c,e,g,j} p_{c,e,g,j}) / (\textstyle\sum_{i-l<j<i} p_{c,e,g,j}). \tag{9}$$

As p is the probability of a contribution to the system, the accumulation of these probabilities provides the expected value of the number of contributions, or contributory density, of the agent up to age i, and is given by:

$$n_{c,e,g,i} = \textstyle\sum_{j\leq i} p_{c,e,g,j}. \tag{10}$$

Eligibility for retirement benefit is based on both a minimum age and a minimum number of contributions, both of which vary with the gender of the worker. Reflecting these conditions, the age at which the worker retires is then given by:

10. Generally, such a cap is nominally set and adjusted by inflation. However, assuming that the cap is kept constant in real terms while the minimum wage increases in real terms would lead, after a certain number of years, to a very simplistic set up where there is only one possible value of contribution and benefit. We opted instead for keeping the ceiling as a fixed proportion of the minimum wage over time.

11. In Brazil, the employer's contribution is based on the total value of the payroll, but we assume that the system's revenue that surpasses the maximum value of contribution is used for other expenditure purposes such as the payment of illness and maternity leave benefits.

12. The function $fc(sc)$ models the value of the contribution in a "cascade-type" fashion, where the contribution rate is 0.08 for the portion of contribution wages that is less than 1.5 minimum wages, 0.09 for the range between 1.5 and 2.5 minimum wages, and 0.11 for the remaining portion (up to the contributory wage cap).

$$\text{if } \#(\Omega_{c,e,g}) > 0\text{: } A_{c,e,g} = min\{i \in \Omega_{c,e,g}\}, \tag{11}$$

where $\Omega_{c,e,g} = \{i: i \geq i0(g)\ \&\ n_{c,e,g,i} \geq n0(g)\}$ and $i0(g)$ and $n0(g)$ are respectively the parameters for the minimum age and the minimum number of contributions for gender g.

In many countries, including Brazil, social security systems include a component that guarantees a minimum benefit to those who, after a certain age, do not meet the criteria for retirement or have a (family) income less than a certain minimum value. Usually financed by general taxes, this benefit has an assistance character and aims to reduce the poverty of the elderly (Gill, Packard and Yermo, 2005). In Brazil, it is called Benefício de Prestação Continuada – BPC (Continuous Cash Benefit) and grants a minimum wage to seniors aged 65 and over whose per capita family income is less than ¼ of the minimum wage. In our exercises, we assume that agents are entitled to receive this benefit when they do not meet the criteria for regular retirement expressed in (12), are aged $i \geq iamin$ and have average wage $sm_{c,e,g,i} < smin(c+i)$ or, if these criteria are not met, start to receive the benefit at age $i > 70$.[13] With these criteria, the age at which the assistance benefit is granted is expressed by:

$$\text{if } \#(\Omega_{c,e,g}) = 0\text{: } A_{c,e,g} = \min\left\{\{i: i \geq iamin\ \&\ sm_{c,e,g,i} < smin(c+i)\},\ 70\right\}, \tag{12}$$

where $iamin$ is the minimum age established in the social security system for granting this type of benefit.

Some simulations are based on the DC system with accumulation of notional assets by the worker. This asset is individual and is accumulated according to the contributions made by the worker and by the application of a remuneration rate that varies over time. Specifically, the expression for the agent's notional asset at age i is given by:

$$x_{c,e,g,i} = \left[x_{c,e,g,i-1} * (1 + y(c+i))\right] + \left[vc_{c,e,g,i} * p_{c,e,g,i}\right], \tag{13}$$

where we assume that the rate of remuneration is given by the variation in productivity expressed in (5), $y(c+i)$, and the last term of the expression corresponds to the contribution when he/she contributes.[14]

13. As we do not model the likelihood that an individual lives in a family whose per capita income is less than ¼ of the minimum wage, our income criterion is that the average wage is strictly less than the minimum wage.

14. Seeking to approximate the DC structure to the DB one, we incorporated the employer's contribution into the employee's assets.

The amount of retirement benefits depends on whether the system is of a DB or DC type. In the former, the benefit is a function of the replacement rate over the agent's average wage at retirement age. In DB systems such as the one in Brazil, the replacement rate is formed by a constant component (differentiated by gender) and another variable component that corresponds to a fraction of the accumulation of contributions at the time of retirement. In the DC system, the benefit amount corresponds to a perpetuity calculated by the value of the asset accumulated by the agent divided by the life expectancy of the agent's own cohort at the time of retirement. Both systems have a benefit floor for agents who do not meet the criteria for regular retirement. For groups that qualify for regular retirement – that is, those with $\#(\Omega_{c,e,g}) > -$, the formulas for calculating the benefit, $b_{c,e,g}$, for each system are given by:

$$\text{DB}: b_{c,e,g} = max(fm * smin(c+i), sm_{c,e,g,A_{c,e,g}} * \left[\alpha(g) + \beta * n_{c,e,g,A_{c,e,g}}\right]), \qquad (14)$$

$$\text{DC}: b_{c,e,g} = max(fm * smin(c+i), x_{c,e,g,A_{c,e,g}} / ve_{c,e,g,A_{c,e,g}}), \qquad (15)$$

where $\alpha(g)$ e β are the parameters of the replacement rate that represent respectively the constant component (for gender g) and the fraction of the accumulation of contributions, $ve_{c,e,g,A_{c,e,g}}$ is the life expectancy of the agent born in cohort c and gender g at retirement age $A_{c,e,g}$. And fm represents a factor that adjusts the benefit floor in both systems as a fraction of the minimum wage. In the simulations, this factor is introduced to adjust the value of the assistance benefit. Thus, for groups that do not qualify for regular retirement – that is, those with $\#(\Omega_{c,e,g}) = 0$, we have:

$$\text{Assistance benefit}: b_{c,e,g} = fm * smin(c + A_{c,e,g}), \qquad (16)$$

where A represents the minimum age at which the worker starts to receive this type of benefit.

Table 1 presents the parameters that condition the rules of the DB and DC systems of reference that we will use in our simulations.

TABLE 1
Parameters of the DB and DC systems of reference

Minimum wage adjustment factor	αmin	0
Maximum contribution wage factor	fM	5
Minimum retirement age	$i0(g)$	65 (men), 60 (women)
Minimum number of contribuitions	$n0(g)$	35 (men), 30 (women)
Minimum age for assistance benefit	$iamin$	65

(Continues)

(Continued)

Minimum wage adjustment factor	αmin	0
Minimum assistance benefit factor	fm	1
Replacement rate formula (DB)	$\{[\alpha(g)]; \beta\}$	$\{[0.2\text{ (men)}, 0.3\text{ (women)}]; 0.02\}$

Authors' elaboration.

2.3 Outcome measures

2.3.1 Outcomes per agent: income, inequity, and inequality

Given the agent's profile and the PAYG system rules, we can define a set of flow measures to compute outcome variables per agent. The flow measures are social security contributions and benefits as well as average income, which seek to incorporate the agent's non-formal earnings throughout the life cycle. Viewed from a cash flow perspective, paid contributions, and received benefits by the agent at age i are expressed by:

$$f_{c,e,g,i} = \begin{cases} -vc_{c,e,g,i} & \text{se } i < A_{c,e,g} \text{ (paid contributions)} \\ b_{c,e,g} * ps_{c,g,i} & \text{se } i \geq A_{c,e,g} \text{ (received benefits)} \end{cases}, \quad (17)$$

where $vc_{c,e,g,i}$, $b_{c,e,g}$ and $A_{c,e,g}$ were defined in section 2.2 and $ps_{c,g,i}$ corresponds to the probability of survival of the agent of cohort c and gender g at age i. For a representative agent of cohort c, contributions and benefits at age i can be expressed as a weighted average by gender and educational category as $f_{c,i} = \sum_{e,g} f_{c,e,g,i}\, z_{c,e,g,i}$.

