

# 107

**DISCUSSION PAPER**

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## **EFFICIENT AND EQUITABLE COMMODITY TAXATION: MICRO-SIMULATIONS BASED ON AN ESTIMATED BRAZILIAN CONSUMER DEMAND SYSTEM**

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## RESUMO

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O presente estudo tem como objetivo calcular a estrutura ótima da taxaço de bens e serviços no Brasil. As simulaçoes baseiam-se em um sistema de demanda de consumo brasileiro estimado com uma forma funcional flexível [*Almost Ideal Demand System*, de Deaton e Muellbauer (1980)]. As fontes de dados são as duas Pesquisas de Orçamento Familiar (POF) do IBGE, realizadas nos períodos 1986/87 e 1995/96, que coletaram dados das nove áreas metropolitanas, do Distrito Federal e do município de Goiânia. O modelo caracteriza-se pela maximizaço de uma função de bem-estar social, sujeita à restriço de receita do governo. O *trade-off* entre equidade e eficiência é considerado pela introduço de um parâmetro que indica o grau de aversão à desigualdade na função de bem-estar social. Além da hipótese de que o único instrumento de política tributária do governo seja a taxaço de bens e serviços, o modelo também admite a concessão de uma transferência uniforme *lump sum* de renda *per capita* do governo para todos os agentes econômicos.

Os resultados mostram que a estrutura de taxaço de bens e serviços caracteriza-se pela seletividade das alíquotas e, em particular, que os bens que as classes de renda inferiores gastam mais, tais como alimentaço e habitaço, deveriam ser subsidiados. Como esperado, o grau de seletividade das alíquotas é mais significativo para valores altos do parâmetro de aversão à desigualdade. Esse resultado é revertido para uma estrutura de *Ramsey tax* quando é introduzida uma transferência uniforme *lump sum*. Por outro lado, quando impomos tetos bastante restritivos aos valores das transferências *lump sum* (que foram encontrados extremamente altos na estrutura ótima irrestrita), o primeiro resultado “anti-Ramsey” é restabelecido. O cálculo do imposto ótimo pode ser de contribuiço valiosa para o atual debate da política tributária no Brasil onde, entre as prioridades, se encontram os objetivos redistributivos.

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# ABSTRACT

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In this study we calculate the optimal commodity tax structure for Brazil. The micro-simulations are based on a complete demand system estimated with a flexible functional form [Almost Ideal Demand System, by Deaton and Muellbauer (1980)]. The data source is a 1995/96 national household budget survey. Preference parameters' estimates are consistent with microeconomic demand theory and allow for a highly accurate optimal commodity tax simulation. The model features the maximization of a social welfare function, subject to a balanced government budget requirement. It is assumed that the only tax policy instrument available to the government is consumption goods' and services' taxation. The trade-off between equity and efficiency is taken into account by introducing the government's aversion to inequality into the social welfare function. We also extend our analysis by allowing for a uniform per capita lump-sum payment to be made by the government to all households.

Our results show that the commodity tax structure is characterized by selective tax rates. More specifically, we found that the commodities in which the lower household's expenditure classes spend most, such as food and housing, should be subsidized. As expected, the degree of selectivity is more significant for higher inequality aversion parameters. Moreover, as the tax revenue goal increases, so do all commodity tax rates. By introducing a uniform lump-sum transfer, however, this result is reversed when the central planner's degree of aversion to inequality is high enough. On the other hand, poll transfer levels are unreasonably high; when we cap them with a binding ceiling, the former pattern is restored. We believe that our empirical findings provide a valuable contribution for the current tax policy debate in Brazil, where distributive goals have a great importance in the agenda.

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## 1 - INTRODUCTION

The purpose of this paper is to calculate the optimal commodity tax structure for Brazil from the estimated demand system. Commodity taxation plays an important role in Brazil. As Varsano *et alii* (1998) point out, commodity taxes raise around 60% of the total tax revenue and its tax burden is 14% of GDP. This renders commodity taxation a subject of considerable policy importance, and one of the main tools available to the government for raising revenue and securing redistribution. Empirical evidence on optimal tax rates will serve as a solid basis for settling the current policy debate in view of the heavy reliance on commodity taxation and the regressive nature of the Brazilian tax system.

Ramsey's (1927) significant contribution on this subject is very well known, and so is its extension to many person case, by Diamond and Mirrlees (1971) and applied to the optimal commodity taxation theory so as to bring out the conflict between efficiency and equity concerns in the design of commodity taxes. Optimal commodity taxation theory studies, therefore, tax structures that raise a given revenue and obtain given distribution objectives, at the lowest cost in terms of efficiency.

Very few studies exist on calculation of optimal commodity tax rates in Brazil. Most of them attempt to estimate the impact on welfare of marginal reforms on the existing tax structure [Sampaio (1993), Sampaio (1996) and Siqueira (1997)]. As Ray (1997) pointed out, "optimal taxation" can be viewed as the limit of a sequence of tax reforms when there is no further possibility of social-welfare-increasing tax changes. To the best of our knowledge, only one study applied the optimal commodity tax design in the same approach as ours: Siqueira (1998) which calculated the optimal commodity taxes for Brazil based on the Linear Expenditure System (LES) estimated by Rossi and Neves (1987). The data source was Estudo Nacional de Despesas Familiares (Endef), a comprehensive survey undertaken from August 1974 to August 1975 in all metropolitan and urban areas, and rural areas in the Southern, Southeastern and Northeastern regions. By using a computable optimal tax model, she attempted to characterize the goods' and services' tax structure within an optimal taxation framework. The model was solved under alternative assumptions regarding: the government's concern with inequality; household preferences; required government revenue level; and constraints on its ability to tax. Restrictions on the structure of commodity taxes were also analyzed: following Heady and Mitra (1986). It was acknowledged that due to the possibility of arbitrage between the urban and rural sectors, the government might be constrained to tax certain goods in both sectors at the same rate. One of the main features in Siqueira's result is the significant quantitative differences in the tax structure of the two sectors: tax rates are higher in the urban than the rural sector. In particular, for both sectors, the tax commodity structure is represented by subsidies on food.

In this study we calculate the optimal commodity taxes based on a Brazilian consumer demand system estimated by Asano and Fiuza (2001) with family



expenditure data covering all consumption categories. Data used for simulation were collected by a 1995/96 national household budget survey, though estimated parameters come from a sample comprising a 1987/88 wave as well. The model used in the estimation was Almost Ideal Demand System (Aids), proposed by Deaton and Muellbauer (1980), which allows for a flexible approximation to general preference structure. The preference parameters estimates are remarkably consistent with microeconomic demand theory, and so, it enables us to conduct a highly accurate optimal commodity tax simulation. The methodology adopted in this study features the maximization of a social welfare function, subject to a balanced government budget requirement, which indicates the value that society places on the welfare of different agents. The only tax policy instrument available to the government is consumption goods' and services' taxation. The trade-off between equity and efficiency is taken into account by introducing the government's aversion to inequality into the social welfare function. The solution, that is, the optimal commodity tax structure is obtained by solving Ramsey's many person equation system using a computable model. We also extend our analysis by allowing for a uniform per capita lump-sum payment to be made by the government to all households. This extension is to investigate the choice among alternative tax structures, following the analysis of Atkinson and Stiglitz (1976); that work introduced a framework to evaluate the appropriateness of different tax bases and showed that in a many person economy featuring an optimal linear income tax, differentiated commodity taxes are unnecessary in a number of special cases.

The paper is organized as follows. In the next section we introduce our methodology and describe empirical results from the Aids estimation. Section 3 presents the data used for the optimal tax rate calculation. Numerical results are discussed in Section 4. Concluding remarks are in Section 5.

## 2 - THE MODEL

The model is based on the traditional Ramsey many person framework, whose equations yield the optimal commodity tax rates. Thus, the government chooses taxes  $t_i (i = 1, \dots, n)$  so as to maximize the social welfare determined by a Bergson-Samuelson social welfare function, that is, a function  $W(v^1, \dots, v^H)$  of the utilities of the  $H$  individuals, given their demand  $x_i^h(p_i, Y^h)$ , where  $p$  is the price vector  $(p_1, p_2, \dots, p_n)$  and  $Y^h$  is individual  $h$ 's total income (see below).

