DISCUSSION PAPER

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ABSTRACT

This paper applies the Holston-Laubach-Williams methodology to estimate the natural rate of interest for Brazil.

Keywords: natural rate of interest; Kalman filter; trend growth.

1 INTRODUCTION

The natural or equilibrium real interest rate provides a benchmark for measuring the stance of monetary policy, with policy expansionary (contractionary) if the short-term real interest rate lies below (above) the natural rate.[1]

Three different concepts of the equilibrium real interest rate that have received substantial attention in the literature are associated with different time horizons.

The first concept is a purely short-run equilibrium. It is often referred to as the natural rate and is well formulated in New Keynesian dynamic stochastic general equilibrium (DSGE) models, where it corresponds to the value of the real interest rate that would be realized if prices are flexible (Woodford, 2003). This short-run equilibrium is influenced by temporary shocks other than monetary policy shocks. Estimates of this natural rate often exhibit greater variability than actual real interest rates, which are influenced by the presence of price rigidities.

Laubach and Williams (2003) introduced another equilibrium rate concept that has received much attention. This concept is of a medium-run nature. Its derivation is based on a mixture of atheoretical time-series methods and a simple Keynesian-style model consisting of an aggregate demand relationship and a Phillips curve relationship. The equilibrium rate is modeled as the function of potential growth and some preference parameters, similar to a fully specified general equilibrium model without imposing the cross-equation restrictions of such models. Equilibrium rate, potential gross domestic product (GDP) growth, and preference parameters are unobserved variables. How much they move depends on technical parameters of the unobserved components time-series specification. More recently, Laubach and Williams (2016) and Holston, Laubach and Williams (2017) – henceforth HLW – have provided updated estimates indicating a sharp decline toward values around 0 percent for the United States and lower values in the euro area. These estimates have had a substantial impact on policy making. Yet they are characterized by a large degree of imprecision, instability, and potential estimation bias (Beyer and Wieland, 2019).

A third concept is the long-run equilibrium rate or steady-state interest rate. The New Keynesian DSGE models that can be used to derive a short-run natural rate also include a long-run equilibrium rate or steady-state rate to which the short-run rate converges over time.

1. We use interchangeably the terms *natural rate of interest* and *equilibrium real rate of interest*. In the literature, some authors prefer to distinguish between the equilibrium real interest rate and the natural rate; as a result, our definition provide guidance that should help the reader understand the analysis, but care must be taken when comparing the discussion herein to that in the broader literature.

This long-run equilibrium rate is a function of steady-state growth (per capita) and household rates of time preference and elasticity of substitution. Since the effects of price rigidities are temporary, the long-run equilibrium rate in New Keynesian DSGE models is equivalent to the equilibrium rate in a model of real economic growth.[2]

This paper applies the Holston-Laubach-Williams methodology to estimate the natural rate of interest for Brazil. Herein, the natural rate is defined to be the real interest rate consistent with real GDP equaling its potential level (potential GDP) in the absence of transitory shocks to demand. Potential GDP, in turn, is defined to be the level of output consistent with stable price inflation, absent transitory shocks to supply. Thus, the natural rate of interest is the real interest rate consistent with stable inflation absent shocks to demand and supply.

For Brazil, estimates of the natural rate of interest have been generated with various methods. Barbosa, Camêlo and João (2016) apply the Hodrick-Prescott fillter to the sum of the international real rate of interest, adjusted for the premium due to country risk and exchange rate risk, in order to estimate the natural rate of interest in a small open economy. Moreira (2018), building on the HLW methodology, as we do, estimates the natural rate of interest for Brazil between the third quarter of 1999 and the first quarter of 2018.[3] Cravo Junior (2021) estimates the natural interest rate using a modified version of the model proposed by Brand and Mazelis (2019), employing Bayesian procedures and the Metropolis-Hastings algorithm.

The paper is organized as follows. Section 2 outlines the empirical model used in the paper. Section 3 describes the estimation procedure employed. Section 4 provides a description of the data used. Section 5 reports the estimation results. Section 6 tests the robustness of the results. Finally, section 7 brings the concluding remarks.

2. See, for example, Smets and Wouters (2007).

3. There are two main differences between our paper and Moreira's. The first one is the sample period: our sample is larger, as it goes from the first quarter of 1996 to the third quarter of 2020. Second, we retain all the initial parameters employed by the HLW code, with the exception of the initial value of the output gap coefficient in the inflation equation, due to convergence issues. Moreira changed additional parameters. These differences help explain the differences between our results and Moreira's.