To compute a measure of the agent's permanent income, it is important to take into account that in developing countries such as Brazil, workers have a high probability of spending periods of their life cycle with only non-formal income. To incorporate this dimension, we compute the income of agent (c, e, g) at age i as a weighted average of formal and non-formal income, where the weight is the probability of contributing to social security, $p_{c,e,g,i}$. Specifically, average income is computed by:

$$r_{c,e,g,i} = \exp\left(s_{c,e,g,i}\right) * (p_{c,e,g,i} * +(1 - p_{c,e,g,i}) * xf_{c,e,g,i}), \quad (18)$$

where $xf_{c,e,g,i}$ is the predicted value of the ratio between non-formal income $(rnf_{c,e,g,i})$ and formal income $[\exp(s_{c,e,g,i})]$.[15]

Based on $f_{c,e,g,i}$ e $r_{c,e,g,i}$ defined in expressions (17) and (18), we calculate a set of outcome variables per agent. One of them is *permanent income*, which is given by the discounted income stream over the agent's life cycle:

$$R_{c,e,g} = \sum_i r_{c,e,g}(1-\lambda)^{-i}, \tag{19}$$

where λ is a fixed discount rate.[16]

Another measure is the *internal rate of return* (irr) of the flow of paid contributions and received benefits, by a group of agents and for the representative agent:

$$\rho_{c,e,g} = irr(f_{c,e,g,i}), \tag{20}$$

$$\rho_c = irr(f_{c,i}), \tag{21}$$

where ρ_c is the aggregate rate of return for cohort c (i.e., for the representative agent of cohort c). To measure the *inequity of rates of return* between agents, we use the standard deviation (sd) of these rates, namely: $s_c = sd(\rho_{c,e,g})$ for educational groups and genders from the same cohort, and $s = sd(\rho_c)$ for the representative agents of different cohorts. We also compute a measure that seeks to assess the effect of pension systems on intragenerational permanent income inequality. To this end, we compute the present value of the contribution and benefit flows (including the assistance benefit):

$$\Delta R_{c.e.g} = \sum_i f_{c,e,g,i}(1-\lambda)^{-i}, \tag{22}$$

and define the measure of the systems' effect on *permanent income inequality* for cohort c by the expression:

$$\Delta\psi_c = cv(R_{c,e,g} + \Delta R_{c.e.g}) - cv(R_{c,e,g}), \tag{23}$$

15. More specifically, non-formal earnings are estimated as the proportion of formal earnings using the wages of those that contribute $(s_{c,e,g,i})$ and of those that do not $(rnf_{c,e,g,i})$. Using a pseudo panel formed by cells of (c,e,g,i), the estimated regression is: $xf_{c,e,g,i} = rnf_{c,e,g,i}/exp(s_{c,e,g,i}) = \delta_{g,e} + c*\alpha1_g + c^2*\alpha2_g + i*\beta1_g + i^2*\beta2_g + u_{c,e,g,i}$, from which $xf_{c,e,g,i}$ is predicted.

16. The discount rate was kept fixed throughout the paper at 0.02 per year.

where $cv(.)$ denotes the coefficient of variation. Note that the first term measures the level of inequality by incorporating the discounted flow from the pension system to permanent income, while the second term measures inequality from permanent income in a hypothetical situation where there is no pension system.

2.3.2 Aggregated outcomes per year: financial stability of PAYG systems

For a set of simulations, we used two aggregated results per year to measure the financial stability of the systems. The bases of these measures are the revenues and expenditures of the system in the period $t = c + i$. The revenue measure aggregates the contributions among all agents of all ages present at t, given by:

$$R_t = \sum_{c,e,g} \sum_{i \in I(c,t)} vc_{c,e,g,i} * p_{c,e,g,i} * z_{c,e,g,i} * pop_{c,g,i}, \quad (24)$$

where all terms were defined in sections 2.1 and 2.2. Expenditures on the payment of benefits (which include the assistance benefit) and the accumulated value of wage contributions are aggregated in a similar fashion:

$$G_t = \sum_{c,e,g} \sum_{i \in I(c,t)} b_{c,e,g} * ps_{c,g,i} * z_{c,e,g,i} * pop_{c,g,i}, \quad (25)$$

$$fsc_t = \sum_{c,e,g} \sum_{i \in I(c,t)} sc_{c,e,g,i} * z_{c,e,g,i} * pop_{c,g,i}. \quad (26)$$

The *system deficit* in period t is presented as the potential increase in the rate required to cancel the deficit, calculated by normalizing the deficit $(G_t - R_t)$ with the potential tax base, i.e., the aggregate contribution salary:

$$A_t = (G_t - R_t)/fsc_t. \quad (27)$$

2.4 Estimation and projection of the agent profile

The parameters that determine the taxpayer's profile presented in section 2.1 were estimated with data from the PNAD between 1992 and 2015. Conducted by the Instituto Brasileiro de Geografia e Estatística – IBGE (the Brazilian Census Bureau), the PNAD is a representative sample of the Brazilian population and provides information for the month of September of each year on a set of people's characteristics (including gender, age, and education) as well as on their insertion in the labor market (including whether they have a formal job in the private sector or contribute to social security as self-employed), and income. As the PNAD is not a longitudinal survey, our analysis is based on a panel of agents defined by birth cohort (c), schooling (e) and gender (g).

The APC model, presented in equations (1) to (3), determines the trajectory of each component of the agent's profile, which is described by 14 variables: wage (2 genders), contribution probability (2 genders) and level-of-schooling group (5 educational categories and 2 genders). For each variable, the model decomposes the profile trajectory into three temporal components (age, period, and cohort), respectively estimating the parameters: $\alpha(i) = (\alpha 1_{i,g}, \alpha 2_{i,g}, \alpha 3_{i,e,g})$, $\beta(t) = (\beta 1_{t,g}, \beta 2_{t,g}, \beta 3_{t,e,g})$, $\gamma(c) = (\gamma 1_{c,g}, \gamma 2_{c,g}, \gamma 3_{c,e,g})$, and $\delta(e) = (\delta 1_{e,g}, \delta 2_{e,g})$, where the vectors $(\alpha(i), \beta(t), \gamma(c))$ and $\delta(e)$ have 14 and 4 dimensions, respectively. The estimates of these parameters were obtained considering the years $t = \{1992, \ldots, 1999, 2001, \ldots, 2009, 2011, \ldots, 2015\}$, a contribution horizon $i = \{20, \ldots, 70\}$, and cohorts $c = \{1942, \ldots, 1982\}$.[17]

For the age profile of the agent, we assumed that the age component $\alpha(i)$ is the same for all future generations. The period component $\beta(t)$ is transient and represents the effect of the economic cycle on the agent's profile. The components of $\beta(t)$ were estimated with condition $E(\beta k(t)) = 0$, $k = 1,2,3$.

We adopted some hypotheses to project the contributory profile for "future" cohorts, that is, those not included in the model estimation. The $\gamma(c)$ component describes the effect of the long-term trend that tends to capture the influence of preferences as well as technological and institutional factors. It was projected with a linear trend inspired by external sources and the APC model's own results.[18] We assume that the profile of schooling in Brazil will reach the OECD profile in 2008 (OECD, 2008) in 2030, productivity growth will remain at 1% – as estimated by Barbosa Filho e Pessoa (2014) for the average growth rate between 1982-2015 – and formalization (i.e., the probability of social security contribution) will follow the same trend estimated in the model of 0.2% per year. These values directly or indirectly define the trend growth of each component of the agents' profile.

17. PNAD was not fielded in the census years of 2000 and 2010. In the last available PNAD of 2015, individuals over 20 years of age – the assumed age of entry in the labor market – were born after 1995, which is the year of the last birth cohort in the data. As the last cohorts tend to have smaller samples and displayed more unstable behavior in the data, we discarded the last 13 cohorts and considered the cohorts born between 1942 and 1982.

18. It is worth noting that the population trends by gender, age, and cohorts were taken from projections made by IBGE up to 2030 and from the UN (2019) – from that year on up to 2100.

3 RESULTS

The model described in section 2 is simulated over the life cycle of agents aged $i \in \{20, \dots, 90\}$ and born in cohorts $c \in \{1942, \dots, 2050\}$, with the agent profile estimated up to the 1982 cohort and projected for the other cohorts. Therefore, in each year t, 70 cohorts live together, from those born in $t - 90$ to those born in $t - 20$. The rules of the simulated systems are in effect from $t = 2016$ and, until this year, it is assumed that a stylized system (PAYG) is in force generating an initial state in 2015 with financial balance. In 2015, there is a contingent of agents who, if active, have a contributory history and, if inactive, are entitled to benefits calculated according to PAYG rules.[19] For these agents (specifically those of cohorts born up to 1994), revenues and expenditures are projected according to fixed rules that are different from those applied in the simulations.

The last projected cohort (c = 2050) enters the labor market in 2070 and generates a flow of resources until 2140. Assuming that cohorts after 2050 have the same profile as the last cohort, revenue and expenditure streams were generated until 2140. Therefore, in 2080 we have the year for which the hypotheses for the projection are fully computed and the subsequent years show the trends of revenues and expenditures relative to the last projected cohort. In summary, the initial period of the projection is 2016, the terminal is 2080, and from there until 2140, only the trends implicit in the hypotheses operate.

