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TEXTO PARA DISCUSSÃO

Uma publicação que tem o objetivo de divulgar resultados de estudos desenvolvidos, direta ou indiretamente, pelo IPEA e trabalhos que, por sua relevância, levam informações para profissionais especializados e estabelecem um espaço para sugestões.

SUMMARY

SINOPSE

A exploração de campos de petróleo no Brasil é realizada mediante um processo de licitação de blocos pela Agência Nacional de Petróleo (ANP), nos quais os concessionários utilizam diversas técnicas financeiras e econômicas para o apreçamento dos ativos em questão. Sendo esses recursos bens públicos e de fundamental valor econômico e estratégico para o país, cabe à agência governamental ter o domínio e controle necessários para compreender a análise de viabilidade econômica de seus ativos públicos, visando a uma eficiência maior e melhor nesse processo.

A firma de exploração e produção (E&P), ganhadora do processo de concessão, detém a opção de desenvolver um campo de petróleo já delimitado. O plano de desenvolvimento do campo deve ser apresentado à ANP até uma data específica, ou os direitos de exploração retornam à agência.

A firma de E&P considera três alternativas mutuamente exclusivas de diferentes custos de investimento para explorar o campo, que representam a escala de produção desse campo. O valor do campo desenvolvido é proporcional ao preço do óleo que evolui segundo uma equação diferencial estocástica.

A oportunidade de investimento no desenvolvimento do campo é análoga a uma opção americana finita cujo *payoff* é o valor do campo desenvolvido (valor do ativo), subtraído do custo da alternativa ótima de desenvolvimento (preço de exercício).

Obtemos o valor da oportunidade de investimento e a regra ótima de desenvolvimento do campo, ou seja, o momento ótimo de desenvolvimento e a escala ótima de produção como função do preço corrente do óleo e de sua incerteza de mercado.

ABSTRACT

Petroleum exploration in Brazil is performed throughout a bidding process coordinated by the National Petroleum Agency (NPA), where the exploration and production (E&P) firms need to evaluate concessions performing financial and economic analyses routinely. Since oil is a public resource with economic and strategic value for the country, the governmental agency should have control over the financial and economic pricing techniques.

The E&P firm holds the investment opportunity to develop a delineated oilfield. The oilfield development investment plan shall be presented to NPA until a specific date or the oilfield rights returns back to NPA. The oilfield can be developed up to a specific time through three mutually exclusive alternatives representing the oilfield production and exploration scale. The developed oilfield is proportional to the price of oil, which evolves according to a stochastic differential equation. The E&P firm considers three mutually exclusive alternatives of scale to exploit the oilfield, with different investment costs.

The investment opportunity is analogous to an American call option with finite time to maturity and payoff equal to that of the developed oilfield (underlying asset) minus the development cost of the optimal alternative (exercise price).

We obtain the investment opportunity value and the optimal development rule for the oilfield, i.e., the optimal development timing and the optimal production scale as function of the current oil price and the economic uncertainty of the market.

1 INTRODUCTION

Petroleum exploration in Brazil is performed throughout a bidding process coordinated by the National Petroleum Agency (NPA), where the exploration and production (E&P) firms need to evaluate concessions performing financial and economic analyses routinely. Since oil is a public resource with economic and strategic value for the country, the governmental agency should have control over such financial and economic pricing techniques.

The real option approach is an effective way to perform economic analysis of large investments with considerable intrinsic and strategic value.[1]

The managerial flexibility in the decision making process, under economic or technical uncertainties, provides important earnings to the valuation of the investment opportunity, especially, for low net present value (NPV) investments, which are similar to *at-the-money options*, and where decision makers face a more difficult analysis in order to undertake or reject the project.[2] Regarding strategic investments, the NPV rule turns out to be difficult to apply and can even fail in trying to quantify the intangible value provided by the investment.[3] Such investments are precisely those where the real option approach aggregates more economic value.

