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TEXTOS PARA DISCUSSÃO INTERNA

Nº 86

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Octávio A. F. Tourinho

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DF-IPLAN: 345

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Tiragem: 100 exemplares

Trabalho elaborado em: Agosto de 1985

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Optimal Foreign Borrowing in a
Multisector Dynamic Equilibrium Model:
A Case Study for Brazil[1]

by

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Instituto de Planejamento Economico e Social - Brazil

August 1985

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[1] This work was done while the author was a Visiting Scholar at the Center for Energy Policy Research at the MIT Energy Laboratory under UNESCO/FINEP fellowship BRA/82/4. Financial support of UNESCO, FINEP and IPEA is gratefully acknowledged. A previous version of this essay appeared, as Working Paper 85-009WP of the MIT Energy Laboratory (June 1985).

[2] While retaining sole accountability for the research and the form in which it is presented, the author wishes to thank Charles R. Blitzer for help in the model specification and suggestions on the data handling procedures, Richard Eckaus, Alan Manne and Joaquim Toledo for comments on a previous draft, and Alex Meeraus and Alan Gelb for making the modeling system of the World Bank available for this research.

INPES, 86/85

ABSTRACT

This paper shows how a dynamic multisector equilibrium model can be formulated to be able to analyze the optimal borrowing policy of a developing country. It also describes how a non-linear programming model with the proposed features was constructed for Brazil, and discusses the optimal solution of a base case scenario for the economy in the next 20 years. The sensitivity analysis emphasizes the response of the model to different interest rates on foreign borrowing, alternative export expansion and imports requirements scenarios, and different hypothesis with respect to future petroleum prices and domestic petroleum production. The main conclusion is that the optimal long run borrowing policy for Brazil is quite sensitive to the expected future interest rates, and may be different from some myopic strategies which are currently being suggested to handle the developing countries' foreign debt problems. The other important conclusion is that in the less favorable scenarios - protectionist foreign environment or higher petroleum prices - it is not optimal to postpone the required domestic adjustments by increased foreign borrowing. The usefulness of the model is not restricted to this set of simulations, since it can be readily adapted to address related issues such as foreign trade, investment and indirect taxation policies.

INPES, 86/85

1. INTRODUCTION

In the wake of the recent international financial crisis, the need was felt to address several interconnected questions of long-range macroeconomic planning in Brazil. They are mainly related to government policy with respect to the foreign debt, the trade balance, and the intersectoral allocation of investment. The choice between high and low foreign indebtedness will have to be made in a context where borrowing is costly and can only be avoided by generating large trade surpluses which constrain the growth of domestic output and consumption. Efforts designed to generate these surpluses through reduced imports and increased exports will affect the size and sectoral composition of output, require shifts in the allocation of capital and labor in the economy, and lead to adjustments in the investment patterns.

These issues can be taken into account in an economy-wide dynamic optimization model, which can be specified to be able to evaluate the savings-consumption tradeoff, and include the restrictions in the foreign accounts and in the factor availabilities. The activity levels and capital stock in each sector, as well as the trade balance and net borrowing requirements can be calculated endogenously to maximize some specified utility function, which insures that the solution of the model has the desirable efficiency properties.

Planning models with similar objectives have been specified and implemented for a number of countries, as surveyed by Taylor [1975], and more recently by Dervis, de Melo and Robinson [1982]. The model described in this paper is similar in some respects to the one built by Blitzer and Eckaus [1983] for Mexico, but emphasizes the aspects that seemed most relevant for the analysis of the Brazilian situation[1]. The innovations of the model proposed here are a rather detailed representation of the foreign debt dynamics, the liberal use of non-linear relationships in the utility function and in the interest rate calculations, and a novel specification of the terminal utility term, all in the quest of attaining more realism in the intertemporal choice aspects of the problem.

Several economy-wide equilibrium models have been built for Brazil in the past, but none stressed the foreign debt problem and the intertemporal choice issues associated with it. Lysy and Taylor [1980] used a Johansen-type model to analyze income distribution issues in the '70s, with a medium-run perspective. Their model incorporates a detailed input-output representation of the productive sector, but does not include any formal utility maximization. Modiano [1983] linked a process representation of the energy system with a detailed econometric model of the rest of the economy, in a model where the market equilibrium for energy goods is found by maximizing consumers' plus producers' surplus. That model is solved forward in time, without dynamic optimization, and does not include a disaggregated representation of the intersectoral flows, as the one here. McCarthy [1983] has built a non-optimizing model for identity-based medium-run policy evaluation which includes the intersectoral flows, and also attempts to account for all the financial flows and budget requirements of the several actors of the economy.

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1 The main differences are a more detailed representation of the borrowing costs, and the use of a more satisfactory formulation of the terminal conditions. In addition, the model presented here was solved with a non-linear programming algorithm which is apt to provide more precise results than the linear programming approximation used in the Mexico model.

This paper also demonstrates that it is now possible to easily solve numerically some of the simpler discrete time non-linear neoclassical optimal control problems that arise in models of economic growth. Thanks to MINOS, a program developed by Murtagh and Saunders [1983], optimization packages are now capable of handling a sizeable number of non-linear relationships, both in the objective function and in the restrictions (the latter at a somewhat higher cost), liberating the empirical model builder from the straight-jacket of linear programming or the purely static equilibrium models.

The plan of the paper is as follows. The next section presents the model formulation, addressing some of the possible extensions. Section 3 describes very briefly some aspects of the implementation, while section 4 presents the "base case" solution. Section 5 contains the sensitivity analysis, with the discussion of the main issues the model was meant to address. The conclusion is in section 6.

2. MODEL FORMULATION

Intertemporal choice problems can be treated consistently in the dynamic general equilibrium framework of optimal growth models. These can be extended by including foreign debt as another state variable, allowing the analysis of the joint determination of the time paths of savings, consumption and borrowing that maximize the discounted value of the stream of utility from future consumption. Two approaches can be used to analyze economies modelled in this way. One, taken by Blanchard [1983], is to work in continuous time and employ optimal control methods to derive analytically the system of differential equations to be satisfied by the optimal paths. The other, which is pursued here, is to solve numerically a discrete-time approximation to the original maximization problem. The tradeoffs are clear: the first allows a more systematic exploration of the underlying properties of the system, while the second permits the use of a more detailed multisectoral representation of the actual economy. The analytical approach does not render numerical optimization useless, because it turns out that the paths of the variables will in general depend very much on the initial conditions of the dynamic system, while on the other hand, the analytical solution is indispensable as a benchmark when implementing an empirical model. In short, the two approaches are highly complementary.

The discrete-time multisectoral model specified below chooses variables in each period such as sectoral production, sectoral investment, sectoral imports and exports, foreign borrowing, and consumption level and composition, subject to various constraints. Those are the material balances, the foreign exchange balance, the labor supply constraint, and the dynamic equations for debt and capital accumulation[2].

The formalization of the main aspects of this programming problem is in the next sub-sections, where the notation is as follows. In the equations, the symbols i and j are used as row and column indices for the vectors and matrices, denoting respectively goods and producing sectors. The parameters are generally denoted by lower-case letters, and exogenous variables by capital letters with bars under them. The indexes t and g denote time periods, all of length n years, and all the variables are specified in yearly terms. Matrix multiplication and internal products are denoted respectively by a star and a dot between the two arrays.

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[2] Borrowing itself is not constrained, nor is the savings rate, since the model chooses these levels based on comparing costs and benefits at the margin.

2.1. Objective Function

The existence of a representative consumer is assumed, and the objective function of the planning problem is composed of two elements: the sum over the plan horizon of the total utility in each period properly discounted (at a rate δ), plus the discounted value of post-terminal utility. The yearly utility (U_t) and the terminal utility are specified in per-capita terms, and are multiplied by population ($\underline{N}_t$) to obtain total welfare.

(1) $$W = \Sigma_t \; (1+\delta)^{-t} \; n \; \underline{N}_t \; U_t + (1+\delta)^{-T} \; \underline{N}_T \; V$$

As discussed in section 2.6, the post-terminal per-capita utility (V) includes a penalty for the accumulated foreign debt, and a premium for the level of capital stock[3], which exist at the end of the last period. It compensates for the truncation of the time horizon, assuming a stationary state prevails in the post-terminal period[4].

A generalized logarithmic function was adopted to evaluate the utility of alternative consumption vectors in each period (C_t)[5], mainly because it gives rise to an extended linear expenditure system (ELES), which has often been used in planning models of this type to characterize consumer demand[6].

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3 Terminal stocks are not exogenously specified as in other models in this tradition.

4 The terminal utility term can be interpreted as fulfilling the role of the primal equilibrium approximation in Svoronos [1985] "sqeezing" algorithm for infinite horizon convex programs. I thank Alan Manne for pointing this out to me.

5 The consumption vector includes non-competing imported consumption, in addition to the several other produced goods.

6 For a discussion of extended linear expenditure systems and their application to several countries, see Lluch, Powell and Williams [1977].