The objective of the simulations is to relate types and designs of PAYG structures – which we will generically call *rules* – with the aggregate financial imbalance of the system, permanent income inequality and the rate of return of agents from different groups and cohorts. Interpreting the effects of rules on the trajectory of the outcome variables of interest is a difficult task because the relationships are complex in themselves, the agents' profiles have long-term trends and are inserted in PAYG schemes in which part of the contribution rate is progressive. Furthermore, the rules and the adjustments required to financially balance the systems affect groups of agents in a heterogeneous way and there is an assistance pillar that introduces discontinuities in the results. Thus, to facilitate understanding of the effects of the different rules, results are presented in three subsections. In the first one, we discuss different rules that determine the value of the benefit in a stylized environment in which some components of the system are excluded to allow a more direct evaluation of effects. In the second subsection, the system is re-evaluated in a more realistic environment where the excluded components are reintroduced. In both subsections, financial

19. The PAYG stylized structure contains essentially two adjustments: the first regarding the size of the first cohort of 1942, which was arbitrarily increased to encompass all previous cohorts; the second regarding the employers' contribution, which was adjusted to balance the system in 2015.

imbalances are allowed over time, that is, it is implicitly assumed that there are external sources to the system that guarantee its sustainability. In the third subsection, the financial imbalance is eliminated within the system through the introduction of alternative rules that require adjustment contributions from active and inactive groups. The introduction of these adjustment contributions alters the flow of payments and receipts of the agents and, therefore, differently affects inequality and returns for different groups of agents born in different cohorts.

3.1 Unbalanced systems *without* pension floor and assistance benefit

To highlight the relationship between rules and results more directly, we consider in this section stylized PAYG structures in which two components of the system that generate non-trivial discontinuities on results are excluded, namely: i) the minimum amount for the retirement benefit; and ii) the guarantee of assistance benefits for the elderly poor.

In the DB system in force in Brazil, the benefit amount is calculated based on a replacement rate that is the sum of two components: $\alpha(g) + \beta.n$, where the first, which varies by gender, does not depend on the number of contributions made until retirement (n) and can be seen as a compensation for groups that have lower contributory densities such as women, and informal or self-employed workers. To isolate the effect of these components, we consider four alternative rules for computing the retirement benefit amount. Specifically, using the notation from section 2.2 (without the subscripts), the simulated rules are:

$$sb0 - \alpha(g)sm + \beta.n.sm; \quad (28)$$

$$sb00 - \beta.n.sm; \quad (29)$$

$$sc0 - x/ve; \quad (30)$$

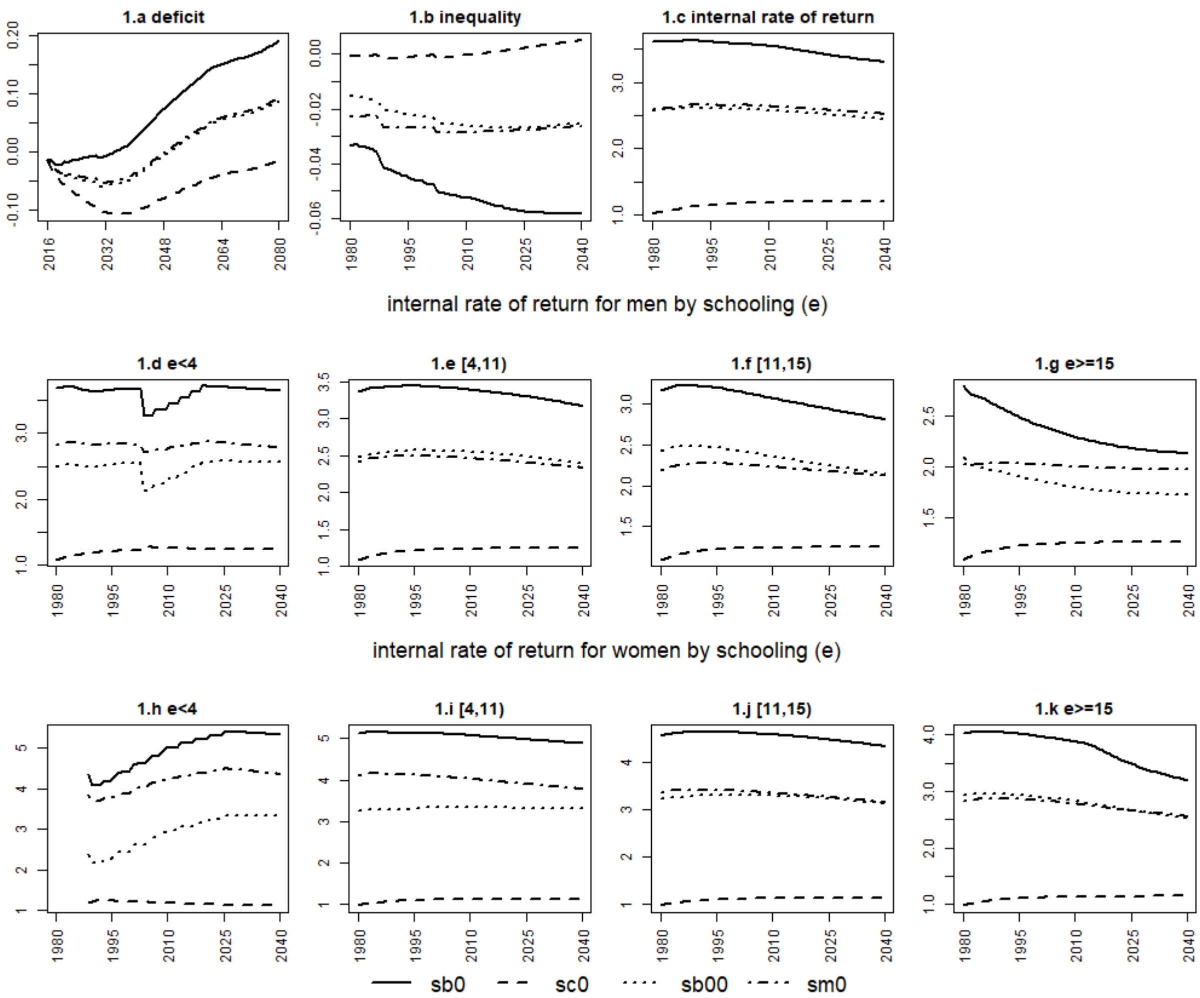
$$sm0 - \alpha(g)sm + x/ve. \quad (31)$$

The *sb0* rule corresponds to the DB system in use in Brazil – i.e., expression (15) without the benefit floor. The *sb00* rule is a variant of *sb0* in which the compensation that is independent of the contributory density of the agent is eliminated. The *sc0* rule corresponds to the benefit of the notional DC system – i.e., expression (16) without the benefit floor – in which the agent receives the capitalization of what he contributed (x) divided by the expected life (ve). And, finally, the *sm0* rule is a new mixed system that adds to the benefit of the DC system, the compensation that is independent of the contributory density of the DB system. Rules *sb00* and *sc0* contain only the component that depends on the contributory density and differ from each other by computing the effect of this density differently. Similarly, rules *sb0* and *sm0* incorporate the compensation component independent of the contributory density and differ from each other for the same reason as in the previous pair of cases.

In figure 1, figures 1.a, 1.b and 1.c present the simulation results of these four rules respectively for the aggregate deficit (A_t), the effect on permanent income inequality ($\Delta\Psi_c$) and the rate of return for the representative agent (ρ_c). Figures 1.d to 1.g (1.h to 1.k) show the rates of return of men (women) for different educational groups ($\rho_{c,e,g}$). Due to the similarity of the results for some educational groups, to simplify the presentation, the return rates of the groups ($e = 1, e = 2$) and of ($e = 3, e = 4$) were aggregated using as weight the proportion of agents with these educational levels in each cohort over time. In all figures of this and other sections, $t = 2016, \ldots.,2080$ and $c = 1980, \ldots, 2040$.

FIGURE 1

Simulation results for unbalanced systems without pension and assistance benefit



Authors' elaboration.

Obs.: 1. Figure 1.a – deficit per year, A_t; figure 1.b – effect on permanent income inequality between groups by cohort, $\Delta\psi_c$; figure 1.c – aggregate rate of return by cohort, ρ_c; the other figures refer to the rate of return by cohort, educational group, and gender, $\rho_{c,e,g}$, as indicated in the figure.

2. The lines correspond to the benefit rules detailed in the text, where: *sb0* – defined benefit system (DB); *sb00* – the restricted variant of *sb0*; *sc0* – notional defined contribution (DC) system; and *md0* – mixed system that combines components from the DC and DB systems.