The supply side of the economy is kept quite simple and we assume given producer prices (infinitely elastic supply). As the consumer price ( $p_i$ ) is the sum of producer price ( $q_i$ ) and taxes ( $t_i$ ), the effect of commodity taxes on consumer welfare comes entirely from changes in consumer prices. Labor is the numeraire and is assumed to be untaxed.

Another important assumption in the model is that, in addition to their labor income, the consumers may also receive uniform lump sum payments from the

governmen. Assumptions regarding consumer behavior and government objectives are described below.

## 2.1 - The Consumer

We assume that there are  $H$  individuals, denoted by a subscript  $h$ . Each consumer is assumed to choose consumption goods ( $x_i$ ) to maximize a well behaved utility function defined over  $n$  commodities given by:

$$u^h = u^h(x_0^h, \dots, x_n^h) \quad (1)$$

where  $x_i^h$  ( $i = 1, \dots, n$ ) is the consumption of commodity  $i$  and  $x_0^h$  is the consumption of leisure. Maximization of (1) is subject to the following budget constraint:

$$\sum_{i=1}^n p_i x_i^h = y^h + I = Y^h \quad (2)$$

where,  $i$  is index over consumption goods;  $p_i$  is consumer price of good  $i$ ;  $x_i^h$  is consumption of good  $i$  by consumer  $h$ ;  $Y^h$  is the household per capita income, including both the fixed labor income  $y^h$  and  $I$ , a uniform lump-sum transfer received by each consumer from the government.

The solution of the consumer problem leads to the Marshallian demand function:

$$x_i^h(p, Y^h) \quad (3)$$

Consumer prices ( $p_i$ ) and lump sum transfers ( $I$ ) are variables subject to government control, while labor income  $y^h$  is fixed. By substituting (3) into (1), we obtain the indirect utility function of the  $h^{th}$  individual ( $h = 1, \dots, H$ ):

$$v^h(p, Y^h) \quad (4)$$

We assume each consumer takes the labor income as exogenous and there are no savings, so that income and total consumption are interchangeable. Hence, it implies that labor supply is inelastic.

## 2.2 - Aids Demand System and its Indirect Utility Function

Our results on optimal tax calculation are based on an Aids, which specifies individuals' expenditure function, from which flexible share equations are derived. Aids is chosen because it provides usual desirable proprieties in the conventional demand system and permits a flexible approximation for the consumer preference structure. The Aids expenditure function is:

$$\log E(p, U) = \log a(p) + U\beta_0 \prod_i p_i^{\beta_i} \quad (5)$$

where  $U$  is the utility index, and:

$$\log a(p) = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j \quad (6)$$

The linear homogeneity of the expenditure function with respect to the price vector requires the following constraints:

$$\sum_i \alpha_i = 1, \quad \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \beta_i = 0 \quad (7)$$

By applying Shephard's lemma to (5), we obtain the share equations:

$$w_i^h = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(Y^h / P) \quad i = 1, \dots, n. \quad (8)$$

where  $w_i^h$  is the expenditure share of good  $i$  for individual  $h$ ;  $p_j$  is the price of good  $j$  ( $j = 1, \dots, n$ );  $Y^h$  is total expenditure. Hereafter, we drop superscript  $h$ , for the sake of simplicity in exposition. The price index is represented by  $P$ , a non-linear function of prices:

$$\log P = \log a(p) \quad (9)$$

Under (7), adding up constraints and homogeneity of the demand functions, corresponding to (8), are all satisfied. The expenditure function in (5) represents the minimal amount of income necessary to achieve a given level of utility  $U$  at prices  $p$ . The parameter  $\alpha_0$  can be interpreted as the subsistence expenditure when all prices are normalized to one.

The model in (8) and (9) is the Aids [Deaton and Muellbauer (1980)]. In Aids, the Hicksian substitution matrix is given by:

$$S = [S_{ij}] = \left[ \left\{ \gamma_{ij} + \beta_i \beta_j \log(Y/P) - w_i \delta_{ij} + w_i w_j \right\} Y / p_i p_j \right] \quad (10)$$

where  $\delta_{ij} = 1$ , if  $i = j$ , else it is 0. Symmetry of the substitution matrix implies symmetry of  $\gamma_{ij}$  ( $\gamma_{ij} = \gamma_{ji}$ ). Negativity of the substitution matrix cannot be imposed in estimation, but it can be checked by examining the eigenvalues of  $S$ . The expenditure elasticities,  $\eta_i$  are given by:

$$\eta_i = 1 + \beta_i / w_i \quad (11)$$

Inverting the expenditure function, we obtain the Aids indirect utility function:

$$v(p, Y) = \frac{\log(Y) - \log a(p)}{\beta_0 \prod_i p_i^{\beta_i}} \quad (12)$$

The value of Aids indirect utility function lies between 0 and 1, and its monotonic transformation can be used as welfare measure.

### 2.3 - Econometric Specification

The Aids parameters, used in our indirect utility function, are estimated by Asano and Fiuza (2001). The model incorporates demographic variables into the share equations, in the following form:

$$w_i^h = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log (Y^h / P) + \sum_k \omega_{ik} Z_k^h \quad i = 1, \dots, n. \quad (13)$$

where  $Z_k$ 's ( $k = 1, \dots, K$ ) are demographic variables, such as family size, education of the household members etc. Underlying to this extension is an adaptation of the subsistence level to incorporate demographic variables:

$$\log a^h(p) = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j + \sum_j \sum_k \omega_{jk} Z_k^h \quad (14)$$

As a reference, mean elasticities estimated for 1996 are displayed on Table 1.

Table 1

#### Expenditure and Price Elasticities — 1996

##### Eigenvalues

-0.257	-0.174	-0.126	-0.108	-0.078	-0.053	-0.000
--------	--------	--------	--------	--------	--------	--------

Expd.	Food	Hous.	Furn.	Clth.	Tran.	Hlth.	Pers. Exp.
Shares	0.326	0.124	0.067	0.089	0.168	0.075	0.150
Elast'y	0.726	0.791	1.304	1.130	1.296	1.103	1.170
(s.e.)	(0.028)	(0.049)	(0.099)	(0.077)	(0.077)	(0.073)	(0.057)

##### Price Elasticities

	Food	Hous.	Furn.	Clth.	Tran.	Hlth.	Misc.
Food	-0.549	0.112	0.033	0.031	0.239	0.022	0.113
(t-val)	(-4.41)	(2.37)	(0.87)	(0.39)	(3.72)	(0.31)	(2.16)
Hous.	0.295	-0.7859	0.058	0.123	0.087	0.200	0.022
(t-val)	(2.80)	(-9.93)	(1.26)	(1.91)	(0.99)	(3.32)	(0.38)
Furn.	0.159	0.1065	-0.695	0.062	0.042	0.075	0.251
(t-val)	(0.87)	(1.13)	(-6.61)	(0.55)	(0.27)	(0.76)	(2.37)
Clth.	0.112	0.170	0.047	-1.035	0.374	0.011	0.320
(t-val)	(0.40)	(1.35)	(0.54)	(-2.58)	(2.49)	(0.04)	(1.91)
Tran.	0.463	0.064	0.017	0.198	-0.998	0.084	0.173
(t-val)	(3.72)	(0.92)	(0.27)	(2.66)	(-6.35)	(1.22)	(2.25)
Hlth.	0.094	0.331	0.068	0.013	0.188	-1.014	0.320
(t-val)	(0.33)	(2.20)	(0.81)	(0.04)	(1.36)	(-3.40)	(1.96)
Pers. Exp.	0.244	0.018	0.113	0.189	0.194	0.159	-0.917
(t-val)	(2.48)	(0.38)	(2.84)	(3.41)	(2.53)	(2.88)	(-13.28)

Source: Asano and Fiuza (2001).

## 2.4 - Social Welfare

The social welfare function describes social welfare as a function of the individual welfare levels  $v^h$ , and its functional form suggested by Atkinson (1970) is:

$$\begin{aligned}
 W &= \frac{1}{1-\varepsilon} \sum_h v^h (p, Y^h)^{1-\varepsilon} && \text{when } \varepsilon \neq 1 \\
 W &= \sum_h \log v^h (p, Y^h) && \text{when } \varepsilon = 1
 \end{aligned}
 \tag{15}$$

where  $v^h$  is the consumer indirect utility expressed as an explicit function of  $p_i$  and  $Y^h$  is total income of individual  $h$  (including transfers received), and  $\varepsilon$  is a non-negative parameter that measures the degree of social aversion to inequality. When  $\varepsilon = 0$  only the total welfare matters and welfare distribution is of no concern (Utilitarianism). The social welfare function therefore embodies a preference for equalizing utility and the strength of this preference increases with the value chosen for  $\varepsilon$ . As  $\varepsilon$  increases, higher weights are attached to changes in the utilities of the less well-off households. When  $\varepsilon$  is infinity only the welfare of the poorest matters (Rawlsian Maxmin).