Finance literature presents several cases of applications of real options to value natural resource investments, such as Tourinho (1979) and Brennan and Schwartz (1985).[4] Paddock, Siegel and Smith (1988) present the first model of real options to E&P of an oilfield, linking an investment opportunity given by the concession rights to an American call option.

Usually after the oilfield is demarcated in the exploratory phase,[5] petroleum companies acting in E&P hold an investment opportunity to develop the oilfield, i.e., hold the option to develop the field by incurring the development costs. In this phase, uncertainty about the economic value of the reserve, which is related to future oil price evolution, is the most relevant. At any time before the option expiration, the E&P firm can develop the field, and therefore holds an equivalent American call option. At the expiration time, the firm can return the oilfield to the government agency if the investment opportunity to develop the field shows to be non attractive.

1. www.realoptions.org shows some of the recent examples of real options applications for investment decisions in several corporations, such as: HP-COMPAQ, McKinsey & Co., General Motors,Chevron/Texaco, Pfizer Inc, Pricewaterhouse Coopers, British Petroleum, Dell Computer Corp, J.P. Morgan, Putman Investments, and Schering Plough, among others.

2. Dixit and Pindyck (1994) present a comprehensive explanation about the differences between the NPV and real option approach to value investment opportunities under uncertainties.

3. Trigeorgis (1996) presents some examples, such as: optimal timing of an investment, option to expand, abandon or suspend the project, strategic options, option to modularity, learning options, and option to investment in information, etc.

4. Schwartz and Trigeorgis (2001) present other applications of real options to natural resources investment opportunities.

5. The exploratory phase is also an investment option, where the holder of the option receives the delineated (and undeveloped) oilfield by paying the exploratory costs. The relevant uncertainties in this phase are the technical uncertainties related to the existence, size and quality of petroleum reserve.

Assume, that the firm receives the value of the developed oilfield, or the value of the operating project, at the same time of the option exercising, and therefore time to build is not considered in the analysis.[6]

This paper follows Paddock, Siegel and Smith (1988), Dixit (1993) and Dias (1998), and calculates the optimal development timing and production scale of an oilfield through mutually exclusive development alternatives by the applying real option theory approach.

Consider an E&P firm that holds the concession rights to explore the oilfield and therefore faces a finite time to undertake the investment. After the expiration time, if the firm chooses non-development, the exploration rights expire and the oilfield is returned to the government.

Assume three mutually exclusive alternatives for the oilfield's development. These alternatives can be seen as implementation of a specific development technology that can lead to a higher use of the reserve capacity (higher and faster extraction rate, lower production costs, higher oil quality, etc) by incurring the related development costs. The higher the use of the reserve capacity given by the chosen development alternative, the higher the investment cost.

This paper is organized as follow: Section 2 presents the investment opportunity model for oilfield development considering oil price uncertainty; Section 3 shows the optimal timing to develop the oilfield and the optimal production scale; and the Section 4 concludes.

2 INVESTMENT OPPORTUNITY MODEL: OPTION TO DEVELOP THE OILFIELD

Let there be k alternatives to develop an oilfield. Each of the k alternatives has a cost $D(k)$. The net present value (NPV) of the investment opportunity to develop the oilfield at time t by choosing the optimal k alternative is defined in Eq.(1), where V is the economic value of the developed oilfield.

$$NPV(P_t, k, t) = V - D(k) \tag{1}$$

The economic value of the developed oilfield at time t can be expressed as:

$$V = P(t)\,.\,q(k)\,.\,B \tag{2}$$

where:

$P(t)$ = oil price per barrel at time t (\$/bbl);

6. For an application of the time to build see Majd and Pindyck (1987) or Dixit and Pindyck (1994, Chapter 10: sequential investments).

$q(k)$ = economic quality of the developed reserve[7] by adopting development alternative k; and

B = number of barrels of reserves in the ground (the reserve volume) in millions of barrels (MMbbl).

