A SUGGESTION FOR BUILDING AN EQUILIBRIUM EMPLOYMENT AND UNEMPLOYMENT THEORY OF THE BUSINESS CYCLE

Marco Antonio Campos Martins

Instituto de Planejamento Econômico e Social - IPEA
Secretaria de Planejamento da Presidência da República,
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INTRODUCTION

The aim of this paper is to present a simple employment and unemployment theory of the business cycle in the context of an equilibrium dynamic monetary model with complete information and overlapping production proccesses.

We do not plan to make any discussion of the outstanding literature on the business cycle. Nonetheless, we shall, below, attempt to show, concisely, how the framework to be developed here relates to the leading model presented by Lucas (1975,1976) on the same subject. We hope that this will be enough for drawing a parallel between our views and the current prominent theoretical efforts in the same field.

The aim of Lucas is to derive a hayekian equilibrium theory of the business cycle (Hayek 1933 p.33n) according to which all profit (or utility increasing) opportunities are fully exploited be the economic agents. To acomplish this task Prof. Lucas (1975) develops an exploratory theory in which serially correlated, "cyclical" movements in real output about trend are triggered by unsystematic monetary shocks in the context of a competitive equilibrium model with efficiently processed incomplete information. However, as he illustrates, the mere introduction of noise into the monetary policy is "not sufficient to induce the sort of responses in real and nominal variables which occur during the observed business cycle. The problem is that in an economy in which all trading occurs in a single competitive market, there is too much information in the hands of traders for them ever to be fooled into altering real decision variables" (Lucas 1975, p. 1120). Indeed, it is crucial, for his argument, that the economic agents make a confusion between relative and general price movements, between real and nominal changes, as they are occurring (see Lucas 1976 pp. 22-23 and 24).

To get an analytically convenient kind of ambiguous informational environment Lucas thinks then of "production and trade as occurring in a large number of markets which are imperfectly linked both physically and informationally", with traders distributed in some way over these markets, at the begin of a period. "This... analytical device first proposed by Phelps (1969)... leads to a real response to a purely nominal disturbance ... However, these real movements are of no longer duration than the duration of the shock: no forces are present to account for the persistence or accumulation of the effects of the initial disturbance" (Lucas 1975 p. 1114). So, to get way from this analytical insuficiency he introduces two of such forces: "informational lags, such as to prevent even relevant past variables from becoming perfectly known, and physical capital, introducing a form of the familiar accelerator effect" (Lucas 1975 p. 1114).

The goal of this paper is also to present a competitive equilibrium theory of the business cycle. However, we shall attempt to fulfill this design in the context of a perfect foresight, certainty model. It seems to us that the inclusion of uncertainty does not alter significantly the theoretical content of the economic models. It serves, basically, for the imposition of a negative real income effect on the economic agents (risk is costly) and a substitution effect against the risky states, with the objective of obtaining analytically convenient results. But uncertainty is just a device. If we could obtain the same results out of a certainty model we might at least gain in analytical simplicity. The central economic ingredient of Lucas's theory, for instance, is not the existence of risk, but the existence of informational lags, which are imposed from outside the model (see Lucas 1975 p. 1121). In this sense, the fundamental task of this paper is to attempt to show how this central ingredient can be substituted by a certainty one, yielding the same basic results.

The thrust of our argument is as follows:

First, observe that the business cycle is well described by serially correlated movements in output about trend: these movements do not exhibit uniformity of either period or amplitude (Lucas 1976, pp. 3-4). The key chacarcteristic of this picture is the recurrent character of these movements (Lucas 1976, p. 11).