Any monotonic transformation of utility index,  $v^h$  may be used for expressing an individual's welfare. In order to evaluate and maximize social welfare, however, we need to aggregate  $H$  individuals' utility through (15). Thus we can't be free from interpersonal welfare comparison.

Let  $w^h = w^h(p, Y)$  be the person  $h$ 's indirect utility given price vector  $p$ , and income  $Y$ . We like to evaluate  $w^h$  due to the change in prices and income. In this study, we employ a money metric utility evaluated at the pre-tax prices, say  $p_0$ . That is, our choice of  $v^h$  is  $e(p_0, w^h(p, Y))$ , where  $e(p, w)$  is the expenditure function, in which  $p$  is price vector, and  $w$  is utility level. Besides its intuitive appeal, this approach has advantages that we can be free from choice of the functional form of indirect utility function, and that it is possible to compare impact of tax reforms, based on alternative functional forms of demand systems.

## 2.5 - Government Budget Constraint

The government tax revenue is written as:

$$R + H I = \sum_{h=1}^H \sum_{i=1}^n t_i x_i^h
 \tag{16}$$

where  $t_i$  is the value of the tax on good  $i$ ;  $x_i^h$  is the consumption of good  $i$  by individual  $h$  and  $I$  is the lump sum transfer.

Following most of the optimal tax literature, we take the revenue requirement as given by a pre-specified ratio of expenditure in the economy. And it is assumed to be spent on nothing that affects the consumer behavior.

## 2.6 - Objective Function

The government's problem is related to the optimal choice of the consumption tax rate ( $t_i$ ) and the lump sum transfers ( $I$ ). The problem is solved by maximizing the welfare social function subject to the budget constraint. Thus the problem can be written as:

$$\begin{aligned} \text{Max } (t_1, \dots, t_n, I) \quad W = W(v^1(\cdot), \dots, v^H(\cdot)) \\ \text{s. t. } R + H I = \sum_{h=1}^H \sum_{i=1}^n t_i x_i^h \end{aligned} \quad (17)$$

where  $W$  is the social welfare function, based on the vector of consumer indirect utilities ( $v^h(\cdot)$ ).

We derive the optimal tax rate structure under alternative assumptions on degrees of inequality aversion ( $\epsilon$ ). The optimal tax calculation will be computed for different values of  $\epsilon$  in order to cover a broad range of distribution judgements.

The first-order conditions for  $t_i$  and  $I$  are the following:

$$\frac{\partial L}{\partial p_i} = \sum_{h=1}^H \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial p_i} + \lambda \left[ x_i^h + \sum_{i=1}^n t_k \frac{\partial x_i^h}{\partial p_i} \right] = 0 \quad i = 1, \dots, n \quad (18)$$

$$\frac{\partial L}{\partial I} = \sum_{h=1}^H \frac{\partial W}{\partial v^h} \frac{\partial v^h}{\partial I} + \lambda \left[ \sum_{i=1}^n t_i \frac{\partial x_i^h}{\partial I} - 1 \right] = 0 \quad i = 1, \dots, n \quad (19)$$

where  $L$  is the Lagrangian. The optimal tax rates are calculated by solving the many person Ramsey equations (18) and (19).

## 3 - DATA

The model was estimated from family-level monthly expenditures on seven consumption categories and their corresponding price indexes: 1. Food; 2. Housing; 3. Furniture and appliances; 4. Clothing; 5. Transportation and communication; 6. Health and personal care; 7. Personal expenses, education and reading.

The data sources for expenditures are two waves of national expenditure surveys conducted in 1987/88 and 1995/96, and sources for price indexes are the monthly national survey consumer prices. Corresponding price indexes were constructed in a way to allow a comparison of prices both across periods and regions. The regions surveyed are the metropolitan areas of São Paulo, Rio de Janeiro, Porto Alegre, Belo Horizonte, Recife, Belém, Fortaleza, Salvador and Curitiba, besides the cities of Brasília-DF and Goiânia. For the sake of simplicity, we will call all of them metropolitan areas.

Regional nominal prices were obtained through a special order to IBGE (the Federal statistical bureau in charge of both the surveys and the national consumer price index calculation), averaged to subitems (the most disaggregate level quoted by IBGE) and then chained by the subitem variations in order to construct a Regional Price Difference Index, which compares prices of a national average consumption bundle (based on the 1995/96 survey) both across periods and regions.

We assume that the tax structure is common for all individuals. As prices differ across metropolitan areas, we also restrict our initial analysis to São Paulo' households.

The optimal commodity taxes are calculated for seven groups of commodities. They are:

#### *Commodity groups*

1. Food = food;
2. Hous. = housing (including rent);
3. Furn. = furniture and appliances;
4. Cloth. = clothing;
5. Trans. = transportation and communication;
6. Hlth. = health and personal care; and
7. Pers. Exp. = personal expenses, education and reading.

## **4 - RESULTS**

### **4.1 - Optimal Commodity Tax Rates**

Table 2 presents the estimates of the optimal tax rates ( $t_i$ ) derived from the estimates of Aids demand system for different levels of inequality aversion ( $\epsilon$ ). The government revenue requirement corresponds to 10% of the consumers' total expenditure. The results can be summarized as follows. At all levels of inequality aversion, except when  $\epsilon = 0.25$ , food category should be subsidized. Regarding the housing group, we note that a positive tax is replaced by a subsidy as the inequality aversion increases (from  $\epsilon = 1.00$  to 1.25). All the other commodities have positive tax rates and their values increase for higher  $\epsilon$ .

Table 2

**Optimal Commodity Tax Rates — Revenue Requirement = 10% of Total Expenditure**

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0,25	0,75	1,00	1,25	1,75
1. Food.	12,2	-2,0	-8,6	-14,6	-24,8
2. Hous.	11,5	5,4	2,1	-1,1	-7,3
3. Furn.	8,1	10,3	12,5	15,3	22,7
4. Cloth.	10,2	16,1	19,6	23,4	31,6
5. Trans.	11,4	20,5	26,1	32,1	45,8
6. Hlth.	10,8	17,3	21,0	24,8	32,8
7. Pers. Exp.	10,9	20,8	26,6	32,9	46,2

For low levels of inequality aversion ( $\epsilon$  equals to 0.25), the optimal commodity tax structure shows a movement towards uniformity. When  $\epsilon$  is near zero, there is no concern for inequality. In this situation, a uniform rate of tax on all goods is equivalent to a tax on labor alone. This corresponds to the conventional prescription if there is a completely inelastic factor, this should bear all the tax [Atkinson and Stiglitz (1972)]. Therefore, as we are assuming that labor supply is completely inelastic, the optimal commodity tax rate is uniform.

As the parameter  $\epsilon$  increases, commodity tax structure presents non-uniformity in its optimal tax rates: the rate of subsidy increases for the subsidized items (food and housing) while tax rates also increases for the taxed items. In particular, tax rates on transportation and personal expenses increase significantly as the degree of inequality aversion becomes higher. As a whole, the results show that in order to achieve redistribution objectives a higher degree of selectivity in commodity tax rates structure is required.

Table 3 displays the sensitivity of tax rates to the government revenue requirement and to the degree of aversion to inequality (values of  $\epsilon$  equal to 0.25, 1.00 and 1.75). As expected, larger revenue requirements brings on an increase in tax rates. In particular, for some degrees of the  $\epsilon$  parameter, the subsidy for certain items is replaced by a tax. When  $\epsilon$  equals 0.25, no item is subsidized. When  $\epsilon$  equals 1.75, larger revenue requirements call for a reduction in the subsidy for food and housing and higher tax rates for the other goods. For values of  $\epsilon$  equal to 0.25 and 1.00, housing is no more subsidized and food is subsidized only when  $\epsilon$  equals to 1.00 and the level of revenue requirement corresponds to 10% of individuals' total expenditure. Nevertheless, it should be stressed that the tax rate structure remains similar among the revenue requirements.

