175

DISCUSSION PAPER

Originally published by Ipea in December 2006 as
number 1252a of the series Texto para Discussão.

A SMALL OPEN ECONOMY AS A LIMIT CASE OF A TWO-COUNTRY NEW KEYNESIAN DSGE MODEL: A BAYESIAN ESTIMATION WITH BRAZILIAN DATA

Marcos Antonio C. da Silveira

ipea

175

DISCUSSION PAPER

Originally published by Ipea in December 2006 as
number 1252a of the series Texto para Discussão.

Brasília, January 2015

A SMALL OPEN ECONOMY AS A LIMIT CASE OF A TWO-COUNTRY NEW KEYNESIAN DSGE MODEL: A BAYESIAN ESTIMATION WITH BRAZILIAN DATA

Marcos Antonio C. da Silveira[1]

---

[1] Da Diretoria de Estudos Macroeconômicos do Ipea.

Federal Government of Brazil

Secretariat of Strategic Affairs of the Presidency of the Republic
Minister Roberto Mangabeira Unger

ipea **Institute for Applied Economic Research**

A public foundation affiliated to the Secretariat of Strategic Affairs of the Presidency of the Republic, Ipea provides technical and institutional support to government actions – enabling the formulation of numerous public policies and programs for Brazilian development – and makes research and studies conducted by its staff available to society.

President
Sergei Suarez Dillon Soares

Director of Institutional Development
Luiz Cezar Loureiro de Azeredo

Director of Studies and Policies of the State, Institutions and Democracy
Daniel Ricardo de Castro Cerqueira

Director of Macroeconomic Studies and Policies
Cláudio Hamilton Matos dos Santos

Director of Regional, Urban and Environmental Studies and Policies
Rogério Boueri Miranda

Director of Sectoral Studies and Policies, Innovation, Regulation and Infrastructure
Fernanda De Negri

Director of Social Studies and Policies, Deputy
Carlos Henrique Leite Corseuil

Director of International Studies, Political and Economic Relations
Renato Coelho Baumann das Neves

Chief of Staff
Ruy Silva Pessoa

Chief Press and Communications Officer
João Cláudio Garcia Rodrigues Lima

URL: http://www.ipea.gov.br
Ombudsman: http://www.ipea.gov.br/ouvidoria

DISCUSSION PAPER

A publication to disseminate the findings of research directly or indirectly conducted by the Institute for Applied Economic Research (Ipea). Due to their relevance, they provide information to specialists and encourage contributions.

© Institute for Applied Economic Research – **ipea** 2015

Discussion paper / Institute for Applied Economic

Research.- Brasília : Rio de Janeiro : Ipea, 1990-

ISSN 1415-4765

1. Brazil. 2. Economic Aspects. 3. Social Aspects. I. Institute for Applied Economic Research.

CDD 330.908

The authors are exclusively and entirely responsible for the opinions expressed in this volume. These do not necessarily reflect the views of the Institute for Applied Economic Research or of the Secretariat of Strategic Affairs of the Presidency of the Republic.

Reproduction of this text and the data it contains is allowed as long as the source is cited. Reproductions for commercial purposes are prohibited.

JEL E32, E52, F41.

SINOPSE

O artigo desenvolve uma versão para dois países do modelo de equilíbrio geral dinâmico e estocástico em Gali & Monacelli (2005), o qual estende para uma pequena economia aberta o modelo novo keynesiano usado como ferramenta para análise de política monetária em economias fechadas. Uma importante característica do modelo é que os termos de troca entram diretamente na curva de Phillips novo Keynesiana como uma segunda variável pressionando os custos e a inflação, de forma que não mais existe a relação direta entre custo marginal e hiato do produto encontrada nas economias fechadas. Diferente da maior parte da literatura, a pequena economia aberta é derivada como um caso-limite do modelo para dois países, em vez de supor que as variáveis externas seguem processos exógenos. Este procedimento preserva o papel desempenhado pelas fricções nominais do resto do mundo na transmissão dos choques externos sobre a economia pesquena. Usando uma abordagem bayesiana, o caso-limite do modelo para uma pequena economia é estimado com dados brasileiros e funções impulso-resposta são construídas para análise dos efeitos dinâmicos dos choques estruturais.

ABSTRACT

We build a two-country version of the DSGE model in Gali & Monacelli (2005), which extends for a small open economy the new Keynesain model used as tool for monetary policy analysis in closed economies. A distinctive feature of the model is that the terms of trade enters directly into the new Keynesian Phillips curve as a new pushing-cost variable feeding the inflation, so that there is no more the direct relationship between marginal cost and output gap that characterizes the closed economies. Unlike most part of the literature, we derive the small domestic open economy as a limit case of the two-coutry model, rather than assuming exogenous processes for the foreign variables. This procedure preserves the role played by foreign nominal frictions in the way as international monetary policy shocks are conveyed into the small domestic economy. Using the Bayesian approach, the small-economy case is estimated with Brazilian data and impulse-response functions are build to analyse the dynamic effects of structural shocks.

SUMMARY

1 INTRODUCTION 7

2 MODEL 9

3 EMPIRICAL ANALYSIS 25

4 CONCLUSION 31

5 REFERENCES 32

6 APPENDIX: STEADY STATE 33

A Small Open Economy as a Limit Case of a Two-Country New Keynesian DSGE Model: a Bayesian Estimation with Brazilian Data[*]

Marcos Antonio C. da Silveira
IPEA-Rio[†]
E-mail: silveira@ipea.gov.br

December 1, 2006

Abstract

We build a two-country version of the DSGE model in Gali & Monacelli (2005), which extends for a small open economy the new Keynesain model used as tool for monetary policy analysis in closed economies. A distinctive feature of the model is that the terms of trade enters directly into the new Keynesian Phillips curve as a new pushing-cost variable feeding the inflation, so that there is no more the direct relationship between marginal cost and output gap that characterizes the closed economies. Unlike most part of the literature, we derive the small domestic open economy as a limit case of the two-coutry model, rather than assuming exogenous processes for the foreign variables. This procedure preserves the role played by foreign nominal frictions in the way as international monetary policy shocks are conveyed into the small domestic economy. Using the Bayesian approach, the small-economy case is estimated with Brazilian data and impulse-response functions are build to analyse the dynamic effects of structural shocks.

Key Words: new Keynesian model, home bias, Bayesian econometrics
JEL Classification: E32, E52, F41

1 Introduction

Under the New Open Macroeconomic literature, dynamic stochastic general equilibrium (DSGE) models with imperfect competition and nominal stickiness have been developed for monetary policy analysis in open economies. Built from *first principles,* these models give rise to a macroeconomic dynamics lead by fundamental shocks, at the same time that they preserve the analytical tractability of the traditional Mundell-Fleming approach. However, a serious failure of the first models is that monetary policy analysis was limited to examine the effects of exogenous monetary shocks on the aggregate macroeconomic variables, so that the interest rate was endogenously determined. More recently, a new generation of models sought to deal more realistically with the way how monetary policy is conducted. These models assume a reaction function for the monetary authority in which the interest rate is the intrumental variable. In this sense, this part of the literature can be regarded as an extension for open economies of the new Keynesian models used to monetary policy analisys in closed economies. This paper follows this second line of research and builds a two-country version of the model by Gali & Monacelli (2005), which extends for a small open economy the new

---

[*] I am grateful to Ajax Moreira (IPEA), Marco Bonomo (FGV RJ), Maria Cristina Terra (FGV RJ) and René Garcia (Universidade de Montreal) for useful comments.

[†] Address: Avenida Pres. Antonio Carlos 51, 17 andar, sala 1714, Rio de Janeiro, Brazil, 20020-010

Keynesian DSGE with Calvo-type staggered price-setting developed initially for closed economies. A distinctive feature of the model is that the terms of trade enters directly into the new Keynesian Phillips curve as a second pushing-cost variable in addition to the output gap, creating in this way a new source of inflationary pressure. Furthermore, real exchange rate fluctuation is embedded into the model by assuming a home bias in households' preferences.

A commom practice in the new Keynesian literature is to build small-open economy models directly by assuming that the dynamics of the rest of the world's economy follows either a non-structural VAR model or the reduced form of a closed-economy structural model. This paper takes an alternative route and derives the small open economy indirectly as a limit case of a two-country model. This is achieved by approaching the relative size of the Home country to zero, at the same time that the Foreign country converges to a closed economy. A first advantage of our procedure is that we derive rigorously the small open economy as part of a integrated world economy, preserving in this way all international trade and financial linkages. Therefore, we do not take the risk of setting aside relevant international channels of monetary transmission.[1] Gali & Monacelli (2005) also derive rigorously the small open economy's structure. However, unlike our model, which use a two-*big* country framework, they assume a world composed by a continuum of symmetric small open economies and obtain the structure of the individual economy by integrating the other ones. Despite the fact that both modelling approaches produce quite similar general equilibrium dynamics for the small open economy, we find our model more flexible as it can be easily generalized for any finite number of small and big economies with arbitrary relative sizes and degrees of openness. In this sense, our approach would be perfectly convenient to analyse monetary policy coordination in a world with more than two big players, such as USA, Japan, European Union and China, as well as the impact of the international monetary policies on the smaller open economies. A second advantage of our procedure is that, as the rest of the world's economy is also microfundamented, it allows us to identify all externaI shocks and understand the transmission channels through which they are conveyed into the small economy. For instance, as the degree of U.S. price stickiness affects the size of the effect of a U.S. monetary policy shock on the U.S. output, it is also crucial to determine the size of the effect of this shock on the Brazilian output, since U.S. income impacts directly on U.S. imports. On the contrary, a non-structural VAR does not provide any information on which external *structural* parameters affect the size of this shock on the small economy. No less important, a third advantage of our procedure is that we can build a canonical representation of the two-country model, which is naturally extended for the small-economy case.[2]

The paper finishes with the Bayesian estimation of the small-economy limit case of the two-country model, in which Brazilian data is used for the small economy and U.S. data for the rest of the world. The parameters of both economies are estimated simultaneously. The Bayesian approach has become widespread in the empirical literature as it allows us to use previous beliefs on the parameters in making inference about them. For the Brazilian case, this Bayesian property is expected to be particularly helpful, since we could compensate the shortness of Brazilian historical series with the information provided by the estimation of analogous models in previous studies with data from other countries, such as Justiniano & Preston (2004) for Australia, Canada and

---

[1] An earlier version of Gali & Monacelli (2005) brings an illustrative example of the type of mistake caused by the attempt to derive directly the small open economy. In this paper, in which they use a two-country framework, the Home new Keynesian Phillips curve is wrongly derivated without the Foreign output gap (the difference between the Foreign outputs under sticky and flexible prices) as a cost-pushing variable feeding Home domestic inflation. This amount to assume flexible prices in the Foreign country, ruling out in this way the current account channel through which a shock on Foreign monetary policy affects the Home output.

[2] This is a representation of the general equilibrium dynamics in terms of the output gap, inflation, interest rate and exchange rate, which is used for evaluation of monetary policy rules.

New Zealand, Smets & Wouters (2004) for U.S and European Union, Caputo et al (2005) for Chile and Liu (2005) for New Zealand. After estimation, we build impulse-response functions in order to examine the effects of domestic and foreign structural shocks on the Brazilian economy's dynamics. The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 carries out the empirical analysis. Section 4 concludes.

2 Model

The world is inhabited by a continuum of infinite-lived households, indexed by $j \in [0,1]$. Each household lives in one of two countries: households on the interval $[0,n)$ live in the Home country, while households on the interval $[n,1]$ live in the Foreign country. The parameter n measures the relative size of the Home country. The small Home country case can be derived by taking the limit of the two-country model as $n \longrightarrow 0$.

Each household owns a competitive-monopolistic firm producing a differentiated good. Thus, there is also a continuum of firms indexed by $i \in [0,1]$, such that firms on the interval $[0,n)$ are located in the Home country, while firms on the interval $[n,1]$ are located in the Foreign country. Firms use only labor for production and there is no investment. Households' labor supply reacts elastically to real wage. Labor market is competitive and internationally segmented.

All Home and Foreign goods are tradable and the law of one price (LOP) holds for all of them. Therefore, the model sets aside the effects of nontradability and international market segmentation on the real exchange rate fluctuation. In this sense, the only reason for PPP (purchase power parity) violation is the introduction of a *home bias* in households' preferences. In addition, prices are set in the producer's currency.

There is a set of states of nature Ω_t for each period t, where $\omega_t \in \Omega_t$. We denote by H_t the set of all histories $h_t \equiv (\omega_0, \omega_1, ..., \omega_t)$ between the initial period 0 and period t. In particular, $H_0 = \Omega_0 = \{\omega_0\}$. Let $\Pr_t(h_{t+s})$ be the probability of history h_{t+s} conditioned on period t being reached through some history h_t. We denote by $H_{t,t+s}$ the set of all histories $h_{t+s} \in H_{t+s}$ such that $\Pr_t(h_{t+s}) > 0$. Following Chari et al (2002), the international financial market structure is complete: every period t, there is market for any Arrow-Debrew security paying a unit of home currency at period t+s in the contingency of some history $h_{t+s} \in H_{t,t+s}$. We denote by $X(h_{t+s})$ the realization of the random variable X_{t+s} at period t+s if history h_{t+s} takes place.

We derive the general equilibrium dynamics for the log-linearized model around the steady state, in which all driving forces of the model remain constant in their long-run equilibrium levels. For convenience and without loss of generality, the log-linearization is carried out on the particular case of a two-country world with symmetric preferences and initial wealth conditions. This procedure is common in the literature as it makes the log-linearization of the model much easier.

As one of our purposes is to derive rigorously the small economy's structure as a limit-case of the two-country model, we maintain the model very simple. The enrichment of the model with other usual assumption, such as habit formation, price indexation, incomplete pass-through and so on, would make the theoretical part of our work much harder and thus we leave it for future research. Throughout the paper, we derive the equilibrium conditions just for the Home country's economy, while the Foreign country's counterparts are shown only if necessary. Starred variables refer to the Foreign country and lowercase variables are in log. Brackets [ ] are used for the Foreign counterpart of an expression.

2.1 Households

This section describes the optimazing behavior of households and aggregate the individual decisions.

2.1.1 Preferences

All households living in a same country have identical preferences and initial wealth. With complete financial markets, this assumption allows us to focus on the problem of a representative household for each country, no matter how the ownership of the firms is shared among households. In this sense, a typical Home household j maximizes the lifetime utility function[3]

$$\sum\nolimits_{t=0}^{\infty} \beta^t E_0 \left[\frac{1}{1-\sigma} \left(C_t^j \right)^{1-\sigma} - \frac{1}{1+\varphi} \left(L_t^{sj} \right)^{1+\varphi} \right], \tag{1}$$

where β is the intertemporal discount factor, φ is the inverse of the wage-elasticity of the labor supply L_t^{sj} and σ is the inverse of the elasticity of intertemporal substitution in consumption C_t^j. As usual, we assume that $0 < \beta < 1$, $\varphi > 1$ and $\sigma > 0$.

The variable C_t^j is defined as the CES composite consumption index

$$C_t^j = \left[(1-\alpha)^{\frac{1}{\mu}} C_{H,t}^{j\,\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} C_{F,t}^{j\,\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \tag{2}$$

where μ is the elasticity of intratemporal substitution between a *bundle* of Home goods $C_{H,t}^j$ and a *bundle* of Foreign goods $C_{F,t}^j$, while α determines the share of the imported (Foreign) goods on the Home household j's consumption expenditure and, as we will see below, is inversely related to the degree of home bias. We assume that $0 < \alpha < 1$ and $\mu > 0$.[4]

The variables $C_{H,t}^j$ and $C_{F,t}^j$ are defined respectively by the CES composite consumption indexes

$$C_{H,t}^j \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n C_{H,t}^j (i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

$$C_{F,t}^j \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 C_{F,t}^j (i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{4}$$

where $C_{H,t}^j(i)$ and $C_{F,t}^j(i)$ are respectively the Home j_{th} household's consumption levels of Home i_{th} good, with $i \in [0, n)$, and Foreign i_{th} good, with $i \in [n, 1]$. The parameter ε is the elasticity of intratemporal substitution among goods produced in a same country. We assume that $\varepsilon > 0$.

The Foreign households' preferences are the same, except for eq.(2), which assumes the form

$$C_t^{j*} = \left[\alpha^{*\frac{1}{\mu}} C_{H,t}^{j*\frac{\mu-1}{\mu}} + (1-\alpha^*)^{\frac{1}{\mu}} C_{F,t}^{j*\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}},$$

where α^* is the share of the imported (Home) goods on the Foreign household j's consumption expenditure, with $\alpha^* \neq \alpha$ in general.

2.1.2 Intratemporal Consumption Choice

For clearness of exposition, it is more convenient to explain how the typical household allocates wealth among goods intratemporally before describing her intertemporal budget constraint and derive her intertemporal consumption choice. The Home j_{th} household takes as given the Home-currency market price of all Home and Foreign goods, denoted respectively by $P_{H,t}(i)$, with $i \in [0,n)$, and $P_{F,t}(i)$, with $i \in [n,1]$. Thus, for any fixed levels of $C^j_{H,t}$ and $C^j_{F,t}$, the optimal $C^j_{H,t}(i)$ and $C^j_{F,t}(i)$ are given respectively by

---

[3] New Keynesian models assume that nominal short-term interest rate is the monetary policy instrument, so that money supply is endogenous. Thus, according to the literature, we do not introduce money demand into preferences explicitly.

