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THE PROCESS OF PUBLIC RESOURCES ALLOCATION FOR INVESTMENT IN HOSPITAL CAPACITIES[1]

Alexandre Marinho[2]

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SUMMARY

SINOPSE

O presente trabalho explicita um modelo simples e abrangente, para explorar possibilidades de aprimoramento na alocação de recursos públicos para investimentos em hospitais. Estuda-se a coexistência de hospitais públicos e privados, que devem atender diferentes estratos populacionais segmentados por renda em um mesmo setor saúde, o que possibilita a ocorrência de excessos de demanda. Nesse arcabouço, são explicitados os impactos que a adoção de diferentes políticas de racionalização da oferta podem ter sobre o desempenho econômico dos hospitais públicos. O modelo é aplicável ao sistema de saúde brasileiro e a outros casos e situações semelhantes.

ABSTRACT

This work develops a simple and comprehensive framework to improve the allocation of public resources to public and private hospitals from the standpoint of public hospital profitability in the presence of income inequalities. The paper illustrates the direct impacts that publicly-funded investments in capacity, the adoption of a public market share, the institution of punishments for insufficient supply and the provision of a stochastic reservation quality have on a public hospital's surplus. The presented framework fits well the enormous Brazilian health system and can be useful in other circumstances and countries where public and private hospitals coexist.

1 INTRODUCTION

This paper uses a comparative static analysis approach to study public investments in hospital capacity and the effect of such investments on a public hospital's economic surplus and on its health care services quality. Based on a elements of health economics, industrial economics and operations research we could bring the investment in hospital capacity into a framework accessible to those interested in public choice matters but just able to deal with standard tools of economic analysis. We use the definition of capacity presented by Färe (1984), and meaning the maximum amount of health care services that the hospitals can produce per unit of time with the corresponding and existing plant and equipment, provided that the availability of variable factors of production is not restricted. According to this definition, capital stock is the only limiting factor to hospital capacity.

Constrained by severe budget restrictions in recent years, public hospitals can no longer afford to run unlimited deficits, and must pay close attention to operational receipts and costs. The public hospital surplus estimates the difference between the total revenues the public hospitals earn from their operational activities and their total operational costs, and for this reason one of the most important tasks of this paper is to consider not only health care services quality but also ways to maximize a public hospital's surplus (or, at least, minimize any possible losses).

In the health care markets literature, beds are a very well established proxy for capacity, although other types of fixed assets (such as equipment, diagnosis devices and plant size) have also been focused on. As discussed in Barnum and Kutzin (1993) a recurring problem in developing countries is the imbalance between demand for and availability of in-patient care beds, evidenced by bed shortages and long waiting lines. Idle capacity is also a major problem, and is no longer a rare occurrence even in countries experiencing some local scarcity as well. Examining the Norwegian health service system Iversen (2000) reports that " (...) there may be idle beds in some counties and long] waiting periods in others"]. The potential consequences of these problems for Pareto efficiency have been well demonstrated by Iversen (1993).

There are other reasons why hospital capacities are very important public policy issues. Since Harris (1973) analysts have recognized that due to their internal markets, hospitals are prone to over-invest (creating excess capacity). Arrow (1963) demonstrated that health care markets are asymmetric and that physicians can induce oversupply. Oversupply, in turn, leads to over-consumption and excessively high total costs.

Nevertheless, in Brazil hospital capacity is not constrained by stringent regulation because efforts to coordinate investment in hospital capacities are in their infancy. As a consequence, hospitalization as a style of care predominates in Brazil, and investments in hospitals take the place of more cost-effective but less hospital-intensive alternatives like disease prevention and health promotion activities.

The features of Brazilian health system make competition, in terms of quality, a secondary dimension of the strategy of Brazilian hospitals. Although mortality rates are a common empirical measure of quality in the health care literature, they do not play a decisive role in the SUS's centralized reimbursement system. We should not conclude from this, however, that the SUS holds a narrow view of health outcomes. The problem, rather, is that the *price system* is not designed to consider service quality a key variable. However, one still can speak of *reservation quality* in the sense of [Färe, Grosskopf and Valdmanis (1989)] as referring to "(...) the hospital having extra beds available for patients as a type of option demand". Since the demand for hospital services is stochastic, hospital's decision makers cannot specify, *ex ante*, the cost-minimizing level of inputs that assures achievement of the desired quality level.

The paper also considers a scenario where the inpatient care market is shared by publicly-owned, private for-profit, and private non-profit hospitals. We focus on the in-patient care market because it accounts for the largest part of hospital costs and requires the greatest inflow of services from the capital stock. This is the case of the enormous Brazilian health system, which has 7,397 hospitals (2,588 public hospitals and 4,809 private hospitals) and 471,171 beds (146,319 public beds and 324,852 private beds) [IBGE (2002)]. In 2002 the system handled 19,967,198 in-patient care admissions (*ibid.*). The system's hospitals, furthermore, can receive public funding through direct contracts with the Brazilian Government for health care services or through credit subsidies, tax-exemptions, or other special investment programs. This flexibility of public funding makes the distinction between public and private (either for-profit or non-profit) hospitals an interesting dimension of our analysis. As we will see, this dimension becomes a crucial issue if the objective of public policy is to assure that public hospitals break even financially.

The model allows some people (the affluent) to have private health plans giving them access to high-quality private hospitals. Despite being private, these hospitals are sometimes full and cannot attend to all their potential patients. Another embarrassing situation occurs when health plans refuse to pay very expensive bills, demonstrating that a person cannot take his or her access to the private sector for granted, even if he or she has a private health plan or can pay out-of-pocket for the services. When a person is not attended in the private sector, the only alternative is the public sector. In our present model, all the people have free access to the public hospital sector, where given the likelihood of demand excess the best quality and readiness of treatment cannot be assured. The public hospital may be overcrowded and quality tends to be inversely related to bed occupancy rates. Occupancy rates deserve great attention in the investment decisions public hospitals make around the world [see *e.g.* Barnum and Kutzin (1993)]. This indicator is a proxy to the likelihood that people have to be attended promptly. Therefore, although no close examination of data is performed in the present paper, some conceptual counterparts to this service utilization indicator play a central role in the model. We believe the above described uncertainties make people prone to purchase private health plans so as to get insured against undesirable states of nature.

Regarding institutional issues in Brazil, which plays a central role in our model, we should say that the Brazilian *Sistema Único de Saúde* (SUS) — SUS is a hybrid health care system where, as we have assigned before, both private and public hospitals can get public funding through direct contracts with the Brazilian Government for health services or through credit subsidies, tax-exemptions, or special investment programs. It still provides health care services to an estimated 95% of the Brazilian population, including roughly 70% of secondary care and 90% of complex care [IBGE (2002)]. Given the underling high costs and expenditures in the system almost all payment for procedures and treatments comes from the Government or from other third parties, rather than directly from care recipients, a situation frequently cited in the literature (*e.g.*, Wedig, Hassan and Sloan (1989)]. Hospitals can choose the number of beds and the volume of procedures they make available to the public sector.

