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ABSTRACT

While often times the Hypothetical Monopolist Test (HMT) utilized in relevant market delineation is implemented with uniform price increases throughout all the goods in the candidate relevant market, since 1984 the versions of the U.S. Merger Guidelines have emphasized that these small but significant and non-transitory increase in prices (SSNIP) should be profit-maximizing, what would result in uniform increases only under very particular conditions. Such increases could then be analyzed–sufficient data existing for such–in the same manner as the simulations of unilateral effects of mergers, introduced in the 1980s and further developed in the 1990s. Thus, in this article, building on structural models of demand and supply and on recent contributions to the literature, we propose a unified framework for merger simulations and for the so-called HMT in its diversity of versions implemented in various countries along the years, and we better detail their differences. To illustrate those differences, we report the results of a Monte Carlo experiment using three demand specifications: isoelastic, linear and linearized Almost Ideal Demand System (AIDS), all of them in a two-stage budget setting. We conclude that the choice of the test version and of the demand specification may affect significantly the size of the relevant market found, depending on the distribution and magnitude of cross and own price elasticities in the potential market.

1 INTRODUCTION

During the early history of applying antitrust legislation, the main concern in the control of structures, based on the Structure-Conduct-Performance (SCP) paradigm, was to avoid collusion among large dominant firms. The main goal of antitrust authorities in examining mergers and acquisitions was thus to prevent the so-called coordinated effects. In the 1980s, however, the unilateral effects models began to attract the attention of economists and antitrust enforcers because of their ability to produce very precise predictions about the impacts of mergers that do not depend on firms' ability to sustain cooperation. At the same time, the multiplicity of collusive equilibria predicted by the so-called folk theorems deprived the coordinated effects analysis of a robustness that could attract enforcers.

The analysis of unilateral effects also diminished among economists the concern for delineating the relevant market. Indeed, the importance of this phase as the first step in analyzing a merger is intimately associated with coordinated effects, particularly because the most widespread methodology among antitrust agencies is the Hypothetical Monopolist Test (HMT), adopted for the first time by the American Department of Justice (DoJ) in its Merger Guidelines (MG) of 1982.

This test is based on the principle that if a monopolist of a particular market does not have the power to impose profitable price increases, then any merger that leads to a situation tending to monopoly (in particular, facilitating the formation of a cartel in that market) will be unable to pose competition problems and hence can be approved. Therefore, all the enforcer should look for is the smallest set of products and geographic regions comprising the merging firms' overlapping (i.e., mutually substitute) products where the merger can cause a profitable increase. This will then be the relevant market of the case.

Once the relevant market is defined, the competition agencies' guidelines for analyzing mergers generally call for closely examining the deal to verify how much it really enhances market power or facilitates it in the newly defined relevant market. Then, circumstances are checked that can deter or counteract the competitive effects (new entrants, competitors' reactions, etc.). Finally, if nothing is found that can allay the concerns about the exercise of market power, whether coordinated or unilateral, efficiencies generated by the deal are examined that can compensate (or more than compensate) the tendency for the new merged firm to increase prices or reduce quality, quantity or variety.

Recently, however, articles by Ivaldi and Lörincz (2005), Davis (2006) and Sabbatini (2006) have laid the foundations to establish a link between simulations of the unilateral and coordinated effects of mergers and the hypothetical monopolist test. The present work aims to formalize this common framework between the HMT, in its several versions, and simulations of the unilateral and coordinated effects of mergers, and to discuss the pros and cons of each approach for one or the other, according to the hypotheses assumed in each case.

This article has four more sections. The next one introduces the general framework for analysis, following the molds of the New Empirical Industrial Organization (NEIO), cf. Bresnahan (1989) and Fiuza (2001). The following section relates this general framework with the various versions of the HMT and with the simulations of unilateral and coordinated effects. The third section implements the computation of the HMT versions presented in the preceding section, as extensions of the now traditional horizontal merger simulations, and reports the results of a Monte Carlo experiment comparing the various HMT versions for each of three demand specifications: isoelastic, linear and linearized Almost Ideal Demand System (LAIDS). The last section concludes.

2 GENERAL FRAMEWORK FOR ANALYSIS

The relevant antitrust market is defined through the hypothetical monopolist test as the smallest market for which a hypothetical monopolist (or cartel) can impose a profitable price increase. Hence, the antitrust analyst starts from the smallest possible market (which at the limit can be composed of only the merging firms) and then proceeds to add products or geographic regions to this monopoly or cartel until the threshold where the increase becomes profitable. In each iteration of the algorithm, the set of products and regions is called a *candidate market.*

For didactic purposes, we assume here that there is a finite upper bound for the number of possible products and regions that the relevant market will encompass. In view of the historical practice in applying the test in various jurisdictions, the existence of a maximum limit does not appear to be a heroic assumption. On the contrary: technically speaking, the common complaint of the merging parties that competition agencies define overly narrow relevant markets does not claim that the agencies fail to consider other substitute products (especially because the interested parties rarely fail themselves to suggest them to the agencies), but rather that they accept the hypothesis of a profitable increase "too soon", that is, in a subset that is too small.

Therefore, assuming the existence of this set $\mathcal{J}$ of products and/or regions, one can represent the system of demand for the products of $\mathcal{J}$ as:

$$q^D = D\left(p, x, y, z, \theta\right) \tag{2.1}$$

where:

- $\boldsymbol{q}^D$ is a demand vector of dimension J X 1, where $J = \#(\mathcal{J})$;
- $\boldsymbol{p}$ is the price vector corresponding to the J products;
- $\boldsymbol{x}$ is the $(J \cdot R)$ X 1 vector of characteristics and other demand shifters referring to all the products;
- R is the number of characteristics and other demand shifters referring to each product;
- y is the income of consumers (it can be a vector if the demand system refers to various geographic areas with observable local income levels);

- $\boldsymbol{z}$ is the G X 1 vector of demand shifting variables referring to consumers (which, as in the income example, can be replicated to the same number of geographic regions);
- θ is the vector of demand parameters K^D X 1.

An important note: if one assumes that firms in the short run do not manage to reposition their products or to influence the specific demand shifters of consumers (that is, $\boldsymbol{x}$ and $\boldsymbol{z}$ are exogenous), the demand system will be independent of the economy's prevailing supply structure. In particular, demand is not affected by collusive agreements or behaviors, which will only shift the supply curve(s). It is also important to observe that the fact of writing each demand as a function of all the products of $\mathcal{J}$ does not necessarily imply that all the cross-derivatives are nonzero, that is, not all the products are necessarily substitutes or complements.

To represent the supply-side, we start from the most general framework possible. Assuming that $\mathcal{J}$ is entirely monopolized by a single firm or a completely cohesive cartel, that firm or cartel would maximize its profit with respect to all the products sold in that market: [1]

$$\max_{p_i, i \in \Im} \Pi = \sum_{i \in \Im} \left[p_i \cdot D_i(\boldsymbol{p}, \cdot) - C_i\left(D_i(\boldsymbol{p}, \cdot)\right) \right] \tag{2.2}$$

The first-order maximization conditions of this hypothetical monopolist with respect to each product i would thus be:

$$D_i(\cdot) + \sum_{l \in \Im} \left(p_l - C_l'(\cdot) \right) \frac{\partial D_l}{\partial p_i} = 0 \text{ , } i \in \mathcal{J}, \tag{2.3}$$

After multiplying and dividing each term of the summation by $p_l D_l(\cdot)$ and then multiplying everything by $\frac{p_i}{\sum_{j=1}^{J} p_j D_j(\cdot)}$, we can rewrite (2.3) as:

$$s_i^0 - s_i^0 \cdot m_i^0 \cdot \eta_{ii}^0 + \sum_{\substack{l \in \Im \\ l \neq i}} s_l^0 \cdot m_l^0 \cdot \eta_{li}^0 = 0 \text{ ,} \tag{2.3A}$$

where:

- $s_i = \frac{p_i D_i(\cdot)}{\sum_{j=1}^{J} p_j D_j(\cdot)}$ is the market share, in value, of product i, defined in $\mathcal{J}$;
- $m_l = \frac{p_l - C_l'(\cdot)}{p_l}$ is the price-cost margin of product l;

1 Note that we are assuming cost separability, and thus absence of economies of scope.

- $\eta_{ii} = -\dfrac{\partial D_i}{\partial p_i}\dfrac{p_i}{D_i(\cdot)}$ is the own-price elasticity of demand for product *i*.

- $\eta_{li} = \dfrac{\partial D_l}{\partial p_i}\dfrac{p_i}{D_l(\cdot)}$ is the cross-elasticity of the demand for product *l* with respect to the price of product *i*;

We can stack the equations and write the quasi-supply system in matrix notation:

$$\begin{pmatrix} s_1^s \\ s_2^s \\ s_3^s \\ \vdots \\ s_J^s \end{pmatrix} + \begin{pmatrix} -\eta_{11}^s & \eta_{12}^s & \cdots & \cdots & \eta_{1J}^s \\ \eta_{21}^s & -\eta_{22}^s & \cdots & \cdots & \eta_{2J}^s \\ \vdots & \vdots & -\eta_{33}^s & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \eta_{J1}^s & \eta_{J2}^s & \cdots & \cdots & -\eta_{JJ}^s \end{pmatrix} \begin{pmatrix} s_1^s \cdot m_1^s \\ s_2^s \cdot m_2^s \\ s_3^s \cdot m_3^s \\ \vdots \\ s_J^s \cdot m_J^s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.4}$$

or:

$$\boldsymbol{s}(\boldsymbol{p}) + \boldsymbol{E}(\boldsymbol{p}) \cdot \left[\boldsymbol{s}(\boldsymbol{p}) \bullet \boldsymbol{m} \right] = 0, \tag{2.5}$$

where *E(p)* is the matrix of elasticities and • is the element-by-element matrix or vector multiplication operator.