Eq.(2) allows the value of the developed oilfield (V) to be conveniently given as proportional to the oil price (P). This approach is similar in concept to that of Gruy, Grab and Wood (1982) , and has been used as an assumption in real option models [see Paddock, Siegel and Smith (1988) and Dixit and Pindyck (1994, Chapter 12, Section 1)]. Consequently, V follows the same stochastic process as P. The proportion factor q is, on average, 33% of the oil price ("one-third" rule of the thumb), but can be a different ratio for different cases of reserves. The higher q is, the higher is the operational profit from this underlying asset.

By knowing the current oil price and the stochastic differential equation that governs its future evolution we can determine both: the optimal timing to develop the oilfield and the optimal production scale.

Hence, the E&P firm is able to maximize the investment opportunity and to set the optimal timing to develop the oilfield, through the optimal alternative, setting the optimal production scale. The option to develop has the same expiration time as the lifetime of the concession. After the expiration time, in the case of non-development, the oilfield reverts to the government.

This investment opportunity is analogous to an exotic American call option, in which the exercise of the option gives the option holder the value of the developed oilfield. Such value depends on the oil price, the economic quality of the reserve and the optimal development alternative chosen.

Let $F(P,t)$ be the option value for development and let the oil price (P) be given by the following stochastic differential equation, known as geometric Brownian motion:

$$\frac{dP}{P} = \alpha dt + \sigma dz \tag{3}$$

where, α is the drift of the process, σ is the volatility parameter and dz is the Wiener increment, defined as: $dz = \varepsilon\sqrt{dt} \quad \varepsilon \approx N(0,1)$.

Assuming that the oil market is sufficiently complete and that there are no arbitrage opportunities in equilibrium, it can be shown that[8] the option value follows the following partial differential equation (PDE), where r is the risk-free interest rate and δ is the convenience yield rate of the commodity:

7. The concept of economic quality of the reserve was introduced by Dias (1998) and is detailed at: http://www.puc-rio.br/marco.ind/quality.html. In summary, it depends on the capital intensity of the development alternative, which provides higher use for the reserve capacity and higher and faster extraction rate with lower operational costs.

8. See Black and Scholes (1973) or Merton (1973) for a financial option pricing methodology, and Dixit and Pindyck (1994) for a real option approach.

$$\frac{1}{2}\sigma^2 P^2 F_{PP} + (r-\delta)PF_P + F_t = rF \tag{4}$$

Eq. (4) is subject to the following boundary conditions Eq.(5 – 8):

$$F(0,t) = 0 \tag{5}$$

$$F(P,T) = \max_{P_k(T)^*} \left[\, NPV(P_t, k, T), \, 0 \, \right] \; ; \; \text{for all } k \tag{6}$$

$$F(P_k(t)^*, t) = NPV(P_k(t)^*, k, t) \;\; ; \;\; \text{for all } k \text{ and } t < T \tag{7}$$

$$\frac{\partial \left[F(P_k(t)^*, t) \right]}{\partial \, P_k(t)^*} = \frac{\partial \left[NPV(P_k(t)^*, k) \right]}{\partial \, P_k(t)^*} = q_k \, . \, B \;\; ; \;\; \text{for all } k \text{ and } t \, < \, T \tag{8}$$

Note that Eq. (4) is the known Black-Scholes equation, but with different boundary conditions. Eq. (5) is characteristic of geometric Brownian motions and usual in option pricing, and says that if the underlying asset goes to a zero value, it remains at zero indefinitely, leading to a worthless option value. Eq. (6) is the option expiration condition, when the option value is either to commit to the investment for the higher positive NPV given by the development alternative *k* or to return the oilfield earning a zero option value. Eq. (7) and Eq. (8) are the value-matching and smooth-pasting conditions. These conditions set the continuity of the option value and its derivative at the optimal price $P_k(t)^*$.

The option pricing problem described by Eq. (4) and the subject boundary conditions constitute a free-boundary problem of optimal stopping time, usual in option pricing theory. The option was solved numerically by applying the finite difference method in explicit form with an optimization procedure[9] as shown in Appendix.