[4] With μ very close to 1, the parameter α is exactly equal to the share of the imported (Foreign) goods in the Home household j's consumption expenditure.

$$\begin{aligned} C^j_{H,t}(i) &= \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C^j_{H,t}, && (5)\\ C^j_{F,t}(i) &= \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C^j_{F,t}, && (6)\end{aligned}$$

where $P_{H,t}$ is the Home-currency price indexes of the goods produced in Home country (a domestic price index) and $P_{F,t}$ is the Home-currency price index of the goods imported from the Foreign country, given respectively by[5]

$$\begin{aligned} P_{H,t} &\equiv \left(\frac{1}{n}\int_0^n P_{H,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}, && (7)\\ P_{F,t} &\equiv \left(\frac{1}{1-n}\int_n^1 P_{F,t}(i)^{1-\varepsilon}\,di\right)^{\frac{1}{1-\varepsilon}}. && (8)\end{aligned}$$

How are $C^j_{H,t}$ and $C^j_{F,t}$ chosen? Given $P_{H,t}$ and $P_{F,t}$ derived in the problem above and given also the Home household j's choice of C^j_t derived in the intertemporal problem below, the optimal consumption allocation between Home and Foreign goods is given by

$$\begin{aligned} C^j_{H,t} &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C^j_t, && (9)\\ C^j_{F,t} &= \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C^j_t, \end{aligned}$$

where P_t is the Home consumer price index (CPI), given by[6]

$$P_t = \left[(1-\alpha)P_{H,t}^{1-\mu} + \alpha P_{F,t}^{1-\mu}\right]^{\frac{1}{1-\mu}}. \qquad (10)$$

Aggregating intratemporal choice Substituting eqs.(5) and (6) into the definitions below for the Home aggregate demand for the Home i_{th} good, with $i \in [0,n)$, and Foreign i_{th} good, with $i \in [n,1]$, denoted respectively by $C_{H,t}(i)$ and $C_{F,t}(i)$, are have that

$$\begin{aligned} C_{H,t}(i) &\equiv \int_0^n C^j_{H,t}(i)\,dj = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}, && (11)\\ C_{F,t}(i) &\equiv \int_0^n C^j_{F,t}(i)\,dj = \frac{1}{1-n}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}, && (12)\end{aligned}$$

---

[5] The optimal choice (5) is the solution of the optimization problem $\min_{C^j_{H,t}(i)} \int_0^n P_{H,t}(i)\,C^j_{H,t}(i)\,di$ subject to the constraint given by eq.(3). In addition, by substituting (5) into the function minimized above, we get $\int_0^n P_{H,t}(i)\,C^j_{H,t}(i) = P_{H,t}C^j_{H,t}$. An analogous result holds the eq.(6).

[6] The optimal choices in (9) and (10) are the solution of the optimization problem $\min_{C^j_{H,t},C^j_{F,t}} P_{H,t}C^j_{H,t} + P_{F,t}C^j_{F,t}$ subject to the constraint given by eq.(2). In addition, by substituting (9) and (10) into the function minimized above, we get $P_{H,t}C^j_{H,t} + P_{F,t}C^j_{F,t} = P_tC^j_t$.

whereas, by substituting eqs.(9) and (10) into the definitions below for $C_{H,t}$ and $C_{F,t}$, we have that

$$C_{H,t} \equiv \int_0^n C_{H,t}^j dj = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\mu} C_t, \tag{13}$$

$$C_{F,t} \equiv \int_0^n C_{F,t}^j dj = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} C_t, \tag{14}$$

where, since there is a representative agent for each country,

$$C_t \equiv \int_0^n C_t^j dj = nC_t^j. \tag{15}$$

In addition, we can prove that $P_tC_t = P_{H,t}C_{H,t}+P_{F,t}C_{F,t}$, where $P_{H,t}C_{H,t} = \int_0^n P_{H,t}(i)\, C_{H,t}(i)\, di$ and $P_{F,t}C_{F,t} = \int_n^1 P_{F,t}(i)\, C_{F,t}(i)\, di$, so that P_tC_t is the Home country's aggregate consumption expenditure, while $P_{H,t}C_{H,t}$ and $P_{F,t}C_{F,t}$ are respectively the Home country's aggregate consumption expenditure with Home goods and Foreign goods.[7] Note that the share of the imported (Foreign) goods in P_tC_t rises with the parameter α and fall with its relative price.

Foreign country Analogous results hold for the Foreign country, so that the Foreign aggregate consumption levels of the Home and Foreign good i are given by

$$C_{H,t}^*(i) \equiv \int_n^1 C_{H,t}^{j*}(i)\, dj = \frac{1}{n}\left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^*, \tag{16}$$

$$C_{F,t}^*(i) \equiv \int_n^1 C_{F,t}^{j*}(i)\, dj = \frac{1}{1-n}\left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^*, \tag{17}$$

where $P_{H,t}^*$ and $P_{F,t}^*$ are the Foreign-currency price indexes of Home and Foreign goods, given by

$$P_{H,t}^* \equiv \left(\frac{1}{n}\int_0^n P_{H,t}^*(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}; \tag{18}$$

$$P_{F,t}^* \equiv \left(\frac{1}{1-n}\int_n^1 P_{F,t}^*(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}, \tag{19}$$

while $C_{H,t}^*$ are $C_{F,t}^*$ are the Foreign composite indexes of Home and Foreign goods, given by

$$C_{H,t}^* \equiv \int_n^1 C_{H,t}^{j*} dj = \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\mu} C_t^*, \tag{20}$$

$$C_{F,t}^* \equiv \int_n^1 C_{F,t}^{j*} dj = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\mu} C_t^*, \tag{21}$$

where, since there is a representative agent for each country,

$$C_t^* \equiv \int_n^1 C_t^{j*} dj = (1-n)C_t^{j*} \tag{22}$$

and the Foreign CPI index is given by

$$P_t^* = \left[\alpha^* P_{H,t}^{*1-\mu} + (1-\alpha^*)\, P_{F,t}^{*1-\mu}\right]^{\frac{1}{1-\mu}}. \tag{23}$$

---

[7] To derive the expression for $P_{H,t}C_{H,t}$, we multiply both sides of eq.(11) by $P_{H,t}(i)$, sum across all Home households and use eq.(7). An analogous derivation applies to get the expressions for $P_{F,t}C_{F,t}$ and P_tC_t.

Home bias It is important to understand how the constantes α and α^* are related to the degree of home bias in Home abd Foreign households' preferences respectively, since home bias is the *only* source of real exchange rate fluctuation in the model. Let´s consider the case for the Home country. For that, suppose without loss of generality that $P_{H,t} = P_{F,t}$ In this case, it follows from (10) and (14) that α is exactly equal to the share of imported goods in Home consumption expenditure. Having in mind this result, it is intuitive that α should fall with the relative size of the Home country, given by the parameter n defined above, and with the degree of home bias in Home households' preferences. A tractable way to formalize these ideas is to decompose α as

$$\alpha \equiv \bar{\alpha}(1-n), \tag{24}$$

where the parameter $\bar{\alpha}$ is given exogenously in the model and its inverse, $\frac{1}{\bar{\alpha}}$, is an index for the degree of home bias in Home households' preferences. Applying the same procedure to the Foreign country, we set $\alpha^* \equiv \bar{\alpha}^* n$.[8] For instance, if the reason for home bias is international trade barries, $\bar{\alpha}$ can be interpreted as an index of openness for the Home country. The particular case with fully opened countries occurs when $\bar{\alpha} = \bar{\alpha}^* = 1$, so that $\alpha = n$ and $\alpha^* = 1 - n$. There is no home bias in this case, since the weight of imported goods in the aggregate consumption of each country is naturally given by the relative size of the Home and Foreign countries. On the other hand, the particular case with fully closed countries has $\bar{\alpha} = \bar{\alpha}^* = 0$, so that $\alpha = \alpha^* = 0$ even if both countries are large ($n = 0.5$ for instance).

Gali & Monacelli (2005) can not use eq.(24) to model α. As they assume a world composed by a continuum of symmetric small open economies and obtain the structure of the individual economy by integrating the other ones, the weight of imports that appears in their eq.(2) counterpart for Home household preferences corresponds to our parameter $\bar{\alpha}$. Therefore, the relative size of the countries n and the degree of openess $\bar{\alpha}$ are not introduced into the model explicitly. This explains why, unlike our model, their model is not flexible to deal with any finite number of small and big economies with arbitrary relative sizes and degrees of openness.

World aggregate demand The LOP holds for all goods, so that $P_{H,t}(i) = \varepsilon_t \ P^*_{H,t}(i)$ for $i \in [0,n)$ and $P_{F,t}(i) = \varepsilon_t P^*_{F,t}(i)$ for $i \in [n,1]$, where ε_t is the nominal exchange rate (Home-currency price of one unit of the Foreign currency). Substituting these results into eqs.(7) and (8), we get $P_{H,t} = \varepsilon_t \ P^*_{H,t}$ and $P_{F,t} = \varepsilon_t \ P^*_{F,t}$. Therefore, summing eqs.(11) ((12)) and (16) (17) and using again the result (7) ((8)) and the LOP, the Home (Foreign) good i's world aggregate demand, denoted by $Y^d_t(i) \ \left(Y^{d*}_t(i)\right)$, is given by

$$Y^d_t(i) \equiv C_{H,t}(i) + C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left(C_{H,t} + C^*_{H,t}\right), \tag{25}$$

$$Y^{d*}_t(i) \equiv C_{F,t}(i) + C^*_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} \left(C_{F,t} + C^*_{F,t}\right). \tag{26}$$

2.1.3 Intertemporal Consumption Choice

Given the CPI index P_t derived above, the period budget constraint of the Home household j is written as

$$\begin{aligned} & C^j_t + x^j_t \frac{V_t}{P_t} + \sum\nolimits_{s=1}^{\infty} \sum\nolimits_{h_{t+s} \in H_{t,t+s}} \frac{Z_t(h_{t+s})}{P_t} B^j_t(h_{t+s}) \\ = \ & \frac{W}{P_t} L^{sj}_t + x^j_{t-1} \frac{DV_t}{P_t} + x^j_{t-1} \frac{V_t}{P_t} + \sum\nolimits_{s=0}^{\infty} \sum\nolimits_{h_{t+s} \in H_{t,t+s}} \frac{Z_t(h_{t+s})}{P_t} B^j_{t-1}(h_{t+s}), \end{aligned} \tag{27}$$

---

[8] In log-linearizing the particular model specification with symmetric preferences, we must set $\bar{\alpha} = \bar{\alpha}^*$.

where W_t is the Home nominal wage, V_t is the total nominal value of all Home firms, DV_t is the total nominal dividends paid out by all Home firms, x_t^j is the Home household j's share on the Home firms's ownership, $B_t^j(h_{t+s})$ is the Home household j's holdings of the history h_{t+s}-contingent claim security and $Z_t(h_{t+s})$ is the nominal price of this asset. All nominal values are measured in Home currency. The Foreign households' corresponding equation is analogous to eq.(27), except for the fact that the contingent claims are denominated in the other country's currency.

Home country's Euler equation and SDF Using eq.(27) to substitute for C_t^j into function (1) and deriving with respect to $B_t^j(h_{t+s})$, we have that the optimal consumption allocation between periods t and t+s, reached through the history $h_{t+s} \in H_{t,t+s}$, meets the marginal condition

$$\frac{Z_t(h_{t+s})}{C_t^{j\sigma} P_t} = \beta^s E_t\left[\frac{Z_{t+s}(h_{t+s})}{C_{t+s}^{j\sigma} P_{t+s}}\right] = \frac{\beta^s \Pr_t(h_{t+s})}{C^j(h_{t+s})^\sigma P(h_{t+s})}. \tag{28}$$

Now, we derive the s-period stochastic discount factor (SDF) for the Home country, denoted by $D_{t,t+s}$, since this variable allows us to get the Home-currency price X_t of any financial asset with Home-currency pay-off $X(h_{t+s})$ in the contingency of history $h_{t+s} \in H_{t,t+s}$ through the condition $X_t = E_t[D_{t,t+s}X_{t+s}]$. We know from the literature in Finance that complete financial markets imply that it exists a unique and strictly positive SDF given by $D_t(h_{t+s}) = \frac{Z_t(h_{t+s})}{\Pr_t(h_{t+s})}$, where $D_t(h_{t+s})$ is the realization of the random variable $D_{t,t+s}$ if history $h_{t+s} \in H_{t,t+s}$ takes place. As a result, it follows from this definition and from the condition (28) that

$$D_t(h_{t+s}) = \beta^s \left(\frac{C^j(h_{t+s})}{C_t^j}\right)^{-\sigma} \frac{P_t}{P(h_{t+s})}. \tag{29}$$

In the particular case of a one-period zero cupon bond (denominated in Home currency), where s=1 and $X(h_{t+1}) \equiv 1$, we have that its price is given by $\frac{1}{R_t} = \beta E_t\left[\left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right]$, where R_t is the one-period nominal spot interest rate. Log-linearizing this equation around the steady state for the symmetric case and using the result (15), we get that the log Home aggregate consumption meets the Euler equation

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(r_t - E_t[\pi_{t+1}] + \ln\beta), \tag{30}$$

where $\pi_t \equiv p_t - p_{t-1}$ is the Home consumer price index (CPI) inflation.

Foreign country's Euler equation Proceeding analogously with the Foreign country, we have that the marginal condition

$$\frac{Z_t(h_{t+s})}{C_t^{j*\sigma} P_t^* \varepsilon_t} = \beta^s E_t\left[\frac{Z_{t+s}(h_{t+s})}{C_{t+s}^{j*\sigma} P_{t+s}^* \varepsilon_{t+s}}\right] = \frac{\beta^s \Pr_t(h_{t+s})}{C^{j*}(h_{t+s})^\sigma P^*(h_{t+s})\,\varepsilon(h_{t+s})} \tag{31}$$

holds in equilibrium for all histories $h_{t+s} \in H_{t,t+s}$. Using this equation, we can prove that the log Foreign aggregate consumption meets the Euler equation

$$c_t^{j*} = E_t\left[c_{t+1}^{j*}\right] - \frac{1}{\sigma}\left(r_t^* - E_t\left[\pi_{t+1}^*\right] + \ln\beta\right), \tag{32}$$

where r_t^* is the log of the one-period spot rate (denominated in Foreign currency) and $\pi_t^* \equiv p_t^* - p_{t-1}^*$ is the Foreign consumer price index (CPI) inflation.

2.1.4 Labor Supply

The marginal conditions for Home and Foreign j_{th} households with respect to labor supply are given respectively by $L_t^{sj\varphi} = \frac{W_t}{P_t}\left(C_t^j\right)^{-\sigma}$, where $j \in [0,n)$, and $\left(L_t^{sj*}\right)^{\varphi} = \frac{W_t^*}{P_t^*}\left(C_t^{j*}\right)^{-\sigma}$, where $j \in [n,1]$. Using these conditions to substitute for L_t^{sj} and L_t^{sj*} into the definitions below and using eqs.(15) and (22) and the fact that it exists a representative agent for each country, we have that the Home and Foreign aggregate labor supplies are given by

$$
\begin{aligned}
L_t^s &\equiv \int_0^n L_t^{sj} dj = nL_t^{sj} = n^{\frac{\sigma}{\varphi}+1}\left(\frac{W_t}{P_t}\right)^{\frac{1}{\varphi}} C_t^{-\frac{\sigma}{\varphi}}, &(33)\\
L_t^* &\equiv \int_n^1 L_t^{sj*} dj = (1-n)\, L_t^{sj*} = (1-n)^{\frac{\sigma}{\varphi}+1}\left(\frac{W_t^*}{P_t^*}\right)^{\frac{1}{\varphi}} C_t^{*-\frac{\sigma}{\varphi}}. &(34)
\end{aligned}
$$

2.1.5 Inflation, Terms of Trade (TOT) and Real Exchange Rate

Now, we derive the relationship between inflation, terms of trade and real exchange rate. The Home country's terms of trade (TOT), defined as $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$, is the Home country's relative price of the imported goods (Foreign goods' bundle) in terms of the domestic goods (Home goods' bundle).[9] Dividing the Home [Foreign] CPI index in eq.(10) [(23)] by $P_{H,t}$ and $P_{F,t}$ $\left[P_{H,t}^* \text{ and } P_{F,t}^*\right]$ and using the LOP and the definition for Home TOT, we get

$$
\begin{aligned}
\frac{P_t}{P_{H,t}} &= \left[(1-\alpha) + \alpha S_t^{1-\mu}\right]^{\frac{1}{1-\mu}} \equiv g\left(S_t\right), &(35)\\
\frac{P_t}{P_{F,t}} &= \frac{P_t}{P_{H,t}}\frac{P_{H,t}}{P_{F,t}} = \frac{g\left(S_t\right)}{S_t} \equiv h\left(S_t\right), &(36)\\
\frac{P_t^*}{P_{H,t}^*} &= \left[\alpha^* + (1-\alpha^*)\, S_t^{1-\mu}\right]^{\frac{1}{1-\mu}} \equiv g^*\left(S_t\right), &(37)\\
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{H,t}^*}\frac{P_{H,t}^*}{P_{F,t}^*} = \frac{g^*\left(S_t\right)}{S_t} \equiv h^*\left(S_t\right), &(38)
\end{aligned}
$$

where $g'\left(S_t\right) > 0$, $h'\left(S_t\right) < 0$, $g^{*\prime}\left(S_t\right) > 0$ and $h^{*\prime}\left(S_t\right) < 0$. Log-linearizing eqs.(35) and (38) around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$, we get

$$
\begin{aligned}
\pi_t &= \pi_{H,t} + \bar{\alpha}(1-n)\Delta s_t, &(39)\\
\pi_t^* &= \pi_{F,t}^* - \bar{\alpha} n \Delta s_t, &(40)
\end{aligned}
$$

where the Home and Foreign log domestic inflation rates (i.e., the percent change of the price index of the goods produced domestically) are given by $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi_{F,t}^* \equiv p_{F,t}^* - p_{F,t-1}^*$. Equation (39) tells us that the size of the positive effect of a deterioration in Home TOT on the gap between the Home CPI and domestic inflation rates increases with the weight of the imported (Foreign) goods in the Home households' preferences, given by $\alpha = \bar{\alpha}(1-n)$, which in turn falls with the relative size of the Home country n and with the degree of home bias $\frac{1}{\bar{\alpha}}$, inversely related to $\bar{\alpha}$.[10] An analogous argument is true for the Foreign country's counterpart in eq.(40). In the particular case of closed countries, when $\bar{\alpha} = 0$, we get $p_t = p_{H,t}$ and $p_t^* = p_{F,t}^*$. In the particular case of a small Home country, when n is very close to 0, we get $p_t^* = p_{F,t}^*$.

---

[9] It follow from the LOP that $S_t^* \equiv \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{1}{S_t}$.

[10] This result is very intuitive when we note that the degree of home bias is inversely related to the degree of openness.