We can say, then, that the above case is the upper limit of products and regions that can conceivably be attained in the candidate market expansion procedures, a limit, as pointed out, ideally having probability zero.

The integration of the same analytical framework for unilateral effects (also known as merger simulation) and coordinated effects now becomes quite straightforward. It is enough to note, as pointed out by Hausman, Leonard and Zona (1994), Berry and Pakes (1993) and Nevo (1998, 2001), that merging partners internalize in their price decisions the effects of the prices of one firm's products over the other's, effects that were not accounted for before the merger. In the unilateral effects model, internalization only takes place among the products of the firm created by the merger. In the coordinated effects model, internalization occurs among the products of the entire cartel.

The HMT used to delineate the relevant market, then, boils down to searching for the smallest perfect cartel that, once formed after the merger, can manage to increase prices profitably, that is, without diverting consumers' demand so much as to cancel out the margin gain on the infra-marginal consumers (consumers who would continue demanding the cartel's products). The difference between the HMT and a formal analysis of the coordinated effects is therefore that in the latter the set of products, regions and producers has already been defined in previous steps, and the antitrust analyst is testing whether, for that given set, the joint/coordinated exercise of market power is possible, likely and cannot be compensated by efficiencies. In contrast, the HMT inverts the question: What is the smallest set for which coordinated exercise is possible? As discussed in a separate review in progress, the crucial change in wording of the HMT between the 1980s and 1990s onward was that in the first period it was also implicitly asked whether coordinated market power exercise was likely, because the prices of the products outside the cartel would also react to small but significant and non-transitory increases in prices (SSNIP).[2]

We are now ready to formalize this integration of analyses. Let {1, 2, ...*f*,...*F*} be the partitions of market $\mathcal{M}$, which correspond to the sets of products and regions referring to the firms and/or alliances (joint ventures or even partial cartels). For example, market $\mathcal{M}$ can be composed of five partitions, four of which refer to firms acting alone and one to a pair acting collusively. Each of these partitions (which for simplicity we call – at the risk of notational abuse – "firms") would maximize its total profit with respect to each of the products in its portfolio – each "firm" has a portfolio of products whose intersection with the market $\mathcal{M}$ is $\mathcal{M}_f$ (the firm can produce and/or sell other products, which are not of interest to us now and will not affect the conclusions). Hence, each firm *f* would have a maximization program:

$$\max_{p_i, i\in\mathcal{M}_f} \Pi = \sum_{i\in\mathcal{M}_f} \left[p_i \cdot D_i\left(\boldsymbol{p}_f, \boldsymbol{p}_{-f}, \cdot\right) - C_i\left(D_i\left(\boldsymbol{p}_f, \boldsymbol{p}_{-f}, \cdot\right)\right)\right], \tag{2.6}$$

where $\boldsymbol{p}_f$ and $\boldsymbol{p}_{-f}$ are the price sub-vectors referring, respectively, to $\mathcal{M}_f$ and to its complement in the universe $\mathcal{M}$, which we call $\mathcal{M}_{-f}$ (the union of the product portfolios of all the other firms in market $\mathcal{M}$).

The first-order maximization conditions of firm *f* with respect to each product *i* are therefore:

$$D_i\left(\cdot\right) + \sum_{l\in\mathcal{M}_f} \left(p_l - C_l'\left(\cdot\right)\right)\frac{\partial D_l}{\partial p_i} = 0\,,\ i \in \mathcal{J}_f\,, \tag{2.7}$$

Thus, (2.3A) can be rewritten as:

$$s_i^0 - s_i^0 \cdot m_i^0 \cdot \eta_{ii}^0 + \sum_{\substack{l\in\mathcal{M}_f \\ l\neq i}} s_l^0 \cdot m_l^0 \cdot \eta_{li}^0 = 0\,, \tag{2.7A}$$

Stacking gives:

$$\begin{pmatrix} s_1^s \\ s_2^s \\ s_3^s \\ \vdots \\ s_J^s \end{pmatrix} + \begin{pmatrix} -\eta_{11}^s & \eta_{12}^s & \cdots & \cdots & 0 \\ \eta_{21}^s & -\eta_{22}^s & \cdots & \cdots & 0 \\ \vdots & \vdots & -\eta_{33}^s & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & -\eta_{JJ}^s \end{pmatrix} \begin{pmatrix} s_1^s \cdot m_1^s \\ s_2^s \cdot m_2^s \\ s_3^s \cdot m_3^s \\ \vdots \\ s_J^s \cdot m_J^s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{2.8}$$

2 . Not that the evaluation of the likelihood of coordinated exercise of market power was complete with the HMT thought experiment, because other elements had to be taken into account, such as supply substitutability, de novo entries and repositionings.

or:

$$\boldsymbol{s}(\boldsymbol{p})+\left[\boldsymbol{E}(\boldsymbol{p})\bullet\boldsymbol{H}(\boldsymbol{g})\right]\cdot\left[\boldsymbol{s}(\boldsymbol{p})\bullet\boldsymbol{m}\right]=0, \tag{2.9}$$

where H is the "ownership matrix", defined as:

$$H_{jk}=\begin{cases}1 \; se\ \exists\ f \mid j,k\in\mathcal{M}_f\\ 0 \; c.c.\end{cases}$$

and g is the type of game being played. Here we restrict ourselves to price-setting games, so that g refers to the ownership structure and degree of collusion among firms. The set G of all the possible g's can thus be seen as the set of all possible partitions of the set of firms. It is interesting to note that, as stressed earlier, the demand for a product in $\mathcal{M}$ is not necessarily affected directly by the price of all the other products. Moreover, it can occur that one firm maximizes its prices by "departments", that is, the price decision of a firm's good is not taken into account in the price decision of any other of that firm's goods. In this case, although two products come from the same firm, for our present purposes they will be considered as coming from different firms, corresponding to a specific partition g.

Having said this, the following particular g's are worth highlighting:

1) **Pre-merger monoproduct**: $H(g) = I_M$ (the f's are the same as the products themselves, so that the $\mathcal{M}_f$ are singletons).

2) **Pre-merger multiproduct**: the f's refer to the firms *stricto sensu*.

3) **Post-merger without collusion**: the new partition g_a (after the merger) has one less firm than the partition g_b (before the merger), so that the set of products of the merged firm f" – which is one of the subsets of $\mathcal{M}$ – corresponds to the union of the $\mathcal{M}_f$'s of the firms that joined to form f". This means that $H(g_a)$ is $H(g_b)$ with some zeros replaced by 1's.

4) **Post-merger with perfect collusion**: $H(g) = i_M \cdot i_M'$ (matrix of 1's).

Having described the scenarios for comparison, we can now formalize the various tests using this single framework:

I) The analysis of *unilateral effects* consists of comparing (3) with (2) or with (1) – or also with some intermediate structure, in which some firms behave like single product firms and others like multiproduct ones.

II) The analysis of *coordinated effects* consists of comparing (4) with (2) or with (1). Recent developments (see Davis [2006] and Sabbatini [2006]) explore other forms of $H(g)$ that are not simple 1's and zeros. These extensions will be the subject of another article.

III) The *hypothetical monopolist test* algorithm consists of examining all the possible $\mathcal{M}$'s, starting with $\mathcal{M}_{f"}$, until the resulting price increase is profitable.[3] It so happens that both the way the price increase occurs and the algorithm's stopping rule vary according to the HMT version being applied.

a. CLASSIC EUROPEAN HMT (OR USA from 1982-84): a new $\boldsymbol{m}_{s+1} = \boldsymbol{m}_s.(1 + t)$ applies to the situation (1), and is solved for the new vector $\boldsymbol{s}$. The relevant market will be the smallest set $\mathcal{M} \subset \mathcal{J}$ such that, for this fixed uniform increase,

   $\boldsymbol{s}_{s+1} \bullet \boldsymbol{m}_{s+1} \geq (\boldsymbol{s}_s \bullet \boldsymbol{m}_s)$ holds.

   Note that the post-merger price vector does not necessarily correspond to a new Nash equilibrium, unless perhaps as a focal point for a collusive strategy restricted by conditions of incentive compatibility, bounded rationality of the cartel's members or imperfect monitoring by the cartel.

b. American HMT post-Horizontal Merger Guidelines (HMG)-1992: seeks the smallest $\mathcal{M}$ such that, comparing the Nash equilibria (1) or (2) with perfect collusion (4), both

   $\boldsymbol{s}_{s+1} \bullet \boldsymbol{m}_{s+1} \geq (\boldsymbol{s}_s \bullet \boldsymbol{m}_s)$ and $\boldsymbol{m}_{s+1} / \boldsymbol{m}_s \geq (1 + t)$ hold.