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Figures 1.a, 1.b. and 1.c show that, while the pure DC system *sc0* has the smallest deficit, the smallest effect (in absolute terms) on permanent income inequality and the smallest rate of return, the pure DB system *sb0* exhibits the opposite pattern. In particular, it should be noted that the DB rate of return is more than three times higher than that of the DC and well above productivity growth, which is an indicator of future deficits in the system.[20] The differences between the rates of return of the DB and DC systems are accentuated for the groups with lower education (figures 1.d to 1.k), especially in the case of women, whose term of the replacement rate that does not depend on the contribution density is higher than that of men.[21] These differences between the two systems were to be expected as the DC system is, by design, more neutral, while the DB tends to be more generous, especially to the less skilled and to women.

Figures 1.a to 1.c also show that the mixed system *sm0*, as well as the *sb00* system, present intermediate results between the DB and DC systems for deficit, permanent income inequality and rate of return. This intermediate pattern of the *sm0* and *sb00* systems is also observed for the returns of the educational groups, both for men (figures 1.d-1.g) and for women (figures 1.h-1.k). However, the *sm0* system tends to present higher returns for groups with less education, especially for women, which is explained by the presence of the term that is independent of contributory density in the calculation of the benefits of the *sm0* system.

It is also possible to observe in figures 1.a to 1.c that the aggregated results of the *sm0* and *sb00* systems are quite similar. Although subject to the parameterization used, this similarity of results implicitly confirms the generosity of the DB system. The mixed system *sm0* can be seen as a DC system in which a compensation is introduced for groups with the lowest propensity to contribute – specifically, the term that is independent of contributory density in the DB system. The incorporation of this component in the *sm0* system makes its results similar to those of the *sb00* system which, like the DC system, does not have this compensation component.

20. Variations in productivity are based on expression (5) and its growth rate between 2016 and 2100 is in the interval [1.2%, 2,0%] per year, that is, less than half of the rate of return of the sb0 system.

21. In general, rates of return are stable across cohorts for most educational groups and genders. An exception is the more educated groups – especially men – in the case of the *sb0* and *sb00* systems, whose rates of return tend to fall. This is due to the progressiveness of the workers' contribution, which is amplified by the rising trend in wages. Another exception is the least educated group of women, which tends to increase returns in the case of the *sb0*, *sb00* and *sm0* systems. This is explained by the anticipation of the retirement age due to the increase in the contribution probability. It is worth noting that the instability of the rates of return of the least educated groups of men and women is due to the operation of two opposite effects: on the one hand, the trend growth of the contribution density among the cohorts reduces the rate of return. On the other hand, this increase in contributory density allows for earlier retirement, increasing the rate of return. The combination of these two effects mainly affects the least educated groups across the cohorts, generating the "roughness" observed in the returns for these groups. This combination of opposing effects is also responsible for the instability of the rates of return in the following subsections.

These results motivate the evaluation of the functioning of the mixed system *sm0*, which reduces the deficit compared to the DB system, has a greater inequality-reducing effect than the DC system and presents an intermediate rate of return between the DB and the DC, including for the different educational groups and genders. However, the abrupt change from the initial DB system to the *sm0* mixed system has the inconvenience of generating a discontinuity in the value of the benefit. For this reason, we propose an adjustment of the rule in which the benefit starts from the DB system and converges in a certain number of years to the mixed system *sm0*. More specifically, the benefit formula in this system (which we denominate DM) is given by:

DM: $\alpha(g)\,sm + [\gamma(\beta * n)sm + (1 - \gamma)\,x/ve],$ (32)

where $\gamma = 1 - \Delta\gamma\,(t - t0)$ is the function that regulates the transition phase. Unless stated otherwise, in the simulations of this system, we set $\Delta\gamma = 1$ and $t0 = 2016.$

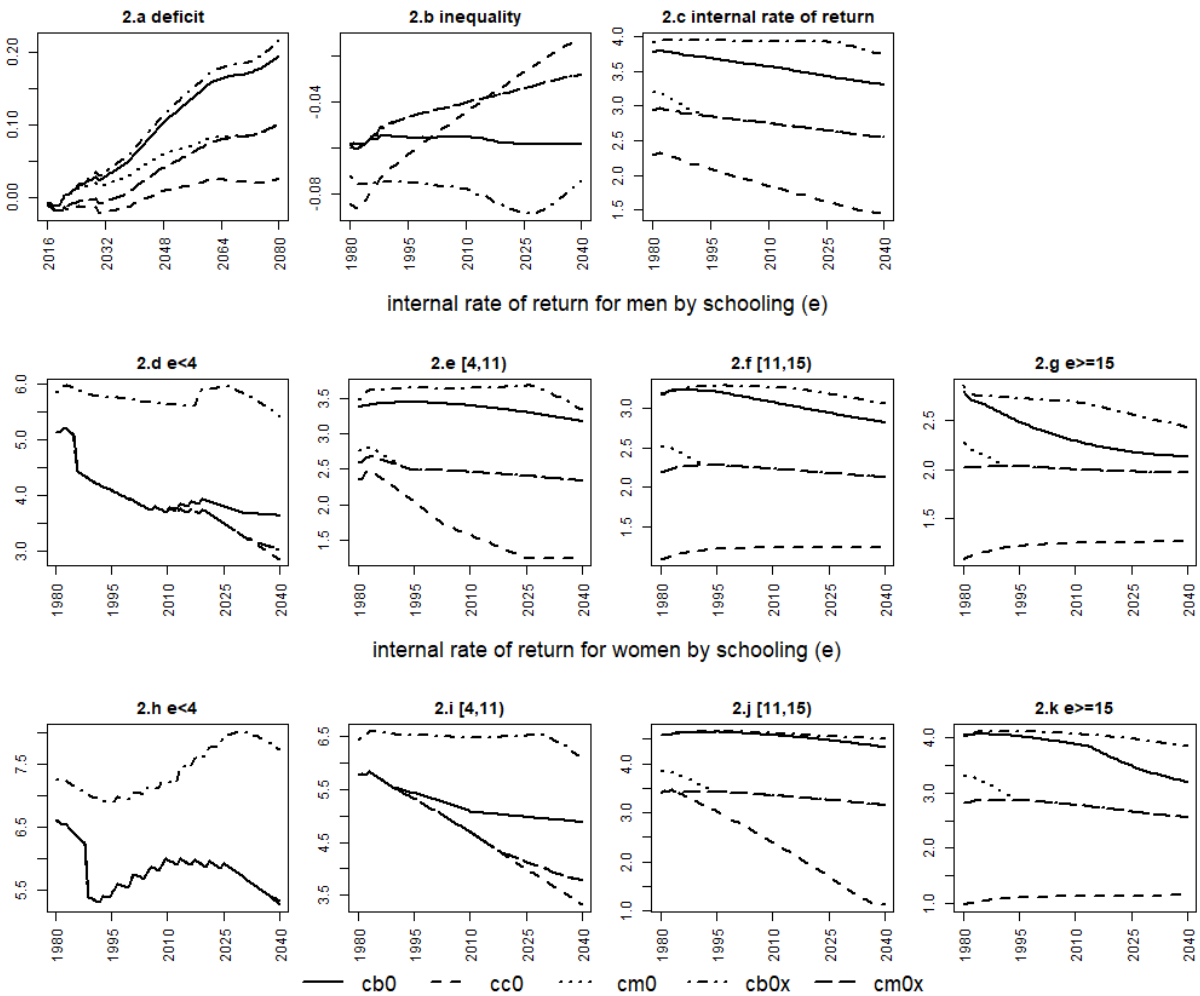
3.2 Unbalanced systems *with* pension floor and assistance benefit

In this section, we evaluate the functioning of the DB, DC and the new mixed DM system presented in the previous subsection in a more realistic environment where the existence of the minimum benefit amount and the guarantee of the assistance benefit for the elderly poor are incorporated. We consider the following rules:

- *cb0* – DB system with benefit calculated by expression (14);
- *cc0* – DC system with benefit calculated by expression (15);
- *cm0* – DM system with benefit calculated by expression (32);
- *cb0x* – DB system considering that the minimum wage is fully indexed by variations in productivity (i.e., $\alpha min = 1$ in equation 6); and
- *cm0x* – DM system with instantaneous transition (i.e., with $\Delta\gamma = 0$ in expression 33).

FIGURE 2

Simulation results for unbalanced systems with pension floor and assistance benefit



Authors' elaboration.

Obs.: 1. Figure 2.a – deficit per year, A_t; figure 2.b – effect on permanent income inequality between groups by cohort, $\Delta\psi_c$; figure 2.c – aggregate rate of return by cohort, ρ_c; the other figures refer to the rate of return by cohort, educational group, and gender, $\rho_{c,e,g}$, as indicated in the figure.

2. The lines correspond to the benefit rules detailed in the text, where: *cb0* – defined benefit system (DB); *cc0* – notional defined contribution (DC) system; *cm0* – mixed system (DM) that combines components from DC and DB systems and has a smooth transition ($\Delta\gamma = 1$); *cb0x* – DB system with full transfer of productivity variations to the minimum wage ($\alpha min = 1$); and, *cm0x* – mixed system (DM) with instant transition ($\Delta\gamma = 0$).