Note that for *k* alternatives for the oilfield development, there are at most $2.k - 1$ threshold curves for the oil price to calculate. Each area between these threshold curves corresponds to one optimal alternative *k* of development (k^*) during the lifetime of the option.

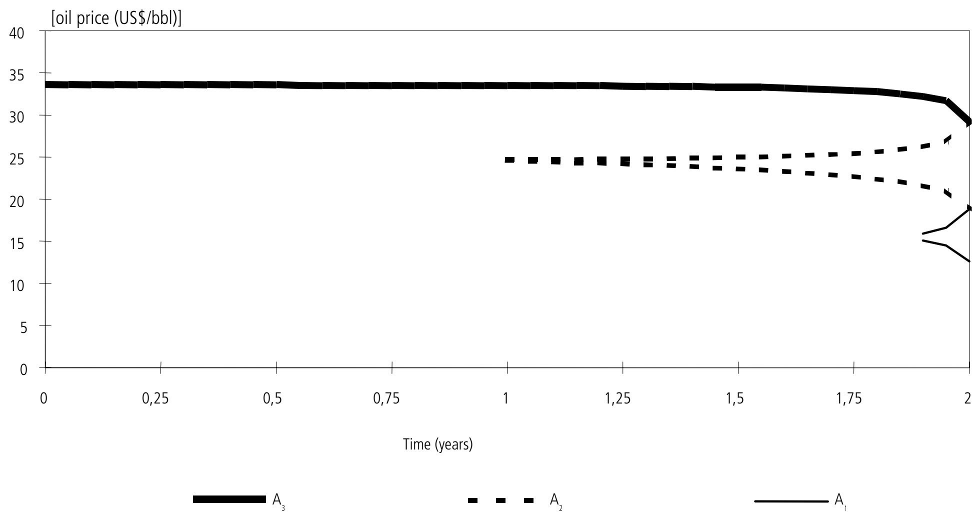
Figure 1 shows the threshold curves for the oil price for an option that expires in two years and for $k = 3$ development alternatives (A_1, A_2, or A_3), representing small, medium and large production scales.

Until the end of the first year, if the oil price is below the threshold curve of alternative A_3, the optimal investment rule that maximizes the option value is to wait and not exercise the option, i.e., not to develop the oilfield. After the first year, if the oil price falls inside the threshold curves of alternative A_2 (or A_1), the manager should develop the oilfield at that moment through alternative A_2 (or A_1) and set a medium

9. For an application of the finite difference method to option pricing, see Brennan and Schwartz (1978). More details about the methodology can be found in Ames (1977) or Smith (1971).

(or small) production scale. The threshold curve of alternative A_3 is optimal only for higher oil prices (~US$34/bbl). At the expiration of the option, if the oil price is below US$13/bbl, the best strategy would not to develop the field and return it to the government.

FIGURE 1
THRESHOLD CURVE FOR OIL PRICE

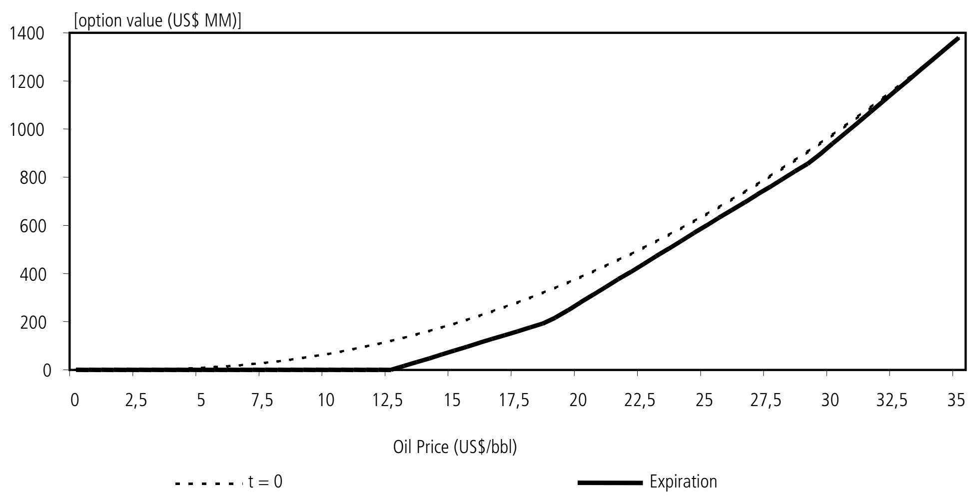


Since we are working with three alternatives, there are five optimal threshold curves to determine, perfoming three areas of option exercise.