We can now also formally introduce the critical loss analysis:

IV) *Critical loss analysis* compares the variation in the quantity predicted by the corresponding experiment (fixed uniform increase or profit maximizing) with that for which the above inequality becomes an equality. It can be applied with both unilateral effects and coordinated effects, and also as an HMT, but it poses the same problems as the HMT – which we discuss in another work in progress – plus symmetry restrictions (when it incorporates the diversion ratio):

$$\eta_{ii}^s = \eta^s, \forall i$$

$$\eta_{ij}^s = \eta_{ji}^s, \forall i, j, i \neq j$$

2.1 THE EXTENT OF $\mathcal{M}$ AND THE HYPOTHETICAL MONOPOLIST TEST

The previous formulations of the HMT can also be generalized for any competitive game between firms before or after a merger. To this end, we resort to the mathematical formalizations of Ivaldi and Lörincz (2005), the most fortunate – to our knowledge – available in the literature. We only need to interpret a single product sold in different geographical regions as different products to be able to apply the following definitions of the two HMT versions to both geographic and product market delineation:

1. **"European" hypothetical monopolist test** (or as interpreted in the U.S. according to the MG of 1982): *Let* $\mathcal{M} \subseteq \mathcal{J}$ *and* $j \in \mathcal{M}$ Let $(\boldsymbol{p}^0, \boldsymbol{q}^0)$ *denote the industry's initial equilibrium price and quantity vectors* [pre-merger] *and* $\pi_j(\boldsymbol{p})$ *the profit generated by product j when the prices are given by*

3 Note that in this process of expanding the set of products, products of firms already present in the previous candidate market and excluded from previous iterations may happen to be incorporated in the following iterations.

p. *Let* $\boldsymbol{p}^{SSNIP}$ *be a price vector whose j-th element is equal to* $(1+t)\ p_j^0$ *if* $j \in \mathcal{M}$*, and equal to* p_j^0 *otherwise, where* $0 < t \leq 0.1$. *Then* $\mathcal{M}$ *is the SSNIP relevant market for product j if and only if:*

$$(i)\ \Delta\pi_M^{SSNIP} \equiv \left(\frac{\sum_{j\in\mathcal{M}} \left[\pi_j\left(p^{SSNIP}\right) - \pi_j\left(p^0\right)\right]}{\sum_{j\in\mathcal{M}} \pi_j\left(p^0\right)} \right) * 100 > 0,\ \textit{and}$$

(ii) for all $\mathcal{M}' \subseteq \mathcal{J}$*, such that* $j \in \mathcal{J}$ *and* $\mathcal{M}'$ *satisfies (i),* $\#(\mathcal{M}) \leq \#(\mathcal{M}')$ *holds.*

This definition is sufficiently general, according to those authors, to account for any type of game, not only a differentiated Bertrand one. Note that the way this version of the HMT has been formulated imposes a single uniform increase in the prices of all the hypothetical monopolist's products, which is optimal only in very particular circumstances. Thus, as the authors point out, this imposition can make the thought experiment of the HMT depart from a comparative statics analysis of equilibria.

2. **Full equilibrium relevant market (FERM) test – for the "equilibrium relevant market price" (ERMP) index:** *Let* $\mathcal{M}$, $\mathcal{J}$, $\mathcal{M} \subseteq \mathcal{J}$ *and* $j \in \mathcal{M}$. *Let* $(\boldsymbol{p}^0, \boldsymbol{q}^0)$ *denote the industry's initial equilibrium price and quantity vectors [pre-merger] and* $\pi_j(\boldsymbol{p})$ *the profit generated by product j when the prices are given by* ***p***. *Let* $(\boldsymbol{p}^{FERM}, \boldsymbol{q}^{FERM})$ *be the new price and quantity vectors when the prices of the products of* $\mathcal{M}$ *are set collusively by the hypothetical cartel. Let the equilibrium relevant market price index for the set of products* $\mathcal{M}$ *be:*

$$ERMP_M \equiv \left(\frac{\sum_{j\in\mathcal{M}} p^{FERM} \cdot q^0}{\sum_{j\in\mathcal{M}} p^0 \cdot q^0} - 1 \right) * 100\,.$$

Consider a small positive value *t*. Then $\mathcal{M}$ is a FERM of product *j* if and only if:

(iii) $\text{ERMP}_M > 100 * t$;

(iv) for all $\mathcal{M}' \subseteq \mathcal{J}$, such that $j \in \mathcal{M}'$ and $\mathcal{M}'$ satisfies (iii), then $\#(\mathcal{M}) \leq \#(\mathcal{M}')$ holds.

In our view this is the closest available definition of the HMT version according to the wording of the MG of 1984. Ivaldi and Lörincz (2005) also provide a version of the FERM test using a consumer surplus index as a criterion, whereby the relevant market would be the smallest within which the reduction in welfare is greater than or equal to – *t*. The use of a consumer surplus index, however, is not supported by the global tradition of delineating relevant markets, and is less immediately accessible to those not proficient in Economics.

Indeed, the wording of the HMT only mentions price increases. But in different markets, where the hypothetical profit-maximizing monopolist would not increase the prices of the various products by the same percentage (the main argument in

Ivaldi and Lörincz's critique to the European version of the HMT), an index has to be chosen to measure the average increase that would be imposed, in the absence of a better distance metric. Since a more scientific criterion regarding what level of price variation would be 'significant' requires more research, the solution is to use a level just as arbitrary as commonly used by competition authorities, five to fifteen percent.

Note that in computing the price and quantity variations between the pre- and post-merger equilibria, even though limiting this to the products of $\mathcal{M}$, the prices of the products of $\mathcal{J} - \mathcal{M}$ (i.e., the substitute products to those of $\mathcal{M}$, but outside the candidate market) *also move in response*, simply because they are substitute products, unless there is perfectly elastic supply at the initial price level, and/or, as we shall demonstrate below, the outsiders' demand curves are isoelastic. This, however, *does not mean the hypothetical monopolist is* maximizing its profit *taking into account the reaction functions* of the outside products as if it were a leader firm in $\mathcal{J}$. It more properly means that supply substitutability is being taken into account.[4]

In our view, then, the criticism of Froeb and Werden (1991) and Werden (1998) about the use of residual demand estimations in Scheffman and Spiller (1987) and Baker and Bresnahan (1985) is unwarranted. According to this critique, those estimations entail Stackelberg leadership. The reaction of the other market competitors, whether it is defined as $\mathcal{M}$ or $\mathcal{J}$, will surely exist in the new equilibrium (except in the cases mentioned above), which *does not* mean that it will be considered in the new profit-maximization conditions of the merged firm. The conditions derived by Baker and Bresnahan are sufficiently general for the estimation of the partial residual demand curves to hold in any type of game, whether sequential or simultaneous. This does not mean that the specification of these curves will be the same. We will have more to say on this subject shortly.

Nevertheless, since the 1992 and 1997 American versions of the HMT require the hypothetical monopolist's price increases not to be accompanied by the prices of the other products, a mathematical formulation closer to the current American HMT concept – call it "Marshallian" – would be:

3. **"Marshallian" profit-maximizing (MPM) hypothetical monopolist test of the relevant market:** *Let $\mathcal{M}$, $\mathcal{J}$, $\mathcal{M} \subseteq \mathcal{J}$ and $j \in \mathcal{M}$. Let $(\boldsymbol{p}^0, \boldsymbol{q}^0)$ denote the industry's initial equilibrium price and quantity vectors [pre-merger] and $\pi_j(\boldsymbol{p})$ the profit generated by product j when the prices are given by* $\boldsymbol{p}$*. Let $(\boldsymbol{p}^{MPM}, \boldsymbol{q}^{MPM})$ be the new price and quantity vectors when the prices of the products of $\mathcal{M}$ are set collusively by the hypothetical cartel of $\mathcal{M}$*, with the prices of the other products of $\mathcal{J}$ remaining constant. *Let the equilibrium relevant market price of $\mathcal{M}$ be:*

4 . Ivaldi and Lörincz (2005) correctly note that the *J – M* restrictions, *ceteris paribus*, make the HMT of the 1992 HMG an out-of-equilibrium exercise. In our view, the FERM version proposed by them is consistent with Marshallian demand, at the same time allowing for a "reaction" of prices of other products, but without incorporating this reaction in the hypothetical monopolist's maximization program, and thus without considering such products as "followers".

$$ERMP_M \equiv \left(\frac{\sum_{j \in \mathcal{M}} p^{MPM} \cdot q^0}{\sum_{j \in \mathcal{M}} p^0 . q^0} - 1 \right) * 100 .$$

Consider a small positive value *t*. Then $\mathcal{M}$ is an MPM relevant market of product *j* if and only if (iii) and (iv) hold, with the $ERMP_M$ redefined as above.

In contrast, but still bearing in mind our disclaimer above, it is true that the interpretation of the 1984 American version of the HMT – which by saying nothing about the absence of a reaction of the other products' prices, would admit that reaction – could leave room for the following mathematical formulation of the HMT, which we call the "dominant firm" or "price-leader firm" formulation:

4. **"Price-leader, profit-maximizer" (PLPM) hypothetical monopolist test of the relevant market:** *Let $\mathcal{M}$, $\mathcal{J}$, $\mathcal{M} \subseteq \mathcal{J}$ and $j \in \mathcal{M}$. Let ($\boldsymbol{p}^0$, $\boldsymbol{q}^0$) denote the industry's initial equilibrium price and quantity vectors [pre-merger] and $\pi_j(\boldsymbol{p})$ the profit generated by product j when the prices are given by* **p**. *Let ($\boldsymbol{p}^{PLPM}$, $\boldsymbol{q}^{PLPM}$) be the new price and quantity vectors when the products of* ***M*** *are set collusively by the hypothetical monopolist of* ***M***, taking into account the price reactions of the other products of $\mathcal{J}$. *Let the equilibrium relevant market price index for the products of $\mathcal{M}$ be:*

$$ERMP_M \equiv \left(\frac{\sum_{j \in \mathcal{M}} p_j^{PLPM} \cdot q_j^0}{\sum_{j \in \mathcal{M}} p_j^0 . q_j^0} - 1 \right) * 100.$$

Consider a small positive value *t*. Then $\mathcal{M}$ is a PLPM relevant market of product *j* if and only if (iii) and (iv) hold, with the ERMPM redefined as above.