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Figures 2.a, 2.b, 2.c show several results that are worth noting. First, it is possible to observe that the incorporation of the social security floor and the guarantee of minimum benefit for the elderly poor does not qualitatively change the comparisons between the DC, DB and DM systems discussed in the previous subsection. The DM system (*cm0* and *cm0x*) remains an intermediary scheme between the DB (*cb0* and *cb0x*) and the DC (*cc0*) systems for almost all variables, the only exception being the effect on inequality in the latter system.

Secondly, differently from the previous subsection, some trends appear in the outcome variables for some rules. This is due to shifts to the right of the distributions of wages, contributions to social security and education, which end up reducing the proportion of agents for whom the social security floor and the guarantee of minimum income for the elderly are restrictive. As this process occurs over time, the new generations – especially the less educated groups and women – are systematically less affected by the existence of the benefit floor and the welfare benefit, thus introducing the observed trends.

The results of figure 2.a reveal an increase in the deficit for the *cb0x* system in relation to the *cb0* system, which was expected, since only in the former is the correction of the value of the minimum wage – and, therefore, of the social security floor and the minimum benefit – equal to the change in productivity. Figure 2.c also reveals that the incorporation of a transition phase from the DB system to the DM system *cm0* generates only a transient impact on the deficit compared to the system without transition (*cm0x*). This result is interesting insofar as it shows that it is possible to modulate the effect on the deficit of a transition from the DB system to a mixed system in which the benefit calculation includes a component that is independent of contributory density and another component entirely based on systems of defined contribution.[22]

The results in figure 2.b show that the DC *cc0* system promotes a significant drop in permanent income inequality for the initial cohorts, but that this reduction gradually attenuates until it practically disappears for the last cohorts. This is because this system drastically reduces the benefits, leading to a high proportion of agents receiving the benefit floor, which substantially reduces inequality for the initial generations. As mentioned before, the shifts to the right of the wage distributions of wages, contributions and education decrease the proportion of groups restricted by the benefit floor until at the end of the time horizon the typical neutrality result of a DC system shown in the previous subsection is obtained. Figure 2.b also shows that the DB schemes *cb0*

22. There is extensive literature that deals with the effects of transitions from a defined benefit to a defined contribution system. In particular, the models show that the form of financing the transition (e.g., type of tax or debt emission) has substantial effects on the economy (e.g., Auerbach and Kotlikoff, 1987).

and *cb0x* are the ones that promote the greatest reduction in inequality among cohorts, and the effect of the integral correction of the benefit floor for productivity in the *cb0x* system is even more expressive. The reduction of inequality by the mixed systems *cm0* and *cm0x* is smaller (in module) than that of DB systems, and the introduction of the transition between systems (*cm0*) has practically no effect on inequality. Unlike the trajectory of the DC scheme *cc0*, the *cm0* and *cm0x* systems do not present a concentration of groups that receive the benefit floor and that, as in the DC system, are particularly affected by the shift to the right of distributions of wages, contributions, and education.

In terms of ranking between systems, the rate of return of the representative agent presented in figure 2.c shows similar results to those of section 3.1, that is, the DB presents a higher rate than that of the DM which, in turn, exhibits a higher rate than that of the DC. However, unlike the previous subsection, there are marked downward trends in aggregate return, which is due to shifts to the right of the aforementioned distributions. It can also be noted in figure 2.c that the difference in return between the mixed systems with and without transition (*cm0* and *cm0x*, respectively) is eliminated along the initial cohorts, which shows that with the transition the return converges smoothly to its long-term trend.

The sets of figures 2.d to 2.g and 2.h to 2.k show decreasing trends in the intergenerational return for men and women respectively, especially for groups with lower education.[23] The comparison of these figures with the ones in the previous subsection confirms, for most educational groups by gender, the effect of introducing additional restrictions due to the benefit floor and the assistance benefit. It is also notable that the introduction of these restrictions increases the rate of return for the lowest education groups, especially for females. Another point to be highlighted in these figures is the superiority of the return associated with the correction of the minimum wage by the evolution of productivity in the *cb0x* system, being particularly expressive for groups with lower education and contributory density. As in the aggregate case, it is also noted that the difference in return between mixed systems with and without transition (*cm0* and *cm0x*) disappears for initial cohorts of more educated groups.

23. An exception is the least educated group of women (figure 2.h), which shows more erratic movements in the rate of return. The explanation lies in a set of factors: i) the trend of increasing contributions implies a tendency to reduce the retirement age until reaching the minimum retirement age; ii) the trend of wage increases results in increasing benefits and tend to approach and exceed the minimum value; and iii) the existence of the assistance benefit for agents who do not qualify for regular retirement. The combination of these factors implies an increase in the return for the cohorts born until 2010, the stability of the return between 2010 and 2025 and a fall in the return from 2025 on.

All simulations presented until this section assume that the deficit is financed by an external source that does not impact the agents' returns. This makes the simulation results not strictly comparable. For example, as we have seen, the DB system generates greater returns than the mixed DM system, but its deficit is much higher. The next section introduces rules for adjusting the contributions of active and inactive groups to make systems financially sustainable and, in that sense, more comparable.

3.3 Balanced systems with pension floor and assistance benefit

In a financially-balanced pension system, revenues and expenditures are equal over the years and, to ensure this equality within the systems, active and inactive generations contribute to a fund that finances imbalances. The additional contribution of the active is a proportion of the contribution wage (τ_t) and the contribution of the inactive is a rate (τb_t) of the benefit received. It is assumed that active and inactive agents who respectively receive more than L and Lb multiples of the minimum wage contribute to this fund, that is, agents with lower income are excluded from the adjustment contribution.

The rates of active and inactive groups are determined per year according to two classes of rules based on Auerbach and Lee (2011), namely:

- $A(\alpha)$ – defines rates τ_t and τb_t so that the proportion of the deficit paid by the inactive generations is equal to α; and
- $X(\beta, \gamma)$ – defines the contribution rate of inactive generations, τb_t, according to expression (36) and calculates the contribution rate of active generations, τ_t, in a residual way to cancel the deficit (expression 37).

The first class of rules distributes the adjustment burden between active and inactive groups according to a single policy parameter (α) and has the advantage of controlling a relevant dimension, which is the relative burden between the two groups. The second class of rules is a stylized form of the system in use in Germany, where the contribution of the inactive is adjusted both by the variation of the ratio between the size of the inactive and the active population and by the variation in wages net of contributions. In this rule, the inactive can be seen as a partner of the social security system, contributing more if the elderly population proportionally increases and contributing less (may be even with negative rates) when the economy grows, and the wages increase. That is, the inactive generations have the system's burden and bonus, and the active generations pay what is necessary to eliminate the deficit.

Using the notation from section 2.2, the following equations define the two classes of adjustment rules. Let:

$$fa_t = \sum_{c,e,g} \sum_{i \in I(c,t)} I(sc_{c,e,g,i}, L * smin_t) * z_{c,e,g,i} * pop_{c,i,g}, \quad (33)$$

and

$$fb_t = \sum_{e,g,c} \sum_{i > A(c,e,g) \cap i \in I(c,t)} I(b_{c,e,g}, Lb * smin_t) * z_{c,e,g,i} * pop_{c,i,g}, \quad (34)$$

where $I(x,y) = x$ if $x \geq y$ and 0 otherwise. Quantities fa_t and fb_t represent, respectively, the total funds arising from wages of the active and benefits of the inactive that are subject to the contribution to financially balance the system in period t.

Adjustment rule $A(\alpha)$:

$$\tau_t = (1-\alpha).x_t \text{ and } \tau b_t = \alpha.x_t, \quad (35)$$

where $x_t = [G_t - R_t]/[(1-\alpha).fa_t + \alpha.fb_t]$.