The parameters of this per-capita utility function are the vector of marginal expenditure shares (β) and the vector of minimum levels of consumption of each good (γ).

$$(2) \qquad U_t = \Sigma_i \; \beta_i \; \log \; (\; C_{i,t} / \; N_t \; - \; \gamma_i)$$

This objective function has several desirable features like, for example, implying a declining marginal utility of consumption for each good, generating a full set of own and cross price elasticities, and internalizing the savings decision. For this functional form, the elasticity of substitution of expenditure between two consecutive time periods is smaller than unity[7], and is increasing with income level, which is consistent with empirical evidence regarding consumer demand in developing countries (see chapter 4 in LLuch, Powell and Williams [1977]). Rubinstein [1977] has also offered a host of arguments why a function of this type should be favored in finance models for analyzing intertemporal choice, particularly under uncertainty.

3.2 Constraints and Technology

The description of the technology, the resource constraints, and the balancing equations follow the standard multisectoral open-economy dynamic formulation (see, for example, Taylor [1975]). A Leontief specification is

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[7] In the limit, when the ratio of the minimum consumption level to actual consumption for the several goods tends to zero, the overall elasticity of substitution tends to that of the standard logarithmic function, which is unity.

used to model production in each sector[8], with four categories of inputs: intermediate goods, non-competitive imports, capital, and labor.

Only labor employed in the formal market is considered scarce and included in the formulation, but its supply is not modelled explicitly, being assumed to grow at an exogenously specified rate. The important problem of absorbing the large numbers of self-employed and sub-employed that exist in the Brazilian informal labor market is not addressed here[9]. As a consequence, the shadow price of labor in the informal sector is implicitly being assumed to be zero, which is an extreme assumption which probably overstates the leverage of capital and formal employment.

For the issues addressed in this paper, the foreign exchange balance restriction plays a crucial role, allowing the model to endogenously calculate the borrowing requirements in each period. Competitive imports are permitted in the material balances equation, and non-competitive imports are required for domestic production. For simplicity, a linear function was adopted to calculate the export revenues and import expenditures of each sector, but the exogenous vectors of foreign prices can be varied to trace the economy's supply and demand schedules in various scenarios. Maximum exports of each commodity in each period were however specified, implying that the market model underlying the export sector is competitive only up to that exogenous limit[10].

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8 Endogenous technological choice in production could have been easily included in the model, had the required data been available, by adding columns corresponding to the different alternative technologies.

9 For 1983, the approximate size of the "informal" market is 6.9 million rural and 9.6 million urban workers. This compares with 7.4 million rural and 23.9 million urban workers employed in the formal market in that year.

10 Unfortunately this specification may lead to some degree of "bang-bang" behavior, in that the model will in general specialize, exporting up to the

2.5. Foreign Borrowing and Debt Service

By making the time path of borrowing endogenous, the model simulates actual policy choice and allows the economy to simultaneously adjust the level of domestic economic activities and of imports. However, there must be provision for payment of interest and repayment of principal either before or after the model's time horizon. A certain fraction of total debt comes due in each period, but it can be rolled over by contracting for additional borrowing, although perhaps at a higher interest rate.

Foreign debt is modelled here using a vintage model in order to properly account for the effect of the situation of the foreign accounts on the total interest payments. To indicate this, let $D_{g,t}$ be the debt contracted in period g held at start of period t, where the index g runs from 0 to t-1. The following equation shows its dynamics, given repayment (R) and borrowing (B) flows, assuming that after an initial amount is borrowed in a period, it can only be repaid later[11]. Since these are both yearly flows, they are entered into the debt balance equation multiplied by the number of years per period. The second equation below defines next year's debt of one period of age as equal to the sum of the yearly flows of this period's borrowing.

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limit in the more attractive sectors, and not at all (or only residually) in the remaining ones. This could have been avoided by incorporating into the model non-linear export revenue functions. From the mathematical and algorithmic point of view this would have been possible, but the functional specifications and estimates of the relevant parameters were not readily available, so that this extension could not be done reliably at the moment.

11 The repayment schedule ($R_{0,t}$) of initial year debt ($D_{0,1}$) is known, and is used in the recursion of equation (12).

(3) $$D_{g,t+1} = D_{g,t} - n\ R_{g,t} \quad \text{for all t and } 0 \leq g \leq t-1$$

(4) $$D_{t,t+1} = n\ B_t \quad \text{for all t}$$

The repayment schedule on any period's borrowing is assumed to be exogenously specified, and is independent of the period in which borrowing occurs. This implies that the fraction of the borrowing done in period g which is amortized in a later period t can be calculated as a function (r) of the difference between these two dates. This simple formulation does not consider changes in the repayment profile of the debt[12], which may however be a matter of policy choice for some countries. In the context of the model this issue is better addressed through sensitivity analysis, since changes in the maturity are usually associated with changes in the interest rates.

(5) $$R_{g,t} = r_{t-g}\ B_g \quad \text{for all g and t}$$

In order to calculate the interest cost, this model recognizes that a large fraction of the debt contracted in the international financial market by developing countries accrues interest on the basis of a floating rate (i.e. LIBOR) plus a spread, which supposedly reflects the country-specific risk. The debt of developing countries also usually has a fixed rate component associated with loans by foreign governments and official institutions, sometimes in preferential terms. Since the likelihood of large interest subsidies to Brazil through fixed-rate loans in the future is small, it was

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12 This is true except for the changes that occur as a result of the gradual shift from the repayment profile of the initial debt to the one implied by the application of the repayment function specified above to future borrowing. To the extent that the function r is derived from the forecast repayment stream on initial debt, this change is small.

assumed that all of the debt is subject to the floating scheme. This is however only a simplification, since the model can be easily extended to allow for this other type of debt.

As with all other variables in this model, interest payments are calculated in real terms, excluding the effects of inflation of the currency in which the debt is denominated. If repayments for actual borrowing are given in nominal terms, higher inflation in the lending country decreases the real value[13] of the debt at any point in time. This effect of foreign inflation can be simulated in the model considering a faster repayment schedule on the real debt.

Therefore, in any given year, interest payments on total debt have two components: the first is proportional to the real interest rate, which is exogenous and can change each period, and the second is a function of the spread rates contracted for in previous borrowing, which is fixed for the life of the loan. As shown in equation (15), the effective interest rate on debt is the sum of the real interest rate (h) and the endogenous spread rate (SH):

$$H_{g,t} = (h_t + SH_g) \; D_{g,t} \qquad \text{for all t and g} \tag{6}$$

It was assumed here that these spread rates are a stationary linear function (with slope α), of the ratio of annual borrowing to an index of real income. This index is approximated by adding the sectoral value added, each calculated as the product of gross output and the initial year shares (a vector va)[14].

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13 i.e. deflated by price index of the currency of denomination.

14 Income as such is not available in the primal of model, since it depends on the shadow prices of the several goods. However, the index of gross domestic product at base year prices, which is used above, can always be constructed.

(7) $$SH_t = \alpha \; (B_t \; / \; Y_t \;) \qquad \text{for all } t$$

The rationale underlying equation (7) is the standard Capital Asset Pricing Model (CAPM) for financial markets[15], coupled with the assumption that higher values of gross borrowing (relative to income) are perceived by lenders as signals of higher volatility of future returns on their loans. This suggests that the spread rate contracted for loans taken out in a given year should be a function of the gross level of borrowing in that same year.

This formulation is cast in a long run perspective, and is offered only as a reasonable way to look at the cost of Eurobond funds from the borrower's point-of-view, and not as an analysis of the rational determination of spread rates in that market. Note also that this formulation allows the country to act somewhat like a monopsonist, since at the optimal solution the marginal costs of foreign borrowing are equated with its marginal productivity.

Alternative explanatory variables for the spread rate function were considered (see Tourinho[1985b]), but the one adopted was preferred both due to its properties and satisfactory empirical fit. The use of net borrowing, instead of gross borrowing, in the spread rate function was avoided because the several equations that were estimated involving that variable had poor statistical results. This negative result can be rationalized if in the Eurobond market the lending decision is evaluated independently of the repayments of previous borrowing, which would be the case if in each period the lenders were not forced to refinance past loans, as seemed to happen until a few years ago.

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[15] It states that securities with higher undiversifiable risk must command a higher interest premium over the riskless rate than the ones with lower risk.

The use of a specification involving debt as an explanatory variable for each period's spread was avoided because, being stock variable, debt is apt to change only slowly and not capture the dynamics of the rate if, for example, borrowing were drastically reduced. The debt service to export ratio, which is also a natural explanatory variable and does not suffer from this criticism, was abandoned because of an unsatisfactory empirical fit. However, an equation using the debt to income ratio as the independent variable fitted well the data, and was used to calculate the spread rate in the post-terminal period, and in the sensitivity analysis.

2.6. Terminal Stocks and Post-terminal Utility

Assuming that a stationary state prevails in the post-terminal period, the expression for the per-capita infinite horizon approximation term (V) can be derived by calculating the indirect utility function associated with the utility function in (1) and (2) at the terminal date. As in equation (8), it can be written as δ^{-1} times a generalized logarithmic function of terminal income, with total minimum expenditure equal to the sum of the minimum expenditures on the several goods (γ_i in (2))[16]. The intuition is clear: it is the present value of an infinite stream of single-period stationary utility (hence the term δ^{-1}) afforded by the supernumerary income flow.