SUS has also rigid public employee salaries, centrally-budgeted investments and a fixed price table that enforces a price regulation system. Since many hospitals are nonprofits, this rigid price regulation system should aim at controlling the treatment intensities and that hospitals try to maximize the social values of health care subject to break even constraints [see Rogerson (1994)]. As a matter of fact, however, health outcomes do not have a central role in this cost-oriented reimbursement system.

In an attempt to connect profitability considerations with eventual income inequalities, the model builds up a link among the public market share, waiting lines and investments in capacity. The entire population has the right to be attended to in a publicly-financed hospital. It is known, furthermore, that most poor do not have health plans. In targeting the poor, the government can set an *ex ante* market share or prioritize a reserve quality for public hospitals to attend to irrespective of the size and variability of demand. However, this procedure comes with a cost because any idle capacity must be paid for. On the other hand, demand excesses are not welcome since they can lead to penalties. The model easily links the public market share and some queuing theory parameters to a public hospital's surplus and other key variables able to emulate the poor share of the population.

Though focusing on Brazil, the model presented below could be useful in many circumstances where public and private hospitals share the market and compete for public resources and incentives. Even if investment decisions are free and undertaken in an independent way, the model would still be useful since its rationale is not constrained by the decision-maker's obligations or commitments since any decision — making unit aims at maximizing some objective function.

The remainder of the paper is organized as follows. Section 2 explains the main features of the model. Section 3 develops a free market framework for the model. Section 4 extends the free market approach by allowing the public hospital to establish an *ex ante* fix market share to attend. An alternative queuing theory framework is developed in Section 5. The general results are displayed in Section 6, and the last section offers a conclusion and some final comments.

2 THE MODEL

The main component of the Brazilian health care system is the Unified Health System — *Sistema Único de Saúde* — (SUS) —, a twofold system financed by public funds. Public and private hospitals can be contracted by the SUS for health care service delivery and are paid through a corresponding fixed price system. Usually, both type of hospitals receive payments per procedure according to a reimbursement system that varies somewhat according to discretionary treatment decisions. The payments vary with the cost and complexity of procedures and treatments rather with the kind of hospital providing the service. Private hospitals under SUS can be either for-profit or not-for-profit oriented. Differences in hospital quality are not taken into account in this very peculiar reimbursement system. The more intensive the care, however, the more the hospital will receive. Many of the private hospitals are nonprofit (known in Brazil as *filantrópicos*). In addition to the SUS, a purely private for-profit sector exists in Brazil. This "off-SUS" sector only provides care for patients who can afford private health plans or out-of-pocket payments for health care services. A remarkable point is that private hospitals (either contracting or not with SUS) are not obligated to provide care for patients beyond their full capacity unless it is a life-or-death situation. Public hospitals, on the other hand, must be prepared to accommodate demand in excess of their capacity under all circumstances, a situation that encourages these hospitals to maintain excess capacity as a preventive measure.

The model considers the local market for internal and long-term care patients as a kind of duopolistic game. All types of hospitals, both public and private (either for-profit or nonprofit), face an uncertain demand in their non-price competitive local markets. The demand for the public hospital is assumed as being independent of the demand that first reaches the private hospital. Both public and the private demands are exogenous as in most hospital literature (*e.g.,* Joskow (1980)]. Public and private hospitals compete for public investment funds financed by general taxpayers and some special contributions. Private hospitals may obtain investment capital from other sources but public hospitals are limited to public investment funding alone. A governmental authority, or regulator, is the decision maker in the process of public investment allocation to hospitals and acts in behalf of the public hospital, trying to maximize the difference (assumed to be positive) between a public hospital's total receipts and its corresponding total costs.

Three basic frameworks are developed here and complement each other in a very interesting way. These frameworks are concerned with the surplus the public hospital can obtain as a function of key-variables in the system. By allowing the hospitals to vary their capacities, as well as their market shares and reserve margins (the probabilities that a patient will not be attended), the model yields very simple insights into welfare and profitability. A social planner or a public manager can even behave as a watchdog and punish public hospitals if planned capacity falls short of demand.

In the first version of the model, strongly based on a paper by Gal-Or (1994) it is assumed that the common prior marginal distribution of the demand for the public hospital is uniform over [0,1]. Thus, this demand represents the share of the total population that seeks the public hospital's services. A similar modeling displays the proportion of the insured population that seeks care in the private hospital. We assume these demands are independently distributed although in the real world it is not always the case.[1]

We believe that both kinds of hospitals try to minimize the likelihood of a patient to be sent away without being attended to. Thus, inspired by Joskow (1990) the model is changed and some queuing theory aspects are brought into its

1. For a extensive discussion, see Cullis, Jones and Propper (2000).

framework. In this setting, the model views demand as a queue or waiting line with the chances of a patient being seen depending on the number of empty beds in the hospitals. As in Iversen (1997) the model allows a public and a private sector to share the market. What is different about this model, however, is that while many authors [*e.g.*, Iversen (1997)] focus mainly on the waiting times associated with treatments, the present paper focuses on aspects of a public hospital's budget, particularly its break even point. This is a new feature that has not been the subject of many specific studies.[2]

As in Braeutigam (1984) the hospitals must choose their capital stock K before observing demand. As is common in the literature [*e.g.*, Gal-Or (1984)] it is assumed that the hospitals do not achieve short-run equilibrium and thus, by imposing constant returns to scale, the model offers a good representation of their long-run behavior. As a consequence, the hospitals are charged at a per-unit capital cost rate of r. Since demand is always stochastic, the model may reveal either excess capacity or lack of capacity relative to demand.

Turning to the real world, there are conflicting forces driving Brazilian hospitals toward supply and demand imbalances. First, since there is a compreheensive social insurance (the public hospital must attend to everybody) and capacity is not constrained, the system engenders a situation in which according to Joskow (1980) hospitals should tend to supply too much. The weak regulation forces hospitals to the classical situation where oversupply is the usual outcome arising from non-price competition and stochastic demand. However, it can be argued that one cannot take for granted that the market as a whole actually oversupplies *ex post* once the waiting lines and queues appear. Because hospitals deliver services, one can state as Easley and O'Hara (1983) that consumers "(…) [may be unable to determine costlessly the] [quality] and even the quantity (...)" [of the output] (…)". Given that many hospitals are for-profit organizations and treatment is not free, quality and even quantity should be below the optimal level. This is because for-profit providers may have an incentive minimize the benefits they provide. The net outcome of these opposing forces cannot be worked out *ex ante*.