The results can be viewed in Figures 1, 2 and 3, which display commodity tax rates for respective revenue requirements equal to 10%, 15% and 20% of individuals' total expenditure.

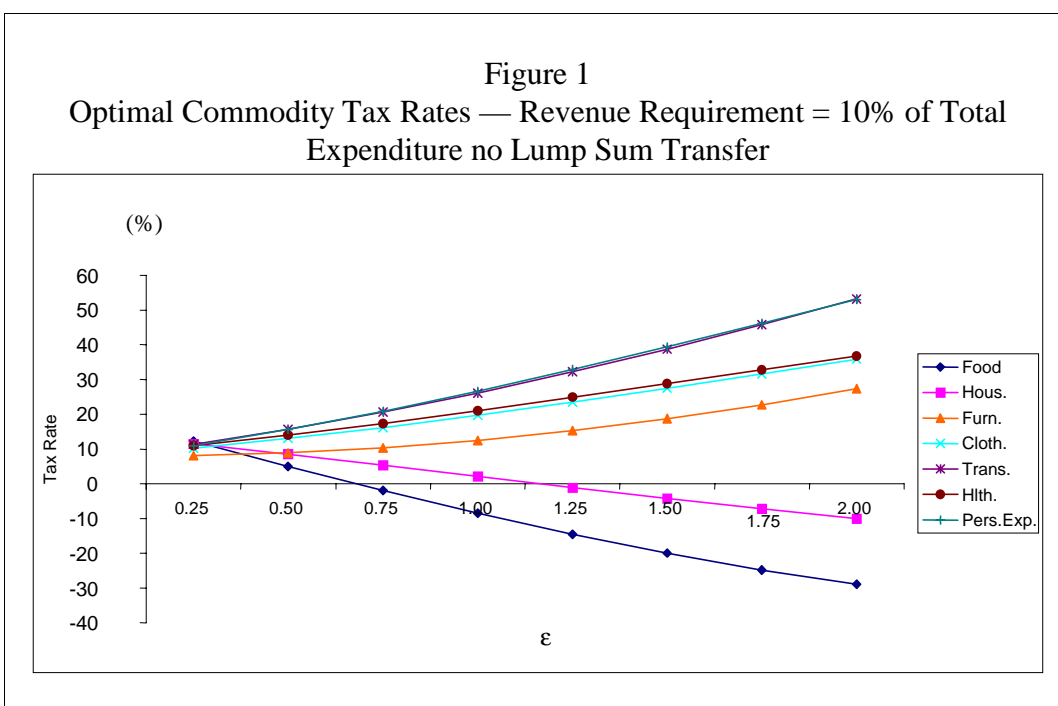


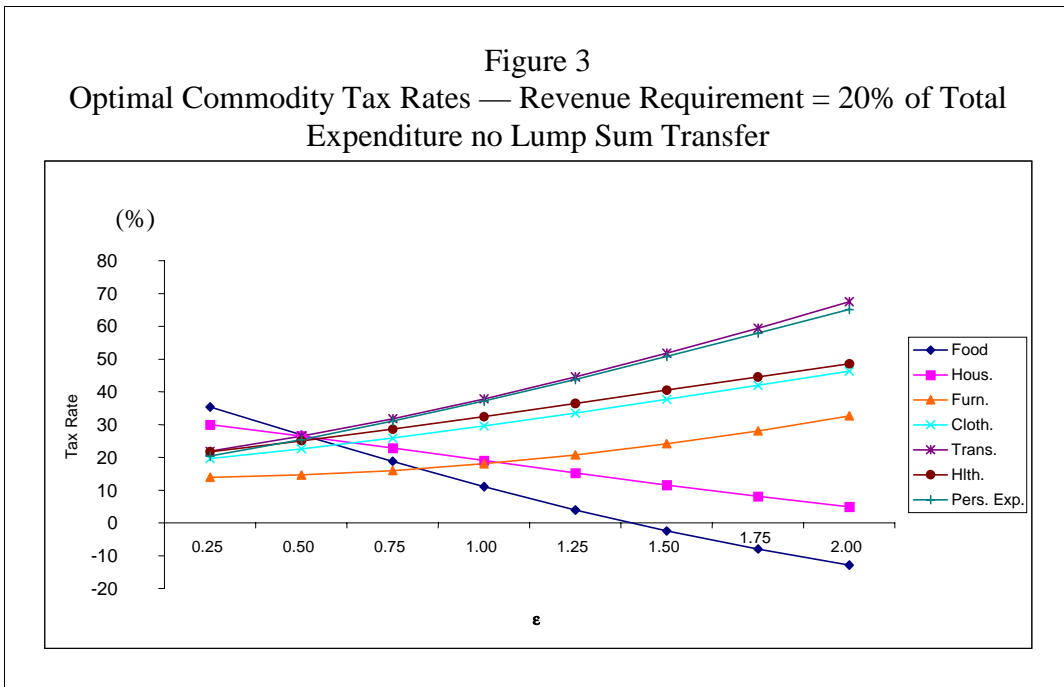
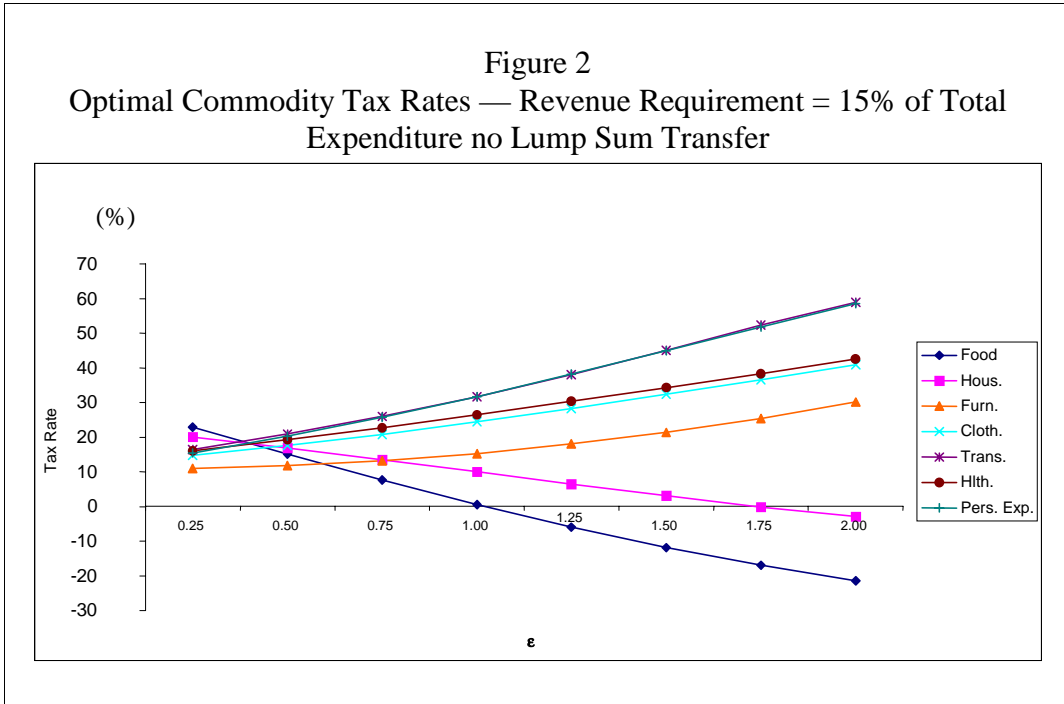
Table 3

**Sensitivity of Taxes to Revenue Requirement and Aversion to Inequality**

(%)

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )								
	0,25			1,00			1,75		
	Revenue Requirement			Revenue Requirement			Revenue Requirement		
	10	15	20	10	15	20	10	15	20
1. Food	12,2	22,9	35,3	-8,6	0,5	11,0	-24,8	-17,0	-8,0
2. Hous.	11,5	20,1	29,9	2,1	10,0	19,0	-7,3	-0,1	8,1
3. Furn.	8,1	11,0	13,9	12,5	15,2	18,0	22,7	25,4	28,1
4. Cloth.	10,2	14,7	19,6	19,6	24,4	29,6	31,6	36,5	42,0
5. Trans.	11,4	16,4	21,9	26,1	31,7	37,8	45,8	52,4	59,4
6. Hlth.	108,0	16,0	21,8	21,0	26,4	32,4	32,8	38,3	44,6
7. Pers. Exp.	10,9	15,5	20,4	26,6	31,7	37,2	46,2	51,8	57,9





#### 4.2 - Optimal (Unconstrained) Uniform Lump Sum Subsidy

Table 4 reports the results for the case where the government sets an optimal uniform lump-sum subsidy to all individuals. Government revenue requirement is assumed to be 10% of consumers' total expenditure. The main results for this case are the strikingly high levels of commodity tax rates and optimal lump sum

subsidies for all levels of inequality aversion parameter. The lump sum subsidy varies from R\$ 1,760 to near R\$ 3,530; this last figure represents almost 30 minimum wages at that time (September 1996). As  $\epsilon$  increases, both the commodity tax rates and the lump sum subsidies increase.