The total terminal income is the sum of the labor income, plus the return (at a rate ρ) on the accumulated capital stock (KT), less the interest cost of terminal debt (DT). The terminal labor income (YT) and the yearly

16 It is a bit tedious to show this. To do it, substitute the linear demand functions of ELES back into the objective, and integrate by parts.

return on terminal capital stock (ρ) are considered here to be exogenous, but can be estimated on the basis of some preliminary runs. This exogeneity was maintained to avoid complicating too much the shadow-price structure of the model. Letting $\underline{NT}$ denote the population at the end of the last period, and recalling that V was specified in per-capita terms, the following equation displays the expression for the per-capita terminal utility.

$$(8) \qquad V = \delta^{-1} \log [(\underline{YT} + \rho\, KT - HT) / \underline{NT} - \Sigma_i \, \gamma_i]$$

The interest payment on the terminal debt in the equation above (HT) is calculated in (9), where the interest rate is assumed to be equal to the real LIBOR plus a spread rate which is calculated as a linear function (with slope σ) of the size of total debt relative to the index of real income defined previously[17]. This implies that the cost of terminal debt, which reduces the post-terminal supranumerary income, is a quadratic function of the debt.

$$(9) \qquad HT = [h_T + \sigma\, (DT / \underline{YT})]\, DT$$

3. MODEL IMPLEMENTATION

The model was applied to Brazil with a horizon covering twenty-four years from 1984 to 2008, which is divided into 6 periods of 4 years. Endogenous variables and balances are estimated at the beginning of each period. The last period ends in 2008, the date at which the terminal

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17 The comments made in connection with equation (16) also apply here, in analogous form. Equation (24) also has a satisfactory empirical fit for the Brazilian data.

condition term of the objective function is evaluated.[18]. The confidence that is accorded to the results for the last periods should take into account the fact that some parameters of the real economy, which correspond to coefficients which are fixed in the model, may change over such a long time span. The economy is divided into nine producing sectors: (1) agriculture, (2) agro-processing, (3) construction, (4) manufacturing of capital goods, (5) other manufacturing, (6) petroleum, (7) utilities (electricity, water, gas), (8) transport and communication, (9) services.

The model was implemented with the use of the GAMS matrix generator and solved by the non-linear programming package MINOS[19]. It has 7 non-linear equations, and its coefficients matrix has 350 rows, 484 columns and 1685 non-zero elements. At the optimum, 48 non-linear variables are super-basic[20].

It should be emphasized that MINOS 5.0, being a non-linear programming package, cannot in general reach exact optimal solutions. Rather, it stops at an approximate optimal solution where the reduced gradient is zero up to some very small tolerance. The algorithm was always able to converge to the optimal solution, as long as the key parameters (discount rate, LIBOR rate, etc.) were in the range where the behavior of the dynamic system underlying

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[18] The accuracy of the infinite horizon approximation, evaluated by of extending the model to a larger number of periods, is discussed in the next section

[19] For an introduction to these programs, see respectively Kendrick and Meeraus [1985], and Murtagh and Saunders [1983]. The author wishes to thank Sethu Palaniappan for providing several hints on how to implement the model in the GAMS language.

[20] Solution time from a "cold start" on a CYBER machine was 17 seconds, but the model can be run in an IBM PC with 640k and numeric co-processor in about 4 hours. The model was developed using the mainframe version of the software because of its faster turn-around time.

the programming model was stable. On non-stable paths it failed in some instances to find a solution with the specified precision.

3.1 Data base construction[21]

Most of the data for implementing the model can be inferred from the intersectoral transactions and final demands table for 1983 (Appendix A), which was constructed by updating the set of preliminary tables for 1975 obtained from the Brazilian statistics institute (IBGE)[22]. The units for all the commodities and factors in the model are consistent with this table and are defined as the quantities that could be bought with one CR$ billion in 1983. Implicit in this definition is the hypothesis that the aggregation within each sector is on the basis of initial year prices. The other important data items for the model are shown in Table 3.1 and were obtained as follows.

The parameters of the utility function (the vectors β and γ) were calculated by constraining the linear expenditure system to reproduce the 1975 consumption vector, and using the income elasticities estimated by Williamson and McCarthy [1981]. These values imply an overall elasticity of substitution equal to 0.54 in 1983. The discount rate of 4% used in the utility function is consistent, given the marginal savings rate of .84 implied by the β's of

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21 The construction of the database for the model is described more fully in Tourinho [1985b].

22 Unfortunately it was not feasible, given the time available for the data collection, to simultaneously maintain the consistency of the 1975 table and attain an exact correspondence with the national accounts aggregates for 1983. The GNP in the updated table is 10% smaller than the value in the accounts, in part because a sizeable discrepancy already existed in the 1975 table.

Table 3.1, with an average net real yearly return on personal savings of 5%. This value is quite reasonable, given that the real coupon rate on indexed passbook savings in Brazil is 6%.

The capital-output and labor-output coefficients were derived from the respective factor shares making use of some depreciation and profit rates data contained in Lysy and Taylor [1980][23].

Table 3.1
Parameters of utility and production functions

SECTORS/GOODS	Marginal consump. shares (%)	Minimum /total consump. 1983 (%)	Capital /gross output ratio	Depre-ciation (yearly) (%)	Labor /gross output ratio
Symbols	β	$[\gamma_i/c_i]$			
Agriculture	2.41	70.8	2.600	1.6	0.1340
Agro-processing	7.63	70.8	1.520	3.7	0.0504
Construction	-	-	0.572	4.2	0.1607
Man. capital goods	5.01	28.4	1.092	4.0	0.0845
Man. other goods	16.46	30.2	1.278	3.6	0.0784
Petroleum	3.97	42.9	2.075	3.6	0.0341
Utilities	2.72	25.4	3.556	3.4	0.1866
Transport & commun.	7.09	22.6	0.935	4.5	0.2470
Services	37.72	46.8	2.354	3.9	0.1923
Non-compet. imports	0.79	52.0	-	-	-
Average			1.769		0.1226

23 A warning is in order about the reliability of the data. Primary data on sectoral capital-output coefficients was not available, so they had to be estimated indirectly as described in the text. Very little information about the marginal investment shares matrix (b) exists in Brazil, so it had to be constructed piecing together information from several sources, and does not have much hard data to support it. A properly estimated linear expenditure system was also not available, but the elasticities that were used are not expected to be very far from the correct ones. Finally, the possibility that any of these parameters may not be stable through time may compound any errors in the estimation. Since no major data collection effort could be made for this project, we had to do our best with the data on hand, but it must be emphasized that further work to validate these coefficients is necessary.

The slope of the function used to calculate the spread rate in each period (equation (16)) and of the post-terminal interest cost function (equation (24)) are respectively $\alpha = 29.69$ and $\sigma = 5.26$, with the spread rate in percentage points. They were obtained from an analysis, using simple linear regression[24], of the real lending rates for Brazil in the Eurobond market between 1974 and 1984. It was assumed that these behavioral relations will continue to hold in the future, in spite of the fact that there is no *a priori* reason why this should be so. If the institutional arrangements for borrowing by developing countries change in the future, new rules to determine the interest cost of debt will have to be included into the model.

The yearly rate of growth of population and labor force were assumed to be 2.5% and 3% respectively, consistently with historical behavior and some recent projections done in Brazil. The rate of labor-augmenting technical progress was assumed to be 2% yearly.

Government expenditures are assumed to grow at the same rate as population growth, which implies they are inelastic with respect to per-capita income. This is not the usual assumption in models for Brazil, but this parameter can be easily varied to check the sensitivity of the solution to it[25].

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24 The R^2 for the two equations were respectively .47 and .50 and both coefficients are significant at the 5% level.

25 Variations in the growth rate of government expenditures in the reasonable range are not expected to have any impact on the conclusions of the sensitivity analysis of section 5.

For the base case scenario[26], the following ad-hoc assumptions were made with regards to the values of the remaining parameters. The real LIBOR rate stabilizes at 5% yearly, slightly lower than the current level, and does not return to the very low levels of the early '70s. The maximum yearly growth rate of agricultural and manufactured exports is equal to 5% and 10% respectively, envisioning an yearly rate of increase of international commerce of 5% and market share increases for Brazilian manufactured goods. It is assumed that domestic petroleum production stabilizes at a level of 600 thousand bbl/day in accordance with recent forecasts, but that the gross output of the sector expands only to incorporate the value of the increased production. This implies a rather extreme assumption that the refining sub-sector does not expand above its base-year capacity, and that in the medium and long term the demand increases are supplied with imports of refined products[27]. The terms of trade are assumed stable at the 1983 level. Sensitivity analysis with respect to these parameters is shown in section 5.

Finally, due to the difficulties with finding a disaggregated estimate of capacity utilization appropriate to the framework of the model, full-employment was assumed to prevail in 1983, in spite of the fact that it was a recession year. In addition, since cyclical departures from full employment are difficult to handle in models of this type, experiments were not made to verify whether the inclusion of slacks in the resource constraints for the

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26 It cannot be emphasized enough that these are only scenarios for parameters whose future evolution we know very little about, and are not projections of the most likely course of events.

27 This may not have been a very good assumption, but it is not completely unreasonable economically in light of the large increases in Middle-Eastern refining capacity which are scheduled to occur in the next few years.

initial period would lead to reasonable results. This assumption however does not seem to be overly problematic for the long-run calculations for which this model is suitable.