Another important result follows from combining the LOP and the results (35) and (37) with the definition of Home country's real exchange rate, denoted by Q_t, so that

$$Q_t \equiv \frac{\varepsilon_t P_t^*}{P_t} = \frac{g^*(S_t)}{g(S_t)}, \tag{41}$$

where Q_t is a increasing function of S_t. Log-linearizing eq.(41) around the steady state for the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$, we get

$$q_t = (1 - \bar{\alpha})\, s_t. \tag{42}$$

Equation (42) tells us that home bias is the only source of PPP violation. Note that $q_t = 0$ every period when there is no home bias ($\bar{\alpha} = 1$). The real exchange rate's volatility increases with the degree of home bias and with the TOT's volatility. Although the LOP holds for all goods individually, the real exchange rate q_t is directly related to the TOT s_t, which fluctuates over time in response to shocks on both countries. Intuitively, home bias implies that P_t and P_t^* are consumer-based price indexes with different weights on the Home and Foreign goods.

2.1.6 International Risk Sharing

According to Chari et al (2002), the complete financial markets assumption allows us to derive an important relationship linking the consumption of both countries. Setting t=0 and s=t, eqs.(28) and (31) can be rewritten as

$$\frac{\beta^t \operatorname{Pr}_0(h_t)\, C_0^{j\sigma}}{C^j(h_t)^{\sigma}} = \frac{Z_0(h_t)\, P(h_t)}{P_0}, \tag{43}$$

$$\frac{\beta^t \operatorname{Pr}_0(h_t)\, C_0^{j*\sigma}}{C^{j*}(h_t)^{\sigma}} = \frac{Z_0(h_t)\, P^*(h_t)\, \varepsilon(h_t)}{P_0^* \varepsilon_0} = \frac{Q(h_t)}{Q_0} \frac{Z_0(h_t)\, P(h_t)}{P_0}, \tag{44}$$

where $Q(h_t) \equiv \frac{P^*(h_t)\varepsilon(h_t)}{P(h_t)}$ in eq.(44) by the definition for real exchange rate. Substituting eq.(43) into eq.(44), we get the international risk sharing (IRS) condition

$$C_t^j = \vartheta Q_t^{\frac{1}{\sigma}} C_t^{j*}, \tag{45}$$

where $\vartheta \equiv Q_0^{-\frac{1}{\sigma}} \frac{C_0^j}{C_0^{j*}}$. We omit h_t because this condition holds for any history between periods 0 and t. Equation (45) tells us that home bias allows for a variable gap between the Home and Foreign (per-capita) households' consumption growth rates, even if international financial market structure is complete. The intuition is that, with home bias, changes in Home TOT produce real exchange rate flotation, which in turn, according to eqs.(43) and (44), gives rise to a gap between the Home and Foreign's intertemporal relative price of consumption.

Combining eq.(45) with eqs.(15) and (22), we get an aggregate version of the IRS condition, given by

$$C_t = \frac{n}{1-n} \vartheta Q_t^{\frac{1}{\sigma}} C_t^*. \tag{46}$$

Following a general procedure in the literature, which is without loss of generality, we assume the same initial conditions - in terms of relative net asset positions - for Home and Foreign households, so that $\vartheta = 1$. Log-linearizing eq.(46) around the steady state, we get

$$c_t = \ln\left\{\frac{n}{1-n}\right\} + \frac{1}{\sigma} q_t + c_t^*. \tag{47}$$

The ratio between the Home and Foreign aggregate consumption levels collapses to zero as Home country becomes a small economy.

2.1.7 Uncovered Interest Parity

As explained in subsection (2.1.3), the Home-currency equilibrium prices of the one-period zero-cupon bonds denominated in Home and Foreign currencies are given respectively by $R_t^{-1} = E_t\left[D_{t,t+1}\right]$ and $\varepsilon_t R_t^{*-1} = E_t\left[D_{t,t+1}\varepsilon_{t+1}\right]$, where $D_{t,t+1}$ is the one-period Home SDF. Combining these equations, we get the uncovered interest parity (UIP)

$$E_t\left[D_{t,t+1}\left(R_t - R_t^*\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right] = 0, \tag{48}$$

which is log-linearized around the steady state to yield $r_t - r_t^* = E_t\left[\Delta e_{t+1}\right]$. In addition, combining this equation with the results (39), (40) and (42), we get

$$r_t^e - r_t^{e*} = E_t\left[\Delta s_{t+1}\right], \tag{49}$$

where $r_t^e \equiv r_t - E_t\left[\pi_{H,t+1}\right]$ and $r_t^{e*} \equiv r_t^* - E_t\left[\pi_{F,t+1}^*\right]$. As should be clear, the UIP condition (48) is not an additional independent equilibrium condition.

Equation (49) can be used to show that the log Home TOT - and consequently the log real exchange rate - is endogenously determined as a function of the current and future gaps between the Foreign and Home expected log real interest rates. Rewritting this equation as $s_t = E_t\left[s_{t+1}\right] + r_t^{e*} - r_t^e$ and solving it forward, we get

$$s_t = \lim_{s\longrightarrow\infty} E_t\left[s_{t+s}\right] + \sum\nolimits_{s=0}^{\infty} E_t\left[r_{t+s}^{e*} - r_{t+s}^e\right],$$

where $r_{t+s}^e \equiv r_{t+s} - E_{t+s}\left[\pi_{H,t+s+1}\right]$ and $r_{t+s}^{e*} \equiv r_{t+s}^* - E_{t+s}\left[\pi_{F,t+s+1}^*\right]$. In the Appendix, we show that $s_t = 0$ in the steady state for the symmetric case. This fact, associated to the stationary structure of the productivity shifters defined in subsection (2.2.1), which are the driving forces of the model, results that PPP holds in the long run, i.e., $\lim_{s\longrightarrow\infty} E_t\left[s_{t+s}\right] = 0$. Finally, substituting this result into the equation above and using the law of iterated expectations, we get

$$s_t = \sum\nolimits_{s=0}^{\infty} E_t\left[\left(r_{t+s} - \pi_{H,t+s+1}\right) - \left(r_{t+s}^* - \pi_{F,t+s+1}^*\right)\right].$$

2.1.8 Current Account

By definition, the Home current account, measured in Home curency, is given by $CC_t \equiv P_{H,t}Y_t - P_tC_t$. Combining this definition with eq.(35), the Home current account as a proportion of the Home output, denoted by NX_t, is given by

$$NX_t \equiv \frac{CC_t}{P_{H,t}Y_t} = 1 - g\left(S_t\right)\frac{C_t}{Y_t}.$$

Log-linearizing the right-hand side of the second equality above around the steady state, we get $NX_t = \bar{\alpha}(1-n)s_t + c_t - y_t$. In addition, combining this result with eq.(68) yields

$$NX_t = \frac{\bar{\alpha}(1-n)\Lambda}{\sigma}s_t, \tag{50}$$

where the signal of $\Lambda \equiv (1-\sigma) + (2-\bar{\alpha})(\sigma\mu - 1)$ gives the direction of the effect of Home TOT on Home current account. In the particular case for fully closed countries, when $\bar{\alpha} = 0$, we get $NX_t = 0$.

2.2 Firms

Each Home and Foreign household owns a competitive-monopolistic firm producing a differentiated good. Therefore, there is also a continuum of firms indexed by $i \in [0,1]$, such that firms on the interval $[0,n)$ are located in the Home country, while firms on the interval $[n,1]$ are located in the Foreign country. Firms maximize profits subject to an isoelastic demand curve and use only a homogeneous type of labor for production. Labor market is competitive and there is no investment.

2.2.1 Technology and Cost Minimization

All Home firms operate an identical CRS technology

$$Y_t(i) = A_t L_t(i), \tag{51}$$

where $Y_t(i)$ is the Home i$_{th}$ firm's output, $L_t(i)$ is the Home i$_{th}$ firm's labor demand and A_t is the Home total factor productivity shifter, which follows the AR(1) process

$$a_t \equiv \ln A_t = \rho a_{t-1} + \xi_t, \tag{52}$$

where $0 < \rho < 1$ and ξ_t are i.i.d Gaussian shocks. All Foreign firms operate a similar technology, except for the fact that $\rho \neq \rho^*$ is possible. The shocks ξ_t and ξ_t^* may be correlated. This technology implies that Home and Foreign nominal marginal costs are given by $MC_t^n = \frac{W_t}{A_t}$ and $MC_t^{n*} = \frac{W_t^*}{A_t^*}$. Defining the Home and Foreign real marginal costs - in terms of Home and Foreign goods respectively - as $MC_t \equiv \frac{MC_t^n}{P_{H,t}}$ and $MC_t^* \equiv \frac{MC_t^{n*}}{P_{F,t}^*}$, we can use definitions (35) and (38) to get

$$\begin{aligned} MC_t &= \frac{W_t}{A_t P_{H,t}} = \frac{W_t}{P_t}\frac{g(S_t)}{A_t}, & (53)\\ MC_t^* &= \frac{W_t^*}{A_t^* P_{F,t}^*} = \frac{W_t^*}{P_t^*}\frac{h^*(S_t)}{A_t^*}, & (54)\end{aligned}$$

so that MC_t rises with Home TOT and MC_t^* falls with Home TOT, since $g(S_t) > 0$ and $h^*(S_t) < 0$.

2.2.2 Labor Demand

Substituting eq.(51) and its Foreign counterpart into the definitions below for Home and Foreign aggregate labor demands respectively, we get

$$\begin{aligned} L_t &\equiv \int_0^n L_t(i)\,di = \frac{Y_t}{A_t}U_t, & (55)\\ L_t^* &\equiv \int_n^1 L_t^*(i)\,di = \frac{Y_t^*}{A_t^*}U_t^*, & (56)\end{aligned}$$

such that Y_t and Y_t^* are the Home and Foreign aggregate output indexes, defined as

$$\begin{aligned} Y_t &\equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_0^n Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}}, & (57)\\ Y_t^* &\equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_n^1 Y_t^*(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}}, & (58)\end{aligned}$$

while $U_t \equiv \int_0^n \frac{Y_t(i)}{Y_t}di$ and $U_t^* \equiv \int_n^1 \frac{Y_t^*(i)}{Y_t^*}di$ measure respectively the dispersion of the Home and Foreign firms' outputs.

2.2.3 Price-Setting

Firms set prices in a staggering way, as in Calvo (1983): every period, a mesure of $1-\phi$ randomly selected firms set a new price, with an individual firm's probability of readjusting each period being independent of the time elapsed since it last reset its price. A Home firm i adjusting price in period t set a new price $P^{o}_{H,t}(i)$ in order to maximize the present value of its stream of expected future profits (dividends), which is given by $V_t(i) = \sum_{s=0}^{\infty} E_t\left[D_{t,t+s} DV_{t+s}(i)\right]$, where $D_{t,t+s} = \beta^s \left(\frac{C_{t+s}}{C_t}\right)^{-\rho} \frac{P_t}{P_{t+s}}$ is the s-period Home SDF at period t, $DV_{t+s}(i) = \left[P_{H,t+s}(i) - MC^{n}_{t+s}\right] Y^{d}_{t+s}(i)$ is the profit at period t+s and $Y^{d}_{t+s}(i)$ is the world demand for Home i_{th} good at period t+s, given in eq.(25).[11]

Log-linearizing the solution of this problem around the steady state, in which inflation is zero, we get[12]

$$p^{o}_{H,t} = \psi + (1-\phi\beta) \sum_{s=0}^{\infty} (\phi\beta)^s E_t\left[mc^{n}_{t+s}\right], \tag{59}$$

where $\psi \equiv \ln \frac{\varepsilon}{\varepsilon - 1}$. The firm sets the new price as a (gross) markup - with size in log equal to ψ - over the weighted average of the current and expected future log nominal marginal costs.[13] This forward-looking behavior arises because the firm recognizes that the new price will be effective for a random number of periods. As the firm faces a isoelastic demand curve, it does not readjust price in response to a shift in this curve if current or expected future marginal costs remain unaltered. This result implies that inflationary pressures must have a cost-pushing origin, which is a central property of the new Keynesian models. We can derive a similar equation for the Foreign country, where $\phi^* \neq \phi$ is allowed. The flexible-price case is a particular one with $\phi = \phi^* = 0$, so that all Home and Foreign firms adjust price every period according to the pricing rule

$$\begin{aligned} \bar{p}_{H,t} &= \psi + mc^{n}_{t}, && (60) \\ \bar{p}^{*}_{F,t} &= \psi + mc^{n*}_{t}. && (61) \end{aligned}$$

2.2.4 New Keynesian Phillips (NKP) Curve

As only a fraction ϕ of firms adjust price each period, we have that $P_{H,t} = \left[\phi P^{1-\varepsilon}_{H,t-1} + (1-\phi) P^{o1-\varepsilon}_{H,t}\right]^{\frac{1}{1-\varepsilon}}$.[14] Log-linearizing this equation around the steady state, we get $\pi_{H,t} = (1-\phi)(\bar{p}_{H,t} - p_{H,t-1})$, which is combined with eq.(59) to yield the NKP curve

$$\pi_{H,t} = \beta E_t\left[\pi_{H,t+1}\right] + \lambda \widehat{mc}_t, \tag{62}$$

where $\widehat{mc}_t \equiv mc_t + \psi$ is the deviation of the log real marginal cost from its steady state level and $\lambda \equiv \frac{1-\phi}{\phi}(1-\phi\beta)$. Proceeding in the same way with the Foreign country, we get

$$\pi^{*}_{F,t} = \beta E_t\left[\pi^{*}_{F,t+1}\right] + \lambda^{*} \widehat{mc}^{*}_{t}, \tag{63}$$

such that $mc^{*}_{t} \equiv mc^{*}_{t} + \psi$ and $\lambda^{*} \equiv \frac{1-\phi^{*}}{\phi^{*}}(1-\phi^{*}\beta)$. As seen in subsection (2.2.1), the real marginal costs are defined as $mc_t = mc^{n}_{t} - p_{H,t}$ and $\widehat{mc}^{*}_{t} \equiv mc^{n*}_{t} - p^{*}_{F,t}$.

---

[11] We assume that the output of each firm not allowed to adjust price meets the market demand for its goods at its unaltered price, so that $Y_{t+s}(i) = Y^{d}_{t+s}(i)$ for any s.

[12] We suppress the index i because all firms adjusting prices optimally take the same decision, since they face identical current and expected future marginal costs.

[13] In the long-run steady state equilibrium, all firms had opportunity to adjust prices, so that they charge the same price. Therefore, we can infer from eq.(59) that the steady state marginal cost is equal to $-\psi$.

[14] By the law of large numbers, the avarage price of the firms not adjusting prices is the last period's domestic price index.

2.3 Monetary Policy

For simplicity, we assume that both Home and Foreign Central Banks follow the Taylor-type rules

$$r_t = \delta_r r_{t-1} + \delta_\pi \pi_t + \delta_y \tilde{y}_t + \xi_{M,t}, \tag{64}$$
$$r_t^* = \delta_r^* r_{t-1}^* + \delta_\pi^* \pi_t^* + \delta_y^* \tilde{y}_t^* + \xi_{M,t}^*, \tag{65}$$

where $\xi_{M,t}$ and $\xi_{M,t}^*$ are Gaussian i.i.d monetary policy shocks and δ_r and δ_r^* are the smoothing interest rate coefficientes. Following the New-Keynesian procedure in setting monetary policy, we introduce the Home and Foreign output gaps into the rules, which are defined as $\tilde{y}_t \equiv y_t - \bar{y}_t$ and $\tilde{y}_t^* \equiv y_t^* - \bar{y}_t^*$, where $\bar{y}_t$ and $\bar{y}_t^*$ are the Home and Foreign output levels in the flexible-price counterpart of the model. These rules have usually been adopted in the most earlier empirical works, although some works use rules with the output growth $\Delta y_t \equiv y_t - y_{t-1}$ instead of the output gap. We do not derive the optimal monetary rules because they are not necessarily used in practice.

2.4 Equilibrium

This section derives the general equilibrium dynamics for the log-linearized version of the model around the steady state, which has the property that the productivity shifters A_t and A_t^* remain constant in their long-run equilibrium levels equal to 1. For sake of simplicity, we follow the literature and assume the symmetric case in which the preference parameters are the same for both countries.[15] The steady state is described in the Appendix.

2.4.1 Demand Side: Goods Markets Equilibrium and IS Curves

As we see in subsection (2.2.3), the markets of all Home and Foreign goods clear in equilibrium, so that $Y_t(i) = Y_t^d(i)$ for $i \in [0,n)$ and $Y_t^*(i) = Y_t^{d*}(i)$ for $i \in [n,1]$, where $Y_t(i)$ and $Y_t^*(i)$ are the outputs of Home and Foreign i$_{th}$ firms, while $Y_t^d(i)$ and $Y_t^{d*}(i)$ are the world aggregate demands for Home and Foreign i$_{th}$ goods. Therefore, substituting the expression for $Y_t^d(i)$ $\left[Y_t^{d*}(i)\right]$ in eq.(25) [(26)] into the definition for the Home [Foreign] output aggregate index (57) [(58)] and using the results (7), (13) and (20) [(8)], (14) and (21)] and the definitions (35) and (37) [(36) and (38)], we get

$$Y_t = C_{H,t} + C_{H,t}^* = (1-\alpha)\, g\,(S_t)^\mu C_t + \alpha^* g^* (S_t)^\mu C_t^*,$$
$$Y_t^* = C_{F,t} + C_{F,t}^* = \alpha h\,(S_t)^\mu C_t + (1-\alpha^*)\, h^* (S_t)^\mu C_t^*.$$

The equations above give the Home and Foreign aggregate outputs as functions of the Home TOT $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ and the aggregate consumption of both countries. Log-linearizing these equations around the steady state for the symmetric case with $\bar{\alpha} = \bar{\alpha}^*$, we get

$$y_t = [1-(1-n)\bar{\alpha}]\, c_t + (1-n)\bar{\alpha} c_t^* + \mu(1-n)\bar{\alpha}(2-\bar{\alpha}) s_t, \tag{66}$$
$$y_t^* = n\bar{\alpha} c_t + (1-n\bar{\alpha}) c_t^* + \mu n\bar{\alpha}(\bar{\alpha}-2) s_t. \tag{67}$$

A deterioration of Home TOT - an increase in s_t - reduces the world demand for Foreign goods and increases the world demand for Home goods. The magnitude of these effects rises with the elasticity of substitution in consumption between Home and Foreign goods μ and falls with the degree of home bias $\bar{\alpha}$. Furthermore, both Home and Foreign outputs increase with the Home and

---

[15] For the empirical purpose of estimating the model with Brazilian data, this assumption is not so restrictive, since the parameters for home bias degree and the elasticity of substitution between Home and Foreign goods, given by $\bar{\alpha}$ and μ respectively, do not affect the Foreign economy's dynamics in the small Home country version of the model with $n \longrightarrow 0$.