According to the parameters of the model, the option to wait and not develop the oilfield may dominate the solution.

Figure 2 shows the option value at the current (t = 0) and at the expiration time. Note that at expiration, we have three different areas of exercising, each representing a certain development alternative for the oilfield. The option at the current time is only exercised for alternative A_3 for oil prices above US$32.50/bbl.

FIGURE 2
THE OPTION VALUE



2.1 MEAN REVERSION MODEL

Consider the more realistic hypothesis for commodities where oil prices (*P*) evolve as the following stochastic process, known as inhomogeneous geometric Brownian motion, or equivalently, the Batthacharya (1978) mean-reverting process:

$$dP = \eta(\bar{P} - P)dt + \sigma P dz \tag{9}$$

Where, η is the reversion speed of the process, σ is the volatility parameter, $\bar{P}$ is the long-run equilibrium mean and *dz* is the Wiener increment defined as: $dz = \varepsilon\sqrt{dt} \quad \varepsilon \approx N\,(0,1)$.

Following the same approach described in the previous section, it can be shown that[10] the option value follows the following partial differential equation, where *r* is the risk-free interest rate and ρ is the risk-adjusted interest rate for the oil price variable. Eq. (10) is subject to the same boundary conditions as Eq. (5-8) described in the previous section:

$$\frac{1}{2}\sigma^2.P^2.F_{PP} + \left[r - \left(\rho - \frac{\eta(\bar{P} - P)}{P}\right)\right].P.F_P + F_t = rF \tag{10}$$

Note that for mean-reverting processes, the convenience yield of the commodity is a function of the oil price, $\delta(P) = \left(\rho - \frac{\eta(\bar{P} - P)}{P}\right)$, a usual characteristic of mean reversion processes. The parameter d is endogenous in our model, and from a market point of view is used in the sense of Schwartz (1997, p.2): "In practice, the convenience yield is the adjustment needed in the drift of the spot price process to properly price existing future prices".

3 RESULTS

3.1 GEOMETRIC BROWNIAN MOTION

Consider three oilfield development alternatives, A_1, A_2 and A_3. The investment opportunity in the oilfield's development expires in $T = 2$ years and the manager has to decide on the optimal timing for development as well the optimal production scale to maximize the investment option value.

Let the following be parameters for the base case:[11] $q_1 = 0.08$, $q_2 = 0.16$, $q_3 = 0.22$, D_1 = US\$ 400 MM, D_2 = US\$ 1.000 MM, D_3 = US\$ 1.700 MM, B = 400 MM bbl, r = 8% p.a., δ = 8% p.a., σ = 25% p.a.[12] and P_0 = US\$ 20/bbl.

10. See Dixit and Pindyck (1994, Chapter 5) for a similar example of a geometric mean reverting process.

11. Some values were estimated using available data about oil prices or using available related literature.

12. Based on the volatility estimation of Dias and Rocha (1999).

Table 1 shows the managerial flexibility value for having a certain number of development alternatives. Note that the option value increases with the number of alternatives.

TABLE 1
MANAGERIAL FLEXIBILITY VALUE

Development alternatives: production scale	Option value (US$ MM)
k = 2 : Medium scale	310.98
k = 1 or 2 : Small or medium scale	322.65
k = 1 or 2 or 3: Small, medium or large scale	323.33

Figure 3 compares the threshold curves of oil prices with a change in the volatility parameters (15% p.a. and 25% p.a.). Note that the lower the volatility, the lower the probability of changes in the oil price evolution. In this situation, the option is exercised sooner (note how the exercise area for the development alternatives become higher) because the option of waiting become less valuable.