One way of taking this characteristic into account is to treat the economic agents as reacting to cyclical changes as "risk", as it has been done by Lucas (Lucas 1975 p. 1121 and 1976 p. 14). Another way, which will be addopted in this paper, is to think of this recurrent character of the business cycle as reflecting mainly the fact that the relevant concept of real resources is related to one or more periods, not to a point in time. The cyclical pattern would then essentially result from the economy's reallocation of some (roughly speaking) given amount of employment, and so of real output, among different points in time within the cycle, in response to external shocks. An equilibrium theory of the business cycle should then be able to explain how that reallocation of resources over time can be efficiently setforth. In order to set the appropriate stage for this kind of analysis we shall present, in the next section, a especially conceived supply function of output, according to which it takes two or more periods to produce each unit of output. This special framework, which will help us to deal with the determination of production over time, will be entirely based on Samuelson's exact consumption-loan model of interest (Samuelson 1958).

Second, instead of thinking of traders (a) as distributed over a large number of "islands" which are imperfectly linked both physically and informationally, and (b) as taking their economic decisions simultaneously, at the begining of a period, we will think of them (a) as located in a single competitive market and (b) as taking their production-consumption decisions at different points in time, not simultaneously.

Finally, instead of assuming that (a) there are informational lags and (b) all traders can review simultaneously their decisions at the begining of each period, we will assume that (a) there is complete information and, (b) as in Martins (1979), the individuals must stick to their nominal economic decisions taken at a point in time, for at least a number of periods.

The remaining of this paper is organized as follows:

THE THEORETICAL FRAMEWORK

AN OVERLAPPING PRODUCTION CYCLES MODEL

The aim of this section is to present a theoretical framework for dealing with the determination of output over time and with the demand functions for a monetary asset. This framework will be entirely based on Samuelson's 1958 exact consumption-loan model of interest (Samuelson 1958).

Samuelson's model deals with the divergence between the competitive and the social optimum opportunity set for consumption, in a simple growth model with overlapping generations and selfish individuals. The source of the divergence lies in the fact that in the context of the model it is impossible to motivate any voluntary private transfer of real resources from incoming (potential savers) to outgoing generations (potential dis-savers) and that leads to the shrinking of the per capita consumption and that leads to the shrinking of the per capita consumption below the social optimum level. This level could nonetheless be attained by a Hobbes-Rousseau type of social contract under which the old generation would have a claim on part of the output produced by the younger one living in the same period. This contract could be materialized by the issuance of a "contrivance money" by a central authority.

As we have shown elsewhere (Martins 1975 and 1979), Samuelson's model is highly useful for monetary analysis, for it generates a demand for securities not backed by physical capital, only by the public trust. Moreover, it could also be used for generating almost any type of consumption pattern at the individual level, as function of the tastes and of the lenght of the life of the individuals, and as function of the rules that the government sets for the issuance of the public debt. It commands then much appeal for serving as foundation for deriving the demand side of a theory of the business cycle. However, the path of the market output, and so the summation of the paths of the individual consumptions are exogeneously set in the context of the original version of the model. This fact rules out from the outset any type of theoretically interesting cyclical phenomena both at the market and at the individual levels. To get the adequate set up it is necessary to let the market output respond at least in part to demand pressures. This will be essentially accomplished by assuming, in this paper, that individuals live for ever and that it takes to any of them more than one period of work to produce one unit of output. To simplify, we will assume that this production process takes exactly two periods. We retain the central hypothesis that output melts away in one period. The model is as follows:

Individuals live for ever. They take however their economic decisions only with respect the next six periods ahead. It takes, to any of them, two periods of work to produce one unit of output. Output can not be stored; it melts away in one period. There are N members, entirely alike, in the population. At each point in time t exactly N_t individuals take production-consumption decisions simultaneously; exactly N_{t-1} individuals have simultaneously taken their decisions one period before; exactly N_{t-2} individuals have simultaneously taken their decisions two periods before, and so on. At each pont in time t the population is then given by.

$$(1) \qquad N = N_t + N_{t-1} + N_{t-2} + \ldots + N_{t-5}$$

In the absence of private contracting and of any adequate social arrangement each individual would choose of its own free will the begining of his production cycle, would consume all his income immediately at the end of this proccess, starving in between. However, there is no need for that: this World is plenty of opportunities.

First there are opportunities for a central coordination of the individuals production and consumption patterns, and also for the issuance of Samuelson's "contrivance money".