Foreign aggregate consumption c_t and c_t^*. In this case, a lower degree of home bias - equivalent to a higher degree of openness - reduces the impact of the domestic consumption of each country on its own output, while increases the impact of the domestic consumption of the other country. In the particular case for fully closed countries, when $\bar{\alpha} = 0$, we have $c_t = y_t$ and $c_t^* = y_t^*$. With a small Home country, when $n = 0$, we have $y_t = (1 - \bar{\alpha})\, c_t + \bar{\alpha} c_t^* + \mu\bar{\alpha}(2 - \bar{\alpha}) s_t$ and $c_t^* = y_t^*$.

Using the IRS condition (47) to substitute for c_t^* $[c_t]$ into eq.(66) [(67)] and combining with eq.(41) and eqs.(35) and (37) [(36) and (38)], we get an expression for y_t $[y_t^*]$ in terms of c_t $[c_t^*]$ and s_t, given by

$$\begin{aligned} y_t &= c_t + \frac{\omega_{\bar{\alpha}} + \bar{\alpha} - 1}{\sigma} s_t, && (68) \\ y_t^* &= c_t^* - \frac{\omega_{\bar{\alpha}}^*}{\sigma} s_t, && (69) \end{aligned}$$

where $\omega_{\bar{\alpha}} \equiv 1 - \bar{\alpha} n + (1-n)\bar{\alpha}(2-\bar{\alpha})(\sigma\mu - 1)$ and $\omega_{\bar{\alpha}}^* \equiv \bar{\alpha} n \left[1 + (2 - \bar{\alpha})(\sigma\mu - 1)\right]$ are positive. Now, these coefficients capture not only the direct effects of changes in Home TOT on output, as we just described above, but also the indirect effects of required movements in Home and Foreign aggregate consumption to meet the IRS condition in equilibrium. As an example, for a fixed c_t, a higher s_t has two conflicting effects on y_t. First, the direct effect, it increases the Home external competitiveness and thereby the world demand for Home goods. Second, the indirect effect, it must be associated with a fall in Foreign aggregate consumption to preserve IRS condition in equilibrium. We can show that $\omega_{\bar{\alpha}} + \bar{\alpha} - 1 > 0$, so that the direct effect is dominant.

Using eqs.(68) and (39) [(69) and (40)] to substitute for c_t and π_{t+1} $\left[c_t^* \text{ and } \pi_{t+1}^*\right]$ respectively into eq.(30) [(32)], we get

$$\begin{aligned} y_t &= E_t\left[y_{t+1}\right] - \frac{1}{\sigma}\left(r_t^e + \ln\beta\right) + \frac{1 - \bar{\alpha} n - \omega_{\bar{\alpha}}}{\sigma} E_t\left[\Delta s_{t+1}\right], && (70) \\ y_t^* &= E_t\left[y_{t+1}^*\right] - \frac{1}{\sigma}\left(r_t^{e*} + \ln\beta\right) + \frac{\omega_{\bar{\alpha}}^* - \bar{\alpha} n}{\sigma} E_t\left[\Delta s_{t+1}\right], && (71) \end{aligned}$$

where $r_t^e \equiv r_t - E_t\left[\pi_{H,t+1}\right]$ and $r_t^{e*} \equiv r_t^* - E_t\left[\pi_{F,t+1}^*\right]$ are the Home and Foreign expected log real (related to domestic inflation) interest rates. Equations (70) and (71) can be naturally interpreted as IS curves rigorously derived from a intertemporal choice problem of optimizing households.

2.4.2 Supply Side: Labor Market Equilibrium and Marginal Costs

The Home [Foreign] labor market equilibrium condition is given by $L_t^s = L_t$ $[L_t^{s*} = L_t^*]$. Substituting eqs.(33) and (55) [(34) and (56)] into this condition and solving it for $\frac{W_t}{P_t}$ $\left[\frac{W_t^*}{P_t^*}\right]$, we get

$$\begin{aligned} \frac{W_t}{P_t} &= n^{-(\sigma+\varphi)} \left(\frac{Y_t U_t}{A_t}\right)^{\varphi} C_t^{\sigma}, && (72) \\ \frac{W_t^*}{P_t^*} &= (1-n)^{-(\sigma+\varphi)} \left(\frac{Y_t^* U_t^*}{A_t^*}\right)^{\varphi} C_t^{*\sigma}. && (73) \end{aligned}$$

Now, substituting eq.(72) [(73)] into eq. (53) [(54)] for the real marginal cost and log-linearizing around the steady state for the symmetric case, we get

$$\begin{aligned} mc_t &= -(\sigma+\varphi)\ln n + \varphi y_t + \sigma c_t + \bar{\alpha}\,(1-n)\, s_t - (1+\varphi) a_t, && (74) \\ mc_t^* &= -(\sigma+\varphi)\ln\{1-n\} + \varphi y_t^* + \sigma c_t^* - \bar{\alpha} n s_t - (1+\varphi) a_t^*, && (75) \end{aligned}$$

where, as explained in Gali & Monacelli (2005), we use the fact that the deviations of $u_t \equiv \ln U_t$ and $u_t^* \equiv \ln U_t^*$ around the steady state are of second order, so that up to a first order approximation we can set $u_t = u_t^* = 0$. As these equations explain the real marginal costs in NKP curves (62) and (63), they allows us to undertand three sources of inflationary pressure. First, a higher output or a lower productivity increases labor demand and pushes real wage and marginal cost up. The magnitude of these effects falls with the real wage-elasticity of labor supply, which is given by the inverse of parameter φ. Second, a higher domestic consumption reduces the marginal utility of real wage and thus shrinks the labor supply, pushing real wage and marginal cost up. This effect increases with the inverse elasticity intertemporal of substitution in consumption σ because this parameters measures the negative impact of the marginal utility with respect to consumption. Third, the purchase power of the Home [Foreign] wage in terms of Home [Foreign] goods increases [decreases] with a higher Home TOT, pushing the real marginal cost - also measured in terms of Home [Foreign] goods - up [down].

Finally, substituting eq.(68) [(69)] into eq.(74) [(75)], we get

$$\begin{aligned} mc_t &= (\sigma+\varphi)\left(y_t - \ln n\right) + \left(1 - \bar{\alpha}n - \omega_{\bar{\alpha}}\right) s_t - (1+\varphi)a_t, && (76)\\ mc_t^* &= (\sigma+\varphi)\left(y_t^* - \ln\left\{1-n\right\}\right) + \left(\omega_{\bar{\alpha}}^* - \bar{\alpha}n\right) s_t - (1+\varphi)a_t^*. && (77) \end{aligned}$$

Now, the coefficients for y_t and s_t capture not only the direct effects of these variables on real marginal costs, as explained above, but also the indirect effects of required movements in Home and Foreign aggregate consumption to meet the IRS condition in equilibrium. The NKP curves (62) and (63), complemented by the real marginal cost equations (76) and (77), fully characterize the supply side of the Home and Foreign economies respectively. Unlike the version of the model for closed economies, there is no more a direct relationship between output and marginal cost - and therefore between output and inflation - for a fixed productivity, since the marginal cost is also affected by the TOT.

2.4.3 Equilibrium with Flexible Prices

By definition, the natural level of a variable - denoted by an overbar - is the one observed in the flexible-price version of the model. As we saw above, since monetary policies react to output gap, we need to derive the general equilibrium dynamics under flexible prices in order to fully characterize its counterpart for the real world economy with sticky prices. Furthermore, as we will see below, a very convenient representation of the equilibrium under sticky prices, known as canonical representation, is formed by the processes for the deviations of the endogenous variables from their natural levels in association with the processes for the natural levels of these variables themselves.

As we saw in subsection (2.2.3), under flexible prices, each Home and Foreign firm reoptimizes its price every period according to the pricing rule (60) and (61) respectively, i.e., as a mark-up over the marginal cost. In addition, all firms of each contry charge the same prices, since they face identical technological and demand constraints. Therefore, the optimal prices are $p_{H,t}^o = \bar{p}_{H,t}$ for all Home firms and $p_{F,t}^{o*} = \bar{p}_{F,t}^*$ for all Foreign firms, where $\bar{p}_{H,t}$ and $\bar{p}_{F,t}^*$ are the Home and Foreign log CPI indexes under flexible prices. Substituting these results into eqs.(60) and (61) respectively, the Home and Foreign log real marginal costs are given by $\bar{mc}_t = \bar{mc}_t^* = -\psi$, such that $\bar{mc}_t \equiv \bar{mc}_t^n - \bar{p}_{H,t}$ and $\bar{mc}_t^* \equiv \bar{mc}_t^{n*} - \bar{p}_{F,t}^*$, where $\psi \equiv \ln \frac{\varepsilon}{\varepsilon-1}$ is the log gross markup in the flexible-price case.

Substituting the results above into eqs.(76) and (77) and combining them with the IRS condition (47) and with the eqs.(68) and (69), we have that the natural level of the Home TOT is given by

$$\bar{s}_t = \frac{(1+\varphi)}{1+\frac{\varphi}{\sigma}\left(\omega_{\bar{\alpha}} + \omega_{\bar{\alpha}}^*\right)}\left(a_t - a_t^*\right), \tag{78}$$

while the natural level of the Home and Foreign outputs are given by

$$\begin{aligned}
\bar{y}_t &= \ln n - \frac{\psi}{\varphi+\sigma} + (1-\Theta_y)\frac{1+\varphi}{\varphi+\sigma}a_t + \Theta_y\frac{1+\varphi}{\varphi+\sigma}a_t^*, && (79)\\
\bar{y}_t^* &= \ln\{1-n\} - \frac{\psi}{\varphi+\sigma} + \Theta_y^*\frac{1+\varphi}{\varphi+\sigma}a_t + \left(1-\Theta_y^*\right)\frac{1+\varphi}{\varphi+\sigma}a_t^*, && (80)
\end{aligned}$$

where $\Theta_y \equiv \frac{\sigma(1-\bar{\alpha}n-\omega_{\bar{\alpha}})}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$ and $\Theta_y^* \equiv \frac{\sigma(\bar{\alpha}n-\omega_{\bar{\alpha}}^*)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$. Now, substituting the processes for a_t and a_t^* in subsection (2.2.1) into eq.(78), we get

$$E_t\left[\Delta\bar{s}_{t+1}\right] = \frac{1+\varphi}{1+\frac{\varphi}{\sigma}\left(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*\right)}\left[(\rho-1)\,a_t - (\rho^*-1)\,a_t^*\right]. \tag{81}$$

Finally, in order to get the Home [Foreign] natural expected real interest rate, denoted by $\bar{r}_t^e$ $[\bar{r}_t^{e*}]$, we combine the results (79) [(80)] and (81) with eq.(70) [(71)], so that

$$\begin{aligned}
\bar{r}_t^e &\equiv \bar{r}_t - E_t\left[\bar{\pi}_{H,t+1}\right]\\
&= \frac{(1-\Theta_r)\,\sigma\,(1+\varphi)\,(\rho-1)}{\varphi+\sigma}a_t + \frac{\Theta_r\sigma\,(1+\varphi)\,(\rho^*-1)}{\varphi+\sigma}a_t^* - \ln\beta, && (82)\\
\bar{r}_t^{e*} &\equiv \bar{r}_t^* - E_t\left[\bar{\pi}_{F,t+1}^*\right]\\
&= \frac{\Theta_r^*\sigma\,(1+\varphi)\,(\rho-1)}{\varphi+\sigma}a_t + \frac{(1-\Theta_r^*)\,\sigma\,(1+\varphi)\,(\rho^*-1)}{\varphi+\sigma}a_t^* - \ln\beta, && (83)
\end{aligned}$$

where $\Theta_r \equiv \frac{\varphi(\bar{\alpha}n+\omega_{\bar{\alpha}}-1)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$ and $\Theta_r^* \equiv \frac{\varphi(\omega_{\bar{\alpha}}^*-\bar{\alpha}n)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$. Under flexible prices, all changes in real variables are induced by shocks on productivity shifters, so that there is no scope for monetary policy to impact on output.

2.4.4 Equilibrium with Sticky Prices - Canonical Representation

The general equilibrium dynamics for the real world with sticky prices has a canonical representation in terms of the output gap and inflation analogous to that of its closed economy counterpart, which provides the structural framework for the comparison of alternative monetary polciy rules. In order to get this representation, we must derive the processes for the deviations of the endogenous variables from their natural levels.

IS Curve Equations (70) and (71) for Home and Foreign IS curves hold under both the flexible and the sticky-price cases. Hence, the Home and Foreign output gaps $\tilde{y}_t \equiv y_t - \bar{y}_t$ and $\tilde{y}_t^* \equiv y_t^* - \bar{y}_t^*$ move according to the differential equations

$$\begin{aligned}
\tilde{y}_t &= E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(r_t - E_t\left[\pi_{H,t+1}\right] - \bar{r}_t^e\right) + \frac{1-\bar{\alpha}n-\omega_{\bar{\alpha}}}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right], && (84)\\
\tilde{y}_t^* &= E_t\left[\tilde{y}_{t+1}^*\right] - \frac{1}{\sigma}\left(r_t^* - E_t\left[\pi_{F,t+1}^*\right] - \bar{r}_t^{e*}\right) + \frac{\omega_{\bar{\alpha}}^*-\bar{\alpha}n}{\sigma}E_t\left[\Delta\tilde{s}_{t+1}\right], && (85)
\end{aligned}$$

where $\bar{r}_t^e \equiv \bar{r}_t - E_t\left[\bar{\pi}_{H,t+1}\right]$ and $\bar{r}_t^{e*} \equiv \bar{r}_t^* - E_t\left[\bar{\pi}_{F,t+1}^*\right]$ are given by eqs.(82) and (83) respectively, while the TOT gap $\tilde{s}_t \equiv s_t - \bar{s}_t$ is the deviation of the Home TOT from its natural level.

New Keynesian Phillips Curve Equations (76) and (77) for Home and Foreign real marginal costs hold under both the flexible and the sticky-price cases. In addition, these variables are given by $mc_t = mc_t^* = -\psi$ in the flexible-price case, which are also their steady state levels. Therefore, the deviations of the Home and Foreign real marginal costs from their steady state values, denoted by $\widehat{mc}_t$ and $\widehat{mc}_t^*$ are given by

$$\begin{aligned} \widehat{mc}_t &= mc_t + \psi = (\varphi + \sigma)\,\tilde{y}_t + (1 - \bar{\alpha}n - \omega_{\bar{\alpha}})\,\tilde{s}_t, && (86) \\ \widehat{mc}_t^* &= mc_t^* + \psi = (\varphi + \sigma)\,\tilde{y}_t^* + (\omega_{\bar{\alpha}}^* - \bar{\alpha}n)\,\tilde{s}_t. && (87) \end{aligned}$$

Substituting eq.(86) [(87)] into eq.(62) [(63)], we get

$$\begin{aligned} \pi_{H,t} &= \beta E_t\left[\pi_{H,t+1}\right] + \lambda\,(\varphi + \sigma)\,\tilde{y}_t + \lambda\,(1 - \bar{\alpha}n - \omega_{\bar{\alpha}})\,\tilde{s}_t, && (88) \\ \pi_{F,t}^* &= \beta E_t\left[\pi_{F,t+1}^*\right] + \lambda\,(\varphi + \sigma)\,\tilde{y}_t^* + \lambda\,(\omega_{\bar{\alpha}}^* - \bar{\alpha}n)\,\tilde{s}_t. && (89) \end{aligned}$$

UIP Equation (49) also holds for both the flexible and the sticky-price cases, so that

$$(r_t^e - \bar{r}_t^e) - (r_t^{e*} - \bar{r}_t^{e*}) = E_t\left[\Delta\tilde{s}_{t+1}\right]. \tag{90}$$

Canonical Representation The canonical representation of the general equilibrium dynamics is composed by the following sets of structural equations: (1) IS curves (84) and (85); NKP curves (88) and (89); (3) monetary policy rules (64) and (65); (4) UIP equation (90). Furthermore, we must add eqs.(82) and (83) for the real interest rate under flexible prices, as well as eq.(52) and its Foreign counterpart for the productivity shifters' dynamics, since these variables appear in the structural equations. For the particular small Home country case, in which $n \longrightarrow 0$, the Foreign country structure is identical to a closed economy's one, regardless of its own home bias degree. Note that $\pi_t^* = \pi_{F,t}^*$ in this case. The canonical form is presented below:

Canonical Form
Home Country's Structure
$\pi_{H,t} = \beta E_t\left[\pi_{H,t+1}\right] + \lambda\,(\varphi + \sigma)\,\tilde{y}_t + \lambda\,(1 - \bar{\alpha}n - \omega_{\bar{\alpha}})\,\tilde{s}_t;\ \lambda \equiv \frac{1-\phi}{\phi}\,(1 - \phi\beta)$
$\tilde{y}_t = E_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(r_t - E_t\left[\pi_{H,t+1}\right] - \bar{r}_t^e\right) + \frac{1-\bar{\alpha}n-\omega_{\bar{\alpha}}}{\sigma} E_t\left[\Delta\tilde{s}_{t+1}\right]$
$\bar{r}_t^e = (1 - \Theta_r)\,\frac{\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma} a_t + \Theta_r \frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma} a_t^* - \ln\beta$
$a_t = \rho a_{t-1} + \xi_t$
$r_t = \delta r_{t-1} + \delta_\pi \pi_t + \delta_y \tilde{y}_t + \xi_{M,t}$
$\omega_{\bar{\alpha}} \equiv 1 - \bar{\alpha}n + (1-n)\bar{\alpha}(2-\bar{\alpha})(\sigma\mu - 1)$
$\Theta_r \equiv \frac{\varphi(\bar{\alpha}n+\omega_{\bar{\alpha}}-1)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$
Foreign Country's Structure
$\pi_{F,t}^* = \beta E_t\left[\pi_{F,t+1}^*\right] + \lambda\,(\varphi + \sigma)\,\tilde{y}_t^* + \lambda\,(\omega_{\bar{\alpha}}^* - \bar{\alpha}n)\,\tilde{s}_t;\ \lambda^* \equiv \frac{1-\phi^*}{\phi^*}\,(1 - \phi^*\beta)$
$\tilde{y}_t^* = E_t\left[\tilde{y}_{t+1}^*\right] - \frac{1}{\sigma}\left(r_t^* - E_t\left[\pi_{F,t+1}^*\right] - \bar{r}_t^{e*}\right) + \frac{\omega_{\bar{\alpha}}^*-\bar{\alpha}n}{\sigma} E_t\left[\Delta\tilde{s}_{t+1}\right]$
$\bar{r}_t^{e*} = \Theta_r^* \frac{\sigma(1+\varphi)(\rho-1)}{\varphi+\sigma} a_t + (1 - \Theta_r^*)\,\frac{\sigma(1+\varphi)(\rho^*-1)}{\varphi+\sigma} a_t^* - \ln\beta$
$a_t^* = \rho^* a_{t-1}^* + \xi_t^*$
$r_t^* = \delta^* r_{t-1}^* + \delta_\pi^* \pi_t^* + \delta_y^* \tilde{y}_t^* + \xi_{M,t}^*$
$\omega_{\bar{\alpha}}^* \equiv \bar{\alpha}n\left[1 + (2-\bar{\alpha})(\sigma\mu - 1)\right]$
$\Theta_r^* \equiv \frac{\varphi(\omega_{\bar{\alpha}}^*-\bar{\alpha}n)}{\sigma+\varphi(\omega_{\bar{\alpha}}+\omega_{\bar{\alpha}}^*)}$
Uncovery Interst Parity
$(r_t^e - \bar{r}_t^e) - (r_t^{e*} - \bar{r}_t^{e*}) = E_t\left[\Delta\tilde{s}_{t+1}\right]$

3 Empirical Analysis

In this section, we calibrate/estimate the structural paramaters with Brazilian data for the Home country and U.S. data for the Foreign country.[16] Next, we use output estimation to compute impulse-response functions in order to analyse the effects of structural shocks on the dynamics of the macroeconomic variables. As Brazilian economy is small, we have $n = 0$ in eq.(24), so that $\alpha = \bar{\alpha}$.