Note that the ERMP_M index can be rewritten as:

$$ERMP_M \equiv \left(\sum_{j \in \mathcal{M}} t_j^{RMT} \cdot s_j^0 \right) * 100 ,$$

where $RMT \in \{FERM, MPM, PLPM\}$.

Note also that just as the first type of test, all the subsequent versions abstain from defining the type of game that is being played, except the last one, which explicitly imposes that the hypothetical monopolist of $\mathcal{M}$ be a price leader in $\mathcal{J}$. The differences, then, can be summarized as follows:

1) European SSNIP test: imposes equal t_i's, which means equal and non-optimal increases, unless very particular conditions hold.

2) FERM test: does not impose equal t_i's, because they are produced by the new equilibrium, which in turn is obtained over the entire set of products $\mathcal{J}$.

3) MPM test: does not impose equal t_i's, because they are produced by the new equilibrium, which in turn is obtained only in $\mathcal{M}$ (i.e., $t_f = 0$, $\forall f \notin \mathcal{M}$).

4) PLPM test: does not impose equal t_i's, because they are produced by the new equilibrium, which in turn is obtained in all $\mathcal{J}$, with the producers of $\mathcal{J} - \mathcal{M}$ being followers of the hypothetical monopolist.

The difference between the MPM and FERM tests is therefore that in the former, in solving the new system of first-order conditions equations of the post-merger firms, only the equations of the products contained in $\mathcal{M}$ are solved for the new prices, with the prices of the remaining products of $J - M$ held constant, while in the FERM, all the J equations of the system are solved simultaneously, without imposing restrictions on any subset of prices. We should stress that the omission of the reaction of the other firms in the MPM test, whether or not they are behaving as followers, can underestimate both the unilateral and coordinated effects (depending on the demand specification), because the price competition of differentiated products makes an initial increase by the merged firm prompt a sequence of further price increases until convergence to the new post-merger equilibrium is reached.[5] Therefore, one should expect the size of the relevant market to be weakly smaller in the FERM test as compared to MPM, because the price increase is more likely to be greater than the threshold earlier in the expansion process, and that is what we obtain in our simulations (see next section).

The difference between the FERM and PLPM tests is that in the former, both before and after the merger the firms of the candidate market $\mathcal{M}$ maximize their profits with respect to their products without considering the price reaction of the products outside $\mathcal{M}$: before the merger, they maximize only with respect to their own products, and thereafter they maximize jointly as a hypothetical monopolist – while in the PLPM test, both before and after the merger, the firms consider the reactions of the products of other firms, with the difference that after the merger, instead of entering through the reaction functions, the products incorporated by the hypothetical monopolist enter through the cross effects (unilateral and coordinated).

But the most important point is that in light of what we discussed at the start of Section 2, both FERM and PLPM encompass methods to simulate the unilateral effects of mergers as a particular case. Indeed, assuming a Cournot price equilibrium (FERM) or price leadership equilibrium (PLPM), the first step of the relevant market delineation algorithm is to test whether there would be a significant price increase with only the merged firm acting as a hypothetical monopolist (as long as it has been established that the parties of the merged firm really competed against each other). If so, the conclusion is that the merged firm itself already dominated 100% of the market, and possible coordinated effects with other firms would only need to be evaluated if the analysis of the unilateral effects found sufficient elements deterring or counteracting the exercise of unilateral dominance. In the absence of any countervailing effects, the examination of the case would stop, and the merger should be challenged.[6] If no significant price increase due to unilateral effects is found, this

5. A point stressed by Langenfeld and Li (2001).

6. This is the only case where the total primacy of the unilateral over the coordinated effects would be fully justified, thus dismissing the customary critique that this amounts to looking for the keys under the lamppost instead of where they were lost (see, for example, Salinger, 2006). Still the antitrust analyst may need or want to proceed in the assessment of coordinated effects in order to account for all constraints when designing remedies.

part of the analysis can be ended, but the antitrust analyst will have to continue investigating possible coordinated effects, and to do this, he or she will need to continue expanding the candidate market, that is, continue with the HMT, going through the subsequent steps to analyze factors that may countervail the creation of conditions conducive to coordination.

2.2 WHAT IS THE DIFFERENCE BETWEEN USING RESIDUAL INSTEAD OF MARSHALLIAN DEMAND?

A remaining question is what elasticity is actually to be used. The controversy becomes whether the demand concept to be used is residual or Marshallian. Note that one can be expressed as a function of the other. To do this, we rewrite formula (2.6) as:

$$\max_{p_i} \Pi = p_i \cdot D_i\left(p_i, p_j\left(p_i\right), \boldsymbol{p}_{-ij}\left(p_i\right), \cdot\right) - C\left(D_i\left(p_i, p_j\left(p_i\right), \boldsymbol{p}_{-ij}\left(p_i\right), \cdot\right)\right), \quad (2.10)$$

with the first-order condition then being:

$$D_i\left(\cdot\right) + \left(p_i - C'\left(\cdot\right)\right) \cdot \frac{dD_i}{dp_i} = 0, \quad (2.11)$$

where $\frac{dD_i}{dp_i} \equiv \left[\frac{\partial D_i}{\partial p_i} + \frac{\partial D_i}{\partial p_j}\frac{dp_j}{dp_i} + \ldots\right]$ is the total derivative of demand with respect to the merged firm's own price. We can rewrite this identity as:

$$\eta_{ii}^{R} \equiv \left[\eta_{ii}^{M} - \sum_{l \neq i} \eta_{il}\omega_{li}\right] \quad (2.12)$$

where:

- η_{ii}^{R} is the total own-price, or residual, demand elasticity;
- η_{ii}^{M} is the partial own-price, or Marshallian, demand elasticity;
- η_{il} is the cross-price demand elasticity;
- ω_{li} is the response elasticity (the elasticity of the reaction curve) of the price of product *i* with respect to the price of product *l*.

Therefore, it is only necessary to substitute one elasticity for the other in the previous formulas for the reasoning to hold. The only – albeit not small – difference is that in using the Marshallian demand, as argued by Froeb and Werden (1991) and as adopted in the 1992 HMG, one is not incorporating supply substitutability into the HMT, as would be the case if using residual demand, since the latter takes into account how the competitors will react to a price increase by the hypothetical monopolist. Noting that we expect both the Marshallian and residual elasticities to be positive, and assuming (as we have been doing so far) that firms compete in prices with differentiated products that are substitutes for each other, and hence that the ω_{li} are positive, we know that the residual demand will always be less than the Marshallian demand, and for this reason, the relevant markets will be defined very restrictively.

This difference will be greater, all else equal, the greater the response elasticities ω_{li} are. These elasticities, in turn, depend negatively on the supply elasticity of the competing products. This result is intuitive: if the "followers" have a lower supply elasticity, they will tend to raise their prices more in response to a price increase by the "leader". In this case, as pointed out by Werden (1998), the residual demand elasticity will be even greater than the Marshallian demand elasticity. All else equal, this will imply that the application of Marshallian instead of residual demand will tend to delineate broader markets.

3 A MONTE CARLO EXPERIMENT

To better understand the influence of the HMT version, we wrote a program in Gauss that simulates both the final price increase of a merger unilateral effect (not reported here) and SSNIPs for a sequence of increasing candidate markets, up to the potential market size. The SNNIPs utilized depend on the HMT type: European style (henceforth SSNIP-EU), FERM, MPM (U.S. style) and PLPM. To keep the problem as simple as possible, we performed the tests assuming that each firm has one good in the potential market, so that we could focus on the effect of a selected set of parameters, specifications and formats on the result of the HMT.

For each test type, we ran a Monte Carlo experiment in three demand specifications: isoelastic, linear and the linearized Almost Ideal Demand System (LAIDS). All of them have a common (isoelastic) upper stage budgeting demand equation. This approach follows closely what Crooke et al. (1999) did for simulations of unilateral effects with variable numbers of firms/products within the market, but there are three main differences: *i)* they used the full version of the Almost Ideal Demand System (AIDS), while we employ the linearized version, which substitutes the Stone index for the equation that defines the log of the price index; *ii)* we introduce more variation in the profile of the elasticities – while they designed a symmetric elasticity matrix with constant own price elasticities, we allow both the own and the cross-price elasticities to vary, such that both the diagonal and off-diagonal elements may be strictly decreasing or strictly increasing, and the two patterns are independent of each other; and *iii)* as our main concern when varying demand specifications is to explore this variety of elasticity matrix patterns – especially whether own and cross-price elasticities are increasing or decreasing – we assume an isoelastic specification for the upper budgeting stage throughout the whole experiment, whereas the paper by Crooke et al. covers these three specifications plus the logit demand. This is because the logit demand requires a lower number of parameters, at the expense of generating a too limited and too structured variety of cross-price elasticities, not the richness that we are able to generate with the three other specifications.