**Table 4**  
**Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy — Revenue Requirement = 10 % of Total Expenditure**

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0,25	0,75	1,00	1,25	1,75
1. Food	174,29	299,01	339,70	373,46	426,80
2. Hous.	124,11	202,13	226,81	247,20	279,85
3. Furn.	29,07	28,37	26,36	24,19	19,96
4. Cloth.	50,47	68,46	72,95	76,32	81,23
5. Trans.	57,83	83,53	90,91	96,82	106,03
6. Hlth.	60,68	86,93	94,16	99,84	108,48
7. Pers. Exp.	50,09	69,25	74,28	78,15	83,92
Lump Sum Subsidy in R\$ Sep. 1996	1762.87	2731.36	3001.13	3212.30	3528.80

For low levels of  $\epsilon$ , commodity tax structure is characterized by high levels on items that are price inelastic, which is similar to Ramsey's rule. One important feature to stress in this result is that this commodity tax structure remains similar as the parameter  $\epsilon$  increases. Therefore, we conclude that when an optimal lump sum subsidy is introduced, there is no room for redistribution through commodity tax rates, except for furnishings.

Tables 5 and 6 presents the results for higher government revenue requirements: 15% and 20% of individuals' total expenditure. In these situations, we have similar results to the one displayed on Table 4.

**Table 5**  
**Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy — Revenue Requirement = 15 % of Total Expenditure**

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0.25	0.75	1.00	1.25	1.75
1. Food	183,04	312,58	354,95	390,15	444,46
2. Hous.	130,51	211,43	237,04	258,18	292,14
3. Furn.	28,88	27,01	24,59	22,05	17,39
4. Cloth.	52,36	70,50	75,02	78,42	83,50
5. Trans.	60,05	86,26	93,82	99,90	109,43
6. Hlth.	63,26	89,96	97,30	103,05	111,92
7. Pers. Exp.	51,87	71,05	76,05	79,88	85,65
Lump Sum Subsidy in R\$ Sep. 1996	1602.05	2540.62	2801.99	3006.29	3313.02

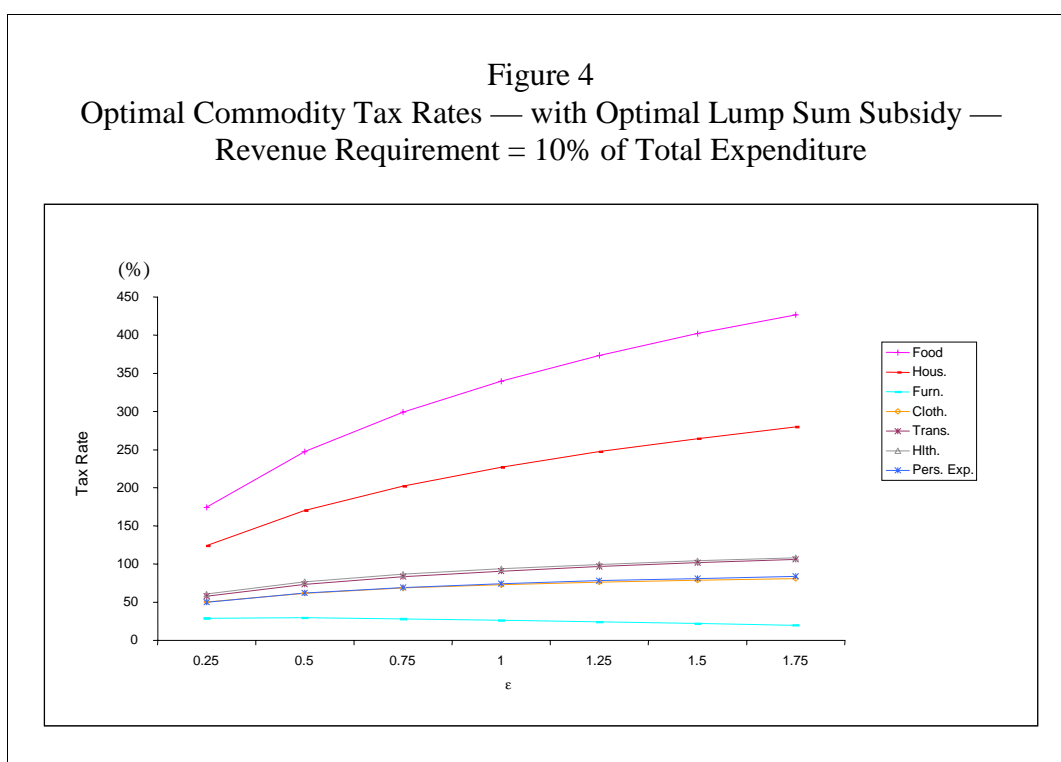
Table 6

**Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy — Revenue Requirement = 20 % of Total Expenditure**

(%)

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0,25	0,75	1,00	1,25	1,75
1. Food	192,73	327,72	372,00	408,82	467,46
2. Hous.	137,56	221,67	248,30	270,25	305,14
3. Furn.	28,46	25,21	22,28	19,31	13,68
4. Cloth.	54,35	72,66	77,22	80,67	85,65
5. Trans.	62,44	89,23	97,01	103,29	113,11
6. Hlth.	66,04	93,20	100,65	106,47	115,19
7. Pers. Exp.	53,74	72,90	77,85	81,61	87,07
Lump Sum Subsidy in R\$ Sep. 1996	1438.08	2345.85	2598.51	2795.69	3089.46

Figures 4, 5 and 6 display optimal taxes corresponding to revenue requirements of 10%, 15% and 20%. Lump sum transfer amounts are collected in Figure 7. Notice that transfer amounts decrease when the revenue requirement increase; this suggests that the need to raise (sunk) revenue sacrifices welfare: raising taxes (as showed by Figures 4 through 6) is not enough to keep transfers stable. In other words, the marginal rate of substitution between taxes and transfers in the welfare function *decreases* when the revenue requirement goes up.