3.1 Calibration and Bayesian Estimation

Parameters $\bar{\alpha}$ and β are calibrated with basis on steady-state relations between endogenous variables. The share of imported goods in the Brazilian aggregate consumption basket, denoted by α, is set to 0.13, which is consistent to the ratio Brazilian imports/GDP and Brazilian net exports/GDP during the sample period.[17] The discount factor β is set to 0.91 (annual basis) in order to get approximately the historical mean of the nominal interest rate in the steady state.[18]

Following an increasing part of the empirical literature, we use the Bayesian approach to estimate the other parameters of the model, which works as follows. First, we assume a prior distribution with density $p(\theta)$ for the vector $\theta \equiv \left(\sigma, \varphi, \mu, \rho, \rho^*, \phi, \phi^*, \delta, \delta^*, \delta_\pi, \delta_\pi^*, \delta_y, \delta_y^*\right)$ of structural parameters to be estimated, which summarizes our previous beliefs on these parameters, that is, all information we have on their location besides the information contained in the database Y^T used in estimation. We follow the usual procedure of assuming that the parameters have independent priors, so that it is enough to specify a marginal prior for each parameter separately. Second, the database Y^T is used to update the prior distribuition according the Bayes rule, giving rise to the posterior distribution

$$p(\theta \mid Y^T) = \frac{L(\theta \mid Y^T) p(\theta)}{p(Y^T)}, \tag{91}$$

where $L(\theta \mid Y^T)$ is the likelihood function. If the posterior is not standard, draws from this distribution can be generated numerically through simulation technics. Third, we compute posterior summary statistics in order to characterize the location of the structural parameters.

We denote by Y_t the vector of observed variables used in estimation, which are proxies for the following endogenous variables in the model: the Brazilian and U.S. output y_t and y_t^*, CPI inflation π_t and π_t^* and nominal interest rate r_t and r_t^*, as well as the bilateral real exchange rate q_t. The database Y^T comprises therefore the T-size historical series of the observed variables, which are described in more detail below. Given some initial values for parameters, we use Kalman filter to evaluate the likelikood function in eq.(91). For that, we need first to write the state-space representation of the dynamics of Y_t, which is given by a set of two vectorial equations: (1) the state equation for the dynamics of the endogenous (state) variables of the model, which is the reduced-form solution (derived with Sim's algorithm) of the log-linearized structural model and (2) the measurement equation $Y_t = \digamma X_t + w_t$, which links the observed variables in Y_t to the endogenous variables grouped in X_t through a known matrix $\digamma$, where w_t is a vector of normally distributed and independent measurement errors. As usual, we assume that w_t are independent from the structural shocks ε_t. To avoid stochastic singulariry in the case where there are more observed variables than shocks, three measurement errors were included in w_t, in such a way they can be interpreted as Brazilian and U.S. inflation shocks and a external risk premium shock. Less rigorously, this amounts to add a IID shock to structural eqs. (88), (89) and (49) respectively.

---

[16] We proceed in this way as a first exercise and leave for future research the task of estimating the model with more representative historical series for the Foreign country, which would be the weighted average of the historical series of the main Brazilian financial and trade partners.

[17] Denoting by M_t and NX_t the Home imports and current account as a proportion of the output, we have that $NX_t = \frac{P_{H,t}Y_t - P_t C_t}{P_{H,t}Y_t} = 1 - g\left(S_t\right)\frac{C_t}{Y_t}$ and $M_t = \frac{C_{F,t}}{Y_t} = \frac{C_{F,t}}{C_t}\frac{C_t}{Y_t}$, with $\frac{C_{F,t}}{C_t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\mu} = \bar{\alpha} h\left(S_t\right)^{\mu}$, where we use definitions (35) and (36), eq.(14) and the fact that $n = 0$ in eq.(24) because Brazilian economy is small. Since $S_t = h\left(S_t\right) = g\left(S_t\right) = 1$ in the steady state with symmetric preferences and initial wealth conditions, we can use the results above to get a calibrated value for $\bar{\alpha}$ from the historical means for NX_t and M_t.

[18] Since the parameter β is common to both countries, we set an intermediary value.

As the posterior distribution in this case is clearly non-standard, we use Metropolis-Hasting algorithm to generate draws from this distribution[19]. The idea behind this procedure is to simulate a Markov Chain θ_t, $t = 1, 2, ...$, that converges to the posterior distribution. Intuitively, the algorithm works because the transition distribution $T(\theta_t \mid \theta_{t-1})$ of the process is built to make it behave like the stochastic version of a stepwise mode-finding algorithm, always stepping to increase the density but only sommetimes stepping to decrease[20]. In order to guarantee convergence, posterior simulation uses only the second half of two parallel chains with 120.000 runs. The scale factor was chose to provide an acceptance rate of about 25%-40%. Convergence diagnosis testes in Brooks&Gelman (1998) were used to evaluate convergence of the Markov chain to the posterior distribution.

3.2 Data

We estimate the model with quarterly data of the Brazilian and U.S. economies for the period from 1999Q3 to 2005Q3. Structural breaks in Brazilian economy prevent us from using longer series[21]. For both countries, the output gap is the detrended (linear) log real GDP (multiplied by 100) and the CPI inflation is the annualized quarterly percentage change in the CPI index. The full IPCA index produced by IBGE is used for Brazil. The U.S. nominal interest rate is the 3-month Treasury Bill annualized percentage period-average daily rate, while the Brazilian nominal interest rate is the annualized percentage period-average daily rate of reference for the 90-day *DI-pré* swap contract traded in BMF. Real exchange rate series is build with the Brazilian and U.S. period-average log CPI indeces and the period-average log nominal bilateral exchange rate. Data are available in ipeadata[22], except for Brazilian swaps, which are obtained from Brazilian Central Bank. All variables are demeaned and, if necessary, seasonally adjusted with the X-12 method. The U.S. inflation and nominal interest rate are also detrended with a linear filter. This previous treatment is necessary because the model is not developed to explain linear tendencies and seasonal movements of the variables.

3.3 Prior Distributions

The prior distributions reflect our beliefs about the values that the parameters can take. Large prior standard deviations result in diffuse distributions, which means we have little information in addition to the data. On the contrary, small prior standard deviations mean we are confident that the parameters take some value around their prior means. Most earlier attempts to estimate new Keynesian DSGE models with the Bayesian approach use data from developed countries, which have a record of macroeconomic stabilization much longer than the Brazilian economy. Therefore, we seek to specify more diffuse priors that those usually found in the empirical literature, since we figure out it should be much harder to deal with the uncertainty on Brazilian parameters. In setting the prior distribution for a given parameter, we need to make two choices: (1) the parametric distribution and (2) the values for the characteristic parameters of this distribution. In many cases, the choice of the distribution for a parameter is directly restricted by the domain in which it can take values. In addition, we can use the Bayesian estimates for other countries as a reference guide. The first columns of Table (1) summarize prior distributions, while Figures (1), (2) and (3) show their graphs in grey.

---

[19] This part is implemented with the use of DYNARE (Matlab version). A detailed explanation of this program is available in www.cepremap.cnrs.fr/dynare/.

[20] Gelman & all. (1995) give a very intuitive description of the algorithm.

[21] The sample period starts at 1999.III with the adoption of the inflation target regime, after the Brazilian exchange rate liberalization.

[22] See www.ipeadata.gov.br.

Beta distributions with mean 0.5 and standard deviation 0.18 were selected for Home and Foreign parameters constrained on the unit-interval: the autorregressive productivity coeficients ρ and ρ^*, the Calvo-rigitity parameters ϕ and ϕ^* and the interest rate somoothing coefficients δ_r and δ_r^*. This is a symmetric and fairly diffuse distribution with a 90% interval between 0.20 and 0.80, which reflects our uncertainty about these parameters.[23]

A Gamma distribution was used for the elasticity of substitution in consumption between Home and Foreign goods μ, since we expect a positive value for this parameter. We set a prior mean and standard at 1 and 0.6 respectively. As usual in the literature, the inverse of the intertemporal elasticity of substitution in consumption σ and the inverse elasticity of labor supply φ are assumed to follow a Normal distribution with prior means and standard deviations such that these elasticities lies on a large 90% interval between 0.55 and 5.63 and between 0.37 and 3.75 respectively.

Following again the literature, we use Normal distributions for the Taylor rules' coefficients regarding inflation and output gap with the prior means set respectively at 1.5 and 0.5. These values are close to the ones commonly used in works for other countries, such as Caputo et al (2005) for Chile. Again, we use larger prior strandard deviations in order to take into account the uncertainty about these rules for the Brazilian economy. Inverse Gamma distributions are used for the volatility of the shocks.

3.4 Posterior Distribution

Table (1) summarizes the posterior densities, while Figures (1), (2) and (3) show their graphs in black. Two facts are notorious. First, posterior densities are less diffuse than their prior counterparts. Second, the prior and posterior means are in most cases considerably apart one another. Both results suggest that the data contain information capable to update our prior beliefs, although in an extent that varies among parameters. Other convenient result is that posterior distributions are roughly symmetric, which makes possible to compare our results with earlier empirical works that use different measures of central tendency. Validating the robustness of our results, our estimates for U.S. economy are, in general, consistent to previous empirical works, such as Rabanal & Ramírez (2001) and Smets & Wouters (2004). The lack of similar empirical works for Brazilian and other emerging economies prevents from evaluating the robustness of our estimates for the Brazilian economy.[24]

The posterior mean of the Brazilian Calvo parameter for price rigidity ϕ is of 0.92, with a 90% confidence interval between 0.87 and 0.98. This estimate can be translated into the avarage duration of price contracts superior to two years and half, so that Brazilian price rigidity would be comparable with that for European and U.S. economies found by Smets & Wouters (2004), but higher than those for Australia, Canada and New Zealand found by Justiniano & Preston (2004). Caputo et al (2005) also estimates a much lower posterior mean for Chile, around 0.12, although their baseline model includes wage rigidity with estimated Calvo parameter around 0.85, which could explain the low value for the Calvo parameter for price rigidity. The estimated Brazilian price stickness sounds strange since we would expect more price flexibility in countries with a higher historical level of inflation rates. A possible explanation is that firms setting prices in a relatively more inflationary environment could have a stronger forward-looking behavior, placing more weight on marginal costs farther into the future, so that frequent readjustments would not be necessary. We leave for future research the estimation of a model with price indexation. As observed in other empirical studies, this new assumption could alter the estimates for Calvo parameters significatively.

---

[23] For robustness test, a beta prior with mean 0.7 and standard deviation 0.1 was also used without changing the results significatively. This prior implies a 90% interval bertween 0.52 and 0.85, which is enough large to include the central moments estimates found in previous studies for other countries.

[24] As far as we are concerned, the only work in this line for a Latin America country is Caputo et al (2005).

The posterior mean of the U.S. Calvo parameter for price rigidity ϕ^* is of 0.94, which amounts to a price duration around three years and half. The posterior density is fairly concentrated around the mean, with a 90% confidence interval between 0.88 and 0.99. These results imply a strong price rigidity for the U.S. economy. In addition, they are in line with the posterior median of 0.91 founded by Smets & Wouters (2004) for the period between 1983:1 and 2002:2.

Both Brazilian and U.S. productivity shocks are very persistent, although still stationary. The 90% confidence interval of the U.S productivity autoregressive coefficient ρ^* is between 0.97 and 0.99. This result is again in line with Smets & Wouters (2004), which estimates basically the same percentiles. The Brazilian productivity shocks are slightly less persistente, with a 90% confidence interval for the Brazilian autorregressive productivity parameter ρ between 0.81 and 0.97.

As we estimate a small-economy version of the model, the elasticity of substitution in consumption between imported and domestic consumption goods μ matters only for Brazilian economy. We estimate for this parameter a posterior mean of 0.10, which is much lower than the estimates for other countries. Caputo et al (2005) found a posterior mean around 0.56 for Chile, while Justiniano & Preston (2004) estimates median higher than 0.20 for Australia, Canada and New Zealand. Despite this counter-evidence, the Brazilian data seem to be very informative with respect to this parameter as its posterior mean is pratically unaffected by the use of higher prior means for μ. The relatively low value for this parameter would indicate that Brazilian output and imports have different composition and therefore a very low substitubility, implying that the TOT channel of monetary policy transmission is not so relevant for Brazilian economy.

The posterior mean of the inverse elasticity of labor supply φ is of 1.59, which means $1/\varphi$ equal to 0.63. In addition, the posterior 5% and 95% percentiles implies that the elasticity φ can take values between 0.38 and 2.17 with probability of 90%. The international evidence suggests lower elasticities. For example, Smets and Wouters (2004) with U.S. data and Liu (2005) with New Zealand data found posterior medians for φ around 2.90 and 2.60 respectively. This result is consistent with the expected higher responsiveness of the labor supply to wages in the countries with lower per-capital personal income, since economic theory postulates that poor workers are less reluctant in allocating free time to work. However, this is another preference parameter shared by Brazilian and U.S. economies, so that its estimate is influenced by U.S. data.[25]

The posterior mean of the inverse elasticity of intertemporal substitution in consumption σ is of 3.41, which means $1/\sigma$ equal to 0.29. Brazilian aggregate demand is responsive to changes in real interest rate, so that the conventional interest rate channel on monetary policy transmission is effective in the Brazilian economy. The data are very informative with respect to this parameter, with the posterior and prior means very far from each other. Although our estimates be above those usually found in earlier empirical works, the great variability that characterize these results does not allow a definitive conclusion.

The estimates of the Brazilian Taylor rule's coefficients for inflation and output gap, denoted by δ_π and δ_y, seem to be consistent to the inflation target regime implemented during the sample period: the posterior 90% confidence intervals lie entirely on the positive line, so that monetary policy reacts countercyclically to inflationary pressures. In addition, the estimates are comparable to those for countries with the same monetary regime: we found posterior means of 0.89 and 0.79 for δ_π and δ_y, while the corresponding values in Caputo et al (2005) are around 1.18 and 0.28

---

[25] We are currently working on a model with different preference parameters for Home and Foreign countries.

respectively. Monetary policy reacts to inflation more strongly than to output gap, although this discrepance is much lower than that in U.S. monetary policy. The posterior mean for the Brazilian interest rate smoothing coefficient δ_r is of 0.38, which is lower than the prior mean of 0.50 and the posterior mean of the U.S. corresponding parameter. Thus, Brazilian data does not suggest a very high persistence for the interest rate.

The posterior mean of the U.S. monetary policy persistence δ_r^* is of around 0.78, with a 90% confidence interval between 0.69 and 0.88. However, this persistence is lower than that founded by Smets & Wouters (2004), which get a posterior median of 0.91. The U.S. coefficients for inflation and output gap δ_π^* and δ_y^* are of 2.11 and 0.49 respectively, implying that U.S. monetary policy reacts more to inflation than to output gap. This qualitative result as well as the magnitude of the estimates are consistent to many empirical works for U.S. economy. However, they are not directly comparable to that found by Smets & Wouters (2004) because they use a different monetary rule.

3.5 Impulse-Response Functions

Figures (4) to (9) show the impulse-response (IR) functions for Brazilian and U.S. endogenous variables in response to unit-size positive temporary productivity, monetary policy and inflation shocks.[26] The green line is the mean IR function, while the blue and red lines are the 5% and 95%-percentiles counterparts respectively. As Brazilian economy is small, domestic shocks do not affect U.S. variables. Stationarity makes all variables converge to their steady-state levels in the long-run. As a result of our estimates for the autoregressive coefficients, productivity shocks have very persistent effects on the economy. On the other hand, monetary policy and inflation shocks are little persistent. Although more persistence can be achieved by incorporating habit formation and price indexation into the model, the theoretical part of the paper.would be more hard, so that we leave these important extensions for future work.

Computational constraints force us to use only 5000 randomly selected draws from the posterior distribution of the parameters. A IR curve for each relevant endogenous variable is then calculated for every draw. Next, the posterior mean and the posterior 5% and 95%-percentiles are calculated at each point of time.