We introduced three sources of randomness into the demand equations: into the price elasticity of the upper stage budgeting; into the cross-price elasticity parameters; and into the own price elasticity parameters. The experiment was designed such that the quantities and prices of the initial equilibrium were the same throughout all random draws, and for each draw the total price elasticities (i.e., own and cross-price elasticities taking account of both upper and lower budgeting stages) were exactly the same for all the four tests and all three demand specifications.

We decided not to assume a priori on the parameters any of the restrictions usually brought from Consumer Theory, in particular adding-up, symmetry and homogeneity, which are always subject to hypothesis testing if there are enough degrees of freedom in the econometric estimations.[7] But then we realized, by varying some basic parameters in the initial runs of this simulation, that the ratios between the own price elasticities and the sum of the cross-price elasticities could significantly affect the results, and that an exaggerated size of the cross-price elasticities could crash the simulation for the linear demand specification. We then introduced some structured variation in this ratio, what ended up revealing some very interesting interactions between this variable and the demand specification in determining the size of the relevant market.

The outcomes of the HMT thus vary according to: (i) the type of HMT; (ii) the demand specification; (iii) the random draws that defined the parameters and the distributions of price elasticities; and (iv) the ratio between the own-price elasticity and the sum of the cross-price elasticities for each good.

3.1 EQUATIONS USED FOR SIMULATION

The upper stage demand equation reads as follows:

$$\ln(Y/P) = \alpha + \beta\ln(X) + \gamma\ln(P) + Z'\theta + \varepsilon \quad (3.1)$$

where:

- Y is the subtotal expenditure in goods included in the whole potential market $\mathcal{J}$;
- X stands for the total income or expenditure of consumers (including outside alternatives);
- $\ln(P)$ is a Stone index of the prices ($\sum_l w_l \ln(p_l)$) in the whole[8] potential market $\mathcal{J}$;
- Z is a vector of demographic variables;
- α, β, γ and θ are the respective parameters (α, β and γ scalars and θ a vector).

7. Note that not imposing the adding-up restriction may be alternatively interpreted as saying that the total potential market is sized $J + 1$, where the demand for "j-plus-oneth" good – an exogenously priced outside alternative – is obtained as a residual equation. This may mean that if we do find that the relevant market is $\mathcal{J}$, the size might have been "censored" (in an econometric sense) in J.

8. The weights are shares of the goods in total expenditure. But since the price of the outside alternative is normalized to one, the weighted sum of the log prices covers nothing more than the goods included in $\mathcal{J}$.

The lower stage demand equation may have one of the following specifications:

1. Isoelastic

$$\ln\left(q_i^{LL}\right) = \alpha_i^{LL} + \beta_i^{LL} \ln\left(Y/P\right) + \sum_{i \in \mathcal{J}} \gamma_{ij}^{LL} \ln\left(p_j\right) + Z'\theta_i^{LL} + \varepsilon_i \tag{3.2}$$

With this specification, the total income and total price elasticities are as follows:

$$\eta_{iX}^{LL} = \frac{\partial \ln\left(q_i\right)}{\partial \ln\left(X\right)} = \beta_i^{LL} \cdot \beta \tag{3.3}$$

$$\eta_{ij}^{LL} = \frac{\partial \ln\left(q_i\right)}{\partial \ln\left(p_j\right)} = \gamma_{ij}^{LL} + \left(1+\gamma\right) \cdot \beta_i^{LL} \cdot w_j \tag{3.4}$$

where w_j are the Stone index weights.

2. Linear

$$q_i^L = \alpha_i^L + \beta_i^L \cdot Y + \sum_{i \in \mathcal{J}} \gamma_{ij}^L \cdot p_j + Z'\theta_i^L + \varepsilon_i \tag{3.5}$$

With this specification, the total income and total price elasticities are as follows:

$$\eta_{iX}^{L} = \beta_i^{LL} \cdot \beta \cdot \frac{Y}{q_i} \tag{3.6}$$

$$\eta_{ij}^{L} = \frac{1}{q_i} \cdot \left[\gamma_{ij}^L \cdot p_j + \left(1+\gamma\right) \cdot \beta_i^L \cdot w_j \cdot Y\right] \tag{3.7}$$

3. LAIDS

$$\varpi_i^A = \frac{p_i \cdot q_i}{Y} = \alpha_i^A + \beta_i^A \ln\left(Y/P\right) + \sum_{j \in \mathcal{J}} \gamma_{ij}^A \cdot \ln\left(p_j\right) + \varepsilon_i \tag{3.8}$$

With this specification, the total income and total price elasticities are as follows:

$$\eta_{iX}^{A} = \beta \cdot \left(\frac{\beta_i^A}{q_i} + 1\right) \tag{3.9}$$

$$\eta_{ij}^{A} = \frac{1}{\varpi_i} \cdot \left(\gamma_{ij}^A - \beta_i^L \cdot w_j\right) + \left(1 + \frac{\beta_i^A}{\varpi_i}\right) \cdot \left(1+\gamma\right) \cdot w_j - 1\left[i = j\right] \tag{3.10}$$

where $1[\cdot]$ is an indicator function.

3.2 THE PARAMETERS USED IN THE EXPERIMENT

The size of the total potential market was set as nine firms/products. The parameters were generated in the following manner:

1) The initial equilibrium price and quantity vectors were set as decreasing arithmetic progressions, respectively from 2.5 to 1.5 and from 3.0 to 2.0; Y was set at 120; this enabled calculation of the Stone index;

2) The parameter γ of the upper budgeting stage was set as: $\gamma = -1.25 + 0.5 \cdot u_3$, where $u_3 \sim U[0,1]$; this generates γ's uniformly distributed in the interval $[-1.25, -0.75]$; β was set at one; and X at 400; we then solved the equation (3.1) for α;

3) The diagonal of the matrix of partial elasticities (i.e., referring to the lower stage only) was generated for the isoelastic specification as a geometric progression centered at 2.75 (value for the fifth product/firm), and the progression rate was set as $r_{po} = 1.05 - 0.1 \cdot u_2$, where $u_2 \sim U[0,1]$ independent of u_3. Thus, the sequence of own price elasticities is decreasing when the draw of u_2 is greater than 0.5, and increasing when the draw is lower than 0.5. Moreover, the elasticities tend to a constant (2.75) as the draw gets closer to 0.5, whereas the ratio between the highest and the lowest elasticities (i.e., how much the elasticities differ from each other) is greater the closer u_2 is to one of the extremes (zero or one). This generated the γ_{ij}'s. The β_I's were set to a decreasing sequence with rate –0.01, starting at 0.1. The vector of α's was then obtained by difference, by solving (3.2) for them.[9]

4) The partial cross-price elasticities were generated in a similar fashion, but from a different draw u_1 independent of u_2 and u_3. The progression rate was set as $r_{pc} = 0.75 + 0.5 \cdot u_1$, but there were two sequences averaged to generate the grid, one vertical and one horizontal, centered at 0.25 and 0.15 respectively (this prevented the elasticity matrix from being symmetric or nearly symmetric, as is usually assumed). As opposed to the diagonal, the off diagonal elements end up being decreasing for u_1 lower than 0.5 and vice versa.

5) In addition, the partial cross-price elasticities were rescaled such that their sum in a row would amount to a fixed ratio of the absolute value of the own price elasticity in the same row. This ratio was kept constant for every 400 draws. The ratios were imposed in an arithmetic sequence with a 0.1 rate (from 0.1 to 0.5). This resulted in 2,000 draws. The so-called "elasticity ratio" was found to have a great impact on the results.

6) Finally the partial elasticities, combined with the parameters of the upper stage and the prices, quantities and expenditures, were input into (3.3) and (3.4) to generate the matrix of total elasticities; by substituting all these values into equations (3.6), (3.7), (3.9) and (3.10), we solved for the γ_{ij}'s and β's of the other demand specifications so as to preserve not only the initial prices, quantities and Herfindahl-Hirschman Index (HHI), but also the initial elasticities (and consequently the price-cost margins) for each draw. This guaranteed that the simulations for all HMT versions would start from the same equilibrium for each draw, no matter what the demand specification was.

9. Demographic variables were actually included, but as they are not subject to variation during the simulations, their effect could well be included in the constants without affecting the results.

The range of the initial equilibrium own elasticities was such that the minimum and the maximum values were uniformly distributed in the intervals [2.25, 2.75] and [3.05, 3.35] respectively. As regards the initial equilibrium cross-price elasticities, Table 1 shows that they vary much more according to the elasticity ratio, such that the minimum value is distributed in the range [0.01, 0.03] for an elasticity ratio equal to 0.1, and in the range [0.07, 0.17] for a ratio equal to 0.5; the maximum cross-price elasticity ranges from 0.03 to 0.08 when the ratio equals 0.1, jumping to the range [0.17, 0.37] when the ratio equals 0.5. But the ranges are not necessarily uniformly distributed.