2 EMPIRICAL MODEL

HLW use the following model to estimate the natural rate of interest.

Output: $y_t = y_t^* + \tilde{y}_t,$ (1)

inflation: $\pi_t = b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \varepsilon_t^\pi,$ (2)

output gap: $\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2}\,,$

$+\frac{a_r}{2}[(r_{t-1} - r_{t-1}^*) + (r_{t-2} - r_{t-2}^*)] + \varepsilon_t^{\tilde{y}},$ (3)

output trend: $y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*},$ (4)

trend growth: $g_t = g_{t-1} + \varepsilon_t^g,$ (5)

other factor: $z_t = z_{t-1} + \varepsilon_t^z,$ (6)

in which y_t is 100 times the (natural) log of real GDP, y_t^* is the permanent or trend component of GDP, $\tilde{y}_t$ is its cyclical component, π_t is annualized quarter-on-quarter inflation, and $\pi_{t-2,4} = (\pi_{t-2} + \pi_{t-3} + \pi_{t-4})/3$. The real interest rate r_t is computed as

$$r_t = i_t - \pi_t^e, \quad (7)$$

in which expected inflation is constructed as:

$$\pi_t^e = (\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3})/4, \quad (8)$$

and i_t is the exogenously determined nominal interest rate.

The natural rate of interest r_t^* is computed as the sum of trend growth g_t and "other factor" z_t.[4]

$$r_t^* = g_t + z_t. \quad (9)$$

The real interest rate gap is defined as $\tilde{r}_t = (r_t - r_t^*)$. The error terms $\varepsilon_t^l, l = \{\pi, \tilde{y}, y^*, g, z\}$ are assumed to be independent and identically distributed (IID) normally distributed, mutually uncorrelated, and with time-invariant variances denoted by σ_l^2.

4. z_t captures other determinants of r^* unrelated to growth, such as households' rate of time preference and fiscal policy.

3 THE ESTIMATION PROCEDURE OF HLW

Given that the model is linear in the unobserved state variables, HLW apply the Kalman filter to estimate the natural rate of output, its trend growth rate, and the natural rate of interest.

Although economic theory provides insights into the various factors affecting the natural rate of interest, measurement of the natural rate of interest has proven more challenging. This arises because the natural rate must be inferred from the data rather than directly observed. One manifestation of this challenge confronted in classical econometric inference is the "pile-up" problem. As emphasized in Stock and Watson (1998), maximum-likelihood estimation of models in which a time series contains a small permanent component and a sizable transitory component tends to drive the estimated role of the permanent component toward zero – that is, the estimated variance of this component "piles up" near zero. Because real interest rates contain (at most) only a small permanent component, the "pile-up" problem is severe in analyses of the equilibrium real interest rate.

The time series and survey evidence suggests that real GDP growth, labor productivity growth, and real interest rates are subject to highly persistent changes masked by volatile transitory shocks. Maximum likelihood estimates of the standard deviations of the innovations to z and the trend growth rate, σ_g and σ_z, are therefore likely to be biased towards zero owing to the "pile-up" problem. HLW therefore use Stock and Watson's (1998) median unbiased estimator to obtain estimates of the ratio $\lambda_g \equiv \frac{\sigma_g}{\sigma_{y^*}}$ and $\lambda_z \equiv \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}$. HLW impose these ratios when estimating the remaining model parameters (including $\sigma_{\tilde{y}}$ and σ_{y^*}) by maximum likelihood.

HLW estimation method proceeds in three steps. In the first step, they follow Kuttner (1994) and apply the Kalman filter to estimate the natural rate of output, omitting the real rate gap term from equation (3) and assuming that the trend growth rate, g, is constant. HLW compute the exponential Wald statistic of Andrews and Ploberger (1994) for a structural break with unknown break date from the first difference of this preliminary estimate of the natural rate of output to obtain the median unbiased estimate of λ_g.