3.5.1 Brazilian Productivity Shock

Figure (4) shows the effects of a positive Brazilian productivity shock. For a given level of output, higher productivity shrinks labor demand, pushing real wage and marginal cost down. Firms allowed to adjust prices react by cutting domestic prices in order to maintain markup unaltered. Consequently, the TOT rises around 1% on impact, which improves Brazilian goods' competitiveness. In addition, despite the small CPI inflation on impact, monetary policy reacts to the negative output gap by cutting nominal interest rate.[27] As a result, domestic consumption goes up in response to lower expected future real interest rates, which reinforce the pressure on output. As a joint effect of higher demand and lower relative price, Brazilian output rises by around 0.28% on impact. On the other hand, Brazilian currency depreciates by around 1% on impact in response to the expected period of monetary loosening. The consequent higher prices of imported goods not only reinforces the increase in TOT, but also explains the small positive CPI inflation observed on impact, despite the fall of domestic prices. After this initial movement, Brazilian currency starts depreciating towards steady state, so that CPI inflation gets negative. Over time, as more firms adjust prices and shock is amortized, economy returns gradually to steady state, which takes more than 20 quarters since productivity shocks are very persistent. Except for CPI inflation, no other variable crosses up the steady state level along the trajectory towards steady state.

---

[26] For lack of space, IR functions to U.S. productivity shocks are omitted, since the associated 5%-95% posterior intervals are much larger than for the other shocks. They can be provided under request.

[27] The positive effect of this shock on the output level under flexible prices is higher than under sticky prices, so that output gap diminishes.

3.5.2 Monetary Policy Shocks

Figure (5) shows the effects of a positive Brazilian monetary policy shock. On impact, Brazilian currency appreciates by around 1%, impairing the Brazilian goods' competitiveness since the TOT decreases around 1.5%. Simultaneouly, domestic consumption falls is response to higher future expected real interest rates. Both effects impact negatively on output, which falls by around 0.3%. In consequence, labor demand shrinks, pushing both real wage and marginal costs down, so that firms start cutting prices. Both domestic deflation and currency appreciation make CPI index fall around 0.20% on impact. Just after the shock, Brazilian currency overshoots and then converges very quickly to steady steady stateae. This explains the positive CPI inflation one period after the shock. In addition, the positive effect of higher nominal exchange rates on TOT overcomes the negative effect of domestic deflation, so that Brazilian external competitiveness starts improving. At the same time, monetary policy gets looser in response to negative output gap, which in turn expands domestic consumption. Both deppreciation and lower interest rates allow the economy to recover from the recession. Except for CPI inflation and exchange rate, variables start converging monotonically to steady state just after the shock, so that they do not cross up the steady state level. In general, the convergence takes around six quarters, so that the effects of the shock can be regarded as relatively little persistent when compared to the productivity shock.

Figure (6) shows the effects of a positive U.S. monetary shock. On impact, U.S. households contract consumption in response to higher future expected real interest rates. The consequent fall in U.S. output by 1% pushes U.S. real wage and marginal cost down, which produces a CPI deflation by 1%. Monetary policy persistence makes the nominal interest rate - and with it the rest of the economy - converge gradually to steady state, which takes around ten quarters to complete.

As a net effect of U.S. deflation and Brazilian currency deppreciation caused directly by the shock, the TOT increases with the higher domestic price of imported goods. Despite the improved Brazilian external competitiveness, Brazilian antiinflationary monetary policy is strong enough to put the economy into a recession, which push real wage down. Over time, U.S. monetary policy persistence causes nominal exchange rate to converge more slowly than output and real wage to steady state. This fact explains why real marginal cost - defined as nominal marginal cost less domestic price - gets positive before returning back to steady state: although real wage remains pratically unaltered two quarters after the shock, its purchase power in terms of domestic goods keeps rising during at least eight quarters more. Brazilian firms react to expected positive future real marginal costs by rising domestic prices, which gives rise to a persistent domestic inflation along the convergence period than annulates the negative effect of the imported goods deflation on CPI inflation.

3.5.3 Inflation Shocks

Figure (7) shows the effects of a positive Brazilian domestic inflation shock. Monetary policy reacts immediately by rising nominal interest rate. The consequent Brazilian currency appreciation reduces the domestic price of imported goods and thereby reinforces the negative impact of domestic inflation on TOT, which falls by around 1% on impact, deteriorating Brazilian external competitiveness and contracting the world demand for Brazilian goods. In addition, monetary tightening also leads domestic consumption to decline in response to higher future expected real interest rates. The consequent fall of the output by 2% shrinks the labor demand, while the lower domestic consumption increases the labor supply. Both effects all together push real wage and marginal cost down. However, the joint effects of this downward pressure on labor market and the Brazilian currency appreciation are not so strong to annulate the primmary and direct effect of the shock, so that not only a positive domestic inflation, but also a CPI inflation by 1%, are observed on impact. Monetary policy is very effective in fighting inflation, which is almost entirely eliminated one period after the shock. In consequence, monetary policy gets looser and Brazilian currency start deppreciating, so that external competitiveness and domestic consumption arises, pushing the output and real wages up. Due to monetary policy persistence, convergence of real variables to steady state takes around five quarters more than inflation. In the particular case of this shock, this relatively low persistence can in part be explained by the absence of price indexation in the model. Again, endogenous variables return monotonically back to equilibrium.

Figure (8) shows the effects of a positive U.S. inflation shock. On impact, U.S monetary policy reacts strongly by raising nominal interest rate by 1%. The consequent fall in U.S. consumption pushes output down by 1%. Due to the nagative impact of the recession on real wage and marginal cost, the primmary inflationary pressure only results in a CPI inflation of 1%. Over time, as inflation gradually falls and households expect lower real interest rates, the output increases and economy returns to stady state.

On impact, U.S. inflation and Brazilian currency depreciation - in response to U.S. monetary tightening - cause an increase in Brazilian CPI index and TOT by 1% and 1% respectively. Despite the improved external competitiveness, the antiinflationary reaction of the Brazilian monetary policy - the nominal interest rate goes up by 1% - makes output fall by 1%, which pushes Brazilian real wage and marginal cost down. Monetary policy is effective in neutralizing the external shock on Brazilian inflation, which takes only to two quarters to return back to steady state. Following the shock, Brazilian currency starts apreciating, pushing TOT down, and monetary policy gets looser with the decline in inflation. Again, the monetary policy effect on output is dominant, so that the economy starts going out of the recession.

3.5.4 Risk Premium

Figure (9) shows the effects of a positive risk premium shock. On impact, Brazilian currency depreciation pushes Brazilian TOT and CPI index up. Brazilian monetary policy reacts to inflation by raising nominal interest rate, which shrinks domestic consumption and allows a partial recovery of Brazilian currency. As a net effect of lower demand and improved external competitiveness, Brazilian output has a small initial increase of 1%. The heated labor market pushes real wage and marginal cost up and firms react to higher expected future marginal costs by increasing domestic price. Over time, the joint effect of domestic inflation and currency appreciation induced by high interest rate brings the TOT - and with it the rest of the economy - back to steady state. The monetary policy is very effective in accelerating the convervenge of inflation and output, which lasts no more than two quarters.

4 Conclusion

We build a two-country new Keynesian DSGE model with Calvo-type staggered price setting, which is an extension of the standard model largely used for monetary policy analisys in closed economies. The small country version of the model follows naturally as a limit case. This procedure has two advantages over the usual way as most part of the literature models a small open economy, which simply assumes that foreign variables follow exogenous processes. First, we do not take the risk of setting aside important channels of international monetary transmissions. In this sense, our model takes into account the effects of foreign frictions, such as price-stickiness, on the way as domestic and foreign real and monetary shocks are conveyed into the small country's economy. Second, we can build inpulse-response functions to see how these shocks affect simultaneously both economies in an integrated way.

The isoelastic demand faced by monopolistic firms implies that price adjustments reflect pressures on costs. Consequently, the NKP curve for open economies must embed the change in TOT as an additional pushing-cost variable feeding the inflation, since there are two distinct channels through which the deterioration of the TOT of some country can press the marginal cost of its firms and produce inflation. First, a higher TOT improves its external competitiveness, shifting the world demand towards its goods. The consequent higher output heats the labor market, pushing the real wage and marginal cost up. Second, *ceteris paribus*, a higher TOT increases the real wage and marginal cost in terms of the domestic goods, leading each firm to adjust its nominal price up in order to increase its relative price - in terms of the other domestic good - and thereby preserve their markup.

Although the law of one price holds for all goods, the assumption of home bias in households' preferences allows the model to capture the important empirical evidence of real exchange rate fluctuation. In this sense, promising avenue for future research would consist in enriching the model with other sources of PPP violation, such as nontradability and international segmentation in the goods' market. The latter assumption would allow the model to incorporate the imperfect pass-through observed in the actual data.

The empirical part of the paper yields promising qualitative results. In general, structural shocks impact on endogenous variables in the right direction, so that the model seems to be helpful as a tool for monetary policy analysis in the Brazilian economy. However, the magnitude and persistence of their dynamic effects are somewhat unrealistic. In order to reproduce actual macroeconomic time series more accurately, additional nominal and real frictions should be introduced, such as price indexation in order to create persistence in inflation and habit formation in consumption and/or real costs of adjustment in capital stock in order to create persistence in output.

5 References

Brooks, S.& Gelman, A. (1998). *General Methods for Monitoring Convergence of Iterative Simulations.* Journal of Computational and Graphical Statistics, 7(4): 434-455.

Calvo, G. A. (1983). *Staggered Prices in a Utility-Maximizing Framework*, Journal of Monetary Economics, 12(3): 383-398.

Caputo, R., Liendo, F. & Medina, J.P. (2005). *New Keynesian Models for Chile in the Inflation Target Period: A Structural Investigation.* Preliminary version. Central Bank of Chile.

Carlin, J.B., Gelman, A., Rubin, D. & Stern, A.S. (1995). *Bayesian Data Analysis.* Texts in Statistical Science. Chapman & Hall.

Chari, V., Kehoe, P. & Mcgrattan, E. (2002). *Can Sticky Price Models Generate Volatile and Persistence Real Exchange Rate?.* Review of Economic Studies, 69(3): 533-563.

Gali, J. & Monacelli, T. (2005). *Monetary Policy and Exchange Rate Volatility in a Small Open Economy.* Review of Economic Studies, 72(3):707-734

Justiniano, A. & Preston, B. (2004). *Small Open Economy DSGE Models: Specification, Estimation and Model Fit.* Manuscript, Columbia University.

Liu, P. (2005). *A Small New Keynesian Model of the New Zealand Economy.* Manuscript. Reserve Bank of New Zealand.

Rabanal, P. & Rubio-Ramírez, J. F. (2001). *Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach.* Federal Reserve Bank of Atlanta Working Paper.

Smets, F. & Woulters, R. (2004). *Comparing Shocks and Frictions in U.S. and Euro Area*

Business Cycles: A Bayesian DSGE Approach. Working Papers Series no.391. European Central Bank.

6 Appendix: Steady State

This appendix characterizes the steady-state equilibrium under the sticky and flexible-price cases, in which the productivity shifters remain at their long-run equilibrium values, i.e., $A_t = A_t^* = 1$. Under flexible prices, we know from subsection (2.3.7) that the Home and Foreign real marginal costs are time-invariant and given by $MC_t = MC_t^* = \frac{\varepsilon-1}{\varepsilon}$. We also know from subsection (2.2.2) that $U_t = U_t^* = 1$ in this case since all firms located in a same country charge identical prices. Substituting these results and eq.(72) into eq.(53) [eq.(73) into eq(54) for the Foreign counterpart] and using the IRS condition (46), we get

$$\begin{aligned} Y_t &= n^{\frac{\sigma}{\varphi}+1} A_t^{1+\frac{1}{\varphi}} \left(\frac{\varepsilon-1}{\varepsilon}\frac{1}{g^*(S_t)}\right)^{\frac{1}{\varphi}} \left(\frac{n}{1-n}\vartheta C_t^*\right)^{-\frac{\sigma}{\varphi}}, && (92) \\ Y_t^* &= (1-n)^{\frac{\sigma}{\varphi}+1} A_t^{*1+\frac{1}{\varphi}} \left(\frac{\varepsilon-1}{\varepsilon}\frac{S_t}{g^*(S_t)}\right)^{\frac{1}{\varphi}} C_t^{*-\frac{\sigma}{\varphi}}. && (93) \end{aligned}$$

In addition, we can substitute the IRS condition (46) and the results (35) and (37) into eq.(66) [results (36) and (38) into eq.(67) for the Foreign counterpart] to get

$$\begin{aligned} Y_t &= \frac{1}{1-n}\left[(1-\bar{\alpha}+\bar{\alpha}n)\,\vartheta g(S_t)^{\mu-\frac{1}{\sigma}} g^*(S_t)^{\frac{1}{\sigma}} + \bar{\alpha}^*(1-n)\,g^*(S_t)^{\mu}\right] C_t^*, && (94) \\ Y_t^* &= \left[n\bar{\alpha}\vartheta g(S_t)^{\mu-\frac{1}{\sigma}} g^*(S_t)^{\frac{1}{\sigma}} + (1-\bar{\alpha}^* n)\,g^*(S_t)^{\mu}\right] S_t^{-\mu} C_t^*, && (95) \end{aligned}$$

where $g(S_t)$ and $g^*(S_t)$ are defined in eqs.(35) and (37) respectively. For any levels of A_t and A_t^*, the flexible-price equilibrium values of S_t and C_t^* are the solution of the system formed by combining eqs.(92), (93), (94) and (95). Therefore, the steady-state levels for S_t and C_t^*, denoted by S and C^*, are the solution of this system for the particular case with $A_t = A_t^* = 1$, which is given by

$$\begin{aligned} & n^{\frac{\sigma}{\varphi}+1}\left(\frac{\varepsilon-1}{\varepsilon}\frac{1}{g^*(S)}\right)^{\frac{1}{\varphi}}\left(\frac{n}{1-n}\vartheta C^*\right)^{-\frac{\sigma}{\varphi}} && (96) \\ = \; & \frac{1}{1-n}\left[(1-\bar{\alpha}+\bar{\alpha}n)\,\vartheta g(S)^{\mu-\frac{1}{\sigma}} g^*(S)^{\frac{1}{\sigma}} + \bar{\alpha}^*(1-n)\,g^*(S)^{\mu}\right] C^*, \\ & (1-n)^{\frac{\sigma}{\varphi}+1}\left(\frac{\varepsilon-1}{\varepsilon}\frac{S}{g^*(S)}\right)^{\frac{1}{\varphi}} C^{*-\frac{\sigma}{\varphi}} && (97) \\ = \; & \left[n\bar{\alpha}\vartheta g(S)^{\mu-\frac{1}{\sigma}} g^*(S)^{\frac{1}{\sigma}} + (1-\bar{\alpha}^* n)\,g^*(S)^{\mu}\right] S^{-\mu} C^*. \end{aligned}$$

The steady state equilibrium exists if the system above has a solution, which depends on the parameters of the model. For the symmetric case, in which $\bar{\alpha} = \bar{\alpha}^*$ and $\vartheta = 1$, we can easily show that, for any n, there is a steady state solution with $S = 1$. For that, it is enough to substitute this value into both eqs.(96) and (97) and verify that both provide the same value for C^*.

.

Table 1

	Prior Distributions					Posterior Distribution			
parameter	distribution	5%^{1}	mean	95%^{1}	std	5%^{1}	mean	95%^{1}	std
Braz. Calvo price stickiness (ϕ)	Beta	0.20	0.50	0.80	0.18	0.87	0.92	0.98	0.03
U.S. Calvo price stickiness (ϕ^*)	Beta	0.20	0.50	0.80	0.18	0.88	0.94	0.99	0.03
Braz. productivity. autoreg. coeff. (ρ)	Beta	0.20	0.50	0.80	0.18	0.81	0.89	0.97	0.05
U.S. productivity. autoreg. coeff. (ρ^*)	Beta	0.20	0.50	0.80	0.18	0.97	0.98	0.99	0.01
elast. subst. btw dom. imp. goods (μ)	Gamma	0.25	1.00	2.15	0.60	0.01	0.10	0.17	0.04
inv. elast. labor supply (φ)	Normal	0.27	1.50	2.73	0.75	0.46	1.59	2.64	0.64
inv. elast. intert. subst. consump. (σ)	Normal	0.18	1.00	1.82	0.50	2.92	3.41	3.96	0.33
Braz. mon. pol. persistence (δ_r)	Beta	0.20	0.50	0.80	0.18	0.21	0.38	0.54	0.10
U.S. mon. pol. persistence (δ_r^*)	Beta	0.20	0.50	0.80	0.18	0.69	0.78	0.88	0.06
Braz. Taylor rule coeff. inflation (δ_π)	Normal	0.27	1.50	2.73	0.75	0.55	0.89	1.19	0.17
U.S. Taylor rule coeff. inflation (δ_π^*)	Normal	0.27	1.50	2.73	0.75	1.24	2.11	3.00	0.53
Braz. Taylor rule coeff. output gap (δ_y)	Normal	0.09	0.50	0.91	0.25	0.47	0.79	1.12	0.21
U.S. Taylor rule coeff. output gap $\left(\delta_y^*\right)$	Normal	0.09	0.50	0.91	0.25	0.17	0.49	0.82	0.21
std ($\xi_{A,t}$)	Inv. Gamma	0.95	2.00	3.85	1.50	2.10	3.80	5.65	0.82
std ($\xi_{A,t}^*$)	Inv. Gamma	0.95	2.00	3.85	1.50	4.27	6.50	8.74	0.98
std ($\xi_{M,t}$)	Inv. Gamma	0.95	2.00	3.85	1.50	1.58	2.16	2.71	0.29
std ($\xi_{M,t}^*$)	Inv. Gamma	0.95	2.00	3.85	1.50	0.71	0.95	1.19	0.12
std ($\xi_{\pi,t}$)	Inv. Gamma	0.95	2.00	3.85	1.50	3.71	4.91	5.94	0.60
std ($\xi_{\pi,t}^*$)	Inv. Gamma	0.95	2.00	3.85	1.50	1.19	1.59	1.99	0.22
std ($\xi_{rp,t}$)	Inv. Gamma	0.95	2.00	3.85	1.50	11.25	14.49	18.07	1.81
						Log Marg. Likelihood -465.2677			