Since private health plans select the cases they will pay for by *cream-skimming* or rejecting long-term and intensive treatments, the public sector bears the burden of most expensive and long-term care. According to Brazilian law, the public sector cannot deny treatment to anyone. The model accounts for the fact that if health insurance plans refuse to pay for a patient's treatment or deny a requested benefit or service, the patient is passed on to the public sector. The same situation arises if private hospitals are full. Once a sick individual arrives to a public hospital he must be given care. It is important to keep in mind that uninsured patients are never attended in private hospitals that are not SUS members.

As in Friedman and Pauly (1981) and in Gal-Or, (1994) the model designs a market share approach in such way that when the demand for the public hospital overcomes its full capacity this hospital faces a penalty at a per unit capital rate of $t > r$ (the per-unit capital cost rate). As in those works, no qualitative consequences of

2. For a extensive discussion, see Cullis, Jones and Propper (2000).

the capital shortage can be observed, although t could result in reductions in quality. We conjecture that penalties are set at rather mild levels and are not fully supported by Brazilian hospitals. In fact, if excess global demand appears, the only possible penalty is a work overload, imposed by the public authority. Since input costs are publicly funded and neither employees nor managers are directly burdened by any extra cost outlays, hospitals have a powerful incentive to keep input stocks when they believe that this penalty would be permanent. It is possible that a great share of the burden of shortage is born by consumers who are turned away (in the hope that they will be able to come back) or have their treatments delayed (sometimes forever). An alternative is to believe that the penalty t is related to some occupancy rate above the normal level and leads to some organizational stress.

Although differing in their objectives and strategic outlook, for-profit and nonprofit hospitals try to maximize some kind of surplus because both of them must break even. As will be seen later, the specific forms of these objectives are not important for the purposes of this paper. Moreover, for the sake of simplicity this paper does not deal in depth with the complex characteristics of the population served by the hospitals.

3 THE FREE MARKET FRAMEWORK (FM)

The first part of the model is in the spirit of Gal-Or (1994). We have much more limited objectives than that extensive paper though some new and interesting features are explored in later sections. The total demand X_i for the public hospital i is divided into two parcels in accordance with the patients insurance status: an insured demand X_{ip}, uniform over [0,1] and an uninsured demand, X_{iu} uniform over [0,1]. We assume these demand are independently distributed. It is supposed that many insured patients go to public hospitals since these organizations do not fully charge people that can afford a health plan and also because treatment in private hospitals is not always available. Let S_i be the surplus (or the payoff) function of the public hospital i facing an uninsured demand X_{iu} for its services and a insured demand X_{ip} not met in a private hospital j. This latter demand represents the insured patients arriving first in the hospital i or turned away from the private hospital j for any reason. Regardless of its origins, this demand is charged a constant per unit capital cost rate r. The public hospital capacity is K_i. Any excess demand beyond K_i is charged on hospital i at an extra penalty rate t. The reimbursement it receives per unit of demand is p. Iversen (1997) studying the interaction between a public and a private sector but focusing on waiting times, assumes that a public hospital receives a fixed budget. However, his wise modeling, which aims at studying flexible cost and revenue balances, is not directly suitable to the present problem. To impose a real loss to the public hospital, we set the condition $t > p$. Obviously, it is assumed that $p > r$.

The private hospital faces an insured demand X_{jp} uniform over [0,1] and independently distributed of the demand for the public hospital i. This demand represents the affluent share of the population (affluent here means they have health plans) that seeks care in the private hospital. Investments in its capacity K_j can be financed either by the private hospital through its own endowments or, alternatively, by public resource inflows. These public resources can take the form of incentives, tax exemptions, credit, direct subsides or any combination of such instruments. Whenever $X_{jp} > K_j$, the public hospital must attend the resulting excess demand.

Since X_i and X_j are stochastic, integrating the surplus functions with respect to these variables over the compact interval [0,1] gives the public hospital's surplus as a function of the public and private capacities. Some comparative static exercises are performed to examine the way the surplus behaves according to the model's policy and strategic variables. We call this, somewhat inadequately, the "free market" framework (FM), since no government or authority interferes with the capacities or shares of the market that the hospitals supply. By doing so, it is possible to make a clear distinction between these situations and those where such decision-making freedom does not exist.

4 THE MARKET SHARE FRAMEWORK (MS)

In this version of the model, we maintain the main aspects of the free market framework. As a new feature, the public hospital is allowed to determine a fixed market share α ($0 \leq \alpha \leq 1$), irrespective of the market size (in other words, it is assumed that the market clears). This assumption may be important because a government can identify a portion of the market that deserves inpatient treatment, regardless of any other considerations. For instance, this share should be equal to the proportion of poor or low-income clients in the market. This feature allows the model to address income inequality issues. By fixing $\alpha = (X_{iu}+X_{ip})/(X_{iu}+X_{ip}+X_{jp})$, which implies $X_{iu}+X_{ip}=(\alpha/(1-\alpha))X_{jp}$, it becomes possible to display the public hospital surplus as a function of the *ex ante* fixed market share the public hospital wants to attend. This very interesting policy tool fits situations where regulators are concerned with equity, especially since the poor have no access to private hospitals and health plans.

5 THE QUEUING THEORY FRAMEWORK (QT)

Queuing theory models have been applied to many service and industrial sectors. Whenever a system cannot attend simultaneously all the customers demanding its services, these customers have to wait. It has been recognized that waiting times are a serious problem in health care markets. A comprehensive work on this subject appears in Cullis, Jones and Propper (2000). Moreover, it is important to keep in mind that waiting lines or queues can be viewed as a public policy option, since the waiting times are a form of cost the consumers must pay for the services. This framework sets consumer welfare and equity considerations in a model where a private and a public sector share the market [see Iversen (1986)]. Instead of paying cash, people face the costs of the time they have to wait until treatment begins. The time spent in queues or waiting lists can heavily burden sick people by inflicting more pain and stress on them. Some cases can quickly deteriorate if treatment is postponed too much. In such situations, the consumer surplus vanishes. In addition, sick people and society in general must cope with the opportunity costs of such people not being able to function as workers and citizens (and as taxpayers). These additional costs vary according to patients' income.

A general and very well established approach in queuing theory is the use of inter-arrival and service times, as in Joskow (1980). This is done by means of an *exponential distribution* and by measuring the number of times an event occurs using a *Poisson process* as will be better explained below. Even this simple model is very hard to apply in a complex system like the Brazilian SUS. To tackle this issue, one must collect data on the time between arrival and the beginning and end of treatment for the various specialties and wards. Despite the expected difficulties involved with such analyses, strong recommendations to perform them can be found in the literature, for instance in Iversen (1986 and 1993), Furukubo, Ohuchi and Kurokawa (2000), and Mango and Shapiro (2001). We summarize this approach below.[3]

Suppose the time between consecutive occurrences of some particular kind of event (for instance, arrivals or length of stay of patients in a hospital) has inter-arrival times represented by a random variable T. This random variable is said to have an *exponential distribution* with parameter λ if its probability density function is:

$f_T(t) = \lambda\ [exp\ (-\lambda t)]$ for $t \geq 0$

and:

$f_T(t) = 0$ for $t < 0$.