3.2 The adjustment of the terminal utility parameters

The post-terminal net return on capital (ρ) had to be estimated by insuring that the Ramsey equation be satisfied at the beginning of the model's post-terminal period. It can be shown, in a simplified version of a similar model, that the equilibrium condition reduces to expression (10)[28], where the rate of decline of the per-capita marginal utility can be approximated locally by the rate of increase of per-capita consumption divided by the overall elasticity of substitution.

$$(10) \qquad \left|\begin{array}{l}\text{marginal rate}\\ \text{of capital}\\ \text{productivity}\end{array}\right| = \left|\begin{array}{l}\text{utility}\\ \text{discount}\\ \text{rate}\end{array}\right| + \left|\begin{array}{l}\text{rate of decline}\\ \text{of per-capita}\\ \text{marginal utility}\end{array}\right|$$

Recall that the base-case discount rate (δ) is 4%, and note that the maximum sustainable post-terminal rate of growth of per-capita consumption is the sum of the rate of labor-augmenting technical progress (2%) plus the rate of increase in the labor market participation rate (0.5%). The elasticity of substitution in the extended linear expenditure system, given the growth rates assumed above for population and per-capita income, would average about 0.66 during the planning period. Therefore, the approximate rate of decline of the per-capita marginal utility is 3.8%, and the discrete-time estimate of the return on capital for the post-terminal period (ρ) is 8.0%.

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28 Note the absence of the population growth rate from (10), due to the fact that in the objective function (1) the maximand is the sum of utility across all individuals.

The pos-terminal yearly net labor income, whose major role is to determine the slope of the terminal condition term in the utility function, was fixed at Cr$ 240 trillion (of 1983), assuming an average growth of per-capita income of 2.5% during the plan period.

Given these values for δ, ρ and YT, the initial runs (see the base case in the next section for an example) showed that the economy managed to attain an averall yearly growth rate of about 5%. Towards the last period of the model, the gross marginal productivity of capital was seen to be close to 12%, implying a net return of 8.2%, given that the average depreciation rate is 3.8%. The rate of decrease of the aggregate marginal utility also was very close to the predicted value of 6.5%, which was calculated by adding the rate of population growth to the rate of decrease of per-capita marginal utility.

The sensitivity analysis with respect to the net return on capital (ρ) in the neighbourhood of 8% showed that the value 8.2% produces, for the range of simulations reported here, a smooth path for consumption, investment and borrowing. The latter value was then adopted for subsequent runs of the model.

The values for these indirectly estimated parameters were held constant in all the simulations reported in the next sections, in spite of the fact that in some cases some fine-tuning would had been in order.

The existence of a convex function for the cost of foreign borrowing generates another equilibrium condition for the model, which requires that the marginal return on capital be equal to the marginal cost of debt. This allows us to calculate the approximate value of the post-terminal debt to income ratio[29] as being equal to .3, which is somewhat smaller than the current .42, but very close to the ratio actually chosen by the model for the last periods of the horizon in these initial runs. This suggests there are no major inconsistencies between the parameters for the post-terminal period and the rest of the model.

Experiments with extending the horizon of the model to 7, 8 and 10 periods showed that the formulation chosen for endogenizing the post-terminal period leads most variables to display the correct long-run behavior in the 6 period model. However, when the values for the foreign sector flows in the last period are compared across the solutions with 6 and 10 periods, significant deviations were observed. They suggest that equation (8) may induce the model to generate a large trade surplus near the end of the horizon, in order for the model to be able to carry an artificially smaller debt to the post-terminal period[30].

4.RESULTS FOR THE BASE CASE SCENARIO

Several important caveats are in order before we begin the discussion of the results. First, these solutions are driven by an optimization operation performed under conditions of perfect foresight, which implies that if the model can calculate that times will be better (or worse) in the future, it will make decisions <u>today</u> which will take that into account. To the extent

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29 Note that the slope of (24) is $[h_T + 2\,(DT/YT)\,\sigma]$, where $h_T = 5\%$ is the post-terminal real LIBOR rate and $\sigma = 5.26$ is the slope of the terminal spread rate function.

30 This is probably due to the fact the post-terminal labor income is exogenously specified in this formulation.

that the model is a deterministic representation of a world which is actually stochastic, the solutions can only be interpreted in a rational expectations framework with identical penalties for over and under-shooting. Second, the model is normative in nature, so the solution should not be seen as a forecast. Third, the behavior of the variables within the periods is supposed to be smooth, which implies in particular that the values for 1984 should be interpreted as those that would produce an optimal path until 1988, rather then reflecting short-run contingencies that affected the economy in that year. Fourth, it is convenient to emphasize that the model's unit is constant 1983 CR$ billions[31], a fact that is particularly relevant when looking at the foreign accounts, where amortization and interest payments must be interpreted in real terms, and exclude the effects of US dollar inflation and exchange rate movements relative to third currencies.

The main aggregates, all evaluated at initial year prices, are shown in Table 4.1. It is divided in four parts: the income accounts, the balance of payments, and the statistics for the capital and labor stocks. These are discussed in turn below.

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31 The average exchange rate for 1983 was CR$ 577 per US$, according to International Financial Statistics [1984].

Table 4.1
Economic aggregates for base case scenario
(evaluated at base year prices)

	1984[1] (Cr$ bil of 1983)	1984[1] (US$ bill)	Index 1984	1988	1992	1996	2000
Gross output	207777.8	360.1	1.0	1.206	1.462	1.771	2.145
Gross dom. income	111319.2	192.9	1.0	1.205	1.461	1.766	2.136
Consumption	76631.5	132.8	1.0	1.193	1.464	1.759	2.130
Investment	27392.1	47.5	1.0	1.286	1.508	1.831	2.222
Government consump.	4652.4	8.1	1.0	1.126	1.267	1.426	1.605
Exports	12927.5	22.4	1.0	1.212	1.704	2.352	3.094
Non-compet. imports	5918.7	10.3	1.0	1.206	1.484	1.819	2.244
Total imports	10284.4	17.8	1.0	1.302	1.826	2.474	3.277
Trade balance	2643.1	4.6	1.0	0.861	1.226	1.879	2.380
Regist. foreign debt	46385.8	80.4	1.0	1.157	1.391	1.601	1.704
Repayments	7596.1	13.2	1.0	0.964	0.940	1.284	1.358
Interest	3491.0	6.1	1.0	1.150	1.347	1.486	1.525
Transfers	973.7	1.7	1.0	1.000	1.000	1.000	1.000
Current Account	-1821.6	-3.2	1.0	1.489	1.337	0.656	0.004
Borrowing	9417.7	16.3	1.0	1.066	1.017	1.162	1.096
Capital stock	358294.8	620.9	1.0	1.191	1.445	1.739	2.097
Capital formation	28946.8	50.2	1.0	1.277	1.502	1.822	2.214
Population[2]	131.374		1.0	1.104	1.218	1.345	1.485
Employment[2]	31.999		1.0	1.113	1.250	1.398	1.565

Note: [1] These are the values calculated by the model for 1984.
[2] these variables are in millions of individuals.

Consumption at the year 2000 is about 2.13 times the current level, corresponding to an average yearly growth rate of 4.75%, while gross output grows slightly faster, at 4.91% per year. Per-capita consumption grows on average at 2.2% per year, a rate very close to its steady-state value equal to the rate of growth of labor productivity.

The trade surplus stays at about US$ 4.5 billion until 1992, and then grows at 9% per year. The larger actual surplus in 1984 (US$ 12.9 billion) can then almost surely be seen as excessive, from the long-run perspective of the model[32]. Accordingly, debt grows at 4% per year until 1996 and then stabilizes at a level of US$ 130 billion.

The current account shows that the inflow of "new" loans (there are no reserve variations) averages about US$ 4.5 billion annually until 1996, showing that the optimal strategy in this base case is to reach a situation of of balanced current account only toward the end of the horizon. The impact of requiring that it be equilibrated before then will be analyzed later.

The weighted average of the shadow-prices of the several commodities, with weights equal to the shares in the initial consumtion bundle, is shown in the last part of the Table 4.2. It is a measure of the marginal utility of income at the several points in time, and is used as a numeraire to calculate the other prices in the economy. It declines because of the time discounting in the objective function, and because of the joint effect of income growth and concavity of the utility function.

Table 4.2
Shadow prices and implicit rates for the base case scenario
(in consumption units)

	1984	1988	1992	1996	2000
Prices Consumption	1.000	1.000	1.000	1.000	1.000
Labor	2.334	2.371	2.655	2.705	2.660
Foreign exchange	1.086	1.027	1.027	1.024	1.022
Marginal yearly rate of capital productivity[1]	0.168	0.148	0.129	0.125	0.127
Average yearly interest rate on foreign borrowing	0.075	0.075	0.073	0.070	0.067
Numeraire[2]	11.700	8.798	6.306	4.720	3.520

Note: [1] This rate is gross of depreciation.
[2] See text for definition of the numeraire.

[32] Recall also it does not incorporate constraints in the level of borrowing.

The aggregate marginal productivity of capital in Table 4.2 is the average of the sectoral return on investment, in terms of consumption units, over one year. These are calculated as the ratio of the marginal utility of the rental one unit of capital in each sector, and the marginal utility of consumption. The productivity is declining due to the increase of the capital to labor ratio in production. Since at the margin the economy can increase the capital stock by purchasing one additional unit of capital abroad, the level of borrowing adjusts so that its marginal interest cost is equal to the marginal productivity of capital, net of depreciation. The decrease in the marginal rate, which is followed by a reduction in the average rate shown in Table 4.2, is accomplished by a reduction in gross borrowing, relative to income. The shadow price of foreign exchange is close to unity in all periods except the first, indicating that no real depreciation would be needed to support the market equilibrium associated with this solution.