It is still worth noting that the initial price-cost margins will be different when we implement the PLPM test, because the market is already in a price leadership setting, and the expansion of the candidate market not only internalizes the diversion ratios among the hypothetical monopolist's products but also changes the status of a follower to a leader. The system of equations (2.8) then becomes:

$$\begin{pmatrix} s_1^s \\ s_2^s \\ s_3^s \\ \vdots \\ s_J^s \end{pmatrix} + \begin{pmatrix} -\eta_{11}^{Rs} & \eta_{12}^{Rs} & \cdots & \cdots & 0 \\ \eta_{21}^{Rs} & -\eta_{22}^{Rs} & \cdots & \cdots & 0 \\ \vdots & \vdots & -\eta_{33}^{s} & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & -\eta_{JJ}^{s} \end{pmatrix} \begin{pmatrix} s_1^s \cdot m_1^s \\ s_2^s \cdot m_2^s \\ s_3^s \cdot m_3^s \\ \vdots \\ s_J^s \cdot m_J^s \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{3.11}$$

where the superscript R stands for residual demand (see equation 2.12) and applies to all products of the firms belonging to the hypothetical monopolist. It is important to stress that the demand specification also affects the reaction elasticities, so we should expect to find a greater difference of the impacts of the demand specification on the size of the relevant market in the PLPM version.

3.3 RESULTS AND DISCUSSION

The simulations were performed in the following way:

For the five, ten and fifteen percent SSNIP versions:

1) For each Monte Carlo draw, all the prices of the merged firm were raised by the respective percentages, while the prices of the rival products were kept constant.
2) If the total profit of the merged firm was increased, the algorithm stopped; otherwise, the next product/firm was added to the merged firm, which then became a hypothetical monopolist of the new set of products (or a partial cartel in the broader potential market). The loop proceeded until the total profit of the hypothetical monopolist increased, or stopped when the set of products to be added reached the maximum established.

For the FERM, MPM and PLPM versions:

1) A new ownership matrix was applied to the initial equilibrium as a first step; given the parameters obtained from the Monte Carlo draw, new price and quantity vectors were obtained to solve for the new equilibrium; for FERM both the initial and the final equilibria are Cournot in prices; for PLPM

both are price leadership subgame perfect Nash equilibria; for MPM, the initial equilibrium is Cournot in prices, but after the merger only the hypothetical monopolist realigns prices, taking as given the prices of the products/firms outside the candidate market, so the final outcome is not necessarily an equilibrium.

2) If the total profit of the merged firm was increased *and* the $ERMP_M$ was found to be above five percent, the algorithm stopped; otherwise, the next product/firm was added to the merged firm, which then would become a hypothetical monopolist of the new set of products (or a partial cartel in the broader potential market). The loop proceeded until both the total profit of the hypothetical monopolist increased *and* the $ERMP_M$ was found significant, or stopped when the set of products to be added reached the maximum established.

But who is the "next" product/firm to be added? Ivaldi and Lörincz (2005) pointed out that applying the FERM test could yield different size delineations depending on the order of expansion of the set of products. It is not unusual for antitrust analysts to adhere promptly to common sense in expanding candidate markets along the HMT towards goods similar in prices and/or quantities. Even Ivaldi and Lörincz (2005), when proposing the FERM test, suggested that the order of expansion followed price similarities between goods. Indeed, in the design of our experiment the price and the quantity vectors are ordered such that the first two firms have the greater share and the following firms to be added to the relevant market are next both in share and in price. Yet, in part of the draws (the ones where $u_1 > 0.5$), the cross-price elasticities are the lowest between the products of the merged firm and they grow as we expand the number of firms. The graphs in Figure 1 show that with this pattern all versions of the HMT tend to produce higher market sizes, and this overshoot is greater when the linear demand specification is assumed and the HMT version is FERM or MPM. We will discuss in greater depth in a separate paper (in progress) an order for the expansion that should produce weakly smaller relevant markets as compared to the usual order (as long as the analyst has access to reasonable estimates of an appropriate measure of the diversion ratio, as we have implicitly constructed here).

Related to the result above is the finding that the elasticity ratio greatly affects the size of the relevant market found, especially when the demand specification is linear or a LAIDS: the larger the ratio, the smaller is the relevant market, regardless of the demand specification and HMT version. This effect can be noticed in both Table 2 – along the rows – and in Figure 1 – by comparing the two columns of graphs. It occurs because the greater the elasticity ratio, the larger all the cross-price elasticities are, and therefore the earlier the algorithm of relevant market expansion is stopped. Table 2 shows that the effect is greatest for the tests with isoelastic demand (except the European style five-percent SSNIP) and for the FERM and MPM tests with Almost Ideal Demand.

Also notice (see Figure 2) that the ranking of average sizes for the relevant market is the same for the three demand specifications: the lowest sizes are always for the isoelastic curve, followed by LAIDS, and the linear curve leads to the greatest sizes in all HMT versions. This result was quite expected, in consonance with the finding of Crooke et al. (1999) that post-merger price increases are greatest for the isoelastic specification, given a single initial set of elasticities, followed by logit, AIDS, and linear curves. Indeed, the greater the price increases, the higher is the probability that the algorithm stops in an early iteration, i.e., when the market size is lower.

A quite remarkable result displayed in Table 2 and Figure 1 is that the performance of the FERM and MPM tests are very close to each other *on average* when the demand is linear, and strictly the same when the demand is isoelastic,[10] but they perform quite differently when LAIDS is assumed. However, as Table 3 shows, the pointwise distribution of pairwise differences is quite skewed for both the LAIDS and the linear specifications (FERM appears to predict larger market sizes more often than smaller). As regards PLPM, it tends to produce smaller sizes for all demand specifications,[11] as was predicted by Werden (1998) – see Section 2.2 above – but the difference is negligible when a LAIDS is used.

Other comparisons of the tests across demand specifications and elasticity ratios can be made by referring to Table 3, which summarizes the distributions of the pairwise differences.

Last but not least, an important accomplishment of this experiment is showing that the old-fashioned uniform SSNIP test using a five percent bracket tends to produce the smallest relevant markets among the set of HMT versions investigated. The five percent SSNIP-EU test is very poor in expanding the candidate market: it persistently predicts a lower size (except for the linear specification with the lowest elasticity ratio, where all tests yield the same result).[12] If we admit that the FERM test is the closest to the spirit of an HMT based in profit maximization, a quantitative application of the European SSNIP test with a five percent bracket may be too conservatively delineating small relevant markets when elasticities are sufficiently high, while the U.S. HMG of 1992 may be not at all far from the ideal market delineation. SSNIP-EU's "undershoot" (admitting FERM as the most appropriate test) is higher and less skewed, the lower are the cross-elasticities relative to the own elasticities, for all demand specifications, with the exception mentioned above. Curiously, the SSNIP-EU with a ten percent bracket performed very closely to both FERM and MPM, thus providing an interesting, easier-to-compute, alternative for the advocates of the uniform SSNIP.

10. The latter result was also expected, because since the elasticities do not vary along the demand curve, the hypothetical monopolist's Lerner rule is completely unaffected by the behavior of the fringe firms' prices.

11. In part because of the earlier termination of the algorithm when the size approaches the upper bound. See footnote 6.

12. It seems that profit-maximizing prices of the post-merger equilibria calculated in the FERM, MPM and PLPM tests are below the five-percent threshold, thus forcing the expansion of the candidate market (in other words, the uniform SSNIP would be above the optimal level). On the other hand, if elasticities are lower, optimal SSNIPs should exceed the five percent threshold, and the market sizes should be closer to the predicted by the SSNIP-EU test. Indeed, to ease minds, we ran the experiments with lower elasticities (centered in 1.5), and the average market sizes for SSNIP 5% decreased to levels between 2.00 and 2.73 (when the demand was isoelastic), while average sizes for the other tests ranged from 2.01 to 3.77. And, more important, SSNIP-EU 5% continued to yield lower sizes than all the others. Additional tables and graphs are available upon request.

4 CONCLUSIONS

This article has sought to demonstrate that the merger simulation techniques for unilateral and coordinated effects and the hypothetical monopolist test are all facets of a single problem, that is, they can all be inserted in a common framework, with particular restrictions imposed in each of them. Even the HMT can be implemented in different ways, according to the current version used by the local antitrust authority.

It occurs that not only the HMT version adopted, but also – and mainly – the demand specification used in the simulation impacts the size of the relevant market found, depending on the magnitude and dispersion of the own-price and cross-price elasticities. We ran a Monte Carlo experiment in the four versions of the HMT presented and with three demand specifications. This experiment bears some similarity with that implemented by Crooke et al. (1999). In that work, the authors showed that the isoelastic demand specification generates a higher post-merger price increase (unilateral effect) than do the AIDS and linear demand setups, in that order. In line with those results, we found that the isoelastic demand specification systematically leads to delineating smaller relevant markets than those found using the LAIDS and linear demand setups (also in this order) in the three HMT versions that calculate new partial equilibria by maximizing the hypothetical monopolist's profits: the Full Equilibrium Relevant Market test by Ivaldi and Lörincz (2005) and the Marshallian and price-leader profit maximizing tests proposed herein.

This was an expected result, because with isoelastic demand, the hypothetical monopolist or merged firm does not lose that much demand by increasing its price and internalizing the demand substitutions. In this case, a price increase – which is always profitable when the cross-price elasticity is positive – is more likely to exceed a threshold, given a same set of own elasticities at the initial equilibrium point. Hence, the relevant market expansion algorithm for aggregating more products is terminated earlier, that is, with fewer products/firms included in the final relevant market. In linear and almost ideal demand systems, the own-price elasticities vary along the demand curve, growing *pari passu* with the prices, and hence discouraging price increases by the hypothetical monopolist with the prospect of losing more customers, and causing the analyst to expand the candidate marketa greater number of times.