Now suppose the patients arriving or being treated in a hospital form a calling population assumed to be an infinite occurrences source. Let $X(t) = n$, a random variable, be the number of occurrences (for instance the arrival of patients in a hospital) by time t ($t \geq 0$), where time 0 represents the instant at which the count begins. This random variable has a Poisson distribution with parameter λt and the following probability density function:

$P\{X(t) = n\} = (\lambda t)^n\ [exp\ (-\lambda t)]/n!$ for n = 1, 2, 3……

It is important to note that if the mean distribution is large, the normal distribution with the same mean approaches the Poisson distribution.

The demand X_i the public hospital i faces is represented by a Poisson process that includes uninsured and privately insured patients. The public hospital demand is not split into these two categories as in the precedent FM and MS models since the public hospitals do not discriminate patients according to their health insurance status. Thus, the order in which the patients are treated depends upon their arrival order, the severity of their case and the risk or urgency diagnoses each of them has been given. The present model does not establish any priority discipline even though in the real world such a first-come-first-served model is not easy to find. But we may still assume that the simple model is representative for patients that are at the same risk or urgency levels. Therefore, along the below calculations it is set $X_{iu}+X_{ip}=X_i$. This simplification does not impose any technical problem, since a stochastic variable resulting from a sum of two random variables that follow a Poisson process follows a Poisson process as well.

[3] Some very preliminary results that arise from the application of the underlying general framework for this model to data from SUS are presented in the Appendix to this paper.

As in the public hospital, the private hospital faces a demand X_j that follows another Poisson process. Again, as in the free market and the market share frameworks, any patient that is not treated in the private hospital must be given care in the public sector. The model takes advantage of the fact that there is a simple and direct relationship between the mean and the standard deviation in the Poisson distribution and hospitals are allowed to choose their turnaway targeting. These targets are set when the hospitals choose how many standard deviations from the demand their capacity will be. Since the model merely sets probabilities for hospitals to meet demand, it is impossible to know if a given patient will in fact be treated. The model does not to set an *a priori* cutoff level for the probability that a patient will be turned away (beyond which the public hospital will be penalized), even though in the real world it is actually possible to do so. However, it is almost impossible to determine such a target level *ex ante*, which makes a purely conceptual modeling more adequate at present. However, this task should be executed *a posteriori* by observing the data along a period of time. Hospitals can then be punished if the actual rate of patients being attended to above the expected capacity level exceeds the rate predicted by the reserve margins. In other words, if the actual bed occupancy rates are above some target level, the hospital should be punished. In the real world, the penalties can take the form of extra workloads or additional stress, in much the same way as in the free market and in the market share frameworks.

The public hospital focuses its attention on the investment decisions it has to make so as to limit the number of patients in the queue. To represent such *desideratum,* a *reserve margin* is set equal to the number (γ) of standard deviations that the number of available beds K_i is to be from the mean demand $E[X_i] = \lambda_i$. Thus $K_i - \lambda_i = \gamma\ (\lambda_i)^{0,5}$ where $(\lambda_i)^{0,5}$ is the standard deviation since we are using a Poisson process. The connection of this reserve margin with the reservation quality concept used by (Färe, Grosskopf and Valdmanis (1989)] straightforward because both of them concern the quantity-quality objectives the hospitals should achieve. This distance determines the likelihood of the public hospital being full for any demand figure. Equivalent parameters β_j and $(\beta_j)^{0.5}$ apply to the private hospital that faces a demand X_j, also a Poisson process. Hence $K_j - \beta_j = (\beta_j)^{0.5}$. The comparative static analysis sheds some light on the impact of the turnaway targeting decision on the investments in capacity and on the public hospital's surplus. This framework will from now on be called the queuing theory framework (QT).

6 THE GENERAL RESULTS

6.1 SOME BROAD CONSIDERATIONS

As we will see below the results that arise from the free market (FM) and from the market share (MS) formulation of the model are very similar to those found in the queuing theory framework (QT). The signs of the partial derivatives of the surplus function with respect to the parameter representing the hospital's investment policy are the same in the three structures. We also found in all three frameworks that the derivatives with respect to the reservation margin parameters (γ and β) and the hospitals capacities (K_i, and K_j) have a similar solution up to a multiplicative constant ($(\lambda_i)^{0.5}$ and $(\lambda_j)^{0.5}$), which represents the standard deviation of the public and the private demands, respectively. Since λ_i and λ_j represent the mean of the demand for the two hospitals, it can be seen that the queuing theory model allows for scale economies, a feature absent in the free market and the market share frameworks. In the queuing theory framework, the larger the demand the more sensitive (though at decreasing rates) the public surplus is to the reserve margins. A social planner should be aware that the *size* of the demand matters and that small hospitals surplus should not respond to risk avoiding policies with the same intensity as expected from larger hospitals.

6.2 SOME COMPARATIVE STATIC EXERCISES

According to the balances between the demand and hospital capacities the following main five situations arise:

a) If $X_{iu} + X_{ip} \leq Ki$ and $X_{jp} \leq K_{j,}$ there is no excess demand nor do the health plans refuse to provide treatments. Thus:

$Si = p(X_{iu} + X_{ip}) - rK_i$

Integrating S_i with respect to X_{iu} and X_{ip} over [0,1] yields the expected surplus of the public hospital i as a function of the investments in the public hospital capacity K_i as well as of the private hospital capacity K_j. Hence the following formulations can be presented:

Surplus function in the free market framework (FM):

$$\text{FM: } S_i(K_i, K_j) = p - rK_i \tag{1}$$

If the public hospital i determines a share α of the market, we have:

Surplus function in the market share framework (MS):

$$\text{MS: } S_i(K_i, K_j, \alpha) = ((p/2)\ \alpha)/(1 - \alpha) - rK_i \tag{2}$$

It is worth noting that these two surpluses given by equations (1) and (2) are equal to each other if $\alpha = 2/3$. This outcome for α appears, for instance, when $X_{iu} = X_{ip} = X_{jp}$ or if $X_{iu} + X_{ip} = 2X_{jp}$. This condition holds in all the others situations in this paper. If the public hospital supplies 2/3 of the market its surplus is equal to the "free-market" solution.

In the queuing theory framework it is possible to obtain the expected surpluses of the hospitals according to the five basic relationships between the capacities and the demand that the public and private hospitals can face.

The public hospital's reserve margin is $K_i - \lambda_i = \gamma\ (\lambda_i)^{0.5}$, which implies $K_i = \lambda_i + \gamma(\lambda_i)^{0.5}$ and $rK_i = r\ (\lambda_i + \gamma(\lambda_i)^{0.5})$. As in all QT's remaining cases $X_{iu} + X_{ip} = X_i$.