Adjustment rule $X(\beta,\gamma)$:

$$\tau b_t = 1 + (\tau b_{t-1} - 1) * [\gamma \Delta sl_t - \beta \Delta rd_t], \quad (36)$$

where Δrd_t: rate of increase in the ratio between the number of inactive and active (old-age dependency ratio); Δsl_t: rate of increase in average wage net of contributions; and

$$\tau_t = [G_t - R_t - (\tau b_t * fb_t)]/fa_t. \quad (37)$$

Applying these rules, paid contributions and benefits received by the agent (c,e,g) who contributes to the adjustment at age are expressed by:

$$f_{c,e,g,i} = \begin{cases} -vc_{c,e,g,i} - \tau_{c+i} * I(sc_{c,e,g,i}, L * smin_{c+i}) & \text{if } i < A_{c,e,g} \\ \left[b_{c,e,g} - \tau b_{c+i} * I(b_{c,e,g}, Lb * smin_{c+i})\right] * ps_{c,g,i} & \text{if } i \geq A_{c,e,g} \end{cases}. \quad (38)$$

Simulations were chosen following the strategy of comparing DB and DM systems in different situations. For rule $A(\alpha)$, we define a set of simulations assuming that active generations finance the entire deficit (i.e., $\alpha = 0$) or that they share the burden in equal proportion with the inactive generation (i.e., $\alpha = 0.5$). For the $X(\beta,\gamma)$ rule, we use the parameterization $\beta = 0.5$ and $\gamma = 1$, which implies that the inactive receive the entire wage gains net of contributions and contribute half of the increase in the dependency ratio. Unless explicitly stated, in all cases, parameters (L, Lb) are fixed in $(3{,}1.15)$.

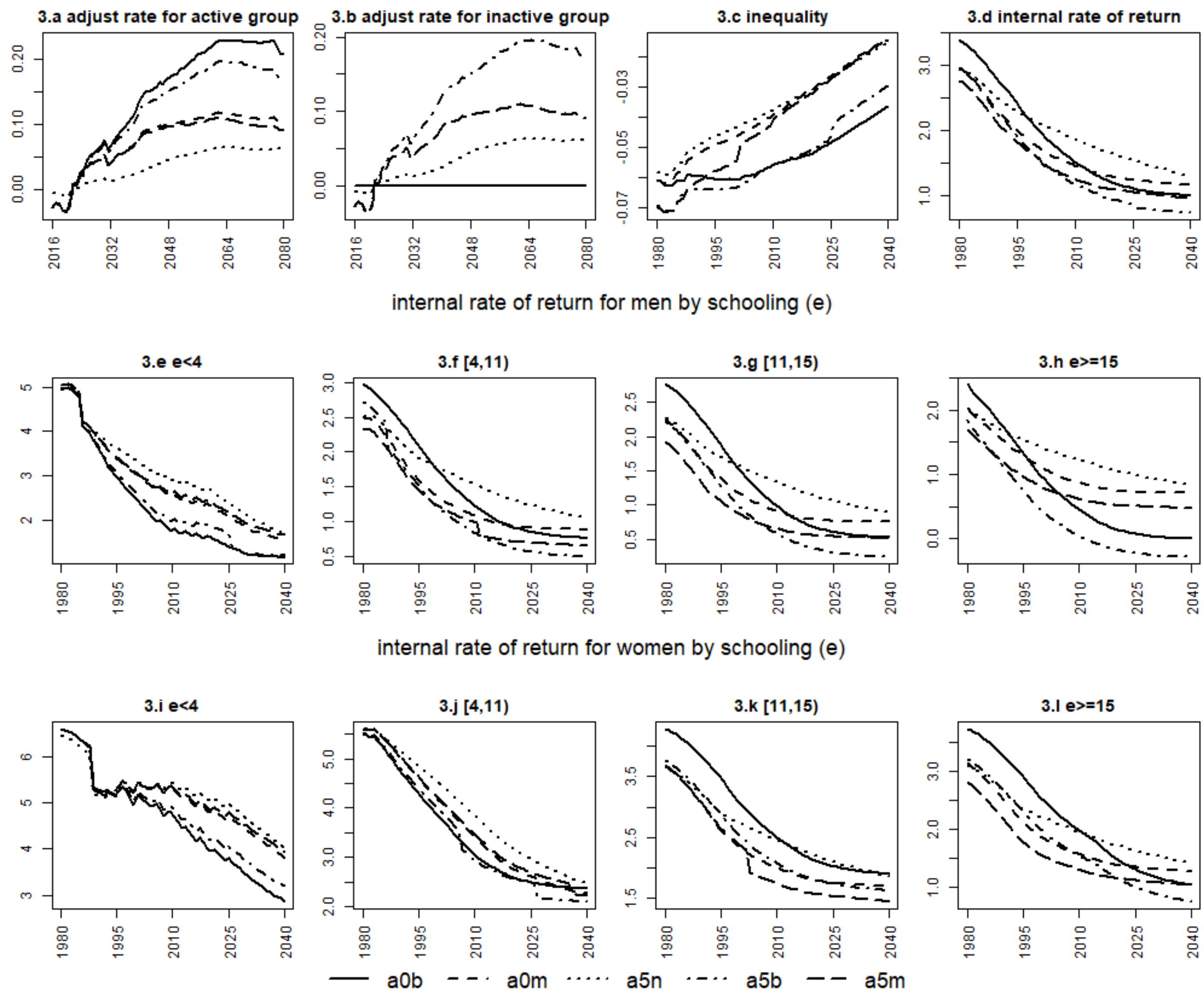
The rules determine for each year the contribution rates of active and inactive generations alive in year t and, as they fluctuate over time, cause the agents of different cohorts to be heterogeneously affected. The simulated rules are:

- *a0b* – DB system with adjustment $A(0)$ (i.e., $\tau b_t = 0, \forall t$);
- *a0m* – DM system with adjustment $A(0)$ (i.e., $\tau b_t = 0, \forall t$);
- *a5b* – DB system with adjustment $A(0.5)$ (i.e., $\tau b_t = \tau_t$);
- *a5m* – DM system with adjustment $A(0.5)$ (i.e., $\tau b_t = \tau_t$);
- *a5n* – DM system with adjustment $A(0.5)$ and lower threshold for the active group (i.e., $\tau b_t = \tau_t$) and $L = 1.15$);
- *xb2* – DB system where the rate of the inactive group is given by $X(0.5{,}1)$ and residual balance is provided by the active group; and
- *xm2* – DM system where the rate of the inactive group is given by $X(0.5{,}1)$ and residual balance is provided by the active group.

Figures 3 and 4 present results for two subsets of these rules in balanced systems where the deficit is zero. The figures are similar to the previous ones, except for the pairs (3.a, 4.a) and (3.b, 4.b), which respectively present the value of the rates $(\tau_t, \tau b_t)$ that are required from active and inactive groups to eliminate the deficit.

FIGURE 3

Simulation results for balanced systems with pension floor and assistance benefit: A rules



Authors' elaboration.

Obs.: 1. Figure 3.a – adjustment rate of the active group per year, $\tau(t)$; figure 3.b – adjustment rate of the inactive group per year, $\tau b(t)$; figure 3.c – effect on permanent income inequality between groups by cohort, $\Delta\psi_c$; figure 4.d – aggregate rate of return by cohort, ρ_c; the other figures refer to the rate of return by cohort, educational group, and gender, $\rho_{c,e,g}$, as indicated in the figure.

2. The lines correspond to the benefit rules detailed in the text, where: *a0b* – DB system with adjustment $A(0)$; *a0m* – DM system with adjustment $A(0)$; *a5n* – DM system with adjustment $A(0.5)$ and $L = 1.15$; *a5b* – BD system with adjustment $A(0.5)$; and *a5m* – DM system with adjustment $A(0.5)$.

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Figure 3.a shows that DB systems *a0b* and *a5b* require higher contribution rates from the active group than DM systems *a0m* and *a5m*. This difference between the systems can also be observed in figure 3.b for the rules involving the contribution of inactive generations, i.e., the *a5b* rule compared to *a5m* and *a5n*. These results are consistent with the difference in deficits presented in sections 3.1 and 3.2 between these two types of systems.

The fund that finances the imbalances of the systems is based on the total contribution of active (inactive) groups, fa_t (fb_t), who receive more than L (Lb) minimum wages. In general, the fa_t portion of the total fund is substantially larger than the fb_t portion, even taking into account the population aging process. This difference explains the proximity of the rates of the pair of rules *a5b* and *a0b* and, especially, of the pair *a5m* and *a0m* (figure 3.a). One implication of this result is to reveal a potential limitation of the $A(\alpha)$ rule, since the contribution of inactive generations does not generate a significant reduction in taxation on active workers even when the distribution of imbalances is carried out in a proportional way, i.e., with $\alpha = 0.5$.

Figures 3.a and 3.b also show the importance of the cut-off point in the distribution of wages above which the active group contribute to the fund that balances the system. As the proportion of agents who earn less than $L = 3$ minimum wages is high, reducing this threshold to the same value as that of the inactive (i.e., $L = Lb = 1.15$) substantially reduces the contribution rates for both the active and inactive groups.[24] In fact, a comparison of the rates of rules *a5m* and *a5n* shows a drop of around 5 percentage points in τ_t and τb_t over almost the entire period of analysis. This result reveals that the distribution of the burden to balance the system between the active and inactive groups can be quite sensitive to the definition of this cut-off point.