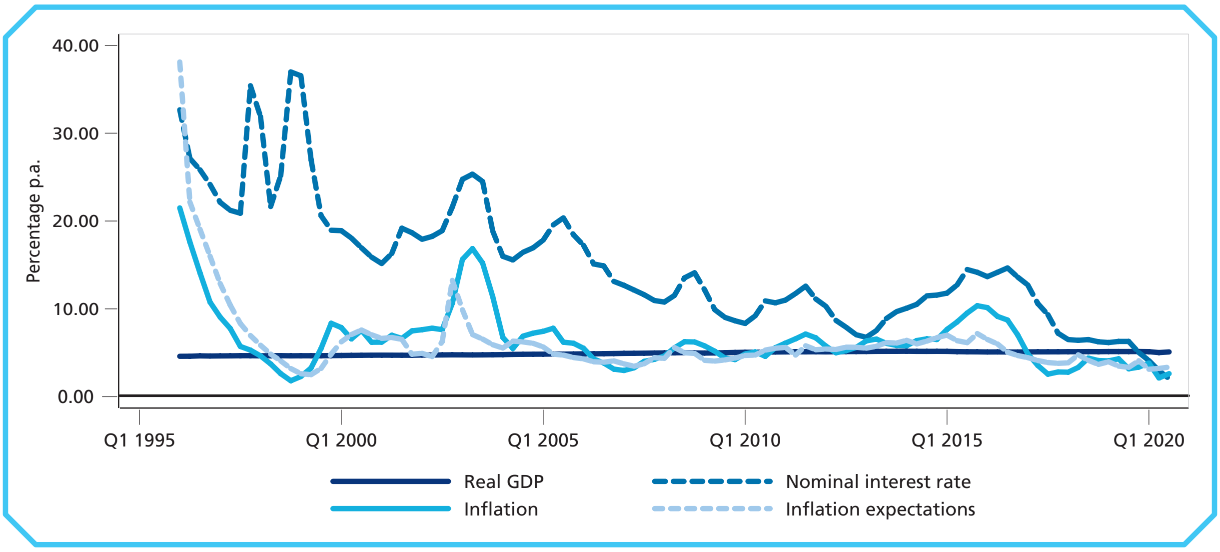
In the second step, HLW impose the estimated value of λ_g from the first step and include the real interest rate gap in the output gap equation under the assumption that z is constant. HLW estimate the model equations and apply the exponential Wald test for an intercept shift in the IS equation at an unknown date to obtain an estimate of λ_g.

In the final step, HLW impose the estimated values of λ_g from the first step and λ_z from the second step and estimate the remaining model parameters by maximum likelihood as described by Harvey (1989). Throughout, they impose the constraints that the slope a_y of the IS equation is negative and the slope b_y of the Phillips curve is positive. They view these as minimal priors on the structure of the model that, in the event, facilitate the convergence of the numerical optimization during estimation.[5] HLW compute confidence intervals and corresponding standard errors for the estimates of the states using Hamilton's (1986) Monte Carlo procedure that accounts for both filter and parameter uncertainty.

4 DATA

The empirical model was estimated using Brazilian quarterly data from the first quarter of 1996 to the third quarter of 2020. The model requires data on real GDP, inflation, nominal interest rates, and inflation expectations. As a measure of real GDP we used the log of the seasonally adjusted chain-weighted volume index published by the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística – IBGE). Inflation is calculated as the accumulated percent change in the last four quarters of the consumer price index (IPCA) of IBGE. As for the nominal interest rate, we used the mean Selic rate target for the three months that make up the quarter, in annual terms. We used the expected inflation four quarters ahead, according to Central Bank's Focus survey from the third quarter of 2001 to the third quarter of 2020. For the period before the start of the Focus survey (first quarter of 1996 to second quarter of 2001), we used four-quarter moving average of past inflation as a proxy for inflation expectations. Figure 1 pictures the data employed by the empirical model.

FIGURE 1
Data employed by the empirical model
(In %)



Source: IBGE.

5. The constraints imposed by HLW are $b_y \geq 0.025$ and $a_r \leq -0.0025$, in addition to $a_{y,1} + a_{y,2} < 1$.

5 EMPIRICAL RESULTS

Table 1 reports the parameter estimates.

The values of λ_g and λ_z indicate time variation in trend growth and the natural rate of interest over the sample.

The coefficient relating the output gap to the real rate gap, a_r, is negative but not statistically significant.

We see that the lower bound restriction on b_y is binding. We set the initial value for b_y at 0.05, and there is no movement away from this value in the numerical routine.

The natural rate of interest is estimated imprecisely, with the sample average standard error of 6.51 percentage points. The imprecision in estimates of the natural rate of interest is even greater when examining the one-sided estimates of r^* that correspond more closely to "real-time" estimates, in that only current and past observations are used in estimating the state.[6] The final observation standard errors shown at the bottom of Table 1 are even wider.

Figure 2A shows the differences between one-sided (filtered) and two-sided (smoothed) estimates of the natural rate of interest.[7] From an econometric view, one-sided and two-sided estimates play a complementary role. The first one serves as the starting point for the second and provides a benchmark to quantify the additional precision that the full sample introduces. Note that two-sided estimates are more precise because they incorporate all the available information from t=1 up to time t=T to estimate the state vector in any intermediate point and, due to their symmetric nature. Note that this symmetry is due to the fact that the filter runs backward from estimates derived forward. In this way, two-sided filtering does not introduce any form of phase-shift in the estimates.