Second observe that each individual may prefer a consumption sequence described, for instance, by 1/3, 1/2, 1/3, 1/2, 1/3, 1/2, ...which never adds up to his maximumm attainable level of production over the same number of periods, instead of 1, 0, 1, 0, 1, 0, ... which does. This opens the possibility for uncompensated income transfers, for maintaining unemployed persons alive, for feeding unproductive burocrats, for the explotation of people. Third there are opportunities for changing the number of periods of the individual production process and even for super-imposing proccesses of different maturities.

Despite all these possibilities we are interested only in setting the stage for the functioning of a very simple, competitive monetary economy. So let us assume, on one hand, that the government issues fiat money which is trustingly held by the public of their own free will, to bridge any gap between payments and receipts, and that this is the only asset avaiable in the economy.

On the other hand let us assume that at the begin of period t all the members of the group of the N_t individuals sell their labor force to firms, for exactly six periods ahead. All these individuals receive the same nominal wage, in advance, regardless the work program which will be set for each of them. So, from the demand for consumption point of view, they will have the same amount of nominal income to spend, and face the same set of current and future prices; they can be analytically handled as if they were just one individual. So, let $C_{t,t+j}$ stands for the t+j-th period of life consumption by all members of the group of the N_t individuals who take their decisions at the begin of period t. They are assumed to value their consumption plan according to the value of a "regularly shaped" utility function U_t $(C_{t,t}$ $C_{t,t+1}$, $C_{t,t+2}$, ..., $C_{t,t+5})$, of non-negative consumptions. That is, this utility indicator is a twice differentiable, strictly concave, strictly monotonic increasing function, and the marginal utility of consumption in any period goes to infinity as the consumption level in that period goes to zero. This last condition guarantees that, if income is positive, consumption in any period in positive.

The demand side of the model will then be represented by the following problem: Maximize U_t $(C_{t,t}, C_{t,t+1}, C_{t,t+2}, \ldots\ldots,$ $C_{t,t+5})$ with respect to $(M_{t,t}, M_{t,t+1}, M_{t,t+2}, \ldots\ldots, M_{t,t+5})$, subject to the following set of budget constraints:

(2) (a) $$P_t\ C_{t,t} + M_{t,t} = W_t + G_t$$

(b) $$P_{t+1}\ C_{t,t+1} + M_{t,t+1} = M_{t,t}$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

(c) $$P_{t+5}\ C_{t,t+5} + M_{t,t+5} = M_{t,t+4}$$

where $M_{t,t+j}$ is the total nominal quantity of money carried by the N_t individuals from period $t+j$ over period $t+j+1$; P_t , P_{t+1} , P_{t+2} , ..., P_{t+5} are the nominal prices of consumption in periods $t, t+1$, $t+2$, ..., $t+5$. At the begin of period t these individuals earn a total of W_t units of nominal wages and anticipates G_t units of nominal government transfer payments. To solve their problem they take P_t , P_{t+1} , P_{t+2} , ..., P_{t+5} , W_t and G_t as given and choose the sequence of the money holdings $M_{t,t}$, $M_{t,t+1}$, $M_{t,t+1}$, $M_{t,t+2}$, ..., $M_{t,t+5}$. With "regularly shaped" utility function, $C_{t,t+j} > 0$. The first order conditions are then:

(3) (a) $$\frac{\delta U_t}{\delta C_{t,t}} = \frac{P_t}{P_{t+1}} \cdot \frac{\delta U_t}{\delta C_{t+1}}$$

(b) $$\frac{\delta U_t}{\delta C_{t,t}} = \frac{P_t}{P_{t+2}} \cdot \frac{\delta U_t}{\delta C_{t+2}}$$

....