The comparison of the time average of normalized shadow prices of commodities in Table 4.3 with 1983 market prices (unity by definition) indicates that the only major changes that would be required for efficiency are increases of 20% and 30% in the prices of the utilities and transport & communications sectors[33]. The prices of traded goods tend to the international price, the only significant discrepancy being the price of agricultural products in the 1984-1988 period.

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33 It is also possible but unlikely that this is an indication that the technological coefficients in production overstated the true costs of these sectors. Alternatively, this could be signaling the need to have alternative production technologies in these sectors.

Table 4.3
Shadow price of commodities in the base case
(in consumption units)

	Agriculture	Agroprocessing	Construction	Man cap goods	Man oth goods	Petroleum	Utilities	Trans & comm	Services
1984	.71	.99	1.17	1.04	1.04	1.09	1.22	1.27	0.97
Average[1]	.94	1.01	1.09	1.02	1.00	1.02	1.20	1.31	0.95

Note: [1] arithmetic average of the values from 1988 to 2004

5. SENSITIVITY ANALYSIS

Five sets of sensitivity runs were performed to assess the response of the model to changes in the parameters characterizing the conditions affecting the foreign sector[34]. The first and second sub-sections trace the changes in the optimal solution due to changes in the discount rate and the cost of foreign borrowing, respectively. The third section tries to look at the effects on the solution of changes in the external markets environment, by varying the allowed growth of exports. The fourth evaluates the importance of the petroleum sector, by first hypothesizing higher levels of domestic petroleum production, and then looking at the effect of an increase in international oil prices. The last sub-section attempts to assess the impact of alternative assumptions about the value of the non-competitive imports requirements coefficients.

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[34] Note that since the values for 1984 are calculated in the model, they change between the scenarios.

It should be clear that this is only a small selection of possible sensitivity runs that could be performed, but they will hopefully illustrate the power and flexibility of the model to analyze in a consistent manner a range of planning issues, especially those relating to the foreign debt.

5.1 Sensitivity to the discount rate

This section discusses the effects in the solution of alternative assumptions with respect to the discount rate for utility. Recalling the discussion in section 3.2, the main dynamic relationship driving the model requires that along the optimal path the marginal cost of foreign debt be equal to the sum of the discount rate and the rate of the decrease of the marginal utility.

Since the former is composed of the real LIBOR rate plus the marginal spread rate, and the latter is roughly constant, increases in the assumed rate of discount for utility will lead to higher levels of optimal borrowing, as the spread rate adjusts to satisfy the Euler equation. The optimal borrowing policy therefore depends essentially on the difference between the real LIBOR rate and the discount rate, through the marginal cost of borrowing. This is illustrated by Figure 5.1, which displays the variation in the total foreign debt as δ is increased to 5%, 6% and 8%[35], while holding the real LIBOR at the base case level (5%).

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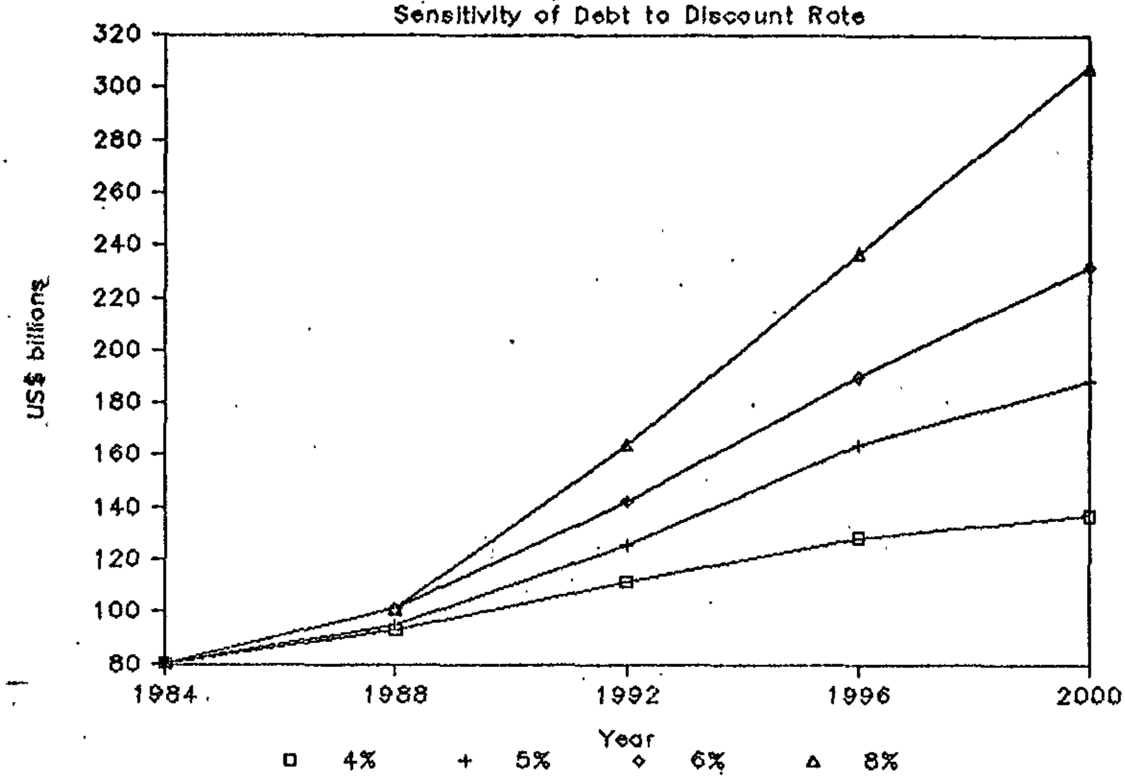
35 The real LIBOR was held at 5%, but the post-terminal return on capital (ρ) was adjusted to 9.2%, 10.2% and 12.2%, as is required satisfy the Euler equation post-terminally.

Figure 5.1

Sensitivity of Debt to Discount Rate



The convexity introduced in the formulation of the model by the endogenous borrowing cost allowed us to avoid having to specify the borrowing rate to be equal to the discount rate. However, the fact that debt at each point in time is sharply increased[36] as the discount rate is raised, shows that this can lead to unrealistically high marginal spreads. This occurs because the specification (and the parameters) of the borrowing cost function in this model do not imply enough convexity to compensate for large discrepancies between the two rates.

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36 In fact, with δ=8% the arbitrary limit on the terminal foreign debt (US$ 435 billion) was reached.

The shift in the intertemporal pattern of consumption towards earlier periods is the counterpart of the increase in indebtedness, and is illustrated in Table 5.1.

Table 5.1
Sensitivity of the yearly growth rate in consumption to changes in the discount rate for utility

Discount rate	1984/88	1988/92	1992/96	1996/2000
4%	4.50	5.26	4.70	4.89
5%	4.85	4.92	4.87	4.61
6%	5.06	4.61	4.50	4.64
8%	5.58	4.22	4.02	4.61

The message to be extracted from this section is that the study of the optimal borrowing policy in the model should center on the results of the sensitivity analysis, as is done in the following sections. The specific values for the foreign sector variables in the base case solution have to be taken with a grain of salt, because they are very dependent on the difference between the discount rate for utility, which is unobservable, and the future interest rate on foreign borrowing, which is unknown. Other variables however are not so sensitive, and their values in the base case are representative of the macro behavior of the model.

5.2. Variations in the fixed part of the interest cost

One would expect that the rational reaction to higher (lower) interest rates would be a reduction (increase) in the level of borrowing, in order to reduce the burden of the debt. This is indeed the response of the model; which can also provide us with an estimate of the size of the changes involved when varying the real LIBOR rates, for example, in the range of 2% to 8%[37].

As the rate increases from 2% to 6%, the curve that describes the behavior of the current account shifts up by roughly US$ 8.5 billion, indicating that (in that range) an increase of one percentage point in the rate induces a <u>reduction</u> of US$ 2.1 billion in the current account deficit. This result must be interpreted in the optimizing framework in which the model operates, since this total derivative of the current account with respect to the interest rate on debt has the opposite sign of the partial derivative that corresponds to the strategy of not responding to the higher rates.

For rates above 6% the net effect of a change in the real LIBOR is much smaller, as the alternatives for response are more limited, and are partly counter-balanced by the added cost of carrying the debt. This can be seen in Figure 5.2, which shows that the response of the trade balance to a change in rates is US$ 2.4 billion per percentage point. Most of the additional foreign exchange revenue is generated by the capital goods sector, which is the only one that had not reached the export limit in the base case.

Figure 5.3 shows that the level of <u>gross</u> borrowing is also only responsive to the interest rate in the range below 6%. Its scope of variation is larger the farther into the future is the period being considered, because the effect of higher (lower) level of borrowing in the early years is compounded in the form of larger (smaller) repayments later.

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37 In all these simulations the changes occur in the first time period and remain in effect until the end, so that they can be seen as "step" responses under perfect foresight.