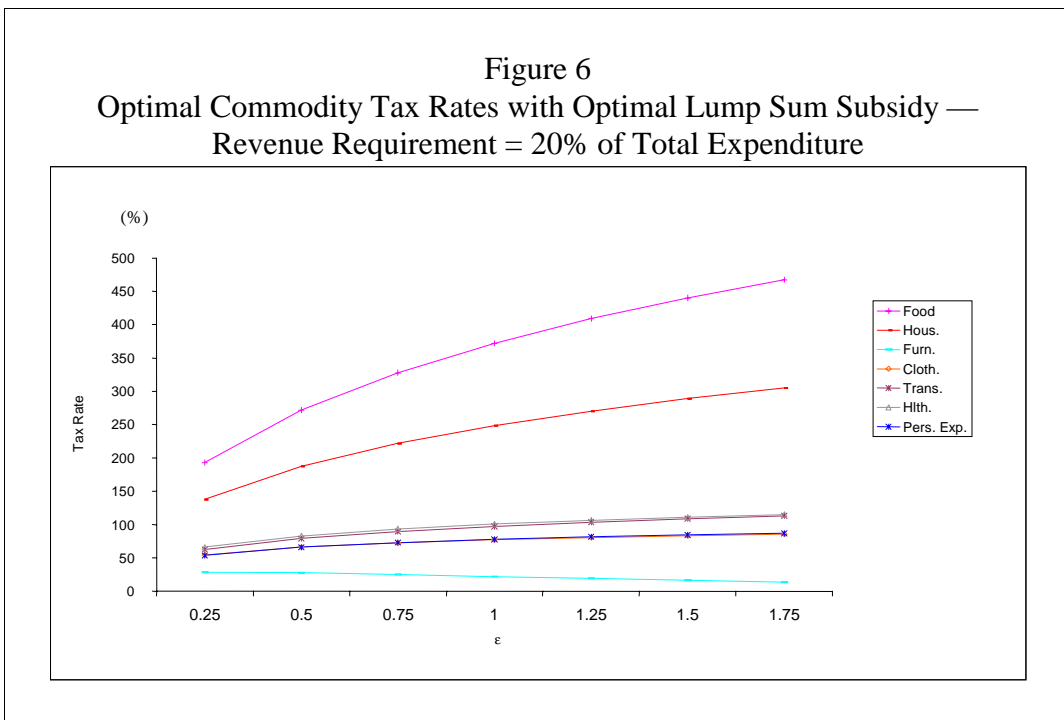
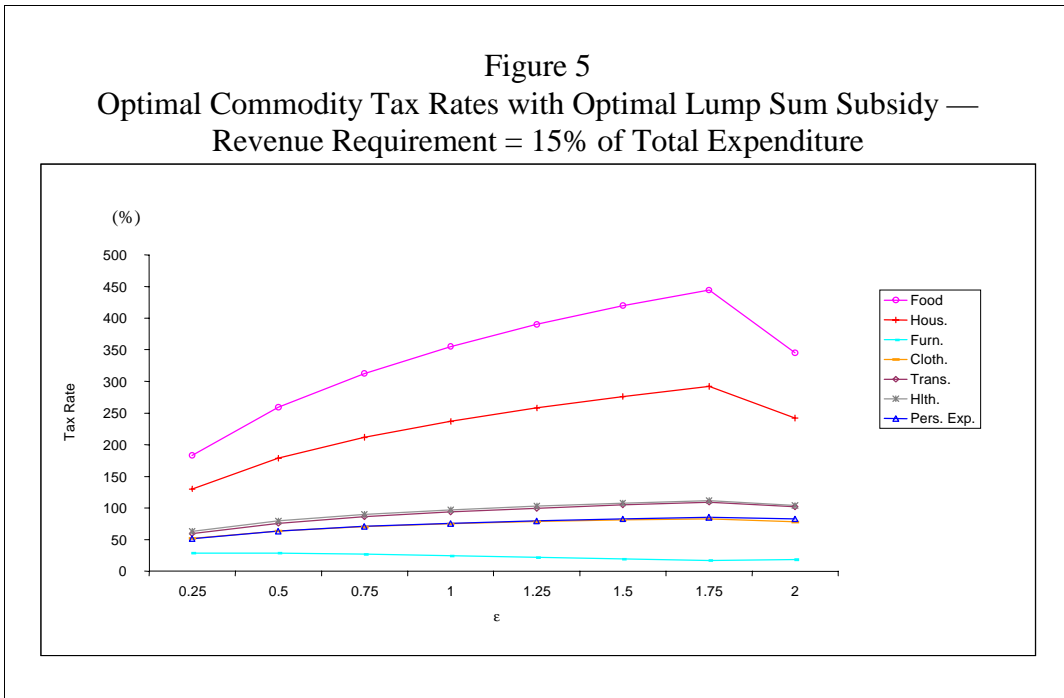
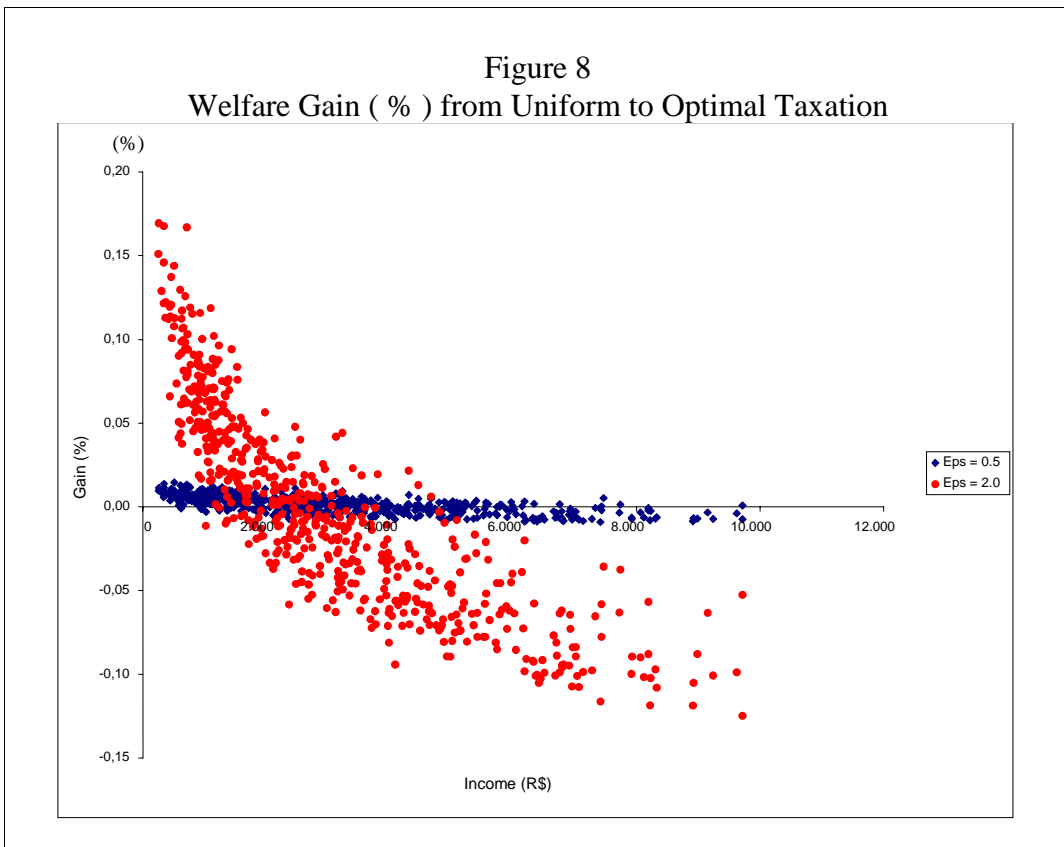
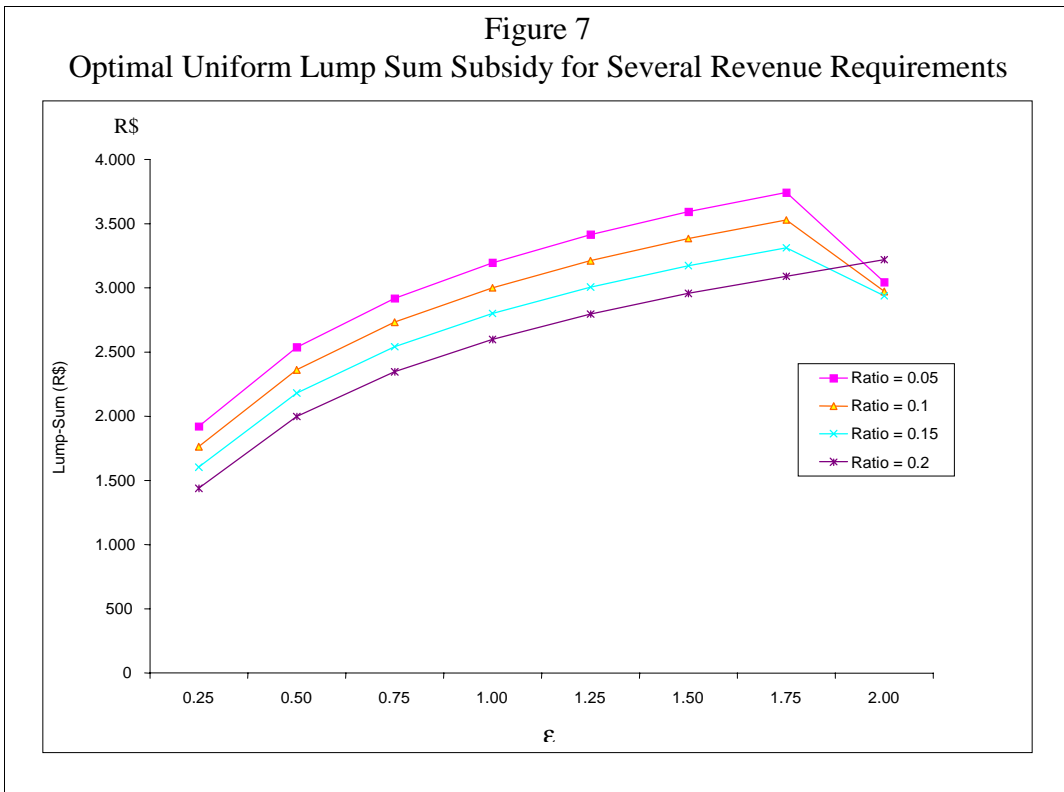


Figure 8 displays the households' welfare gains from moving from a uniform structure to an optimal commodity taxation (assuming the revenue requirement equals to 15% of the total expenditure). This case is presented to draw a



comparison of the households' welfare between the uniform commodity taxes, which are equivalent to a linear income tax, and the results of optimal commodity taxation. For the lower income households the welfare gain is higher. As the household income increases the welfare gain is lower and eventually negative. It is worth noting that the stronger the commitment to equity (that is, the higher the government's aversion to inequality), the higher is the welfare variation at each income level.