In contrast, in the traditional version of the SSNIP test, as set forth in the U.S. Department of Justice's MG of 1982, in the European Commission's *Notice on the definition of the relevant market for the purposes of Community competition law* of 1997 and in the Brazilian *Guia de Análise de Atos de Concentração Horizontal* of 2001, by construction the price increase is uniform and significant for all the products/firms of the hypothetical monopolist. In this version, the expansion of the candidate relevant market only occurs when the price increase is not profitable, noting that a uniform increase only maximizes profits given very particular combinations of own- and cross-price elasticities and of initial prices and quantities. The five-percent version ends up delineating persistently smaller relevant markets, with a minor exception, while neither the ten-percent SSNIP-EU version nor the version based on the U.S. HMG of 1992 are at all far from the ideal (FERM) market delineation.

As a whole, the average market sizes found by implementing the various versions of the tests analyzed depend heavily on the demand specification assumed. Results can be very similar or significantly distinct. The differences in predicted relevant market size between the isoelastic and LAIDS setups almost vanish under various configurations of own- and cross-price demand elasticities. The linear demand specification always predicts greater markets, no matter which test version is applied.

Another important finding of the experiment is that the traditional order of expanding the relevant markets for products that are closer in terms of market share and/or price can bring different results if the cross-price elasticities are increasing or decreasing. In the first case, the analyst may be including more products than necessary to attain the minimum market size that enables a SSNIP by the hypothetical monopolist.

5 TABLES AND FIGURES

TABLE 1

Initial equilibria: descriptive statistics

Elasticity ratio	Minimum Cross-Price Elasticity			
	Average value	Median	Minimum value	Maximum value
0.1	0.02	0.02	0.01	0.03
0.2	0.05	0.05	0.03	0.07
0.3	0.08	0.08	0.04	0.10
0.4	0.10	0.10	0.06	0.13
0.5	0.12	0.13	0.07	0.17

Elasticity ratio	Maximum Cross-Price Elasticity			
	Average value	Median	Minimum value	Maximum value
0.1	0.05	0.04	0.03	0.08
0.2	0.09	0.09	0.07	0.15
0.3	0.14	0.14	0.10	0.22
0.4	0.19	0.18	0.14	0.29
0.5	0.24	0.23	0.17	0.37

Source: own calculations.

TABLE 2

Average size of the relevant market found by the different versions of the Hypothetical Monopolist Test

Linear demand

Elasticity Ratio	SSNIP 5%	SSNIP 10%	FERM	MPM	PLPM*
0.1	**9.00**	**9.00**	**9.00**	**9.00**	**8.00**
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
0.2	**6.50**	**9.00**	**9.00**	**9.00**	**8.00**
	(0.52)	(0.00)	(0.00)	(0.00)	(0.00)
0.3	**4.87**	**7.41**	**7.33**	**7.37**	**7.08**
	(0.65)	(0.52)	(0.51)	(0.51)	(0.49)
0.4	**4.08**	**5.93**	**5.84**	**5.90**	**5.47**
	(0.54)	(0.61)	(0.58)	(0.58)	(0.50)
0.5	**3.49**	**5.15**	**5.02**	**5.14**	**4.48**
	(0.51)	(0.63)	(0.64)	(0.63)	(0.50)

Almost ideal demand system

Elasticity Ratio	SSNIP 5%	SSNIP 10%	FERM	MPM	PLPM*
0.1	**4.06**	**6.62**	**9.00**	**9.00**	**8.00**
	(0.82)	(1.11)	(0.00)	(0.00)	(0.00)
0.2	**3.11**	**4.63**	**5.89**	**5.94**	**5.81**
	(0.58)	(0.72)	(0.61)	(0.61)	(0.64)
0.3	**2.67**	**3.77**	**4.47**	**4.53**	**4.36**
	(0.52)	(0.63)	(0.57)	(0.58)	(0.61)
0.4	**2.39**	**3.27**	**3.73**	**3.81**	**3.59**
	(0.49)	(0.49)	(0.56)	(0.57)	(0.58)
0.5	**2.21**	**3.02**	**3.31**	**3.39**	**3.16**
	(0.41)	(0.50)	(0.46)	(0.49)	(0.46)

Isoelastic demand

Elasticity Ratio	SSNIP 5%	SSNIP 10%	FERM	MPM	PLPM*
0.1	**4.99**	**7.69**	**7.85**	**7.85**	**7.50**
	(0.67)	(0.50)	(0.36)	(0.36)	(0.52)
0.2	**3.29**	**4.65**	**4.77**	**4.77**	**4.39**
	(0.45)	(0.61)	(0.60)	(0.60)	(0.60)
0.3	**2.64**	**3.56**	**3.64**	**3.64**	**3.41**
	(0.48)	(0.54)	(0.56)	(0.56)	(0.50)
0.4	**2.28**	**3.11**	**3.18**	**3.18**	**2.90**
	(0.45)	(0.40)	(0.41)	(0.41)	(0.54)
0.5	**2.11**	**2.79**	**2.87**	**2.87**	**2.62**
	(0.31)	(0.45)	(0.45)	(0.45)	(0.53)

Source: own calculations.

Note: standard deviations in brackets below the respective means.

(*) the size of the PLPM relevant market was censored at J–1, because otherwise there would be no firm left as a follower.

TABLE 3

Market size pairwise differences between Hypothetical Monopoly Test (HMT) versions across elasticity ratios: descriptive statistics

Linear demand

Elasticity ratio:	0.1			0.2			0.3			0.4			0.5		
Variable	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness
Ferm_Plpm	1.000	0.000	n.a.	1.000	0.000	n.a.	0.258	0.438	1.113	0.365	0.482	0.563	0.548	0.498	-0.192
Ferm_Snp10	0.000	0.000	n.a.	0.000	0.000	n.a.	-0.075	0.264	-3.239	-0.093	0.290	-2.824	-0.125	0.331	-2.276
Ferm_Snp5	0.000	0.000	n.a.	2.498	0.520	0.117	2.465	0.499	0.141	1.758	0.429	-1.206	1.530	0.500	-0.121
Mpm_Ferm	0.000	0.000	n.a.	0.000	0.000	n.a.	0.033	0.178	5.293	0.065	0.247	3.542	0.120	0.325	2.348
Mpm_Plpm	1.000	0.000	n.a.	1.000	0.000	n.a.	0.290	0.454	0.929	0.430	0.496	0.284	0.668	0.472	-0.714
Mpm_Snp10	0.000	0.000	n.a.	0.000	0.000	n.a.	-0.043	0.214	-3.761	-0.028	0.216	-2.346	-0.005	0.123	-2.616
Mpm_Snp5	0.000	0.000	n.a.	2.498	0.520	0.117	2.498	0.501	0.010	1.823	0.383	-1.694	1.650	0.478	-0.631
Plpm_Snp5	-1.000	0.000	n.a.	1.498	0.520	0.117	2.208	0.464	0.658	1.393	0.489	0.442	0.983	0.131	-7.387
Snp10_Plpm	1.000	0.000	n.a.	1.000	0.000	n.a.	0.333	0.472	0.714	0.458	0.499	0.171	0.673	0.470	-0.738
Snp10_Snp5	0.000	0.000	n.a.	2.498	0.520	0.117	2.540	0.499	-0.161	1.850	0.358	-1.968	1.655	0.476	-0.655

LAIDS

Elasticity ratio:	0.1			0.2			0.3			0.4			0.5		
Variable	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness
Ferm_Plpm	1.000	0.000	n.a.	0.078	0.268	3.172	0.110	0.313	2.502	0.135	0.342	2.144	0.153	0.360	1.940
Ferm_Snp10	2.380	1.108	0.217	1.260	0.477	0.602	0.693	0.499	-0.404	0.458	0.499	0.171	0.290	0.454	0.929
Ferm_Snp5	4.940	0.820	-0.164	2.778	0.467	-0.651	1.800	0.470	-0.584	1.343	0.475	0.666	1.098	0.329	1.838
Mpm_Ferm	0.000	0.000	n.a.	0.055	0.228	3.919	0.060	0.238	3.719	0.083	0.275	3.046	0.075	0.264	3.239
Mpm_Plpm	1.000	0.000	0.643	0.133	0.339	2.176	0.170	0.376	1.764	0.218	0.413	1.375	0.228	0.420	1.305
Mpm_Snp10	2.380	1.108	0.217	1.315	0.491	0.478	0.753	0.476	-0.600	0.540	0.499	-0.161	0.365	0.482	0.563
Mpm_Snp5	4.940	0.820	-0.080	2.833	0.474	-0.472	1.860	0.470	-0.434	1.425	0.495	0.305	1.173	0.378	1.740
Plpm_Snp5	3.940	0.820	-0.080	2.700	0.501	-0.506	1.690	0.500	-0.394	1.208	0.424	1.147	0.945	0.320	-1.172
Snp10_Plpm	-1.380	1.108	-0.217	-1.183	0.463	-0.599	-0.583	0.514	0.112	-0.323	0.468	-0.762	-0.138	0.345	-2.113
Snp10_Snp5	2.560	0.559	-0.282	1.518	0.500	-0.070	1.108	0.318	2.307	0.885	0.327	-2.205	0.808	0.395	-1.566