1: 5% and 95% percentiles

std ($\xi_{A,t}$); std ($\xi_{A,t}^*$) : standard deviations of Brazilian and U.S. productivity shocks

std ($\xi_{M,t}$) : std ($\xi_{M,t}^*$) : standard deviations of Brazilian and U.S. monetary pol. shocks

std ($\xi_{\pi,t}$) : std ($\xi_{\pi,t}^*$) : standard deviations of Braz. and U.S. inflation (measurement errors) shocks

std ($\xi_{rp,t}$) : std. dev. of risk premium (measurement error) shocks

Figure 1: Prior and Posterior Densities

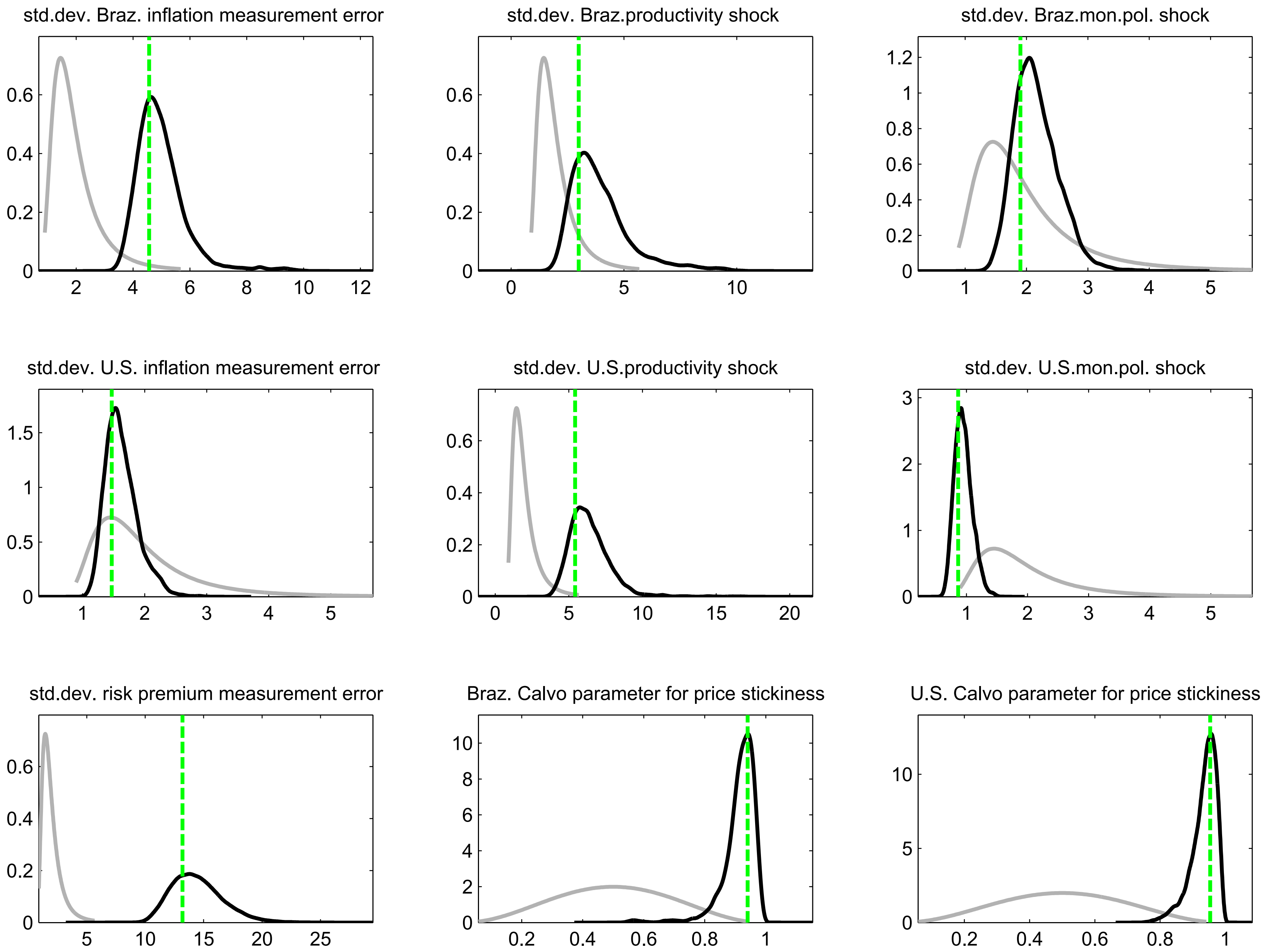


Figure 2: Prior and Posterior Densities

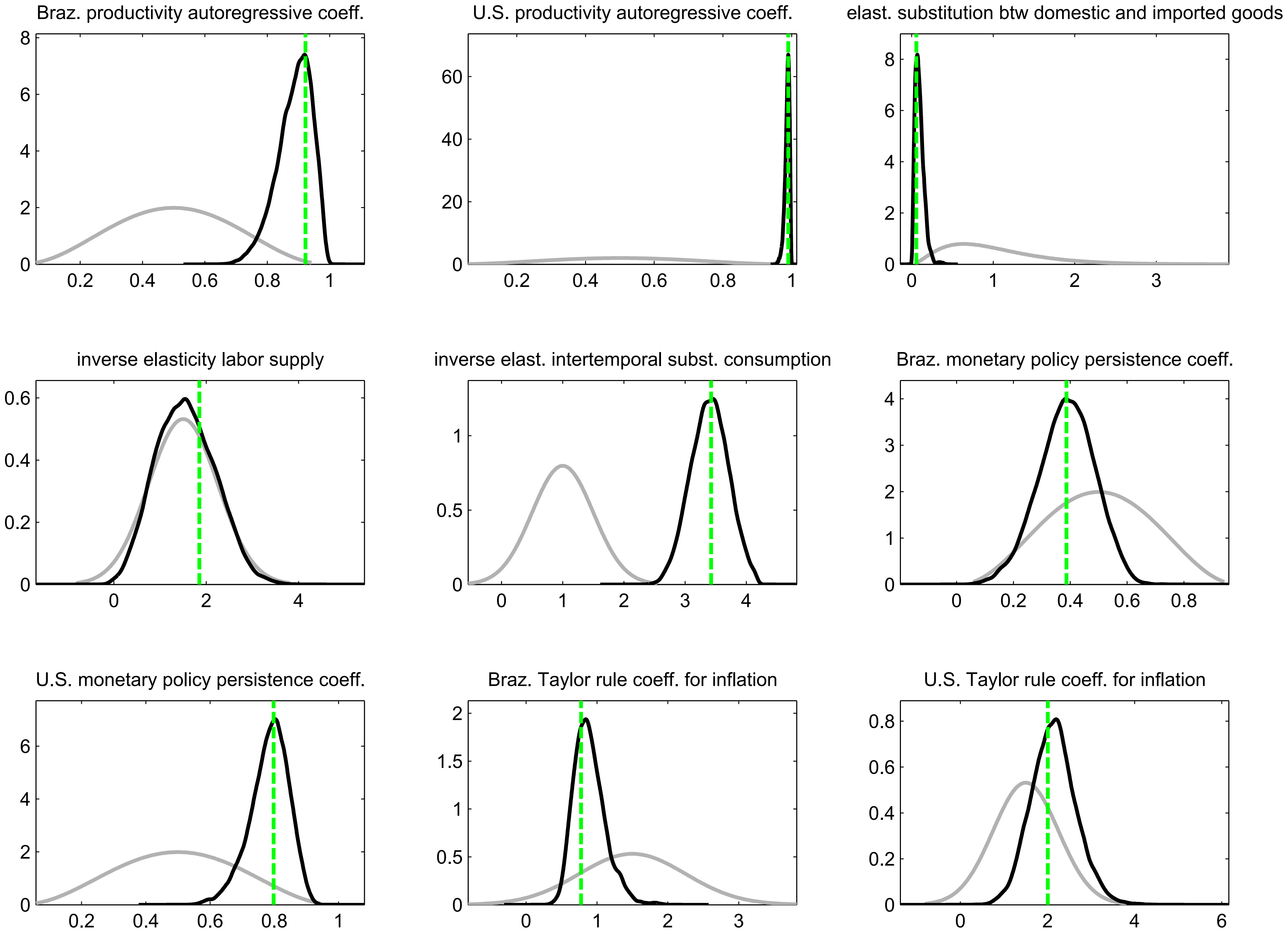


Figure 3: Prior and Posterior Densities

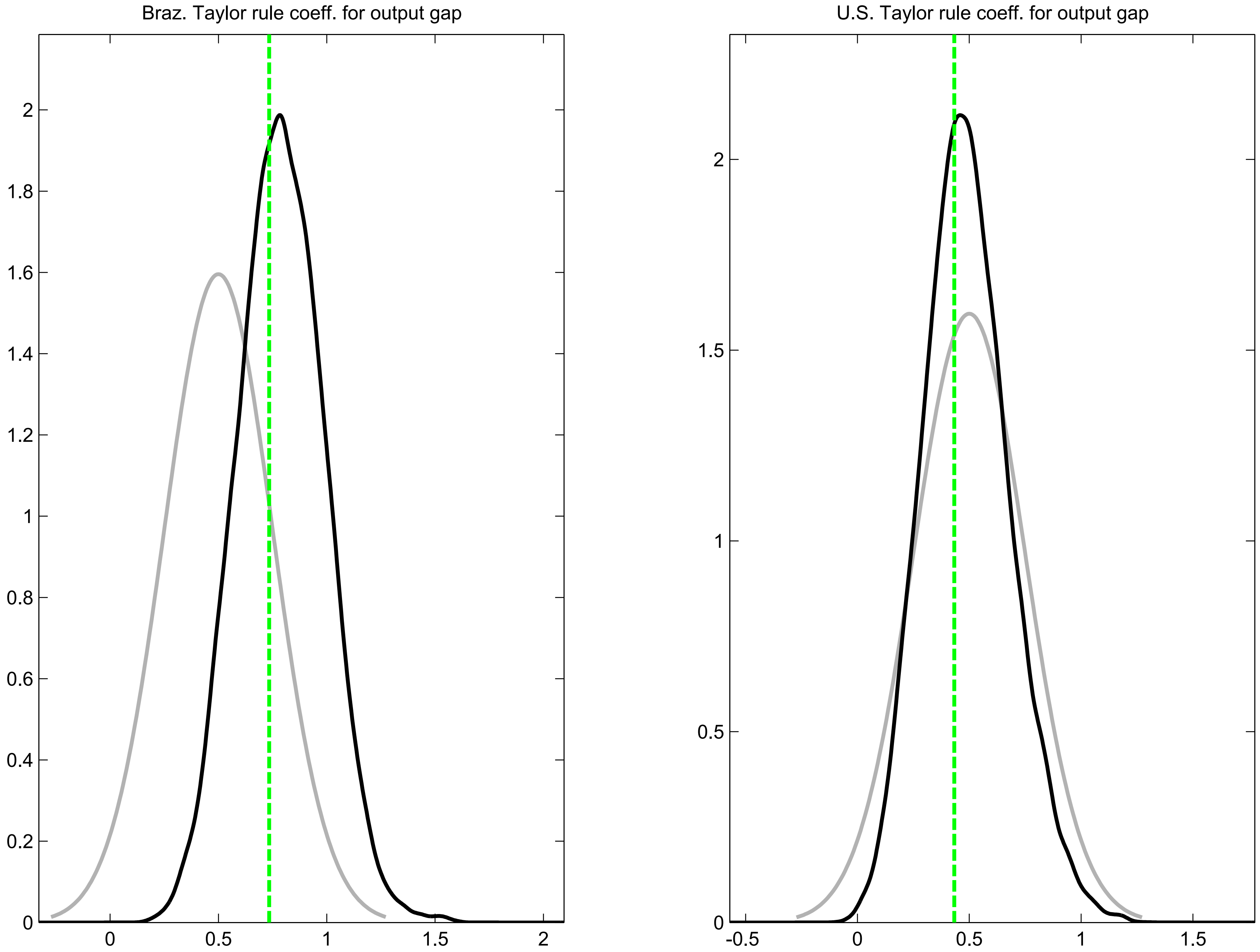


Figure 4: Impulse Response Functions to Brazilian Productivity Shock

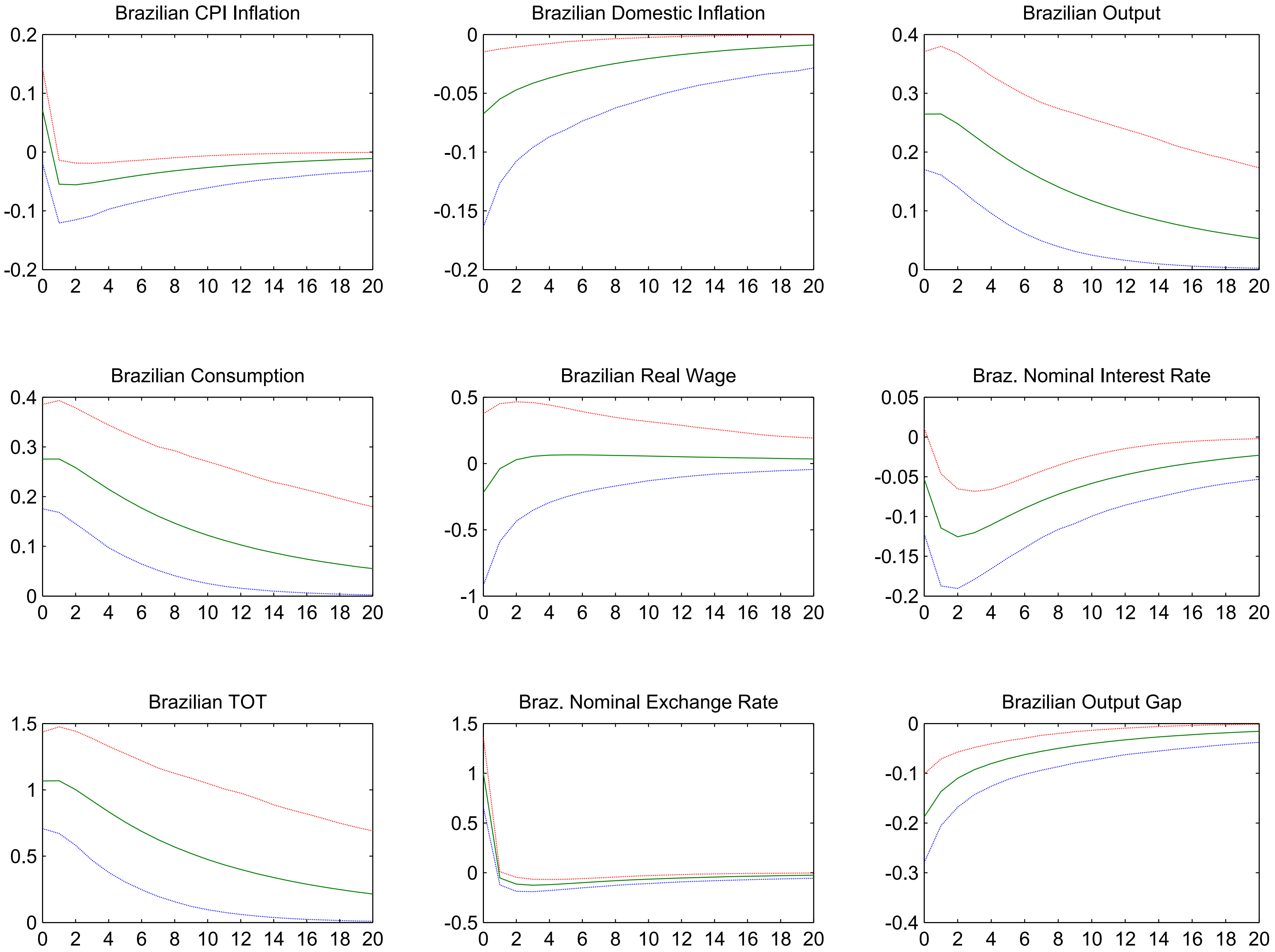


Figure 5: Impulse Response Functions to Brazilian Monetary Policy Shock

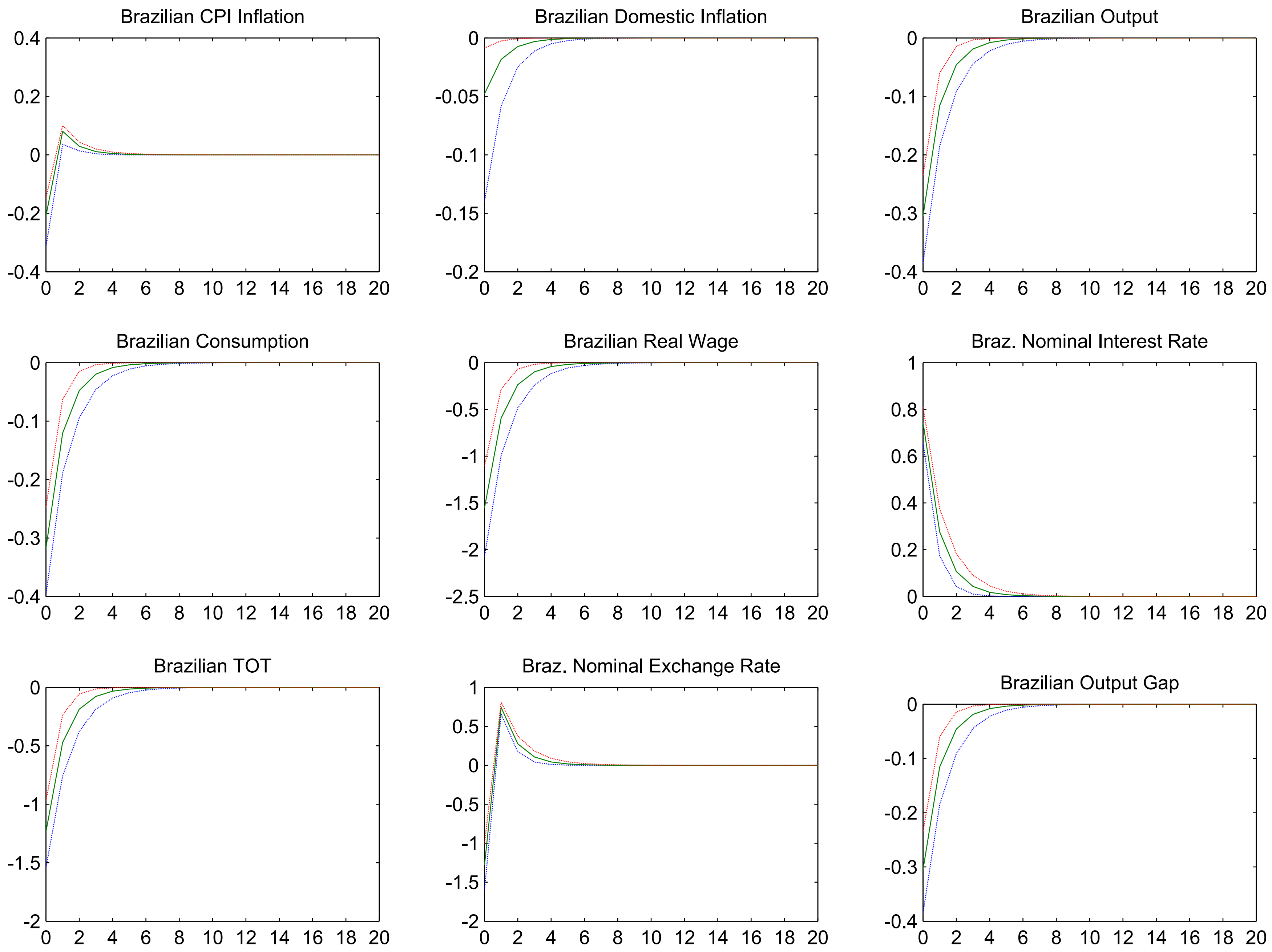


Figure 6: Impulse Response Functions to U.S. Monetary Policy Shock

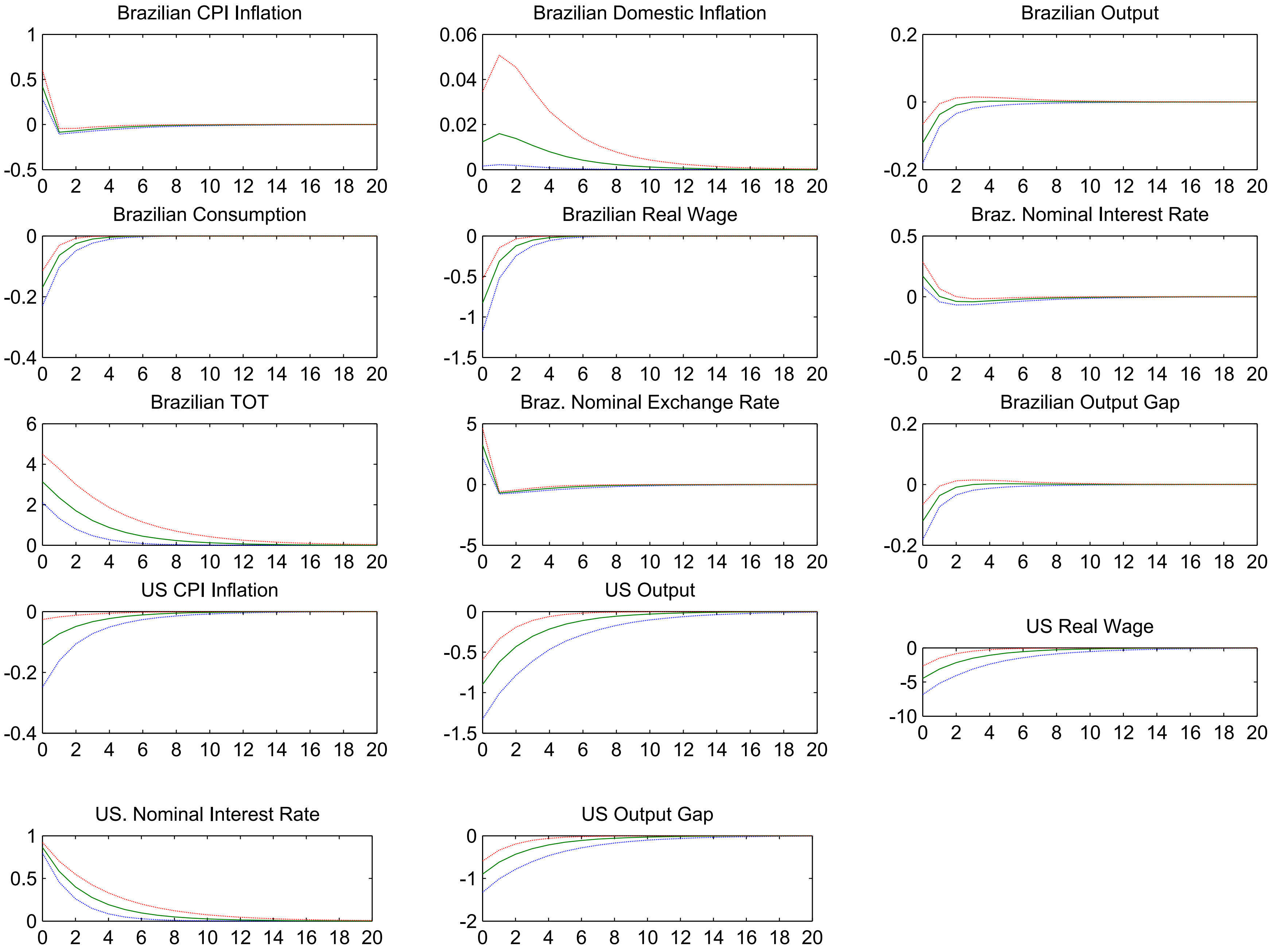


Figure 7: Impulse Response Functions to Brazilian Inflation Shock

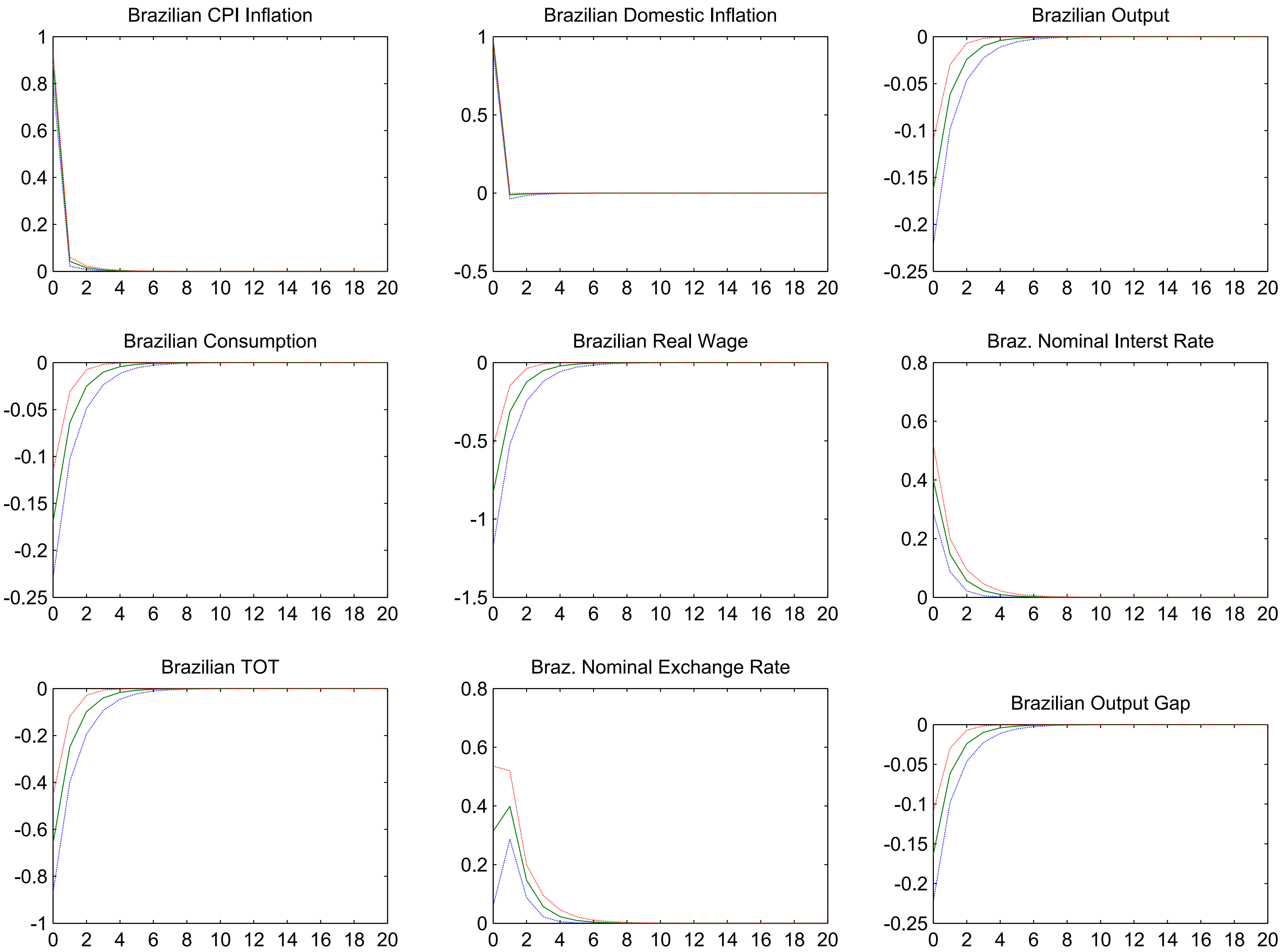


Figure 8: Impulse Response Functions to U.S. Inflation Shock

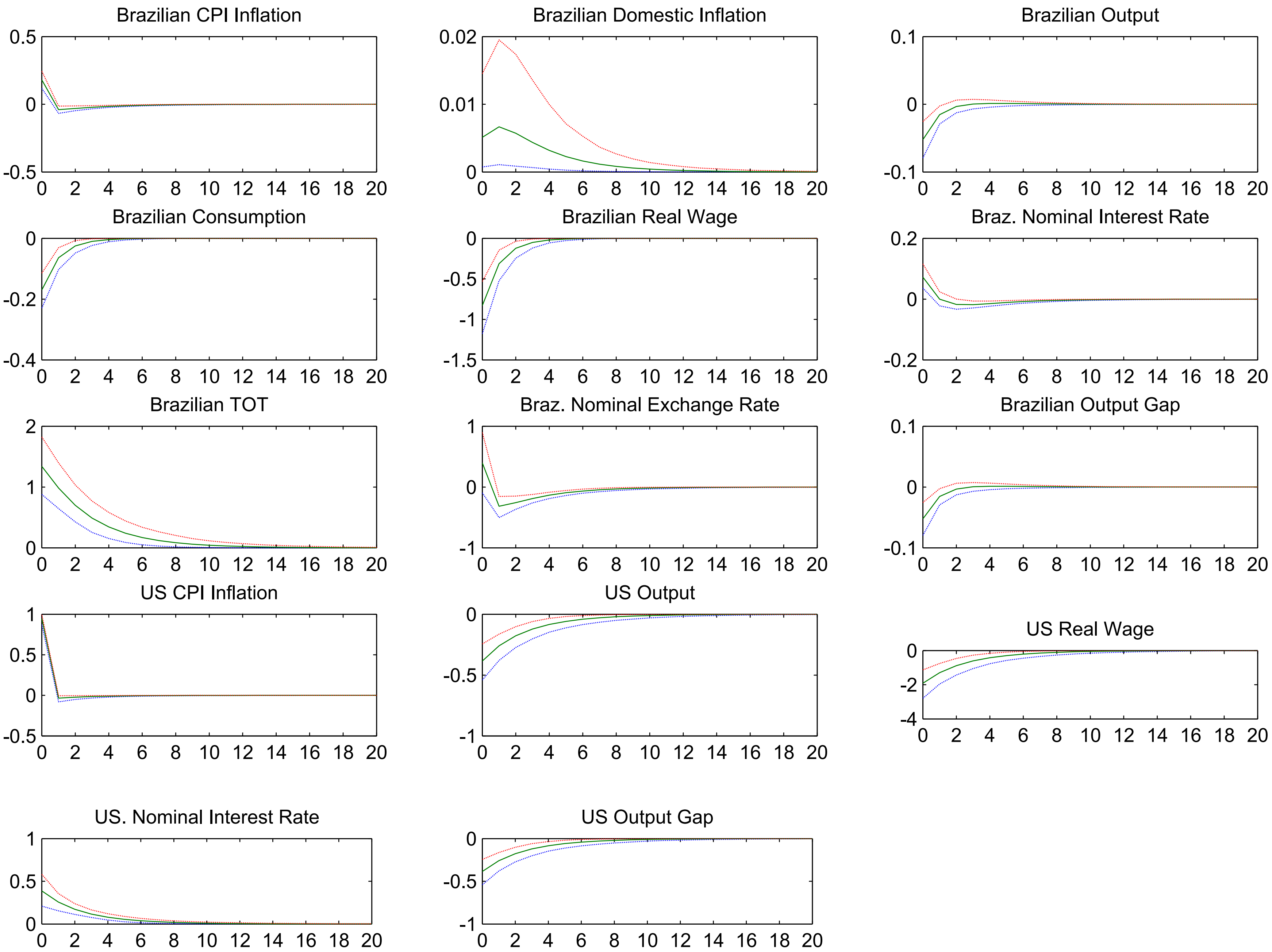