Figure 2B exhibits the estimates of the output gap and figure 2C the real interest rate gap.

6. Because the full sample is used in estimating the model parameters, the analogy, however, is not exact.

7. One-sided estimates of states at time *t* are based solely on information available at time *t*. No data after period *t* is used to calculate estimates of the unobserved state variables. Two-sided estimates use data both before and after time *t* to compute expected values of the state variables at time *t*.

TABLE 1
Parameter estimates

Parameter	Values
λ_g	0.099
λ_z	0.013
$a_{y,1}$	0.635
(1.727)	
$a_{y,2}$	0.123
(0.569)	
a_r	-0.038
(0.542)	
b_π	1.208
(26.667)	
b_y	0.05
(0.501)	
$\sigma_{\tilde{y}}$	1.294
σ_π	1.149
σ_{y^*}	0.919
σ_g	0.090
σ_z	0.442
σ_{r^*}	0.451
Standard error (sample average)	
r^*	6.517
g	1.517
y^*	1.657
Standard error (final observation)	
r^*	9.439
g	1.673
y^*	2.231

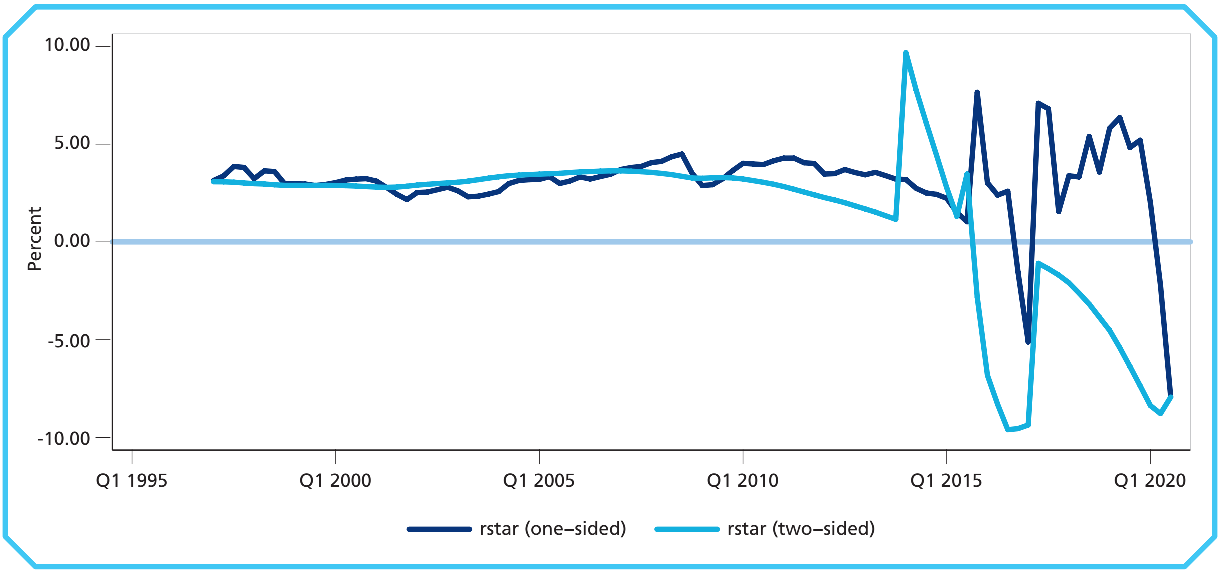
Author's elaboration.
Obs.: T-statistics are indicated in parenthesis.