Since $\mathrm{E}[X_i] = \lambda_i$ we have $E[S_i] = E[pX_i - rK_i] = p\lambda_i - r(\lambda_\iota + \gamma(\lambda_i)^{0.5})$ and it is possible to obtain the expected surplus of the public hospital i as a function of the parameters λ_i and λ_j. Then:

Surplus function in the queuing theory framework (QT):

$$\text{QT: } S_i(\lambda_i, \lambda_j) = (p - r)\ \lambda_i - r\gamma(\lambda_i)^{0.5} \tag{3}$$

Taking the derivatives with respect to the key variables in equations (1)-(3) gives us:

$\partial S_i/\partial K_i = -r$. If both hospitals have excess capacity, the public hospital should not invest any more. Rather, it should reduce its capacity;

$\partial S_i/\partial K_j = 0$. The public hospital's investment decisions do not depend on K_j.

As these two derivatives show, the net marginal gain of the investment in the public hospital i compared to the private hospital j is $-r - 0 = -r$. Since $r > 0$, a policy maker acting in behalf of the public hospital should not allow further public investment.

$\partial S_i/\partial \alpha = (p/2(1-\alpha)^2)$. The public hospital can increase profits by increasing its market share since it maintains excess capacity.

$\partial S_i/\partial t = 0$. The public hospital surplus does not depend on the per unit capital penalty t since no such penalty is applied.

$\partial S_i(\lambda_i, \lambda_j)/\ \partial \gamma = -r(\lambda_i)^{0.5}$. Since there is a global excess surplus, the probability that no patients will be turned away is inversely related to the expected public hospital surplus. In other words, the greater the probability that all patients will be attended, the smaller the expected surplus will be. This happens because this hospital will likely experience some idle capacity.

b) If $X_{iu} + X_{ip} \leq Ki$; $X_{jp} \geq K_j$, and $K_i - (X_{iu} + X_{ip}) > (X_{jp} - K_{jp})$; there is either excess demand or the health plans refuse to provide treatments. The excess supply in the public hospital, however, exceeds the private shortage. Then:

$S_i = p(X_{iu} + X_{ip}) - rK_i + p(X_{jp} - K_{jp})$ and:

FM: $S_i(K_i, K_j) = p(1.5 - K_j) - rK_i$ (4)

MS: $S_i(Ki, Kj, \alpha) = p(1/2(1-\alpha) - K_j)$ (5)

QT: $S_i(\lambda_i, \lambda_j) = (p - r)\ \lambda_i - r\gamma(\lambda_i)^{0.5} - p\beta(\lambda_j)^{0.5}$ (6)

The derivatives are as follows:

$\partial S_i/\partial K_i = -r$. The public hospital still oversupplies and should reduce capacity.

$\partial S_i/\partial K_j = -p$. If the private hospital j expands its capacity, the public hospital faces a loss equal to its marginal revenue p.

As these two derivatives show, the net marginal gain of the investment in the public hospital i is $-r - (-p) = p - r$. Since $p > r$ if some investment is made, authorities should allocate this investment to the public hospital even though it maintains excess supply. An additional unit of investment in the private hospital takes a per unit capital receipt p from the public hospital while additional units of investment cost only r to the public hospital. Clearly, however, no investment should be made when $\partial S_i/\partial K_\mathrm{i} = -r$.

$\partial S_i(K_i, K_j, \alpha)/\partial \alpha = (\mathrm{p}/2(1-\alpha)^2)$. Since the public hospital has excess capacity, it should increase its market share, represented by α.

$\partial S_i(K_i, K_j)/\ \partial t = \partial S_i(K_i, K_j, \alpha)/\partial t = 0$. The public hospital faces no penalty.

$\partial S_i(\lambda_i, \lambda_j)/ \partial\gamma = -\, r(\lambda_i)^{0.5}$. The larger the probability that a patient will be treated at a public hospital, the smaller this hospital's surplus will be since it maintains excess capacity. The idle capacity in the public hospital can accommodate the excess demand in the private hospital and still have extra capacity to spare. The reserve margin should not increase.

$\partial S_i(\lambda_i, \lambda_j)/\partial\beta = -\; p(\lambda_j)^{0.5}$. The larger the probability that a patient will be attended to at the private hospital, the smaller the public hospital surplus will be. This is because the public hospital profits from any excess demand the private hospital is unable to attend to. A public authority should not allow the private reserve margin to increase.

c) If $X_{iu} + X_{ip} \leq Ki$; $X_{jp} \geq K_j$ and $K_i - (X_{iu} + X_{ip}) < (X_{jp} - K_{jp})$, there is excess demand for the private hospital and/or private health plans refuse to offer treatment to some patients. The excess supply in the public hospital, however, cannot accommodate all of the excess demand in the private hospital. In this case $K_i + K_j < (X_{jp} + X_{iu} + X_{ip})$; thus, despite utilizing its full capacity, the public hospital suffers a penalty t for the excess demand it cannot attend to. This penalty must be subtracted from its per unit revenue p. Thus:

$S_i = (p - r)\, K_i + (p - t)\, (X_{iu} + X_{ip} + X_{jp} - K_i - K_j)$ and so:

$$\text{FM: } S_i(K_i, K_j) = (p - r)K_i + (p - t)[1.5 - K_i - K_i - K_j] \quad (7)$$

$$\text{MS: } S_i(K_i, K_j, \alpha) = (p - r)\, K_i + (p - t)[(1/2(1 - \alpha)) - K_i - K_j)] \quad (8)$$

$$\text{QT: } S_i(\lambda_i, \lambda_j) = (p - r)\, (\lambda_i + \gamma(\lambda_i)^{0.5}) + (p - t)\, (-\gamma(\lambda_i)^{0.5} - \beta(\lambda_j)^{0.5}) \quad (9)$$

The derivatives follow:

$\partial S_i/\partial K_i = t - r$. Since $t > r$, each additional unit of capital the public hospital invests avoids a loss $t - r$.

$\partial S_i/\partial K_j = t - p$. Each unit of capacity the private hospital invests withdraws a marginal receipt p from the public hospital. The private investment, however, allows the public hospital to avoid a per unit penalty t that would be imposed on it for the excess demand it would be unable to attend to.

As these two derivatives show, the net marginal gain of the investment in the public hospital i is $t - r - (t - p) = p - r$. Since $p > r$, decision-makers should channel public investment to the public hospital. Moreover, since the public hospital cannot accommodate all of the excess demand from the private hospital, the public hospital must invest to avoid penalties.

$\partial S_i(Ki, Kj, \alpha)/\partial\alpha = (p - t)/2(1 - \alpha)^2$. Since $t > p$, the public hospital can increase its total surplus by reducing its market share.