FIGURE 3
THREASHOLD CURVES FOR OIL PRICE AND THE VOLATILITY EFFECT

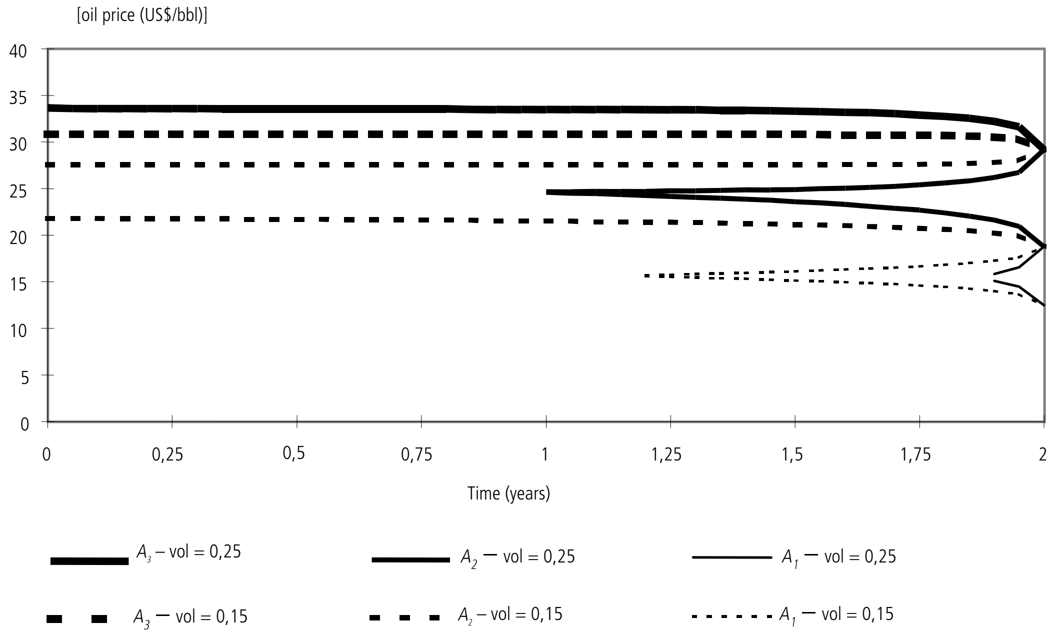


Table 2 shows the option value and the optimal investment rule for different volatilities and current oil prices.

TABLE 2
OPTION VALUE (US$ MM): GEOMETRIC BROWNIAN MOTION

Volatility (% p.a.)	Current oil price (US$/bbl)		
	15	25	30
15	85.89 "wait"	600 "exercise *A2*"	942.21 "wait"
20	102.55 "wait"	600 "exercise *A2*"	948.65 "wait"
25	122.29 "exercise *A2*"	605.21 "wait"	958.72 "wait"

3.2 MEAN REVERTING PROCESS

We use the base case parameter from the previous section added to the specific paramenters of the mean reverting process: ρ = 12% p.a., η = 0.3466,[13] $\bar{P}$ = US$20/bbl.[14]

The option value considering all three-production scales gives US$ 313.86 MM for a current oil price of US$ 20/bbl.

Figure 4 compares the threshold curves for oil price and the volatility effect (25% p.a. and 45% p.a.). Note that the optimal exercise of alternative *A*1 is just at the expiration date, since before that date the option to wait and not develop the oilfield is higher than the NPV given by this alternative. This is due to the mean-reversion expectation of the oil price to the long-run equilibrium mean (US$20/bbl).