FIGURE 2

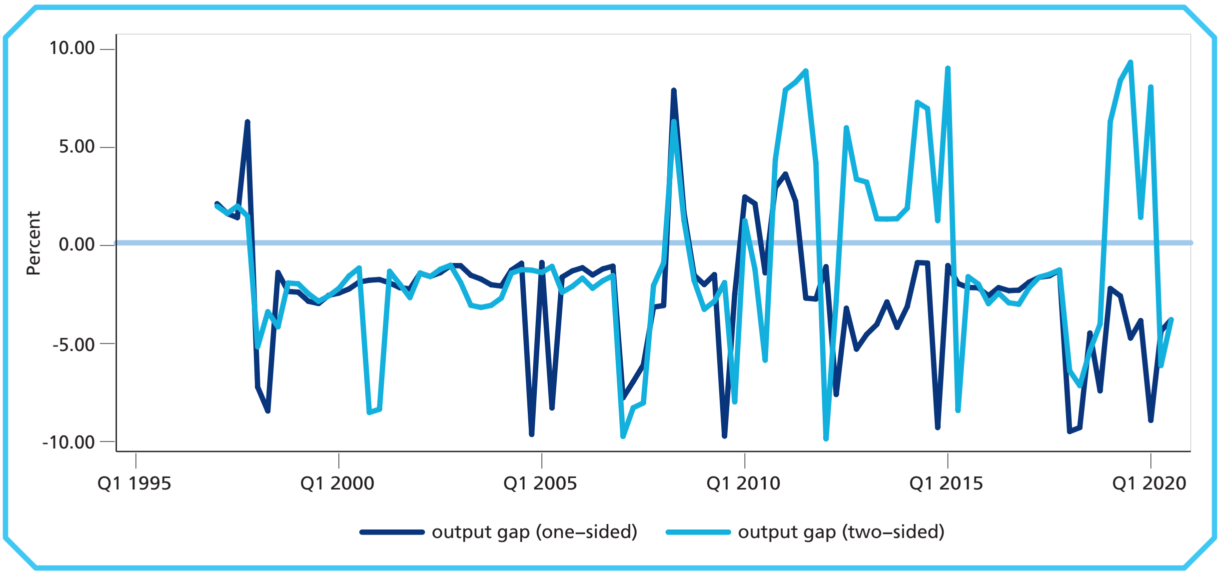
One-sided and two-sided estimates of the natural rate of interest (rstar), the output gap, and the real interest rate gap

(In %)

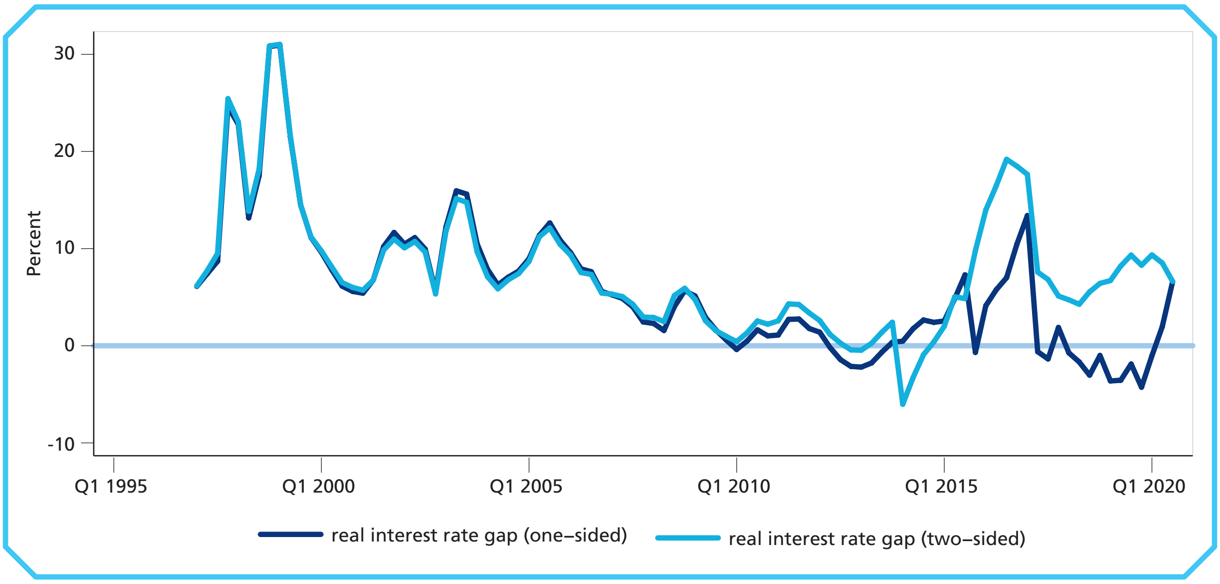
2A – Natural rate of interest (rstar)



2B – Output gap



2C – Real interest rate gap



Author's elaboration.