$\partial S_i(K_i, K_j)/ \partial t = K_i + K_j - 1.5$. The public hospital faces a portion (not all) of the excess demand for the private hospital. The expected value of the total demand amounts $E(X_{iu}) + E(X_{ip}) + E(X_{jp}) = 1.5$. If the total investment is below this expected value for demand, the derivative is negative and the surplus falls when t increases. This is because a shortage probably occurs and the public hospital may face a penalty. If the total investment is equal to the expected value, the derivative is equal to zero and the surplus is unaffected by penalties. If the total investment is above 1.5, no penalties are imposed and the derivative becomes positive, since each unit increase in the penalty increases the losses the public hospital avoids.

$\partial S_i(K_i, K_j, \alpha)/\partial t = K_i + K_j - [1/(2(1 - \alpha))]$. Similarly to the previous situation, the public hospital can avoid losses if the total investment is high.

An interesting cross-derivative is $\partial S_i(K_i, K_j, \alpha)/\partial t\partial\alpha = -1/2(1 - \alpha)^2$. As the penalty increases, the sensitivity of the surplus to penalties falls with the expansion of market share. Alternatively, the sensitivity of the surplus to the market share decreases as the penalty increases. Nonetheless, the sensitivity of the surplus depends on the market share 〈 and it is drastically reduced if the public hospital maintains a very large market share.

$S_i(\lambda_i, \lambda_j)/ \partial\gamma = (t - r)\ (\lambda_i)^{0.5}$. Since $t > r$, the greater the probability that a patient will be attended to in the public hospital, the larger the surplus the public hospital enjoys.

$\partial S_i(\lambda_i, \lambda_j)/ \partial\beta = (t - p)\ (\lambda_j)^{0.5}$. Since $t > p$, the greater the probability that a patient will be attended to in the private hospital, the larger the surplus in the public hospital.

By subtracting the last derivative from the first, we arrive at the following result:

$$[\partial S_i(\lambda_i, \lambda_j)/ \partial\gamma] - [\partial S_i(\lambda_i, \lambda_j)/ \partial\beta] = (p - t)\ (\lambda_j)^{0.5} + (t - r)\ (\lambda_i)^{0.5} \quad (10)$$

One can assume $\lambda_i > \lambda_j$ because the public hospital must attend to the entire uninsured demand. The uninsured in Brazil represent roughly 70% of the total population [IBGE (2002)]. Since $t > p$, the first parcel of this expression is negative. However, since $t > r$, the second parcel is positive. Because $p > r$, if p replaces r, the expression becomes less positive and takes the form $(p - t)\ (\lambda_j)^{0.5} + (t - p)\ (\lambda_i)^{0.5}$. By replacing λ_i for λ_j in the second parcel, the expression becomes even less positive and takes the form $(p - t)\ (\lambda_j)^{0.5} + (t - p)\ (\lambda_j)^{0.5} = 0$, which is a minimum. Then, we can take for granted that the expression (10) is positive. The economic surplus of the public hospital is more sensitive to the parameter γ (representing the reserve margin of the public hospital) than to the corresponding private hospital's parameter β.

d) If $X_{iu} + X_{ip} > K_i$; $X_{jp} < K_j$; and $(X_{iu} + X_{ip} - K_i) + X_{jp} > K_j$, the public hospital faces excess demand and the private hospital faces excess supply. Thus, one of the two following situation arises:

d.1) $X_{iu} \leq K_i$; the public hospital capacity faces an excess supply for uninsured clients from the private sector but there exists a total excess demand. The rational path for the public hospital is to attend to the uninsured demand and part of the insured demand up to its full capacity. The remaining insured patients that cannot be attended to in the public hospital should be taken to the private hospital until this hospital becomes full. This way, the uninsured patients the private hospital cannot attend to are treated in the public hospital, with a per capita penalty t enforced on it. Hence:

$$S_i = (p - r)\ K_i + p(X_{ip} + X_{jp} - (K_i - X_{iu}) - K_j) - t((X_{ip} + X_{jp}) - (K_i - X_{iu}) - K_j)$$

$$\text{FM: } S_i(K_i, K_j) = (p/2) - rK_i + (p - t)\ (1 - K_j) + (K_i - 0.5) \quad (11)$$

$$\text{MS: } S_i(K_i, K_j, \alpha) = (p/2(1 - \alpha)) - rK_i - (p - t)K_j + t(K_i - 0.5) - t \quad (12)$$

$$QT: S_i(\lambda_i, \lambda_j) = (p - r)\ \lambda_i + (t - r)\ \gamma\ (\lambda_i)^{0,5} + (t - p)\ \beta\ (\lambda_j)^{0,5} \quad (13)$$

The derivatives are:

$\partial S_i/\partial K_i = t - r$. Facing excess insured demand, the public hospital can alleviate the penalty t by investing in capacity. To do so, it must pay the usual unit capital cost r.

$\partial S_i/\partial K_j = t - p$. The private hospital investment allows the public hospital to get rid of the per capita penalty t but at a cost of the unit demand price p.

These two derivatives show that the net marginal gain of the investment in the public hospital is $t - r - (t - p) = p - r$. Because $p > r$, the policy-maker should set the public hospital as the preferred investment recipient since the public net gain is positive.

$\partial S_i(K_i, K_j, \alpha)/ \partial\alpha$ = p/2 $(1 - \alpha)^2$. By increasing its market share, the public hospital boosts its profits since it is faced with excess demand.

$\partial S_i(K_i, K_j)/ \partial t = \partial S_i (K_i, K_j, \alpha)/\partial t = K_i + K_j - 1.5$. The public hospital faces excess demand. Thus, if both hospitals invest more than the expected value of the total demand, which reaches $E(X_{iu}) + E(X_{ip}) + E(X_{jp}) = 1.5$, the public hospital avoids the penalty. The more the hospitals invest, the more in penalties the public hospital avoids. On the contrary, if the hospitals present a total capacity below 1.5, a shortage may occur and the surplus falls as the penalty t increases.

$\partial S_i(\lambda_i, \lambda_j)/ \partial\gamma = (t - r)\ (\lambda_i)^{0,5}$. Since $t > r$, the greater the probability that a patient will be attended to in the public hospital, the larger the surplus this hospital will enjoy since there is excess demand for its services.

$\partial S_i(\lambda_i, \lambda_j)/ \partial\beta = (t - p)\ (\lambda_j)^{0,5}$. Since $t > p$, the greater the probability that a patient will be attended to in the private hospital, the larger the public hospital's surplus. The latter's hospital's savings (due to penalties avoided) is represented by $(t - p)$ per unit multiplied by the standard deviation of the private hospital.