As can be seen in figure 3.c, all rules generate a decrease in permanent income inequality. However, this figure also shows that all rules display pronounced trends that attenuate across cohorts. This pattern is essentially due to two factors acting in opposite directions. On the one hand, the increase in the contribution of active and inactive agents observed in figure 3.a and 3.b raises the proportion of agents who receive the minimum wage and this accumulation of agents at the minimum generates a reducing effect on inequality. On the other hand, shifts to the right in the distribution of wages, contributions, and education work to reduce the concentration of agents receiving the minimum. The trends observed in figure 3.c show that this second effect more than offsets the first effect across generations.

24. The proportion of active workers who receive less than 3 minimum wages is in the range [70%, 80%] depending on the year.

Another point to be observed in figure 3.c is that the DB system tends to reduce inequality more than the DM system. However, both systems show similar trends across cohorts. In fact, maintaining the trends observed in figure 3.c, the effect of the two types of systems tends to cancel out for the last cohorts, with the difference that this occurs for later generations in the DB system than in the DM system.

Figure 3.d reveals that all rules show a downward trend in aggregate return across cohorts, with the rate of return converging towards the unit value in most rules for the final cohorts.[25] Another point to be highlighted in figure 3.d is a pronounced accentuation of the downward trends of the DB and DM systems comparable to those analyzed in section 3.2 (figure 2.c) – specifically *cb0* for the DB case and *cm0* for the DM case.

In other words, the introduction of the contribution of active generations to eliminate the growing deficits of the DB and DM systems (figure 2.a.) intensifies the downward trends in the representative agent's return already observed in the context without financial equilibrium. It is also noticeable in figure 3.d that the rules that tax the inactive in the same type of system have a systematically lower return than those that do not tax them (i.e., *a5b* compared to *a0b* or *a5m* compared to *a0m*). These patterns in rates of return are also verified for almost all educational groups of both men (figures 3.e to 3.h) and women (figures 3.i to 3.l).[26] These results show that the rate of return is strongly influenced not only by increases in the taxation of the active generations, but also by the adjustment contribution of the inactive ones.

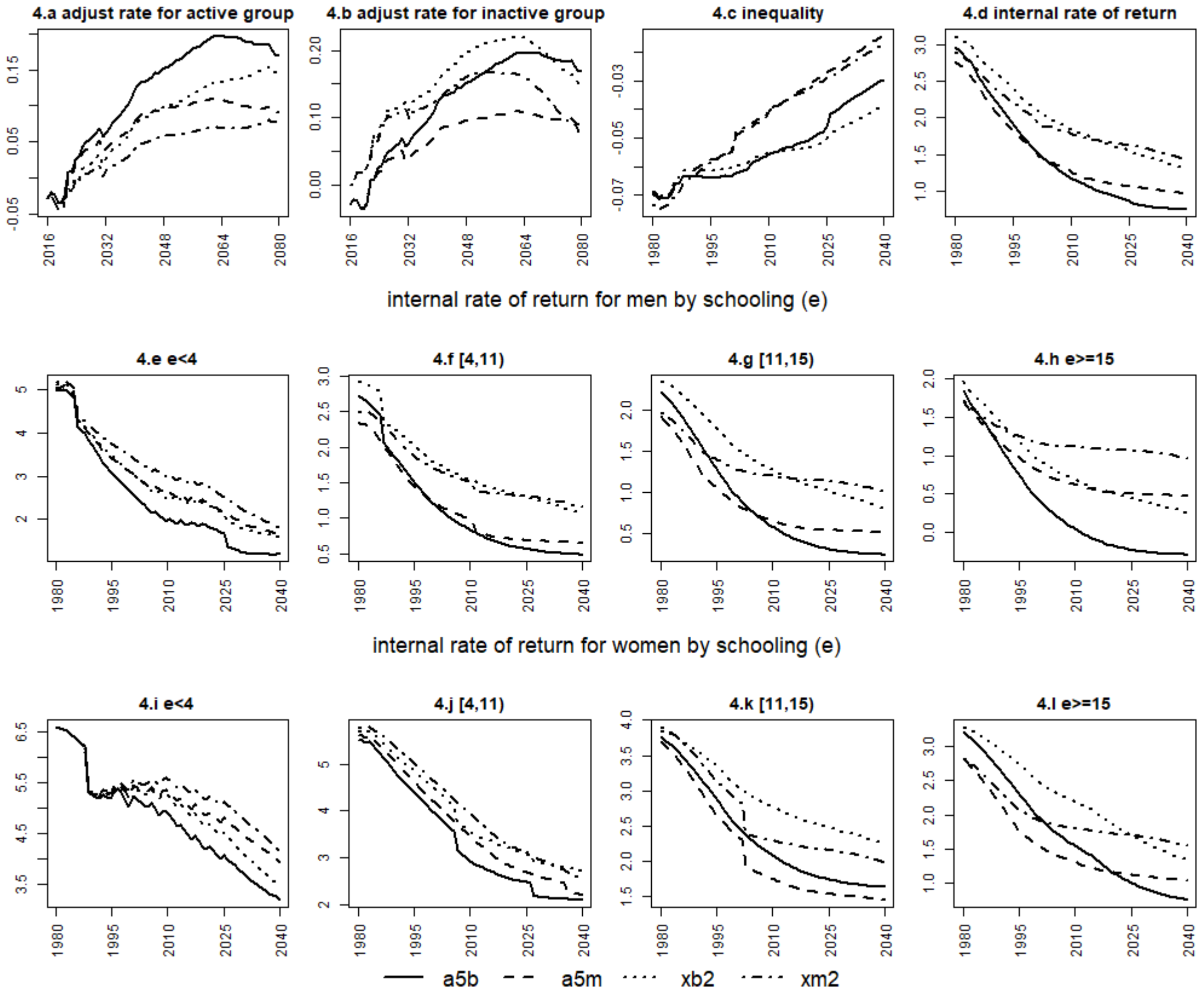
We selected rules *a5b* and *a5m* to compare with rules *xb2* and *xm2*, which are based respectively on a DB and a DM system whose benefits are affected by changes in both the dependency ratio and the wages net of contribution. The results are shown in figure 4, whose figures are arranged in the same way as in figure 3.

25. This convergence is expected as the internal rate of return is close to the long-run growth rate in productivity and the rate of return of the notional assets assumed in the simulations.

26. The exceptions are the groups of men and women with less education, for whom there are practically no differences in returns for the same type of system when taxation is introduced for the inactive generations (i.e., *a5b* against *a0b* or *a5m* against *a0m*). This is because of the benefit floor, which equalizes the benefits for these groups.

FIGURE 4

Simulation results for balanced systems with pension floor and assistance benefit: A and X rules



Authors' elaboration.

Obs.: 1. Figure 4.a – Adjustment rate for the active group per year, τ_t; figure 4.b – adjustment rate for the inactive group per year, τb_t; figure 4.c – effect on permanent income inequality between groups by cohort, $\Delta\psi_c$; figure 4.d – rate of return by cohort, ρ_c; the other figures refer to the rate of return by cohort, educational group, and gender, $\rho_{c,e,g}$, as indicated in the figure.

2. The lines correspond to the benefit rules detailed in the text, where: *a5b* – DB system with adjustment $A(0.5)$; *a5m* – DM system with adjustment $A(0.5)$; *xb2* – DB system with adjustment $X(0.5,1)$; and *xm2* – DM system with adjustment $X(0.5,1)$.

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The simulations in figure 4 make it possible to compare the rules (A, X) and the systems (DB, DM), that is, they allow the analysis of the partial effects of changing the adjustment rule given the system – i.e., comparing *a5b* with *x2b* or *a5m* with *x2m* – or from changing the system given the adjustment rule – i.e., comparing *a5b* with *a5m* or *x2b* with *x2m*. As pointed out earlier, the contribution rate of the inactive group in rule X is given by its calculation formula and the contribution rate of the active group is determined in a residual fashion to financially balance the systems. The comparison of the rates of the inactive group in figure 4.b shows that, keeping the system fixed, rule X presents higher rates than system A. It is also possible to observe in this figure that, keeping the adjustment rule fixed, the rates of the DB system are larger than those in the DM system. The comparison of the contribution rates in figure 4.a shows that, in general, the highest rate is for system *a5b* and, in descending order, for rules/systems *x2b*, *a5m* and *x2m*. This ordering shows that, relative to the inactive group, there is a reversal between rules X and A, with the first presenting lower rates than the second. Furthermore, as in the previous subsection, the DB system has higher contribution rates from the active group than the DM system.