Figure 5.2

Sensitivity of Trade Balance to LIBOR

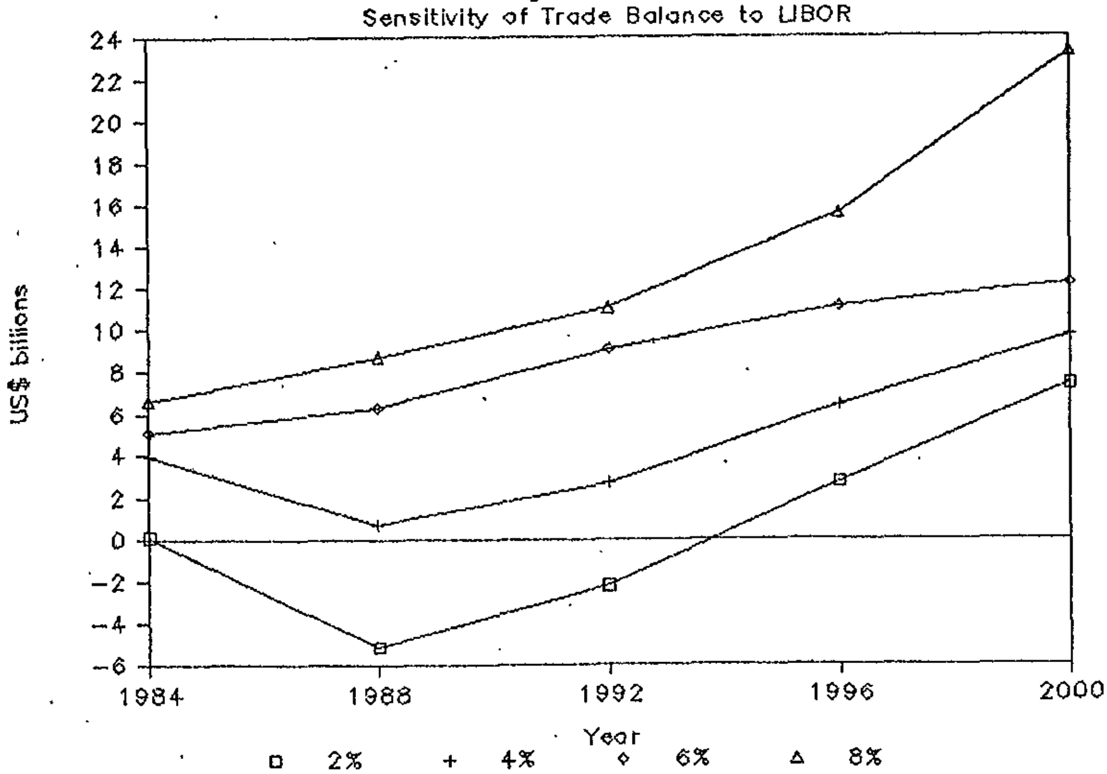
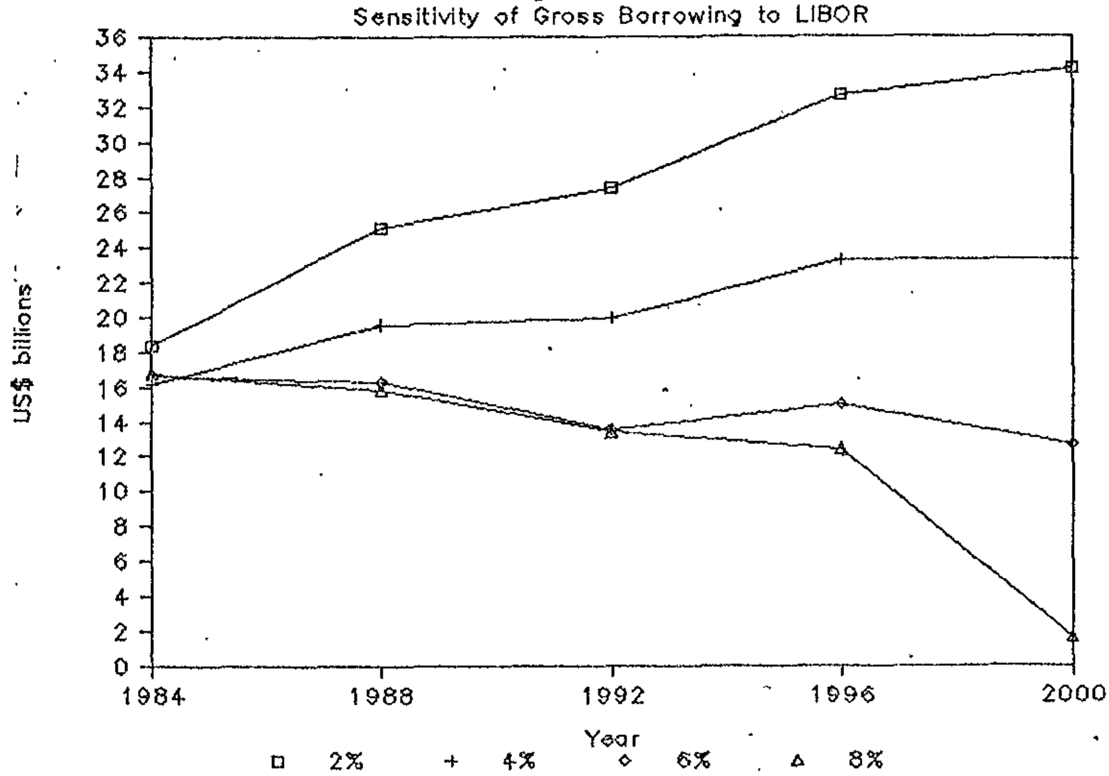


Figure 5.3

Sensitivity of Gross Borrowing to LIBOR



The optimal level of the total debt in the year 2000 can be anywhere between US$ 226 billion and US$ 100 billion (in real terms), depending on the level of interest rates. This highlights the point that the country's strategy with respect to the foreign debt should be cast in a *long range* framework of analysis, and that the discussion of the optimal policy with respect to the debt must consider explicitly the real rates which are expected to prevail in the future[38]. If they are expected to continue at high levels, it may not be optimal to try to pursue strategies that will lead to the continued increase of the debt.

The objective function changes by only 1% between the two extreme cases of the real LIBOR rate. The impact in the domestic side of the economy is further illustrated in Table 5.2, that shows how consumption (evaluated at 1983 prices) is affected by the changing rates. In general, as rates increase both initial and terminal aggregate consumption are reduced, falling by approximately 5% and 1% respectively when the two extreme cases are compared.

Table 5.2
Comparison of consumption in the first and last periods for alternative real LIBOR scenarios
(% deviations from the base case)

Real LIBOR -->	2%	4%	6%	8%
1984	+3.4	+0.7	-0.7	-1.9
2000	+1.3	+0.4	-0.4	-3.3

Summing up, while the optimal strategy with respect to the debt changes

38 This consideration seems sometimes to be missing in the negotiations to reschedule the Brazilian debt.

quite a bit with a change in interest rates, this economy seems to be flexible enough to absorb (in the long run) relatively large variations in them without much impact on welfare. It is interesting to note how limited is the long-term impact of widely different borrowing strategies, indicating that the real issues in the LDC borrowing problem must be more related to the short and medium term liquidity restrictions.

This last argument can be confirmed by Table 5.3, which compares the base case scenario with another where non-negative current account balance is imposed for the whole horizon. In this alternative scenario the real debt is not allowed to grow beyond the initial level, and the model responds to this by reducing consumption and increasing the net exports in the early years. This loss of utility is compensated, in present value terms, by higher consumption towards the end of the horizon, when the required exports are much smaller. The impact therefore is very similar to that of a reduction of the discount rate, causing a reduction of about 2% in the yearly aggregate consumption level until 1992.

The shadow-price of foreign exchange displays a sharp increase of 70% in the first period, drops to a level 10% above the base cain the second, and stabilizes at the base case level by the end of the horizon.

Table 5.3
Comparison of "base case" and "balanced current account" scenarios
(% deviations from base case)

ITEM	1984	1988	1992	1996	2000
Consumption	-2.5	-2.0	-0.1	+0.3	+2.1
Trade balance	+68.9	+88.9	+25.8	-21.3	-40.0

5.3. Export Markets

To evaluate the impact in the optimal solution of the conditions faced by Brazilian exports in penetrating foreign markets, two simulations were performed altering the rates at which manufactured and agricultural exports could grow[39]. In the protectionist scenario the maximum yearly rates of growth for agricultural and manufactured exports were set to 3% and 7.5% respectively. The optimistic scenario maintains the base case rate for the growth of agricultural goods exports (5%) and considers a rate of growth for manufactured exports (13.5%) that would lead to a doubling, by the end of the horizon, of the base case increase in the level of this type of exports[40]. Note that because these bounds are in terms of rates, no large departures from the base case can occur in either scenario in the early periods, since the different rates can only affect significantly the levels further out in the future.

In the optimistic scenario large trade surpluses occur only in the last two periods, which indicates that the model prefers to increase imports at the higher rate of export growth, rather than use resources to reduce the debt. On the other hand, in the protectionist scenario the model knows that times will be difficult in the future and does not delay the adjustment by borrowing. Rather, debt is smaller than in the base case until the next to

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39 These rates apply cumulatively over the initial export level, and are not a restriction on the speed at which exports can grow at each point in time.