FIGURE 4
THRESHOLD CURVES FOR OIL PRICE AND THE VOLATILITY EFFECT
[oil price (US$/bbl)]

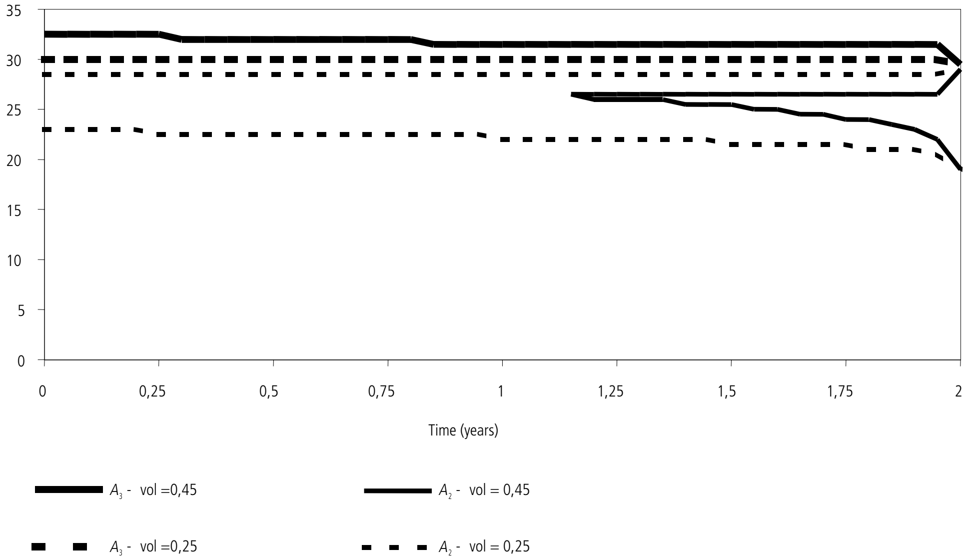


Table 3 shows the option value and the optimal investment rule for different volatilities and current oil prices.

TABLE 3
OPTION VALUE (US$ MM): MEAN-REVERTING PROCESS

Volatility (% p.a.)	Current oil price (US$/bbl)		
	15	25	30
15	126.21 "wait"	600 "exercise *A2*"	940 "exercise *.A3*"
20	140.92 "wait"	600 "exercise *A2*"	940 "exercise *A3*"
25	158.45 "wait"	600 "exercise *A2*"	940 "exercise *A3*"

13. Half-life of roughly 2 years. Bradley (1998, p. 59) finds a half-life close to our base case (of 1.39 years).

14. Baker, Mayfield and Passons (1998, p. 129) estimate the long run oil price as $18.86/bbl (in 1995 dollars) and used (p. 138-140) US$20/bbl as the initial long run level in their model. This value is also adopted in Bradley (1998, p. 59-61) and shown in Cortazar and Schwartz (1996, Figure 4).

Comparing Tables 2 and 3, we verify the difference in the option value due to the hypothesis about the oil price evolution.

Note that in the mean-reverting process, the option is exercised immediately when the current oil price is above the long-run equilibrium mean (US$20/bbl). This is due to the expectation of the mean-reverting process, leading to a small probability for oil prices to be far from the mean.[15] Immediate exercise in this case maximizes the option value.

However, for oil prices lower than the long-run equilibrium mean, the option is not exercised even for lower volatility. In this case, the expectation that prices will revert to the mean increase the option to wait. These results vary according to the parameters of the model reversion speed, current price, long-run mean, expiration time) and should be taken with caution.

4 CONCLUDING REMARKS

Petroleum exploration in Brazil is performed throughout a bidding process coordinated by the NPA, where the E&P firms need to evaluate concessions performing financial and economic analyses routinely. Since oil is a public resource with economic and strategic value for the country, the governmental agency should have control over the financial and economic pricing techniques.

Our goal is to assist NPA with the asset-pricing problem regarding the oilfield exploration bidding process by analyzing the investment opportunity of developing an oilfield. This paper calculates the investment option value and also the optimal development timing and production scale, considering the oil price uncertainties and applying the real option approach.

The investment opportunity in the oilfield is analogous to an exotic American call option whose payoff is the value of the developed reserve minus the cost of the optimal development alternative.

As results we obtain the optimal investment rule for development. This rule depends on the available production scale and on the oil price uncertainty.

