



**Monetary policy under uncertainty:
robustness and stabilization properties
of policy rules in Brazil**

Ajax R.B. Moreira and Marco A.F.H. Cavalcanti

IPEA/JICA Workshop

August 14-15, 2001

Rio de Janeiro, Brazil



**Monetary policy under uncertainty:
robustness and stabilization properties
of policy rules in Brazil**

Ajax R.B. Moreira and Marco A.F.H. Cavalcanti

IPEA/JICA Workshop

August 14-15, 2001

Rio de Janeiro, Brazil

Monetary policy under uncertainty: robustness and stabilization properties of policy rules in Brazil

Ajax R. B. Moreira

Institute of Applied Economic Research (IPEA)

Marco A.F.H. Cavalcanti*

Institute of Applied Economic Research (IPEA)

May 2001

KEYWORDS: Monetary policy rules, optimal monetary policy, parameter uncertainty, model uncertainty, Taylor rule.

JEL Classification: E52

Correspondence:

Ajax Moreira: Instituto de Pesquisa Econômica Aplicada, Av. Pres. Antonio Carlos 51/1508, Rio de Janeiro – RJ, CEP 20020-010, Brasil. Tel.: 55-21-38048145, e-mail: ajax@ipea.gov.br.

Marco Cavalcanti: Instituto de Pesquisa Econômica Aplicada, Av. Pres. Antonio Carlos 51/1716, Rio de Janeiro – RJ, CEP 20020-010, Brasil. Tel.: 55-21-38048120, e-mail: cavalcanti@ipea.gov.br.

* We are grateful for very helpful comments by Elcyon C. Lima. Any remaining errors are of course our sole responsibility.

Abstract

Based on three versions of a small macroeconomic model for Brazil, this paper presents empirical evidence on the robustness and stabilization properties of monetary policy rules. We find that: (i) parameter uncertainty should make policymakers react less aggressively to the economy's state variables, although this effect seems to be relatively small; (ii) optimal rules derived from each model perform very poorly under alternative models, whereas a simple Taylor rule is relatively robust; (iii) even within a specific model, the Taylor rule may perform better than the optimal rule under particularly unfavorable realizations from the policymaker's loss distribution function.

1. Introduction

The literature usually characterizes monetary policy as the management of a short-term instrument, such as the monetary base or interest rates, with the objective of minimizing expected deviations of inflation and the output gap from prespecified target levels (see, *inter alia*, Svensson (1996, 1997), Blinder (1998), Walsh (1998)). This is an optimal control problem in a stochastic framework that may be subject to four types of uncertainty:

- (a) uncertainty arising from the random disturbances in the macroeconomic model's equations (*additive* uncertainty);
- (b) uncertainty about the true values of the model's parameters (*multiplicative* uncertainty);
- (c) uncertainty about the correct measurement of the economy's state variables;
- (d) uncertainty about model specification.

It is a well-known result, due originally to Simon (1956) and Theil (1957), that in a linear-quadratic framework under purely additive uncertainty the optimal policy rule will be *certainty equivalent*, i.e. it will be identical to the one that would be obtained in a deterministic model. But as we move to other forms of uncertainty answers are not so straightforward, and there is considerable disagreement in the literature as to the direction and magnitude of the effects of such uncertainties on policy rules.

Consider the presence of parameter uncertainty, for instance. Clearly, the optimal rule will no longer be certainty equivalent; but how should the rule be modified in this case? A strand in the literature supports the classic result by Brainard (1967), according to which parameter uncertainty should attenuate the coefficients in the optimal reaction function, thus making authorities more "conservative" in the conduct of monetary policy.¹ But other studies argue that this effect may be relatively modest or even have the opposite sign, i.e. policy should be more aggressive under uncertainty.² The effects of parameter uncertainty on monetary policy rules are therefore an empirical issue.

Uncertainty on the model's correct specification – which equations should be included in the model, which variables should enter each equation, etc. – has even more ambiguous implications for the analysis of policy rules. This is clearly the worst form of uncertainty for policymaking, as the optimal rule derived from a specific model may perform very poorly if that model differs significantly from the "true" model. But how bad might a specific rule's performance be in the case of model misspecification? Should we expect fully optimal monetary policy rules to be less robust to model uncertainty than simple rules? These questions, once again, may only be resolved on empirical grounds.³

¹ See, for example, Sack (1998) and Martin and Salmon (1999) in the context of fully optimal rules and Hall et al. (1999) for *efficient* simple rules.

² Some studies that suggest Brainard's "conservatism principle" to have little empirical significance are Peersman and Smets (1998), Estrella and Mishkin (1999) and Rudebusch (2000). Shuetrim and Thompson (1999) show that under certain conditions parameter uncertainty might entail more aggressive policy.

³ For some empirical evidence on these issues, see Levin et al. (1999a,b), Rudebusch and Svensson (1998), Peersman and Smets (1998) and Drew and Hunt (2000).

Based on three versions of a small macroeconomic model for Brazil, this paper attempts to shed further light on these issues. First, it provides additional empirical evidence on the relation between parameter uncertainty and policy rules, which may improve our understanding of the extent to which Brainard's "conservatism principle" may be expected to hold in practice. Second, it compares the relative performance of optimal rules and a simple Taylor rule over three different model specifications, thus allowing an informal analysis of each rule's robustness and contributing to the literature on the choice between optimal and simple rules.