(c) $$\frac{\delta U_t}{\delta C_t} = \frac{P_t}{P_{t+5}} \cdot \frac{\delta U_t}{\delta C_{t+5}}$$

For given values of P_t , P_{t+1} , P_{t+2} , ..., P_{t+5} , W_t and G_t , the above system determines the consumption program ($C_{t,t}$, $C_{t,t+1}$, $C_{t,t+2}$, ..., $C_{t,t+5}$) and the demands for $M_{t,t}$, $M_{t,t+1}$, $M_{t,t+2}$, ..., $M_{t,t+5}$. In this paper we shall investigate only a simple, particular solution.. Assume that the utility function is represented by.

$$(4) \quad U_t = \log C_{t,t} + \log C_{t,t+1} + \log C_{t,t+2} + \ldots + \log C_{t,t+5}$$

The marginal first order conditions then become:

$$(5) \quad (a) \quad \frac{C_{t,t+1}}{C_{t,t}} = \frac{P_t}{P_{t+1}}$$

$$(b) \quad \frac{C_{t,t+2}}{C_{t,t}} = \frac{P_t}{P_{t+2}}$$

....

$$(c) \quad \frac{C_{t,t+5}}{C_{t,t}} = \frac{P_t}{P_{t+5}}$$

By collapsing the set of budget equations 2.a, 2b and 2.c, and the set of first order equilibrium conditions 5.a, 5.b and 5.c, we can easily find the demand functions for consumption and money holdings:

$$(6)\quad (a)\quad P_{t+j}\ C_{t,t+j} = (W_t + G_t)/6 \quad , j = 0, 1, 2, \ldots 5$$

$$(b)\quad M_{t,t} = 5\ (W_t + G_t)/6$$

$$(c)\quad M_{t,t+1} = 4\ (W_t + G_t)/6$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$(d)\quad M_{t,t+4} = (W_t + G_t)/6$$

$$(c)\quad M_{t,t+5} = 0$$

According to such a solution the t-th individuals spend exactly one sixth of their perceived total nominal income in each one of the periods. Moreover, as shall be clear in the next section, the value of this income will be entirely determined by the past behavior of the money supply.

THE NOMINAL SUPPLY OF MONEY AND

THE AGGREGATE NOMINAL EXPENDITURES

Now let us integrate the government budget constraint into the analysis. This, together with the demand function for cash balances determine the paths of the nominal wages and the aggregate nominal expenditures, irrespectively of any considerations regarding the production side of the model:

So, assume that the government transfer payments are fully financed by the issuance of money,

$$(7) \qquad G_t = M_t - M_{t-1}$$

where M_t is the total stock of money at the begin of period t. This must also be equal to the sum of the individuals money holdings in the same instant; i.e:

$$(8) \qquad M_t = M_{t,t} + M_{t-1,t} + M_{t-2,t} + M_{t-3,t} + M_{t-4,t}$$

Clearly, for a given history of the money supply, M_{t-1} , $M_{t-1,t}$, $M_{t-2,t}$ $M_{t-3,t}$ and $M_{t-4,t}$, the choice of G_t by the government determines not only M_t but also $M_{t,t}$, which is the quantity of money accumulated by the t-th individuals, at the begin of period t. The set of equations 6.a - 6.e can then be more conveniently expressed in terms of $M_{t,t}$:

The t-th individuals will spend one sixth of their total nominal earnings $W_t + G_t$ in each one of the periods spanning from t to t+5. Their total nominal expenditure per period can be found by just re-writing 6.a and 6.b as

$$(9) \qquad P_{t+j}\, C_{t,t+j} = \frac{1}{6}\,(W_t + G_t) = \frac{1}{5}\, M_{t,t}$$

Nominal weges can be immediately found by re-writing (9) as

$$(10) \qquad W_t = \frac{6}{5}\, M_{t,t} - G_t$$

By the same token, $M_{t,t+j}$ will be given by

$$(11) \qquad M_{t,t+j} = \frac{5 - j}{5}\, M_{t,t} \qquad\qquad j = 1,\ 2,\ 3,\ 4 \text{ and } 5$$

Finally, the aggregate nominal expenditure, E_t , at the begin of period t, will be equal to the summ of (9) accross all groups of individuals, i.e. :

$$(12) \qquad E_t = \frac{1}{5} (M_{t,t} + M_{t-1,t-1} + M_{t-2,t-2} + \dots + M_{t-5,t-5})$$

where the values of the $M_{t-j,t-j}$, $j = 0, 1, 2, \dots 5$, are fully determined by the set of G_{t-j} and the correspondent past history of the money supply, as illustrated by (7) and (8).