### 4.3 - Constrained Lump Sum Subsidy

As the optimal lump sum subsidies presented in the last section were remarkably high in its values, we constrained the optimal lump sum in order to obtain politically more realistic results. We call each constraint "bonus rates". The government revenue requirement is assumed to be 10% of individuals' total expenditure. Tables 7, 8 and 9 report the result for bonus rates equal 0.5, 1.0 and 1.5 (50%, 100% and 150% of the minimum observed income). The constrained optimal lump sum values remain quite stable as the inequality aversion increases.

Table 7

#### Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy — "Bonus Rate" = 0.5 of Optimal Lump Sum Subsidy

(%)

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0,25	0,75	1,00	1,25	1,75
1. Food	21,12	7,29	0,99	-4,74	-14,33
2. Hous.	18,48	12,65	9,65	6,69	1,19
3. Furn.	10,22	11,43	12,74	14,51	19,20
4. Cloth.	13,53	18,83	21,79	24,88	31,19
5. Trans.	15,16	23,66	28,57	33,83	44,97
6. Hlth.	14,76	20,74	23,95	27,22	33,67
7. Pers. Exp.	14,28	23,45	28,57	33,92	44,80
Lump Sum Subsidy in R\$ Sep. 1996	127.19	127.19	127.19	127.12	126.90

Table 8

#### Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy — "Bonus Rate" = 1.0 of Optimal Lump Sum Subsidy

(%)

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0.25	0.75	1.00	1.25	1.75
1. Food	30,38	16,80	10,69	5,17	-4,23
2. Hous.	25,59	20,04	17,23	14,49	9,29
3. Furn.	12,27	12,65	13,40	14,48	17,48
4. Cloth.	16,83	21,59	24,15	26,77	31,90
5. Trans.	18,83	26,81	31,26	35,92	45,45
6. Hlth.	18,63	24,20	27,10	29,99	35,47
7. Pers. Exp.	17,56	26,11	30,73	35,46	44,78
Lump Sum Subsidy in R\$ Sep. 1996	254.37	254.37	254.35	254.37	251.95

Table 9

**Optimal Commodity Tax Rates and Optimal Uniform Lump Sum Subsidy –  
“Bonus Rate” = 1.5 of Optimal Lump Sum Subsidy**

(%)

Commodity Group	Degree of Inequality Aversion ( $\epsilon$ )				
	0,25	0,75	1,00	1,25	1,75
1. Food	40,00	26,59	20,61	15,22	6,22
2. Hous.	32,86	27,56	24,90	22,33	17,61
3. Furn.	14,22	13,94	14,25	14,85	16,70
4. Cloth.	20,06	24,37	26,63	28,90	33,27
5. Trans.	22,43	29,95	34,05	38,26	46,70
6. Hlth.	22,46	27,69	30,35	32,96	37,85
7. Pers. Exp.	20,75	28,77	33,01	37,28	45,53
Lump Sum Subsidy in R\$ Sep. 1996	381.56	381.56	381.56	381.54	381.39

As expected, in each case, we have significant higher tax rates (and lower subsidies) than the results presented in Table 2 (the case with no lump sum transfers). The payment of an optimal uniform subsidy to all individuals, a more efficient tool to achieve redistribution, implies in the increase of commodity tax rates in order to finance it. However, in comparison to the case of unconstrained optimal lump sum (last section), the present result shows that the introduction of a constrained optimal lump sum gives some room to the commodity taxation as redistributive instrument. As the inequality aversion increases, taxes for food and housing are substantially reduced. In particular, for inequality aversion equal to or above 1.25 food is subsidized.

For a bonus rate equal to 0.5 (50% of the minimum observed income), the optimal lump sum subsidy is around R\$ 127 yearly, an amount slightly above the minimum wage at that time (R\$ 121 on September, 1996). As expected, when the bonus rate increases (and so does the optimal lump sum subsidy), optimal commodity tax rates also increase. The results can be viewed in Figures 9, 10 and 11.



Figure 9  
 Optimal Commodity Tax Rates with Optimal Uniform Lump Sum Subsidy —  
 Revenue Requirement = 10% of Total Expenditure  
 (Bonus Rate = 0.5 )