Again, since both derivatives are equal to those in the preceding item (*c*), the public hospital's parameter γ is more influential on its own surplus than the private hospital's counterpart parameter β.

d.2) $X_{iu} > K_i$; The public hospital cannot attend to even the full uninsured demand. This situation is similar to the item (*d*.1) but now the hospital is slapped with an additional penalty on its uninsured excess demand $(X_{is} - K_i)$. The rational path the public hospital should follow is to attend to the uninsured demand and send insured demand to the private hospital up to the point where the latter reaches full capacity. The public hospital faces a penalty on the total excess demand, which includes insured and uninsured patients.

$$S_i = (p - r)\ K_i + (p - t)\ [X_{iu} + X_{ip} - K_i - (K_j - X_{jp})]$$

$$\text{FM: } S_i(K_i, K_j) = (p - r)\ K_i + (p - t)((1{,}5 - (K_i + K_j)) \quad (14)$$

$$\text{MS: } S_i(K_i, K_j, \alpha) = (p - r)\ K_i + (p - t)\ ((1/2(1 - \alpha) - K_i + K_j) \quad (15)$$

$$\text{QT: } S_i(\lambda_i, \lambda_j) = (p - r)\ (\lambda_i + \gamma(\lambda_i)^{0.5}) + (t - p)\ [\gamma(\lambda_i)^{0.5} + \beta(\lambda_j)^{0.5}] \quad (16)$$

The derivatives are as follows:

$\partial S_i/\partial K_i = t - r$. Given the existence of excess insured demand, each unit of the public hospital investment reduces the per unit impact of the penalty t on the total surplus. The hospital, of course, must pay the usual per unit capital cost r, as in the item (d.1).

$\partial S_i/\partial K_i = t - p$. Again, as in the item (d.1), the private hospital investment allows the public hospital to avoid the penalty t at a per unit cost to the public hospital of p (due to the loss of demand).

As shown by the two preceding derivatives, the net marginal gain of the investment in the public hospital is $t - r - (t - p) = p - r$. Since $p > r$, the public hospital should be given priority in investment decisions.

$\partial S_i(K_i, K_j, \alpha)/\partial\alpha = (p - t)/2\ (1 - \alpha)^2$. Because there is excess demand for the public hospital, further investment increases its profits. This derivative is less than its corresponding derivative in the item (1), since now the public hospital must pay a per unit penalty t on its uninsured excess demand.

$\partial S_i(K_i, K_j)/\partial t = K_i + K_j - 1.5$. As in the item ($d$.1), the public hospital faces excess demand. So if both the hospitals invest more than the expected value of the total demand (1.5) the public hospital will not incur penalties. The more the hospitals invest, the more the public hospital avoids in fines. If the sum of the hospitals' capacities falls below 1.5, we can expect a shortage that reduces the surplus in accordance with the increases in the penalty t.

$\partial S_i(K_i, K_j, \alpha)/\partial t = K_i + K_j - [1/2(1 - \alpha)]$. The result is similar to that of item (d.1). The more the hospitals invest, the more the public hospital avoids in fines. Thus, the greater the fines increase, the more the public hospital saves.

Again, as in item (c), an interesting cross-derivative is $\partial S_i\ (K_i, K_j, \alpha)/\partial t\partial\alpha = -1/2(1 - \alpha)^2$. As the penalty increases, the sensitivity of the surplus to penalties falls with increases in market share. It is also true that the sensitivity of the surplus to the market share decreases along with the penalty increase. The sensitivity of the surplus depends on the market share α, becoming less significant as α expands.

$\alpha S_i(\lambda_i, \lambda_j)/\partial\gamma = (t - r)\ (\lambda_i)^{0.5}$

$\alpha S_i(\lambda_i, \lambda_j)/\ \partial\beta = (t - p)\ (\lambda_j)^{0.5}$

These results are equal to those already seen in the preceding items (c) and (d.1).

e) If $X_{iu} + X_{ip} > Ki$; $X_{jp} > K_j$; there is excess demand in the whole system. The public hospital must accommodate all the excess demand and a penalty t is imposed on its operations beyond full capacity.

$S_i = (p - r)K_i + (p - t)(X_{is} + X_{ip} + X_{jp} - (Ki + Kj))$

$$\text{FM: } S_i(K_i, K_j) = (p - r)\ K_i + (p - t)\ (1 - K_i) + (p - t)\ (0.5K_j) \tag{17}$$

$$\text{MS: } S_i(K_i, K_j, \alpha) = (p - r)\ (K_i) + (p - t)\ [(\alpha/(2(1 - \alpha)) - K_i)] \tag{18}$$

$$\text{QT: } S_i(\lambda_i, \lambda_j) = (p - r)\ [\lambda_i + \gamma(\lambda_i)^{0.5}] + (t - p)\ [\gamma(\lambda_i)^{0.5} + \beta(\lambda_j)^{0.5}] \tag{19}$$

The following derivatives are of interest:

$\partial S_i/\partial K_i = t - \mathrm{r}$. Since there is a global excess demand, the hospital public investment saves *t* and spends *r* per unit of investment in capacity.

$\partial S_i/\partial Kj = t - p$. The investment of the private hospital spares *t* and costs *p* per unit of public hospital capacity. Since $t > p$, investments in the private hospital benefit the public hospital *i*.

Since $p > r$, the net marginal gain of the investment in the public hospital is $(t - r) - (p - r) = p - r > 0$. The public hospital should be the first investment target because its surplus is more sensitive to its own investments than to an expansion in private hospital capacity. Since this result prevails in all the preceding situations, a broader policy recommendation emerges: whenever investment in capacity can benefit the public hospital, a publicly financed investment policy must first target the public hospital even under the hypothesis that it has excess capacity that can accommodate the private sector's excess demand. Moreover it should happen even if both hospitals have exhausted their capacities.

$\partial S_i(K_i, K_j, \alpha)/\partial\alpha = (p - t)/(2(1 - \alpha)^2$. The public hospital's surplus decreases as its market share increases.

$\partial S_i(K_i, K_j)/\partial t = K_i + K_j - 1.5$. Both public and private hospital face excess demand. As in item (*d*), if the hospitals invest more than is necessary to accommodate the expected total demand (1.5), the public hospital avoids the penalty. The more the hospitals invest, the more the public hospital avoids in penalties. However, if the hospitals invest less than 1.5, demand exceeds capacity and the surplus decreases as the penalty *t* increases.

$\partial S_i(K_i, K_j, \alpha)/\partial t = K_i + K_j - [1/2(1 - \alpha)]$. The result is equal to the result in items (*d*.1) and (*d*.2). Above a certain total capacity level, the public hospital avoids the penalty entirely and its situation improves as the penalty increases.

Again, $\partial S_i(K_i, K_j, \alpha)/\partial t\partial\alpha = -1/2(1 - \alpha)^2$. The sensitivity of the surplus to penalties decreases with the expansion of market share. But this cross derivative depends on the market share α and become less important as the market share increases.