6 ROBUSTNESS

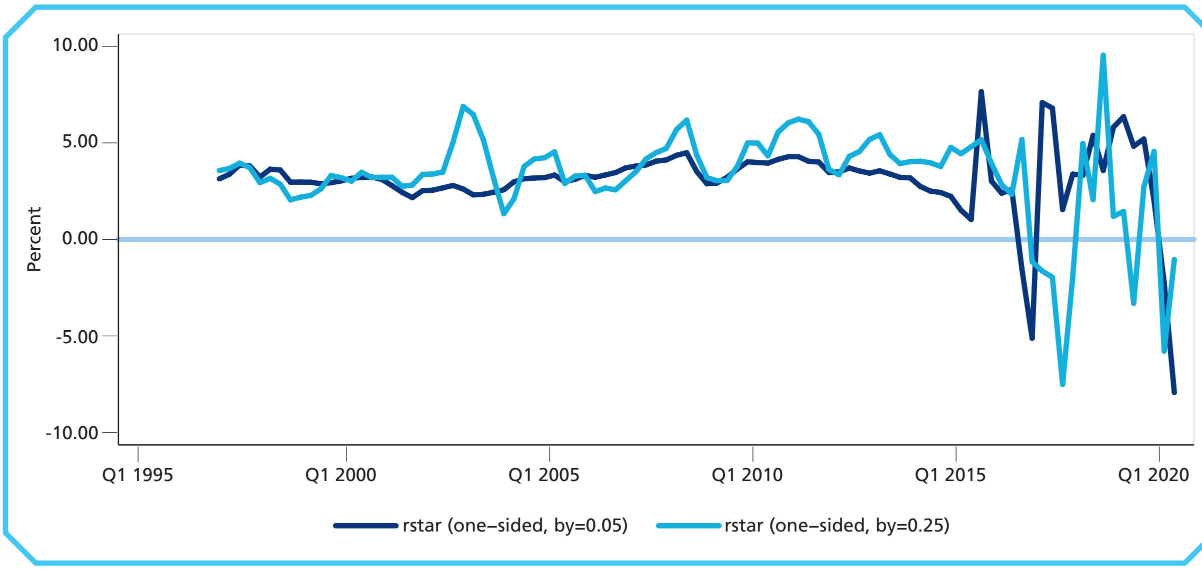
Figures 3 and 4 display the estimates we obtain for the natural rate of interest and the output gap when a different value for the lower bound of b_y is used [$b_y \geq 0.25$], instead of the benchmark value [$b_y \geq 0.05$].

FIGURE 3

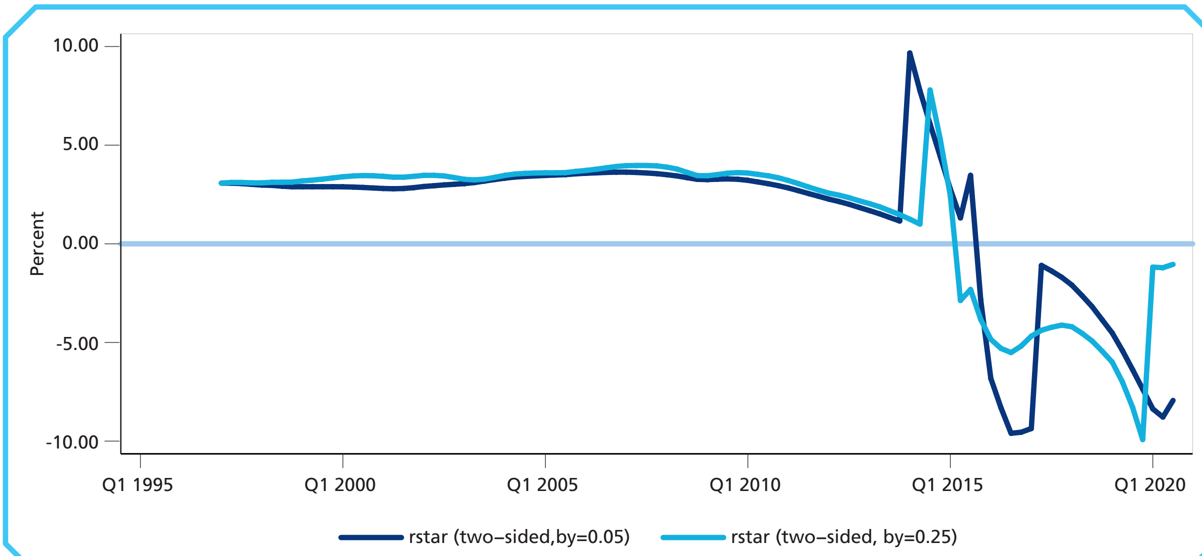
One-sided and two-sided estimates of the natural rate of interest (rstar) for different values of the lower bound restriction on the coefficient b_y

(In %)