As can be seen in figure 4.c, the effects on inequality of rules *xb2* and *xm2* are very similar to their corresponding counterparts, *a5b* and *a5m*. Again, rules based on the DB system (*a5b* and *xb2*) have a systematically greater effect (in module) than rules based on the DM system (*a5m* and *xm2*). As before (figure 3.c), the main difference between these two systems is that relative to the DM scheme, the effect cancels out for the DB system for later cohorts.

In figure 4.d, it is possible to see that rules *xb2* and *xm2* present the same pattern of falling return of the representative agent as rules *a5b* and *a5m*. This result confirms what was mentioned earlier about the compressing effect that the introduction of contributions of active and inactive generations has on the representative agent's return. It is observed, however, that rule X exhibits greater returns than rule A both for the DB system (*xb2* compared to *a5b*) and for the DM system (*xm2* compared to *a5m*).

As can be seen in figures 4.e to 4.h and 4.i and 4.l, except for the group of women with less education, the other educational groups of both sexes experience a pattern of decline in returns similar to that of the representative agent. It is also possible to see in these figures the higher returns of the X rules in relation to their corresponding A rules for educational groups of both sexes. These results indicate that rule X can generate greater returns to agents than rule A across cohorts.

Finally, to assess the impacts of the balanced systems on intergenerational equity, table 2 presents the standard deviation of rates of return between cohorts for both the representative agent and different educational groups by gender. As seen earlier, these rates of return show downward trends, but even in this situation, this measure can capture the differences in return inequality between systems. Table 2 shows that, in general, the simulation of the DM system with rule X (i.e., *xm2*) is the one with the smallest dispersion, while the simulation of the DB system with rule $A(0)$ (i.e., *a0b*) is the one with the greatest dispersion. The other rules/systems constitute intermediate cases, but with a tendency for those based on rule A and system DB to exhibit higher intergenerational inequity. It is interesting to note that the systems/rules with the lowest (highest) standard deviation tend to require the lowest (highest) taxation on the active group. One explanation for this pattern is that adjustments made earlier in the life cycle are less discounted than those that occur later (Auerbach and Lee, 2011).

TABLE 2

Standard deviation of the rate of return across cohorts by education and gender group

	Representative	Men				Women			
System	Agent	e = {1,2}	e = {3,4}	e = 5	e = 6	e = {1,2}	e = {3,4}	e = 5	e = 6
a5b	0.70	1.16	0.69	0.64	0.65	0.95	*1.17*	0.69	0.77
a5m	0.55	0.98	0.55	0.42	0.35	0.68	1.11	0.73	0.53
a0m	0.55	1.01	0.52	0.46	0.40	0.71	1.13	0.62	0.57
a0b	*0.78*	*1.19*	*0.74*	*0.75*	*0.77*	*1.06*	1.06	*0.81*	*0.89*
xb2	0.55	1.03	0.54	0.48	0.49	0.82	**1.02**	**0.51**	0.60
xm2	**0.43**	**0.93**	**0.42**	**0.26**	**0.18**	**0.59**	1.07	0.63	**0.35**

Authors' elaboration.

Obs.: The systems/rules are detailed in the text. Bold (italic) numbers indicate the lowest (largest) standard deviation for each column in the table.

4 SUMMARY AND CONCLUSIONS

In this article, we analyze the effects of various PAYG public pension systems on financial balance, permanent income inequality, and equity in rates of return across different groups of individuals and cohorts. To do so, we use an empirical model with overlapping generations in which we introduce a set of dimensions that are particularly relevant for developing countries. Specifically, we allowed each generation to be heterogeneous in terms of gender and education and incorporated labor informality (seen as periods of non-contribution to social security) and the existence of an assistance pillar that grants a minimum benefit to agents who do not comply with the rules of eligibility for regular retirement.

We used an identifiable age-period-cohort model (Deaton and Paxson, 1994) to project wage trajectories, the probability of contribution to pension systems, and the distribution of agents among educational groups. These projections were combined with rules from several PAYG structures in their modalities of DB, notional DC – whose remuneration of pension assets is linked to the variation of wages in the economy – and a new mixed system (called DM) that merges elements of the DB and DC systems. Specifically, the retirement benefit in the DM system is calculated by a formula that sums up a component of the DB system that does not depend on the contributory density of the agents – and, therefore, favors the groups whose contribution is more intermittent, such as women and the less educated – with the component of the DC system that depends on the contributory density of the agents.

The simulations of the systems were implemented also considering situations without and with financial balance over time. The results of the simulations without imposition of balance show that, in general, the DB system presents a greater reduction in permanent income inequality and greater returns for the representative agent and for women and for less educated groups. However, this generosity of the DB system has the counterpart of generating larger deficits over time. The results for the DC system have very different characteristics, as it tends to reduce inequality less, has lower rates of return and smaller deficits. The DM system presents intermediate results between the previous two systems for all variables of interest. In this sense, it corresponds to a PAYG structure that combines the redistributive capacity of the DB system with the neutrality and greater financial stability of the DC system.

The effects of introducing a social security floor and an assistance pillar are relevant. The systems' deficit tends to increase and there are trends in the effects on inequality and rates of return between cohorts, notably for groups with less education. This is because of the development process itself – reflected in shifts to the right in the distributions of wages, contribution to social security and education –, which systematically reduce the proportion of the most vulnerable agents who depend on the benefit floor and the assistance pillar. The results also show a relevant impact of minimum wage adjustments indexed to productivity variations in the economy. In fact, the transfer of productivity gains to the minimum generates an increase in the deficit, a greater effect of reducing intragenerational inequality and a higher rate of return, especially for the less educated groups.

If, on the one hand, the DB system generates higher rates of return and greater inequality reduction, on the other, it displays greater deficits. Given this trade-off, the imposition of financial equilibrium internally to the systems makes the comparison between them more leveled. We thus performed simulations of two classes of adjustment rules in which both active and inactive generations contribute to balance the systems. In one of them, the contributions of these two groups depend only on a parameter that regulates the proportion of the amounts of wages (active group) and benefits (inactive group) that balance the systems. Inspired by the current German system, in the other class of rules, benefits vary according to the old-age dependency ratio and changes in the economy's wages – that is, the inactive generations absorb the burden and the bonus of demographic and economic changes – and taxation on the active generations occurs in a residual way to balance the systems. For both classes of rules, the balancing measures are gauged by the annual contribution rates of the active and inactive groups.

Consistent with larger deficits of the DB system, the results for the two classes of balancing rules show that this system requires tax rates from both the active and inactive groups that are higher than those of the DM system. The rates, however, are sensitive to the cut-off point in the distribution of wages above which agents are charged, being substantially reduced with the inclusion of lower-wage groups to help "pay the bill" for the sustainability of the system. Although subject to the parameterization used, another result to be highlighted is that, even in a situation of population aging, charging inactive individuals in the first class of rules does not seem to have significant effects on the rates of contribution of the active workers, which is due to the lower total amount of benefits relative to the total amount of wages.

Naturally, the collection of contributions from active and inactive generations to sustain the systems reduces the flow of income throughout life and, in this sense, represents a factor that counteracts the shifts to the right in the distributions of wages, contribution to social security and education. As the results show, the combination of these two factors generates pronounced trends in the effects on permanent income inequality and on the rates of return between cohorts, with noticeable attenuation of the decrease in inequality and reduced intergenerational returns.

Other important results emerge in the context of balanced systems. The first is that the class of rules that modulate benefits based on the evolution of demographics and wages can alleviate the additional taxation that falls on active generations. Furthermore, the rate of return of this class of rules tends to be higher than that based on the proportional distribution of the burden between active and inactive groups. Finally, the results show that the former class of rules combined with the mixed DM system presents the smallest intergenerational dispersion of returns, while the latter class presents the largest dispersion.

From the point of view of the discussion on structural reforms of PAYG systems in developing countries, our results suggest at least two points that deserve to be highlighted. The first is that the new mixed system that combines redistributive elements of the DB system with the neutrality features of the notional DC system generates intermediate results between these two systems in terms of deficit, reduction of inequality and aggregate return for the representative agent and between educational groups by gender. In this sense, if well designed, it can represent a compromise between the financial imbalances that the country is able to support from its PAYG system and the distributive/equitable role that society wants this system to have. The second point is that, if the social choice is to keep the PAYG system internally balanced, the new mixed system in which benefits are subject to the burdens of demographic transition and the bonuses of productivity growth tends to generate lower contribution rates for active and inactive generations and, from the point of view of horizontal equity, it presents the smallest dispersion of intergenerational returns.

Naturally, our results are conditional on the long-term scenarios that base our projections, the designs of the system types (DB, DC and mixed) and the adjustment rules to balance the systems. However, the algorithm used can be modified to consider other long-term scenarios – including introducing randomness on them – and parameters that make up the systems and rules. The exploration of these exercises is left for future research.

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