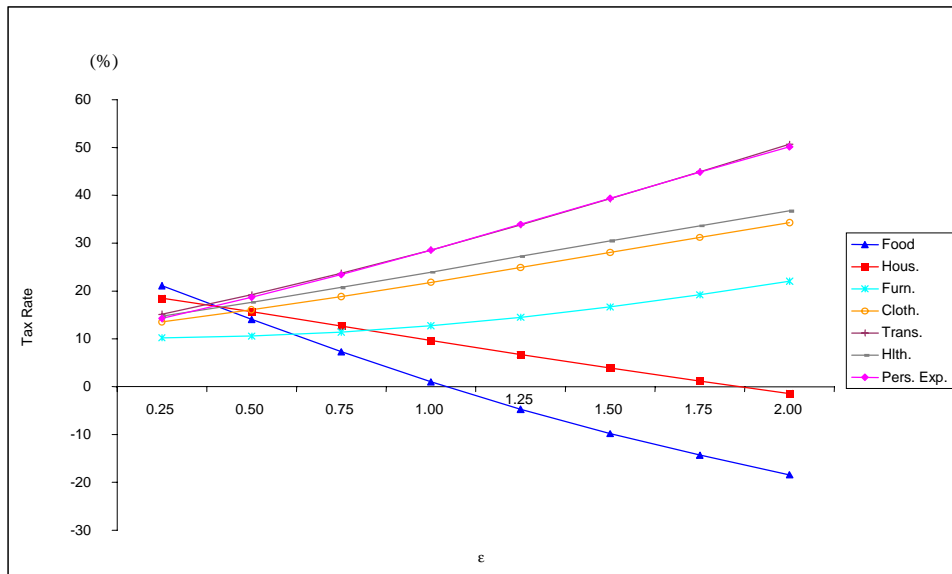
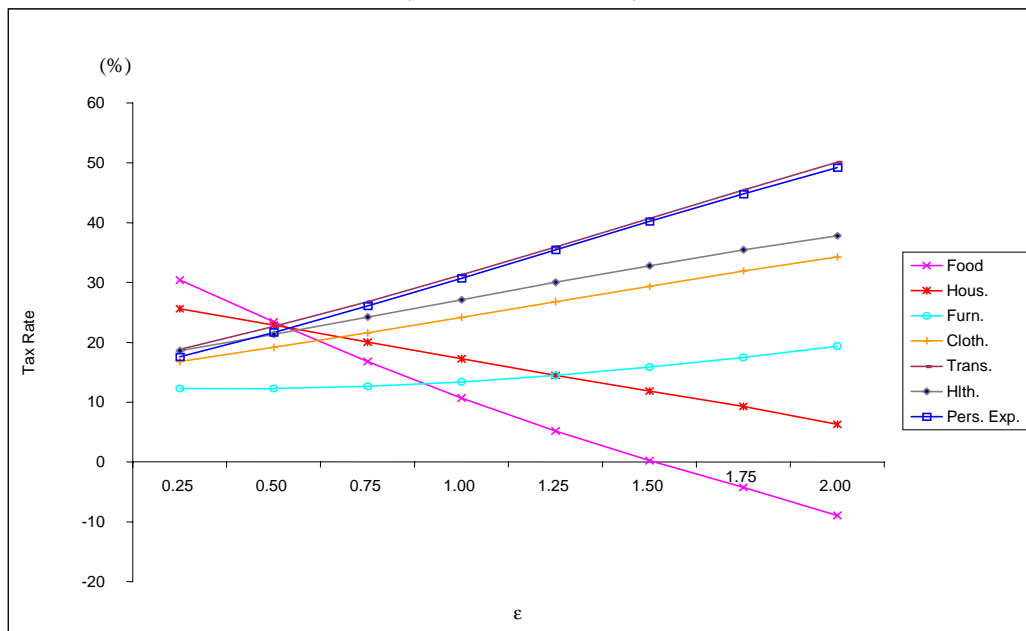
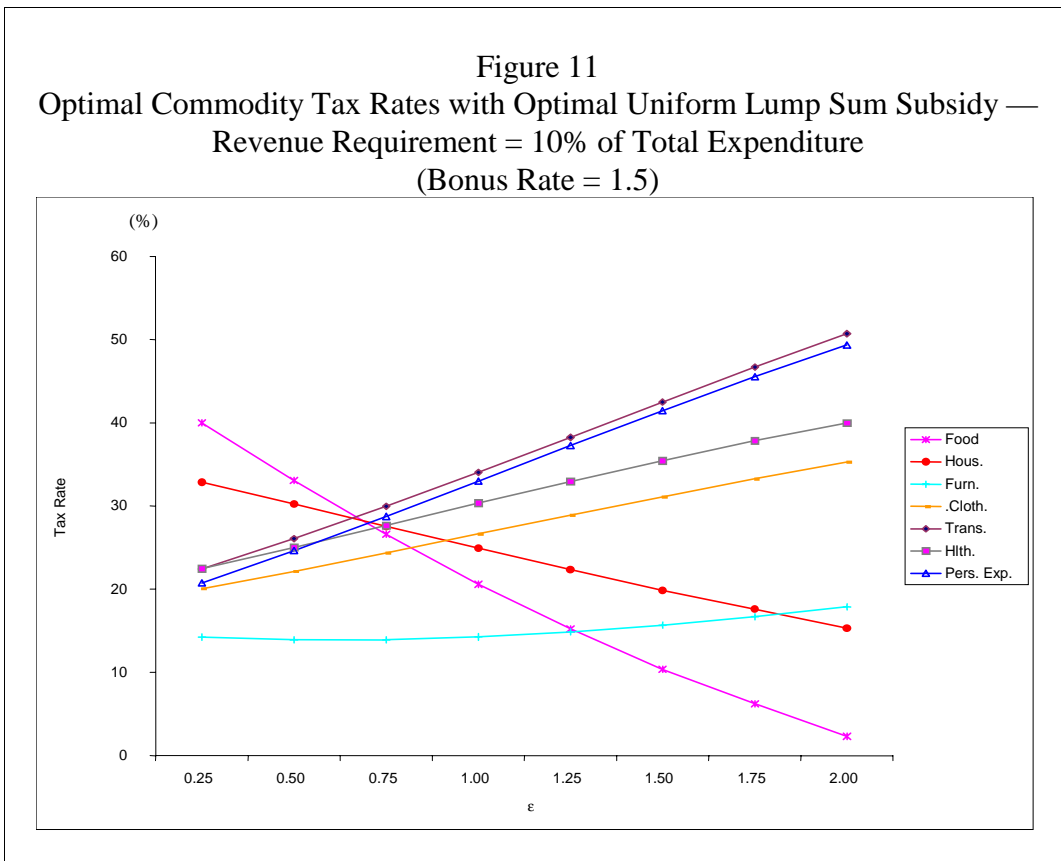


Figure 10  
 Optimal Commodity Tax Rates with Optimal Uniform Lump Sum Subsidy —  
 Revenue Requirement = 10% of Total Expenditure  
 (Bonus Rate = 1.0)





## 5 - CONCLUSION

This paper has used the framework of the many person Ramsey equations to investigate the structure of optimal commodity taxes in Brazil. Unlike a number of empirical studies on commodity taxation based on very restrictive consumer preferences, our micro-simulations were based on a complete demand system estimated with a flexible functional form, the Aids. Preference parameter estimates were consistent with microeconomic demand theory and allow for a highly accurate optimal commodity tax simulation.

Our results showed that commodities which the lower income classes spend most on, such as food and housing, should be subsidized. Moreover, optimal tax rates are quite sensitive to the government's redistribution objectives. As the aversion towards inequality increases, the degree of selectivity is more significant.

We have also extended our analysis by allowing for a uniform per capita lump sum payment to be made by the government to all households. The optimal lump sum transfer reverses the results such that Ramsey taxes are collected in order to finance the transfer; therefore the redistributive role of taxes is canceled out and redistribution is achieved by the transfer alone. If we constrain the lump sum

transfer, however, tax structures follow similar patterns to the first case, as long as the constraint is binding.

Some final remarks should be stressed. Separability of labor supply and commodity demand has been assumed throughout the discussion. Although this is a useful assumption regarding consumer demand systems, the empirical evidence shows that when joint decision of leisure and commodity choice is taken into account along with flexible functional forms, separability is decisively rejected [Blundell and Walker (1982)] and Browning and Meghir (1991)]. Asano (1997) has tested the separability assumption between labor and consumption using Japanese data. He estimated the Aids allowing for a joint choice of leisure and consumption commodities, thus relaxing the separability assumption. The results implied “definite rejection of weak separability of labor supply and commodity choice, and non-rejection of homogeneity and symmetry restrictions on the demand system” (p. 65). We believe that an estimation based on an extended demand system with flexible form including labor supply could generate different results regarding optimal commodity taxation. It is worth remarking that this exercise for Brazil requires a richer data set, as our data source does not provide such information. Solutions on combining cohorts from the expenditure survey with corresponding cohorts from some compatible labor survey are to be pursued as possible future extensions.

One aspect that should be stressed is the absence of demographic variables on the estimated consumer demand system, which, if included, can change the results of our study. It is quite widely conceded in the literature that optimal tax rate calculations depend heavily on the utility and demand specification. The inclusion of demographic variables into the demand system used for the calculations would be of interest, as many countries have a system of transfers based on household composition. Empirical studies — e.g., Ray (1989) and Ebrahimi and Heady (1988) — have emphasized that demographic variables impact significantly the calculated optimal tax rates.

Another limitation of our study is that it has not considered administrative costs and ignores a range of relevant institutional features. Nevertheless, we believe that our empirical findings provide a valuable contribution for the current tax policy debate in Brazil, where distributive goals have a great importance in the agenda.

## APPENDIX

### Derivatives Utilized in the First Order Conditions, Based on Aids and Atkinson (1970) Specifications

The first order conditions (18) and (19) in Section 2 contain the following derivatives:

$$\frac{\partial W}{\partial v^h} = [v^h(p, Y)]^{-\varepsilon} \quad (\text{A.1})$$

$$\frac{\partial v^h}{\partial p_i} = \frac{\left[ \frac{-d \log a^h(p)}{dp_i} \right] \beta_0 \prod p_i^{\beta_i} - \left[ \log \frac{Y^{h*}}{a^h(p)} \right] \frac{\beta_i}{p_i} \beta_0 \prod p_i^{\beta_i}}{\left[ \beta_0 \prod p_i^{\beta_i} \right]^2} \quad (\text{A.2})$$

$$\frac{d \log a^h(p)}{dp_i} = \alpha_i + \frac{1}{2} \sum_j \gamma_{ij} \log p_j + \sum_k \omega_{ik} Z_k^h \quad (\text{A.3})$$

$$x_i^h = w_i \cdot \frac{Y^h}{p_i} \quad (\text{A.4})$$

$$\frac{\partial x_i^h}{\partial p_i} = \left\{ \gamma_{ij} + \beta_i \cdot p_i \left[ \frac{-d \log a^h(p)}{dp_i} \right] \right\} \cdot \frac{Y^h}{p_i^2} + \frac{x_i^h}{p_i} \quad (\text{A.5})$$

$$\frac{\partial v^h}{\partial I} = \left[ \frac{1}{Y^{h*} \beta_0 \prod p_i^{\beta_i}} \right] \quad (\text{A.6})$$

$$\frac{\partial x_i^h}{\partial I} = \frac{w_i^h + \beta_i}{p_i} = \frac{x_i^h}{Y^{h*}} + \frac{\beta_i}{p_i} \quad (\text{A.7})$$

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