3A – One-sided estimates



3B – Two-sided estimates



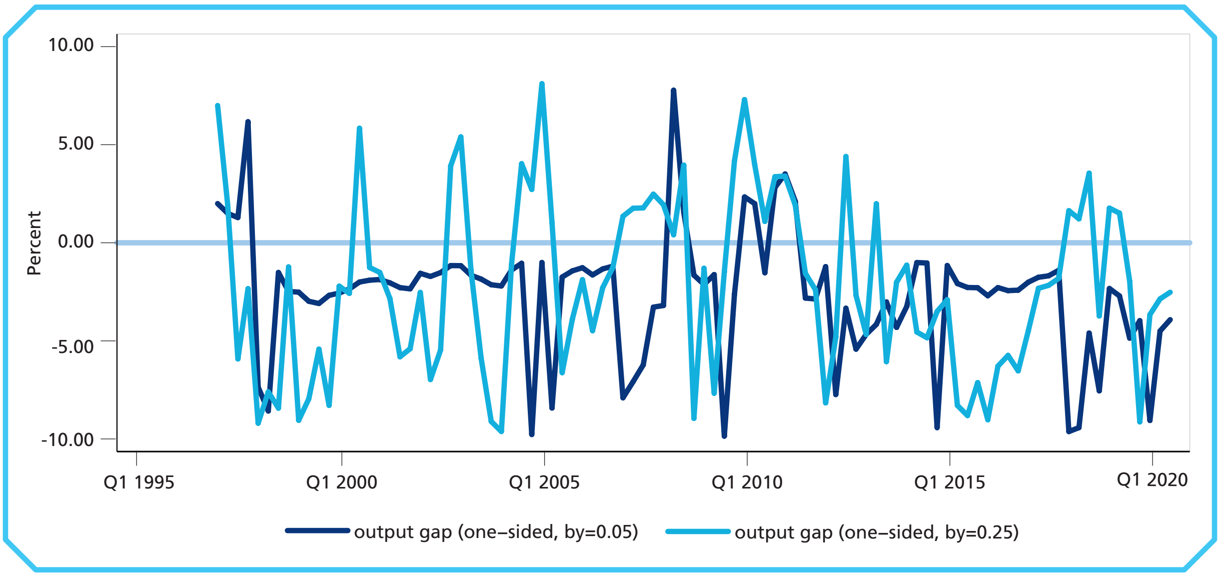
Author's elaboration.

FIGURE 4

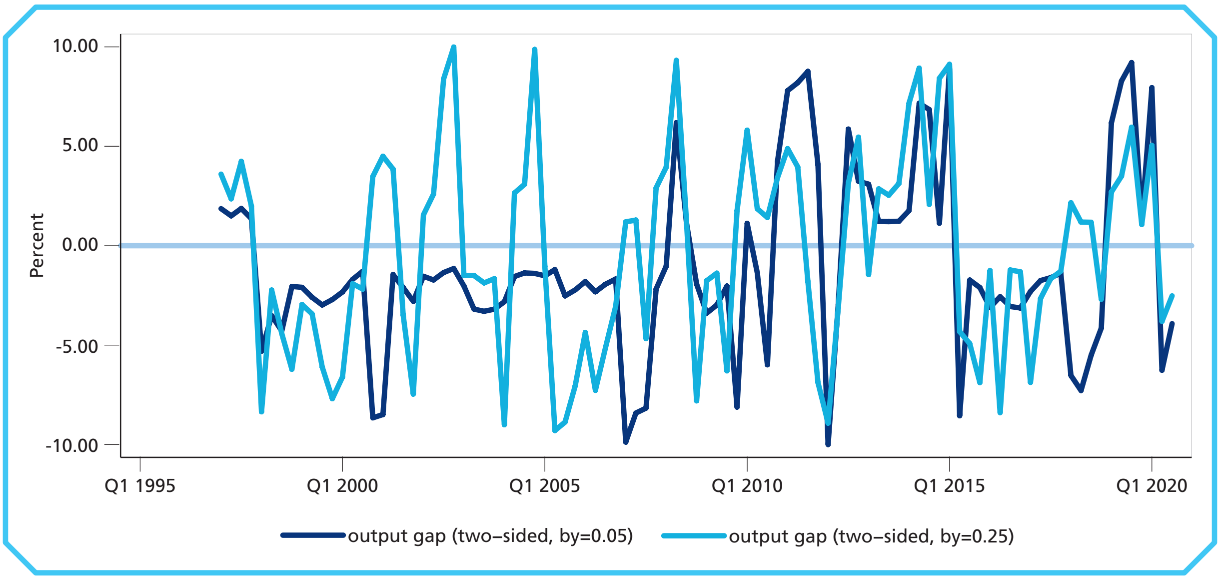
One-sided and two-sided estimates of the output gap for different values of the lower bound restriction on the coefficient b_y

(In %)

4A – One-sided estimates



4B – Two-sided estimates



Author's elaboration.