In the next section we deal with the supply of output and with the derived demand for labor over time.

THE SUPPLY FUNCTIONS OF OUTPUT AND THE DERIVED DEMAND FOR LABOR

At the begin of period t the firms hire, under contract , for six periods, all the avaiable N_t units of labor force, and distribute them efficiently over these periods, with the aim of maximizing profits. Assume that these contracts can not be sold in the market. There are three basic production strategies, of the same lenght, open to the firms: the first is to keep any unit of the labor force full-employed over three complete two-periods production cycles, starting at the begin of period t; this strategy would yield a receipt equal to P_{t+2} plus P_{t+4} plus P_{t+6} of nominal income. The second would be to use the labor force of that individual only over the production cycles which begin at t and at $t+3$ respectively, leaving it unemployed in the remaining of the six-period contract. Observe that according to this strategy the individual would not perform any work from the begin of period $t+6$ until the end of the contract. Hence, the firm might presumably obtain an extra receipt from this worker by selling the remaining of his job contract over the market. We have however ruled out this possibility by assuming, above, that this kind of contract is not marketble. Therefore this strategy would yield a receipt just equal to P_{t+2} plus P_{t+5} of nominal income. Finally the third strategy would be to use the worker only over the production cycles which begin at $t+1$ and $t+4$ respectively, leaving it unemployed during the rest of the contract; this strategy would yield P_{t+3} plus P_{t+6} of nominal income. Competition will drive profits to zero. Hence, in equilibrium, the total amount, N_t , of the labor force avaiable at the begin of period t, will be distributed over the six periods contract time spann in such way that the total sales by the firms, within this time spann, equals the total wage W_t , paid in advance. I.e. , the distribution of the labor force over the contract time must satisfy the following equilibrium condition:

$$(13) \quad (\alpha_t+\beta_t)\, P_{t+2} + \gamma_t\, P_{t+3} + \alpha_t\, P_{t+4} + \beta_t\, P_{t+5} + (\alpha_t+\gamma_t)\, P_{t+6} = \\ = W_t \;/\; N_t$$

where the numbers α_t , β_t and γ_t , non-negative and smaller than unit, are the proportions of the labor force avaiable at the begin of period t, allocated, respectively, in each one of the production strategies considered above; their sum equals to unit. This equilibrium condition represents indeed the derived demand for labor as a function of the current wage rate and future prices of output.

Now observe that in view of the fact that the job contracts can not be sold over the market, the individuals avaiable for hiring at the begin of period t are exactly the same who signed their last contracts six periods before. This means that

$$(14) \qquad N_t = N_{t-6}$$

and that the distribution of the population over the groups of N_t, N_{t-1}, N_{t-2}, ... and N_{t-5} individuals will be entirely set outside the model.

Finally, note that the current supply of output Q_t will be determined by the decisions taken by the firms in the past. It will be given by.

$$(15) \qquad Q_t = (\alpha_{t-2} + \beta_{t-2}) N_{t-2} + \gamma_{t-3} N_{t-3} + \alpha_{t-4} N_{t-4} + \beta_{t-5} N_{t-5} + (\alpha_{t-6} + \gamma_{t-6}) N_{t-6}$$

Let us now deal with the forecasting problem and then derive the market equilibrium conditions.

THE FORECASTING PROBLEM AND THE MARKET EQUILIBRIUM

The aim of this section is to derive the complete market equilibrium and to pave the ground for some interesting simulations.