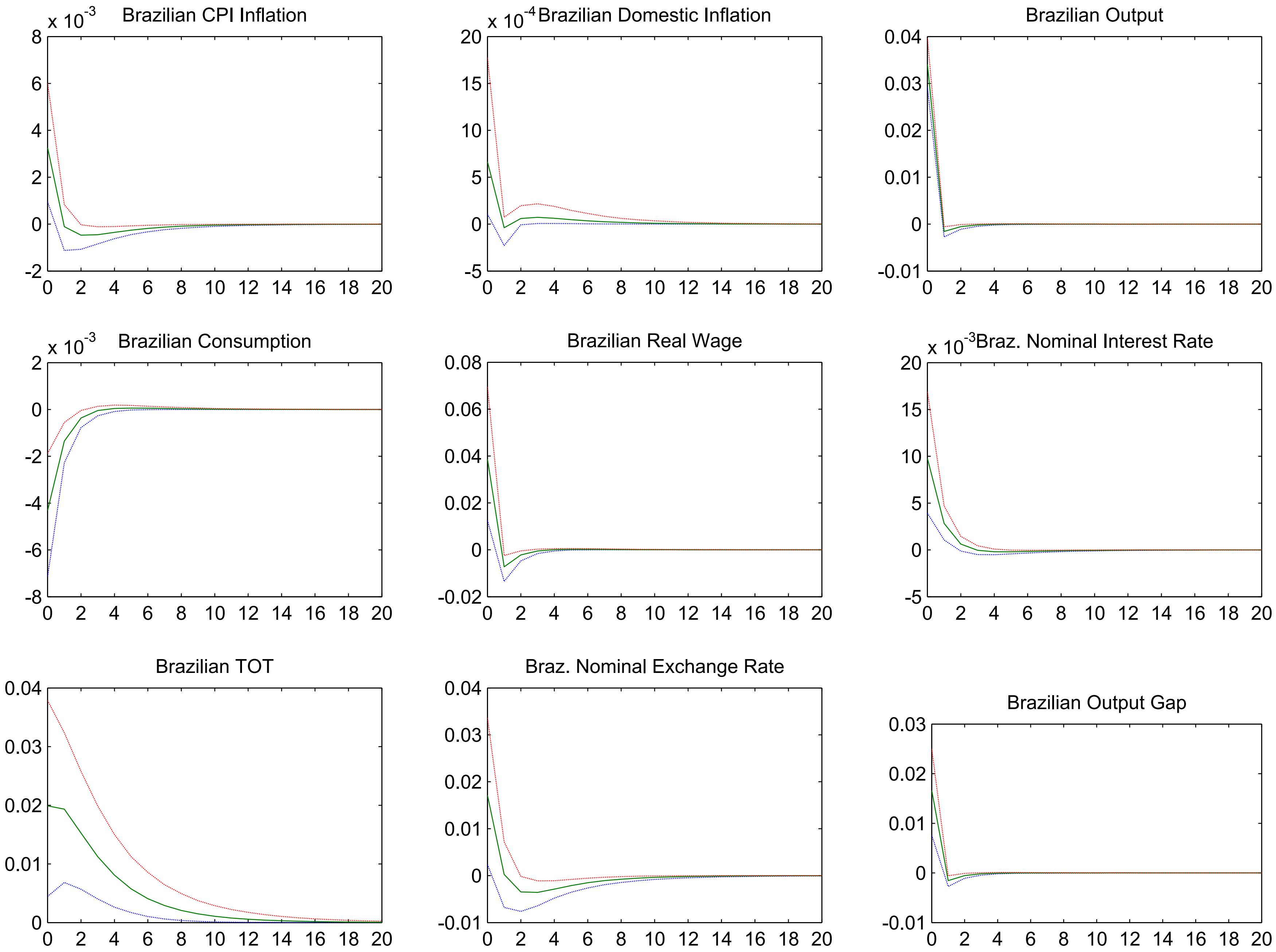


Figure 9: Impulse Response Functions to Risk Premium Shock



Ipea – Institute for Applied Economic Research

PUBLISHING DEPARTMENT

Coordination
Cláudio Passos de Oliveira

Supervision
Everson da Silva Moura
Reginaldo da Silva Domingos

Typesetting
Bernar José Vieira
Cristiano Ferreira de Araújo
Daniella Silva Nogueira
Danilo Leite de Macedo Tavares
Diego André Souza Santos
Jeovah Herculano Szervinsk Junior
Leonardo Hideki Higa

Cover design
Luís Cláudio Cardoso da Silva

Graphic design
Renato Rodrigues Buenos

The manuscripts in languages other than Portuguese published herein have not been proofread.

Ipea Bookstore

SBS – Quadra 1 – Bloco J – Ed. BNDES, Térreo
70076-900 – Brasília – DF
Brazil
Tel.: + 55 (61) 3315 5336
E-mail: livraria@ipea.gov.br

Composed in Adobe Garamond 11/13.2 (text)
Frutiger 47 (headings, graphs and tables)
Brasília – DF – Brazil

Ipea's mission
Enhance public policies that are essential to Brazilian development by producing and disseminating knowledge and by advising the state in its strategic decisions.

ISSN 1415-4765
9 771415 476001

ipea Institute for Applied Economic Research

Secretariat of
Strategic Affairs

BRAZILIAN GOVERNMENT
BRASIL