Isoelastic demand

Elasticity ratio:	0.1			0.2			0.3			0.4			0.5		
Variable	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness	Mean	Std Dev	Skewness
Ferm_Plpm	0.348	0.477	0.643	0.378	0.485	0.507	0.228	0.420	1.305	0.280	0.450	0.984	0.243	0.494	0.425
Ferm_Snp10	0.165	0.398	1.320	0.123	0.328	2.311	0.080	0.272	3.108	0.070	0.255	3.383	0.080	0.272	3.108
Ferm_Snp5	2.858	0.522	-0.164	1.478	0.500	0.090	1.003	0.288	0.080	0.895	0.307	-2.587	0.758	0.429	-1.206
Mpm_Ferm	0.000	0.000	n.a.	0.000	0.000	n.a.	0.000	0.000	n.a.	0.000	0.000	n.a.	0.000	0.000	n.a.
Mpm_Plpm	0.348	0.477	0.643	0.378	0.485	0.507	0.228	0.420	1.305	0.280	0.450	0.984	0.243	0.494	0.425
Mpm_Snp10	0.165	0.398	1.320	0.123	0.328	2.311	0.080	0.272	3.108	0.070	0.255	3.383	0.080	0.272	3.108
Mpm_Snp5	2.858	0.522	-0.164	1.478	0.500	0.090	1.003	0.288	0.080	0.895	0.307	-2.587	0.758	0.429	-1.206
Plpm_Snp5	2.510	0.506	-0.099	1.100	0.442	0.472	0.775	0.430	-1.130	0.615	0.487	-0.474	0.515	0.500	-0.060
Snp10_Plpm	0.183	0.387	1.650	0.255	0.436	1.128	0.148	0.355	1.996	0.210	0.408	1.429	0.163	0.465	0.528
Snp10_Snp5	2.693	0.462	-0.837	1.355	0.484	0.542	0.923	0.334	-1.378	0.825	0.380	-1.717	0.678	0.468	-0.762

Source: own calculations.

FIGURE 1

Average size of relevant market for each demand specification, according to hmt version and asymmetry in the cross-price elasticity distribution, for selected elasticity ratios

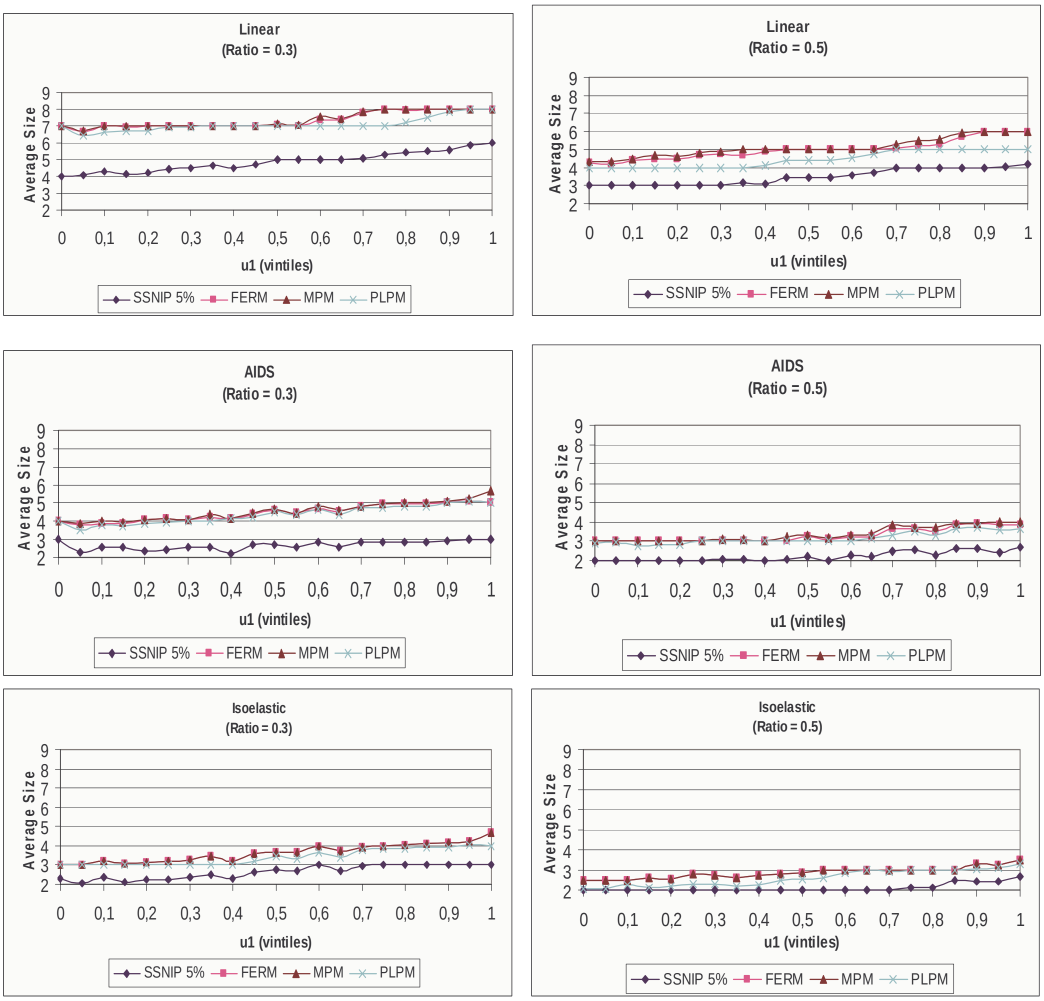
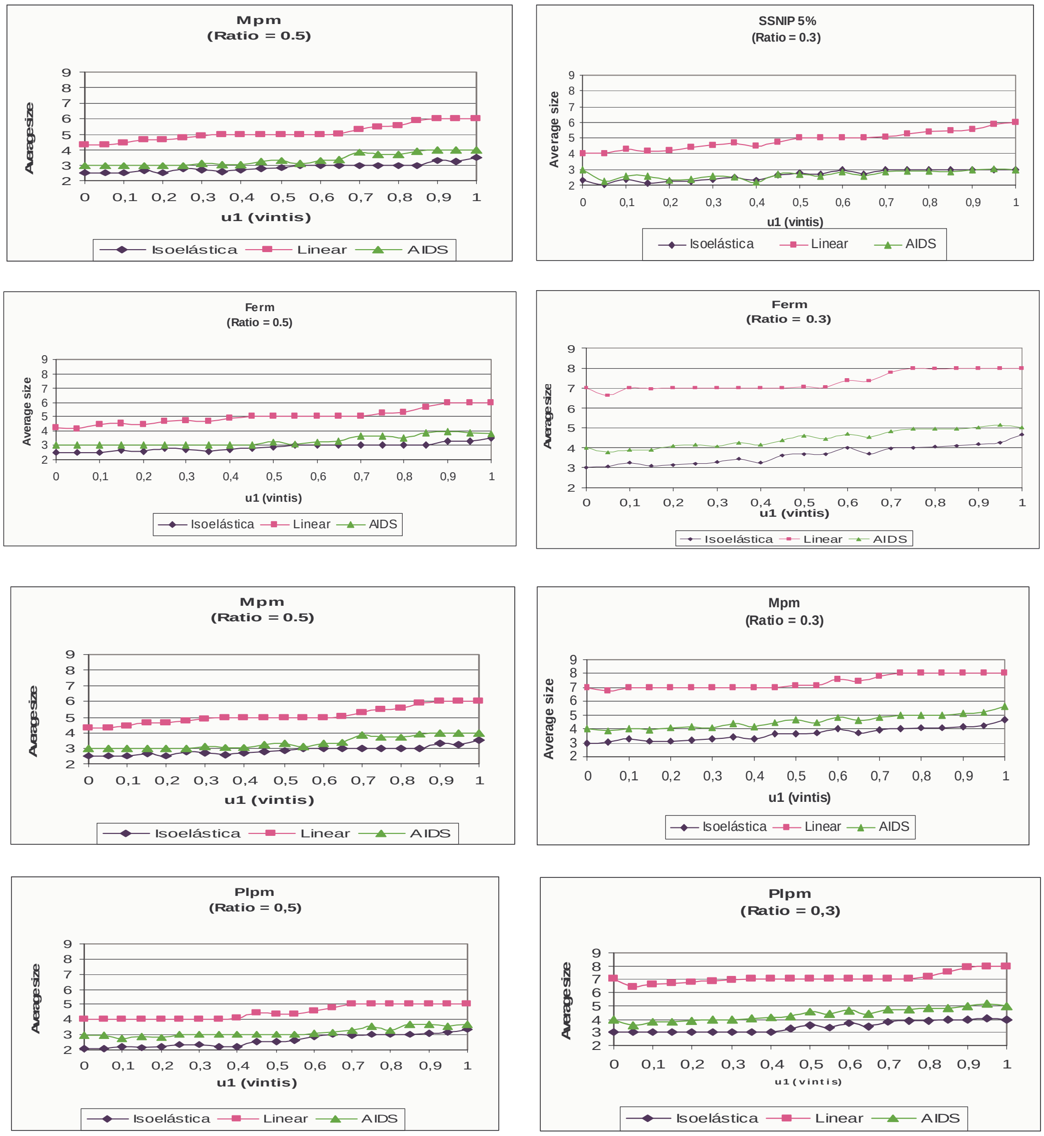


FIGURE 2

Average size of relevant market for each hmt version, according to demand specification and asymmetry in the cross-price elasticity distribution, for selected elasticity ratios



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