40 More optimistic scenarios than this do not affect much the results, as the model is not willing to reduce consumption to take advantage of them, as will be seen below.

last period, when it is allowed to grow, leading to a post-terminal value 80% larger than in the base case. The surplus in the trade balance is however clearly decreasing, a trend opposite to the one in the base case.

The analysis of the shadow-prices uncovers the exchange rate policy implied by the allocation above, and indicates that a real devaluation of 40% would be necessary to bring about the changes in consumption required to reduce imports in the protectionist case. If we were to measure real income in terms of the foreign currency, this reduction is an indication of the cost imposed on the economy by a more difficult external markets situation.

5.4. Effect of assumptions about the petroleum sector

The model allows us to evaluate the impact on the optimal plan of a discovery today of a large (by Brazil's standards) oil field. This was simulated by solving the model with hypothetical domestic petroleum production levels reaching 700, 800 1000 and 1200 thousand bbl/day in 1992, 1996, 2000 and 2004 respectively, rather than stabilizing at 600 thousand bbl/day as in the base case. In this simulation it was also assumed that the gross output of the whole sector (including refining) expands proportionately to this increase in domestic production. It should be emphasized however, that this scenario can only illustrate the effect of knowing <u>for sure</u> that future production can be higher. The resulting changes in aggregate consumption and income are reported in Table 5.4 in percentage terms, and show that this optimistic scenario would have a significant, but not overwhelming, impact in the growth of this economy.

The tilting of the time profile of consumption is quite interesting, and can be explained by looking at the last row of the table, and realizing that it is necessary to invest more in the petroleum sector (especially in the early periods) to support the higher production level. Consequently, the total terminal capital stock is also 6.2% higher than in the base case.

Table 5.4
Comparison of "base case" and "large oil finding" scenarios
(% deviation with respect to base case levels)

ITEM	1984	1988	1992	1996	2000
Income	-	+1.0	+2.4	+2.5	+3.0
Consumption	-1.4	-1.9	+1.0	+0.4	+1.2
Investment	+5.1	+11.1	+4.8	+8.2	+7.8

In this new scenario there are no major changes in the foreign sector, with borrowing, the current account, and the level of the debt having similar values in the two cases. The foreign exchange savings due to the reduction of petroleum imports is instead used to increase (dramatically) the imports of capital goods. This can be inferred from the sectoral gross output changes displayed in Table 5.5, which shows the growth of the petroleum and transportation sectors, and the shrinkage of the capital goods sector.

Table 5.5
Comparison of sectoral gross output in year 2000 for
"base case" and "large oil finding" scenarios
(% deviation with respect to base case levels)

Agric culture	Agropro cessing	Constr uction	Capital goods	Other goods	Petroleum	Util ities	Transp & comm	Serv ices
-1.6	-	+5.8	-47.6	-2.7	+158.0	-2.3	+9.2	-

To see how the model responds to an adverse shift in the terms of trade, a simulation was performed considering that the real petroleum price would be anticipated to grow at an yearly rate of 4%, instead of remaining stable through the whole horizon. Table 5.6 summarizes the impacts of this scenario on the quantity index (weighted at initial year prices) of the main aggregates.

Higher petroleum prices have a very strong long run effect on consumption, relative to the other sensitivity analyses that were performed. The quantum of petroleum imports is reduced by 13% immediately, a margin which is extended to 17% by the end of the horizon. The <u>value</u> of <u>total</u> imports however, is 37% higher by the year 2000, due to the higher price. The trade balance surplus in 2000 drops from US$ 10 billion to US$ 2.7 billion, in spite of the sizeable increase in exports, which expand in all sectors at the maximum allowed rate. It is this export drive that explains the stability of gross output, shown in Table 5.6, on the face of smaller domestic demand (both relative to the base case). It is convenient to recall that the only substitution mechanism in this model works through output composition changes, which probably overstates the impact on consumption, and understates the import reduction and the elasticity of domestic demand with respect to prices[41].

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41 The implied long-run aggregate price elasticity of demand is 0.3, about half of the consensus value of 0.6.

Table 5.6
Comparison of real values of economic aggregates for the base case and high petroleum price scenarios
(% deviations from base case)

Gross output		Consumption		Imports		Exports	
1984	2000	1984	2000	1984	2000	1984	2000
-	+1.0	-1.4	-7.9	-6.7	-10.1	-	+20.0

The impact of higher petroleum prices on the shadow prices can be seen in Table 5.7, which shows short and long run increases in the optimal foreign exchange rate of 25% and 15%, respectively. Domestic factor prices are also adversely affected, with a reduction of 25% in the long run average real wage and an increase of half a percentage point in the real interest rate.

Table 5.7
Comparison of the aggregate shadow prices for the base case and high petroleum price scenarios
(in consumption units)

Scenario	Foreign exch.		Labor		Capital rental	
	1984	1988-2000[1]	1984	1988-2000[1]	1984	1988-2000[1]
High petroleum	1.253	1.156	2.385	1.990	0.173	0.133
Basic case	1.086	1.024	2.334	2.647	0.168	0.128

Note: [1] these are arithmetic averages of the values from 1988 to 2004.

The level of the foreign debt in the last period is virtually the same in the two cases which indicates that, given the other parameters of the base case, it is not optimal to borrow to defer adjustment to the higher prices.

5.5 Sensitivity to the imports coefficients

This section will discuss the impact in the solution of considering values for the non-competitive imports coefficients 20% above and 20% below those in the base case. There is a dual motivation for these scenarios. One is the fact that imports requirements may change in the future, as a result of technological shocks and substitution, and the other is the possibility that the crude procedure used to calculate the coefficients may have led to a biased estimate of their true value.

The analysis of the effect on consumption in Table 5.8 shows that it is approximately symmetric in the two cases and that, even in this model without technological choice, it is very modest. It is however of the same order of magnitude as the effects obtained in the previous sections, indicating that it is as important a factor as the others examined before. In the first period the relative change in consumption is approximately equal to the proportional change in income due to the change in imports, ceteris paribus. In the other periods it is double that amount.

After 1988, the effort of the model to compensate for the increased (decreased) import requirements by generating larger (smaller) trade balances and additional (less) borrowing is also illustraded in the table. In the first period there is less flexibility and apparently the model is not able to compensate for the direct impact on the trade balance, which induces it to make a larger adjustment in borrowing. This initial difficulty in handling the larger non-competitive imports coefficient is reflected in the change in the implicit exchange rate, which is larger in the first period.

Table 5.8
Sensitivity of the solution to the non-competitive imports coefficients
(% deviations from base case values)

	NCI Scenario	1984	1988	1992	1996
Consumption	-20%	0.6	1.7	1.2	1.3
	+20%	-0.6	-1.2	-1.3	-1.7
Trade balance	-20%	30.6	-5.6	-16.8	-8.5
	+20%	-29.8	1.6	10.1	12.9
Gross foreign borrowing	-20%	-8.6	-5.1	-2.9	-3.5
	+20%	8.4	5.9	5.9	3.9
Shadow price of foreign exchange	-20%	-4.7	-1.4	-1.6	-1.1
	+20%	5.3	2.7	2.3	0.9

5.6 Alternative spread rate function

It was pointed out in section 2.5 that the choice of functional form for the calculation of the spread rate on each period's foreign borrowing[42] was to some extent based on the notion that in the Eurobond market the flow effect of the volume of borrowing dominates the effect of the stock of debt. The truth of this hypothesis has not been verified, so this section shows the effect on some of the foregoing scenarios of solving the model using equation (7') instead of (7), to calculate the spread rate[43].

(7') $$SH_t = \sigma (D_t / Y_t) \quad \text{for all } t$$

---

[42] Recall however that, for simplicity, the formulation in terms of the stock of debt was chosen for the post-terminal period.

[43] In (7') the slope σ is the same as in the terminal interest cost function (equation (9)).

Table 5.9 shows that this alternative formulation does not affect consumption, except for an increase in the base case level in the period 1988 to 1992. Debt and borrowing at each point in time are slightly larger in this case, but do not change enough to affect the qualitative conclusions derived in the previous sections. When the alternative formulation is compared with the original one, the inflow of new loans in the two unfavorable scenarios is on average 15% higher.

Table 5.9
Sensitivity to alternative specification of the spread rate function
(% deviations from values in the runs of the original model)

Scenario	Variable	1984	1988	1992	1996	2000
Base case	Consumption	0.5	1.7	0.0	0.5	0.9
	Foreign debt	0.0	1.2	13.0	13.6	16.8
	Current account	8.5	71.7	17.7	65.7	na
	Gross borrowing	1.6	20.0	13.3	17.8	81.8
High petroleum price	Consumption	0.3	0.0	0.0	0.1	-0.1
	Foreign debt	0.0	1.8	2.2	3.1	5.5
	Current account	17.7	10.5	15.3	22.7	13.3
	Gross borrowing	2.5	2.2	4.1	6.9	7.4
Protectionist export markets	Consumption	0.3	0.1	0.1	0.2	0.0
	Foreign debt	0.0	1.8	2.6	3.7	5.8
	Current account	20.2	10.1	11.2	16.6	13.6
	Gross borrowing	2.6	2.7	4.7	7.0	7.8

Table 5.10 compares the two models in terms of the relative changes (with respect to the corresponding base case) that occur in the two scenarios above. It shows that the effects on the main aggregates are similar in both models. In particular, given the utility discount rate and the foreign interest rate of the base case, it is still not optimal to delay the required internal adjustments to adverse conditions in the foreign markets.