As we have discussed above, the paths of the nominal expenditures, and of the nominal wages, are entirely determined by the path of the government transfer payments. So, assume that the individuals know the values of G_t, G_{t+1}, G_{t+3}, ..., at the begin of period t. This unambiguously determines $M_{t,t}$, $M_{t+1,t+1}$, $M_{t+2,t+2}$, $M_{t+3,t+3}$, ... (through 7 and 8); W_t, W_{t+1}, W_{t+2}, W_{t+3}, ... (through 10); and E_t, E_{t+1}, E_{t+2}, E_{t+3}, ... (through 12). Let us assume from now on that all these sequences are known at the begin of period t.

On the good market, the aggregate nominal expenditures must equal the total nominal sales, at each point in time. Since the supply of output is given by (15), this means that

$$16 \quad (a) \quad (\alpha_{t-2}+\beta_{t-2}) N_{t-2} P_t + \gamma_{t-3} P_t + \alpha_{t-4} N_{t-4} P_t + \beta_{t-5} N_{t-5} P_t + + (\alpha_{t-6}+\gamma_{t-6}) N_{t-6} P_t = E_t$$

(b) $$(\alpha_{t-1}+\beta_{t-1}) N_{t-1} P_{t+1} + \gamma_{t-2} N_{t-2} P_{t+1} + \alpha_{t-3} N_{t-3} P_{t+1} + \beta_{t-4} N_{t-4} P_{t+1} + (\alpha_{t-5}+\gamma_{t-5}) N_{t-5} P_{t+1} = E_{t+1}$$

(c) $$(\alpha_t+\beta_t) N_t P_{t+2} + \gamma_{t-1} N_{t-1} P_{t+2} + \alpha_{t-2} N_{t-2} P_{t+2} + \beta_{t-3} N_{t-3} P_{t+2} + (\alpha_{t-4}+\gamma_{t-4}) N_{t-4} P_{t+2} = E_{t+2}$$

(d) $$(\alpha_{t+1}+\beta_{t+1}) N_{t+1} P_{t+3} + \gamma_t N_t P_{t+3} + \alpha_{t-1} N_{t-1} P_{t+3} + \beta_{t-2} N_{t-2} P_{t+3} + (\alpha_{t-3}+\gamma_{t-3}) N_{t-3} P_{t+3} = E_{t+3}$$

....

In the above system, all past variables are known at the begin of period t, the sequence of N_t will be determined by (14), and the sequence of E_t will be given by the government transfer payment policy. Equations 16(a) and 16(b) determine P_t and P_{t+1} ,respectively. There is, however, no way of determining the values of the employment distribution parameters (i.e. α, β and γ) from period t on, and of the future price levels, from period $t+2$ on, only with the knowledge of this set of informations. We must still deal with the forecasting problem: In the first place we need to learn how to forecast the sequence of future prices. But this would not be enough: if we knew P_{t+2} we could solve only for the sum $\alpha_t + \beta_t$ (through 16.c), not for α_t separately. Now observe that α_t represent the part of the supply of output, in period $t+2$, which is planned at the begin of period t. So, in order to close the model we need, in the second place, to learn how to forecast the demand for output, at least in period $t+2$, to begin with.

With respect the first of these two problems, assume that the firms forecast always perfectly. Profits will then be systematically equal to zero and the equilibrium condition (13) will always hold ex-post. In other words, the sequence of the price level can be straightforwardly forecast, for five periods ahead, by the following set of equations, similar to (13):

(17) (a) $$(\alpha_{t-6}+\beta_{t-6}) P_{t-4} + \gamma_{t-6} P_{t-3} + \alpha_{t-6} P_{t-2} + \beta_{t-6} P_{t-1} + $$
$$+ (\alpha_{t-6}+\gamma_{t-6}) P_t = W_{t-6} / N_{t-6}$$

(b) $$(\alpha_{t-5}+\beta_{t-5}) P_{t-3} + \gamma_{t-5} P_{t-2} + \alpha_{t-6} P_{t-1} + \beta_{t-5} P_t + $$
$$+ (\alpha_{t-5}+\gamma_{t-5}) P_{t+1} = W_{t-5} / N_{t-5}$$