7 CONCLUDING REMARKS

Through our analysis we find a number of difficulties in estimating the natural rate of interest – even putting aside real time considerations, there are many reasons to be concerned about the robustness of natural rate of interest estimates. Estimates of the natural rate of interest prove to be highly sensitive to the specification of the natural rate equation. Estimates of the natural rate of interest can be highly dependent on the amount of variability allowed in trend growth and the component of the natural rate of interest determined by forces other than trend growth. We also encounter what amounts to an identification problem, in the form of sensitivity to the initial values of the state space model. Essentially, it is very difficult to decompose the natural rate of interest into contributions from potential trend growth and other components that may be linked to fiscal policy and consumer preferences.

Ultimately, in light of all of these problems, our results suggest that statistical estimates of the natural rate of interest will be difficult to use reliably in practical policy applications. Estimates could be useful in historical analyses of the economy and policy, with the caveat that different models may well yield very different estimates. But certainly the real time estimation problems make it very difficult to rely on the natural rate of interest in current policy analysis.

REFERENCES

ANDREWS, D. W. K.; PLOBERGER, W. Optimal tests when a nuisance parameter is present only under the alternative. **Econometrica**, v. 62, n. 6, p. 1383-1414, 1994.

BARBOSA, F. de H.; CAMÊLO, F. D.; JOÃO, I. C. A taxa de juros natural e a regra de Taylor no Brasil: 2003-2015. **Revista Brasileira de Economia**, v. 70, n. 4, p. 399-417, 2016.

BEYER, R. C. M.; WIELAND, V. Instability, imprecision and inconsistent use of equilibrium real interest rate estimates. **Journal of International Money and Finance**, v. 94, p. 1-14, 2019.

BRAND, C.; MAZELIS, F. **Taylor-rule consistent estimates of the natural rate of interest**. Frankfurt am Main: ECB, 2019. (Working Paper Series, n. 2257).

CRAVO JUNIOR, L. F. de C. **Estimativas da taxa natural de juros da economia brasileira**. 2021. Dissertação (Mestrado) – Universidade Estadual do Rio de Janeiro, Rio de Janeiro, 2021.

HAMILTON, J. D. A standard error for the estimated state vector of a state-space model. **Journal of Econometrics**, v. 33, n. 3, p. 387-397, 1986.

HARVEY, A. C. (Ed.). **Forecasting, structural time series models and the Kalman filter**. Cambridge, United Kingdom: Cambridge University Press, 1989.

HOLSTON, K.; LAUBACH, T.; WILLIAMS, J. C. Measuring the natural rate of interest: international trends and determinants. **Journal of International Economics**, v. 108, p. S59-S75, 2017.

KUTTNER, K. N. Estimating potential output as a latent variable. **Journal of Business & Economic Statistics**, v. 12, n. 3, p. 361-368, 1994.

LAUBACH, T.; WILLIAMS, J. C. Measuring the natural rate of interest. **The Review of Economics and Statistics**, v. 85, n. 4, p. 1063-1070, 2003.

_______. Measuring the natural rate of interest redux. **Business Economics**, v. 51, n. 2, p. 57-67, 2016.

MOREIRA, J. R. R. **Estimativas para a taxa natural de juros no Brasil após a adoção do regime de metas para inflação**. 2018. Dissertação (Mestrado) – Universidade Federal do Rio Grande do Sul, Porto Alegre, 2018.

SMETS, F.; WOUTERS, R. Shocks and frictions in US business cycles: a Bayesian DSGE approach. **The American Economic Review**, v. 97, n. 3, p. 586-606, 2007.

STOCK, J. H.; WATSON, M. W. Median unbiased estimation of coefficient variance in a time-varying parameter model. **Journal of the American Statistical Association**, v. 93, n. 441, p. 349-358, 1998.

WOODFORD, M. (Ed.). **Interest and prices**: foundations of a theory of monetary policy. Princeton: Princeton University Press, 2003.

COMPLEMENTARY BIBLIOGRAPHY

CLARK, T. E.; KOZICKI, S. Estimating equilibrium real interest rates in real time. **The North American Journal of Economics and Finance**, v. 16, n. 3, p. 395-413, 2005.

KILEY, M. T. What can the data tell us about the equilibrium real interest rate? **International Journal of Central Banking**, v. 16, n. 3, p. 181-209, 2020.

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