INPES, 86/85

Table 5.10
Comparison of variables in standard and alternative[1] models
in unfavorable scenarios
(% deviations from base case)

Scenario	Variable	Model	1984	1988	1992	1996	2000
High petroleum price	Consumption	Alternative	-1.6	-4.6	-3.0	-4.5	-8.8
		Standard	-1.4	-2.9	-3.1	-4.0	-7.9
	Foreign debt	Alternative	0.0	-3.5	-23.5	-28.3	-23.9
		Standard	0.0	-4.1	-15.4	-21.1	-15.7
	Borrowing	Alternative	-4.9	-33.2	-31.7	-17.5	-29.9
		Standard	-5.8	-21.5	-25.6	-9.1	18.7
Protectionist external markets	Consumption	Alternative	-1.3	-2.2	1.7	1.4	3.2
		Standard	-1.1	-0.6	1.6	1.8	4.0
	Foreign debt	Alternative	0.0	-4.5	-20.9	-20.4	-11.2
		Standard	0.0	-5.1	-12.9	-12.8	-2.0
	Borrowing	Alternative	-6.4	-28.6	-18.9	-2.5	-19.0
		Standard	-7.3	-16.6	-12.3	7.3	36.6

Note: [1] The alternative model uses equation (7′) instead of (7).

6. CONCLUSION

This paper has shown how a dynamic general equilibrium model can be formulated to be able to analyze a country's policy towards its foreign debt in the proper intertemporal framework. The utility function is non-linear, and the effects of the truncation of the model's horizon are reduced, but not completely eliminated, by including a term in the objective function which captures the post-terminal value of capital and cost of debt. The model also includes a detailed accounting of the debt dynamics by vintages, and recognizes that the interest rate charged on the foreign debt is dependent on the level of borrowing. The specification allows the model to behave as a monopsonist facing rising marginal borrowing costs in the market for foreign loans. A foreign exchange balancing equation incorporates the interaction between the capital account and the trade balance. The production function however, follows a standard dynamic Leontief specification.

The model was applied to Brazil to generate optimal scenarios for the next 20 years. A significant amount of data manipulation was required to assemble the database, which is valuable in itself, for allowing other economy-wide models for Brazil to be built more easily in the future. The model was implemented using an algebraic modeling system, and solved by a general purpose non-linear optimization package. It was observed that these programs, only recently available, greatly simplified the construction and solution of this economy-wide non-linear dynamic model.

The analysis of the sensitivity of the results to the number of time periods included in the horizon of the model showed that the formulation adopted for the terminal conditions performed satisfactorily, in spite of not being able to eliminate all the end effects.

In addition to the base case, several simulations were performed to evaluate the optimal responses of this long-run model of the Brazilian economy to perfectly anticipated "step" changes in some of the parameters. We have seen that while aggregate consumption may not be very sensitive to changes in the real LIBOR rate, the optimal borrowing policy certainly is, displaying a marked positive response when that rate falls below 5%. Above that level, the model tries to reduce debt as fast as possible. This response of the model shows that the attempt to devise new mechanisms of debt rescheduling which would in effect lead to high levels of net borrowing, may be a myopic policy from the long term perspective of the model, if interest rates are to continue at high levels.

It was seen that the impact of requiring current account balancing is akin to a reduction of the discount rate for utility, and would require a very large devaluation in the short run, relative to the base case.

The response of the model to relaxing the maximum future export levels is not very potent, probably due to the implicit costs, in terms of consumption, of larger trade balances. A protectionist scenario would however require a devaluation of 40% and lead to slightly lower debt until the last two periods, because the model foresees that times will be difficult in the future and does not delay the necessary adjustments.

The effects of petroleum discoveries that would double domestic production in the long run are an increase of 3% in yearly income and a tilting of the time profile of consumption, favoring later periods. Higher prices for petroleum (growth of 4% yearly, instead of being stable) lead to markedly lower consumption (reduction of 8% by 2000) and exploitation of all the export opportunities, but does not affect significantly the size of terminal debt. This seems to indicate that, given the model's assumptions, it is not optimal to borrow to delay adjustment of the economy to the higher prices.

A reduction of 20% in the non-competitive imports coefficients allows an average increase of 1.4% in the level of long run consumption, relative to the base case, while an increase of 20% has a symmetric effect.

These sensitivity analyses are only a sample of the issues that can be addressed with a model such as this, but they will hopefully have made the point that in can provide interesting quantitative insights as to the relative importance of the several factors affecting a developing country's debt problems. Applications of the model are not limited to the analysis of the debt issue, since it can also be used as is, or with minor modifications, to tackle trade and investment policy questions.

Several possible extensions are possible, mainly exploring the possibility of using a non-linear formulation in other parts of the model. For example, the production function could be reformulated to allow the inclusion of some measure of technological substitution and/or complementarity between capital and energy in some of the sectors. Another possibility is to make the export revenue functions non-linear, to account for some degree of imperfect competition in export markets. None of these could be included in this version of the model due to time limits for this phase of the research project.

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APPENDIX A

SECTORAL TRANSACTIONS TABLE FOR BRAZIL IN 1983
(in CR$ billions)

SECTORS / PRODUCTS	SECTORS / PRODUCTS	Agriculture	Agro-process.	Construction	Manuf. cap.gds	Manuf. other	Petroleum	Utilities	Transp. & comm.	Other services	INTERMED DELIVERY	Private consump.	Governmt consump.	Investment	Stock change	Exports	Imports	FINAL DEMAND	GROSS DEMAND
Agriculture	Agriculture	2253.2	7196.1	15.4	412.6	2121.3	4.9	2.1	0.9	152.1	12158.6	3797.1	32.2	329.1	1211.9	1540.9	0.0	6911.3	19069.8
Agro-processing	Agro-processing	602.8	3549.6	0.8	64.2	365.8	1.8	0.1	22.5	944.8	5552.3	12080.7	71.1	0.0	442.6	2973.5	0.0	15567.9	21120.2
Construction	Construction	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	16629.9	0.0	0.0	0.0	16629.9	16629.9
Manuf. cap. goods	Manuf. cap. goods	13.2	131.7	645.1	2610.6	2385.8	100.5	114.1	501.5	421.6	6844.1	3220.8	103.0	4737.0	132.2	1961.8	0.0	10154.8	16998.9
Manuf. other goods	Manuf. other goods	1249.6	1335.3	5358.8	4275.0	12724.9	420.8	64.5	203.0	710.1	26341.8	11197.3	740.3	841.2	814.9	5481.3	0.0	19075.0	45416.8
Petroleum	Petroleum	507.6	167.6	1077.2	735.5	3184.0	7575.3	221.1	2411.0	1462.5	17341.8	3195.2	299.7	0.0	28.0	670.6	4534.7	-341.2	17000.6
Utilities	Utilities	26.1	228.6	39.4	310.9	1000.5	68.1	645.1	41.4	704.0	3064.1	1677.6	81.0	0.0	0.0	0.0	0.0	1758.6	4822.6
Transport. & commun.	Transport. & commun.	240.5	492.3	202.2	396.7	1127.8	1030.0	10.7	197.6	423.8	4121.7	4219.6	325.8	72.0	5.1	0.0	0.0	4622.4	8744.1
Other services	Other services	921.5	1181.8	2225.0	1539.9	3712.4	210.3	34.7	634.7	1150.8	11619.0	32598.6	2863.0	2081.8	116.7	0.0	0.0	37660.1	49279.1
Non-compet. imports	Non-compet. imports	58.3	276.5	284.3	642.7	1591.0	0.0	16.5	420.5	242.9	3532.7	338.5	24.6	1267.3	59.3	0.0	5222.4	-3532.7	.0
TOTAL INTERMEDIATE	TOTAL INTERMEDIATE	5872.8	14559.5	9848.3	10988.1	28133.5	9411.6	1108.8	4433.1	6220.4	90576.0	72325.4	4540.7	25958.3	2810.7	12628.1	9757.1	108506.0	199082.1
Indirect Taxes	Indirect Taxes	-418.0	-864.5	1757.3	7.5	-379.6	256.7	111.2	487.4	1901.2	2859.2								
Wages	Wages	1960.7	1018.4	2137.6	1434.6	3533.7	579.7	900.0	1164.7	7703.9	20433.3								
Expenses w/labor	Expenses w/labor	388.9	380.7	426.9	855.3	3362.9	215.4	258.8	59.4	2083.7	8031.9								
Auton. employment	Auton. employment	595.3	45.9	534.2	1.8	24.9	.0	.0	995.4	1773.2	3970.8								
Return to capital	Return to capital	10670.3	5980.3	1925.5	3711.6	10741.5	6537.2	2443.8	1604.1	29596.6	73210.8								
VALUE ADDED	VALUE ADDED	13197.1	6560.7	6781.6	6010.8	17283.4	7589.0	3713.8	4311.1	43058.6	108506.0								
GROSS PRODUCTION	GROSS PRODUCTION	19069.8	21120.2	16629.9	16998.9	45416.8	17000.6	4822.5	8744.1	49279.1	199082.1								

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