(c) $$(\alpha_{t-4}+\beta_{t-4}) P_{t-2} + \gamma_{t-4} P_{t-1} + \alpha_{t-4} P_t + \beta_{t-4} P_{t+1} + $$
$$+ (\alpha_{t-4}+\gamma_{t-4}) P_{t+2} = W_{t-4} / N_{t-4}$$

(d) $(\alpha_{t-3}+\beta_{t-3})\, P_{t-1} + \gamma_{t-3}\, P_t + \alpha_{t-3}\, P_{t+1} + \beta_{t-3}\, P_{t+2} +$

$+ (\alpha_{t-3}+\gamma_{t-3})\, P_{t+3} = W_{t-3} / N_{t-3}$

(e) $(\alpha_{t-2}+\beta_{t-2})\, P_t + \gamma_{t-2}\, P_{t+1} + \alpha_{t-2}\, P_{t+2} + \beta_{t-2}\, P_{t+3} +$

$+ (\alpha_{t-2}+\gamma_{t-2})\, P_{t+4} = W_{t-2} / N_{t-2}$

(f) $(\alpha_{t-1}+\beta_{t-1})\, P_{t+1} + \gamma_{t-1}\, P_{t+2} + \alpha_{t-1}\, P_{t+3} + \beta_{t-1}\, P_{t+4} +$

$+ (\alpha_{t-1}+\gamma_{t-1})\, P_{t+5} = W_{t-1} / N_{t-1}$

the value of P_t will be determined by 17(a), given P_{t-4}, P_{t-3}, P_{t-2}, and P_{t-1}, α_{t-6}, β_{t-6} and γ_{t-6}, W_{t-6}, and N_{t-6}; the value of P_{t+1} will be determined by 17(b) in a similar way, and so on.

Now assume that the individuals always forecast perfectly too. Their real consumption over any six period decision time span must then be systematically equal to their contribution to real output over the same time. In view of the production strategies avaiable to firms, the former will be given, at the begin of period t; in per capita terms, by the following expression:

$$(\alpha_t+\beta_t) + \gamma_t + \alpha_t + \beta_t + (\alpha_t+\gamma_t) = 2 + \alpha_t$$

while in view of the budget constraints (2) and of (9), the latter will be given by:

$$\frac{1}{N_t}(C_{t,t} + C_{t,t+1} + C_{t,t+2} + \ldots + C_{t,t+5}) = \frac{1}{N_t}(\frac{W_t + G_t}{6\ P_t} +$$

$$= \frac{W_t + G_t}{6\ P_{t+1}} + \frac{W_t + G_t}{6\ P_{t+2}} + \ldots. + \frac{W_t + G_t}{6\ P_{t+5}}) =$$

$$= \frac{1}{N_t}\ (\frac{M_{t,t}}{5\ P_t} + \frac{M_{t,t}}{5P_{t+1}} + \frac{M_{t,t}}{5P_{t+2}} + \ldots. + \frac{M_{t,t}}{5\ P_{t+5}})$$

equating these two expressions we obtain:

$$(18) \quad (2 + \alpha_t) = \frac{M_{t,t}}{5P_t} + \frac{M_{t,t}}{5P_{t+1}} + \frac{M_{t,t}}{5P_{t+2}} + \frac{M_{t,t}}{5P_{t+3}} + \frac{M_{t,t}}{5P_{t+4}} + \frac{M_{t,t}}{5P_{t+5}}$$

This last equilibrium condition can be readily interpreted as the real supply price of labor. It determines α_t, for a given value of $M_{t,t}$, as set by the government policy, and for a given sequence of prices, P_t , P_{t+1} , P_{t+2} , ... P_{t+5} , as estimated by (17). Clearly, for a given value of the current variables, N_t , $M_{t,t}$ and P_t , the real supply of labor α_t , will be negatively correlated with future price levels. Finally, by substituting the value of α_t into 16,c we can immediatily calculate β_t , and so γ_t . This closes the model.

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