The presence of managerial flexibility, i.e., the option to choose the timing for developing the oilfield and also to set the appropriated production scale, increases the investment option value.

We determine the oil price threshold curves for exercising the option. These curves correspond to specific production scales, and instead of one threshold curve as in standard financial American call options; we have areas for option exercise. Each area corresponds to a specific optimal alternative for developing the oilfield.

According to the oil price volatility and the evolution process for oil price, the area for exercising the option can degenerate and the option to wait can dominate immediate development. An increase (decrease) in the volatility parameter, decreases (increases) the area for exercising the option for the stochastic process, geometric

15. The mean-reverting process is a stationary process with a limited variance. The reversion speed is the force that pulls back the oil price to the long-run equilibrium mean. The higher the reversion speed, the sooner the reversion.

Brownian motion and mean-reverting process. This is due to the fact that the option to wait for better conditions before committing to the investment is higher (lower) in such cases.

Finally, the mean-reverting process, a natural implementation for commodities, presents different results compared to the geometric Brownian motion. Specific production scales may never be optimal before expiration.

APPENDIX

FINITE DIFFERENCE METHOD IN THE EXPLICIT FORM FOR NUMERICALLY SOLVING THE PARTIAL DIFERENTIAL EQUATION (PDE)

The finite difference method transforms the partial differential equation Eq. (4) and its respective boundary conditions into a difference equation that can be solved numerically.

By using a specific discretization mesh (time step and oil price step), the explicit form converges exactly to the solution of Eq. (4) and it is easier and faster (especially for a low number of state variables) than the implicit forms or the Monte Carlo simulation techniques associated with optimazation procedures.

Regarding the free-boundary problems defined in Eq. (4), the explicit form and the discretization mesh can easily handle the optimization algorithm used solve these optimal stopping time problems, like the "backward induction" style of a stochastic dynamic programming approach. Implicit forms, however, have to deal with a simultaneous system of equations together with the optimization procedure.

Numerical solutions for partial differential equations can be found in Ames (1977) or Smith (1971). Dixit and Pindyck (1994, Chapter 10) apply the same procedure (explicit form together with an optimization algorithm) to solve an option-pricing problem about sequential investments.

Let $F(P,t)$ at point (P,t) be represented by $F_{i,j}$, where $P = i\Delta P$ for $i \in (0,m)$ and $t = j\Delta t$ for $j \in (0,n)$

Assume following partial derivative approximations:

$$F_{PP} \approx [F_{i+1,j+1} - 2F_{i,j+1} + F_{i-1,j+1}]/(\Delta P)^2;\; F_P \approx [F_{i+1,j+1} - F_{i-1,j+1}]/2\Delta P;$$

$$F_t \approx [F_{i,j+1} - F_{i,j}] / \Delta t$$

We use the central difference approximation to variable price (P), and forward difference approximation to variable time (t). Applying these expressions in Eq.(4) we have the following difference-equation:

$$F_{i,j} = p_i^+ F_{i+1,j+1} + p_i^- F_{i-1,j+1} + p_i^0 F_{i,j+1} \qquad \text{(A1)}$$

$$p_i^+ = \frac{\frac{1}{2}\sigma_P^2 i^2 + \frac{(r-\delta)i}{2}}{r + \frac{1}{\Delta t}} \qquad p_i^- = \frac{\frac{1}{2}\sigma_P^2 i^2 - \frac{(r-\delta)i}{2}}{r + \frac{1}{\Delta t}} \qquad p_i^0 = \frac{-\sigma_P^2 i^2 + \frac{1}{\Delta t}}{r + \frac{1}{\Delta t}}$$

We can apply the same procedure to the boundary conditions of Eq. (4). It can be shown[16] that the solution of equation (A1) converges to the solution of Eq.(4) if all the "p_i" coefficients are non-negative numbers. Therefore, we have to choose discretization time-step, and price-step in order to guarantee that condition.

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16. See the theorem in Ames (1977) page 65.

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