$\partial S_i(\lambda_i, \lambda_j)/\partial\beta = (t - r)\ (\lambda_i)^{0.5}$

$\partial S_i(\lambda_i, \lambda_j)/\partial\beta = (t - p)\ (\lambda_j)^{0.5}$

The derivatives are also equal to those we foundin the items (*c*), (*d*.1) and (*d*.2).

7 FINAL COMMENTS

This paper develops a schematic view of various policy results, demonstrating that no definitive strategy should be adopted when a public decision maker has to deal with the problem of investment allocation to both public and private hospitals. These results are obtained within a framework where a public and a private hospital share a market in which the demand for health care services may exceed the potential supply. In some situations, no investment in capacity should be made and the public market share should not expand. Sometimes capital accumulation appears to be profitable and an increase in the public hospital's market share makes this hospital better off.

Investment in capacity can be viewed in a very broad sense, encompassing "beds capacity" (the classical approach) or including other fixed assets such as equipment, diagnosis devices and plant size. All of these items set limits to a hospital's operational capacity.

The paper presents a flexible model that analyzes the consequences of income inequality within a free market structure, a market share approach and a queuing theory design. As in most Western countries, a significant share of the non-poor population is entitled to private health plans and individuals purchase private health plans because they seek the highest quality coverage. In our model, however, even the non-poor can be passed on to the public sector and placed on waiting lines if the private hospitals are full. This undesirable situation can arise even if the health plans owned by the non-poor do not cover several diseases their clients develop or some procedures they must undergo.

The model also allows income inequality to play an indirect role in determining hospital capacities. Though poor and even near-poor people are not used to having health plans, public hospitals are part of a publicly-financed system offering everyone the right to full public coverage regardless of health plans or income status. The rich-to-poor ratio of the population that seeks public hospitals first is biased *pro-poor*, since affluent people seek out private hospitals first. Thus, since the regulator can establish *a priori* targets for public hospital market share or reserve margins, these targets may be biased towards the poor as well. It is possible to improve the underlying income-related health status of the entire population if the focus on the poor is effective. Hence, it is reasonable to believe that some pro-poor biased queuing should be established in the real world. The introduction of such biased priority-discipline enables the regulator to emphasize the desired equity component of the net benefits that should prevail in the health care market.

A broad recommendation is that the public hospitals should be the first target of any profitable public investment in capacity. This result is valid even when the public hospital maintains excess capacity and can accommodate excess demand in the private sector or when both hospitals are full. It also happens despite the fact that the penalties established in the models have the support of the public hospital (or of its workers or the patients). It is important to keep in mind that such a broad recommendation applies *only* to public resources that can be invested in either the public or private sector. Therefore, if it is possible to sum of the private and the public budgets the model heads in the right direction.

Another general conclusion is that public hospitals and social planners must be aware that average public and private occupancy rates should be the main reference for investment decisions. However, too frequently in Brazil (and elsewhere), the main variables that public sector health authorities use when evaluating possible investments in public hospital capacity pertain to the operational performance of individual hospitals. Such variables, like turnover rates, average length of stay and average occupancy rates, are almost always assessed *within* the public sector or within a single hospital. Ideally, however, such indicators should refer to the entire set of hospitals in a given area rather than to every individual hospital (for more comments on this issue see [Färe, Grosskopf and Valdmanis (1989)].

Another fundamental reference to capacity acquisition should be private investment policy. Private investment and capacity deserve a great deal of investigation by public decision-makers, since they can play a major role in determining the breakeven point of the public hospitals.

As is usually the case in economics, the supply side of the market is not all that matters. The expected size and the variability of demand for health care services must also factor into public investment policy. The model sheds some light on the returns-to-scale implications of the stochastic nature of demand. A simple and direct connection between public hospital profitability and investment policy and the standard deviation (and the size) of demand could be established in the present paper. Hospitals facing small demands for their health care services have operational surpluses that are not so sensitive to reserve margins (a parameter representing the likelihood that a patient will be turned away) as those hospitals facing larger demands.

A future research agenda should include a specification of some social welfare function representing a "social utility" to be maximized by a "social planner" so as to have a framework where the model cares about social health care benefits minus costs. Another relevant issue is to consider the possible implications of agency relationships between patients and hospitals that should make the demand for health care services an endogenous variable.

APPENDIX

APPLYING SOME QUEUING THEORY TOOLS TO DATA FROM SUS

The queuing theory model we used in this paper is known in Operations Research literature as Markovian/Markovian/**s**ingle model (M/M/s model). This name comes from the fact it assumes a Markovian distribution of interarrival times distributed according to an *exponential* distribution independent and identically distributed (*i.i.d.*) (and so the input process is Poisson) and a Markovian distribution of service times according to another exponential distribution (also *i.i.d.*). The number of servers is s, any positive integer. In the following calculations we assume a single server because just per bed indicators are computed. We take each new one admission as a different patient. By applying this model to SUS data for 2002 we can generate some interesting results. Despite these results are exploratory *vis-à-vis* a more accurate perception of SUS we believe they display some very preliminary but attractive insights.

The results are as follows:

The number of patients (A) = 12,500,000;

The number of beds (B) = 487,058 beds;

The patients per bed ratio (R) = (A/B)/365 = 0.07 per day;

The average length of stay (in days) (T) = 5.60 days per patient;

The mean service rate for overall SUS is S = (1/T) = 0.18 patients per bed per day;

The expected fraction of time a bed in SUS is busy (the *utilization factor*) (U) = $R/S = R \times T = 0.39 = 39\%$;

The probability that a random arriving patient will *not* find an empty bed = U = 39%;

The probability that a random arriving patient will find an empty bed = $1 - U$ = 61%;

The expected number of patients in SUS (equal to the number of patients in queue + number of patients being attended) (Ns) = $(R)/(S - R)$ = 0.64 patients per bed per day;

The expected waiting time in SUS (equal to the expected time in queue + average length of stay) (W) = Ns/R = 9.10 days;

Probability that a random arriving patient will wait more than 9 days to complete a treatment in SUS = 38%;

The expected waiting time in queue (excludes the average length of stay): (Wq) = $W - T = U \times W$ = 3.5 days. The time a patient spends in queue is *less* relevant than the time he or she spends in treatment (5.6 days);

Probability that a random arriving patient will wait more than 3 days in queue = 28%;

The expected number of patients in queue (excludes patients being attended) (Nq) = 0.26 patients per bed per day;

The expected number of patients being attended (Na) = $Ns - Nq$ = 0.38 patients per bed per day. The number of patients being attended is *more* relevant than the number of patients in queue;

Probability that a random arriving patient will wait more than a day in queue (treatment not included) = 35%;

Probability that a random arriving patient will wait more than a day to complete a treatment in SUS (includes the treatment) = 100%.

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