With regard to the latter point, a distinguishing feature of our paper is that it also evaluates each rule's performance "under risk", i.e. under unfavorable situations drawn from the model's estimated probability distribution. As the optimal rule only ensures that *expected* losses will be minimized, even within a specific model it might be possible for a simple rule to perform better than the optimal rule under particularly unfavorable realizations from the policymaker's loss distribution function. Our approach allows us to investigate this possibility.

The paper is organized as follows. Section 2 briefly overviews the main issues involved in the analysis of monetary policy rules under uncertainty. Section 3 presents and estimates three macroeconomic models for Brazil. Section 4 calculates the optimal policy rules for each model and analyzes their robustness and stabilization properties. It also compares these rules' performance with a simple Taylor rule. Section 5 summarizes our results and presents our final comments.

2. Monetary policy rules and uncertainty: an overview

Research on monetary policy rules has been concerned with two types of rules: optimal and simple rules. An optimal rule is the solution to a stochastic control problem in which policymakers aim to achieve some economic policy objective by managing their control instrument under the restrictions given by the macroeconomic model. The rule specifies the policymakers' setting of the instrument in response to all available information on the state of the economy as given by expected, current and lagged values of the system's variables.

A simple rule, on the other hand, is a reaction function in which policymakers do not use all available information when setting values for the policy instrument. It may be obtained as the solution to an optimization problem as before, subject to the additional restriction that policy should respond to only a limited subset of variables in the system. In this case, the rule is defined as an *efficient* or *optimized* simple rule. But simple rules may also be derived in a completely *ad hoc* manner, by exogenously specifying values for the reaction function coefficients, as in the well-known original "Taylor rule" (Taylor (1993)).

The analysis of monetary policy rules raises a number of interesting issues. One set of questions we will be focusing on refers to the effects of various forms of uncertainty on policy rules. According to the *certainty equivalence principle*, due originally to Simon (1956) and Theil (1957), the solution to an optimal control problem in a stochastic framework will be identical to the one that would be obtained in a deterministic model as long as the following conditions are met: (a) the loss function is quadratic; (b) the model is linear; (c) the model structure is known; (d) there are no measurement errors. In other words, if the only source of uncertainty comes from the model equations'

innovations (i.e. purely *additive uncertainty*), the optimal reaction function will not be affected by such uncertainty. In practice, however, the model structure is unknown and economic variables are imperfectly observed. This means that policymakers must rely on econometric estimates of parameters in possibly misspecified models and on indicators of relevant economic variables.

The main implication of these additional sources of uncertainty relates to the validity of the certainty equivalence principle. In the case of uncertainty on the true parameter values in a given model (i.e. *multiplicative uncertainty*, as it affects the system's multipliers), the optimal control problem still admits a closed-form solution, as shown by Chow (1975), but the optimal reaction function is no longer independent of the degree of uncertainty about the estimated parameters. In other words, certainty equivalence no longer holds.

In this context, the best-known result is due to Brainard (1967), who showed how parameter uncertainty might proportionately reduce the policy reaction function coefficients, thus making policymakers more cautious in the conduct of monetary policy. The idea that more uncertainty regarding the effects of monetary policy instruments should make policymakers more "conservative" is very intuitive and appears to be consistent with observed practices, which explains much of its popularity (Blinder (1998)). However, Brainard's "conservatism principle" may be of little empirical significance and may even be reversed under specific patterns of correlation among parameters. Some studies argue that monetary policy should indeed exhibit significantly less aggressive responses under parameter uncertainty, as shown by Sack (1998) and Martin and Salmon (1999) for optimal rules and by Hall et al. (1999) for efficient simple rules, but others show that this effect may be relatively modest (Peersman and Smets (1998), Estrella and Mishkin (1999), Rudebusch (2000)) or even have the opposite sign, i.e. more aggressive policy under parameter uncertainty (Shuetrim and Thompson (1999)). The effects of parameter uncertainty on monetary policy rules are therefore an empirical issue.

In the case of uncertainty related to the measurement of the economy's state variables, certainty equivalence depends on the specific way in which uncertainty enters the model. If it arises from the noisy observation of the relevant state variables, the optimal rule will still be certainty-equivalent, as shown by Chow (1975) in a linear-quadratic backward-looking framework and by Svensson and Woodford (2000) in a model with rational expectations. But if it refers to a signal-extraction problem, in which the relevant state variables are not observed and policymakers are restricted to act on observable indicators of these variables, then certainty equivalence no longer holds and optimal responses should be "attenuated" as proposed by Brainard (Swanson (2000)).

As regards efficient simple rules, the effects of this type of uncertainty are theoretically ambiguous but the empirical evidence also supports the conservatism principle (Smets (1998), Orphanides (1998), Peersman and Smets (1998), Drew and Hunt (2000)). In this paper we will not explicitly analyze this type of uncertainty, but by applying two different measures of the output gap to our macroeconomic model we may be able to indirectly investigate some of the consequences of using a mismeasured state variable.

Of course, uncertainty on the correct model structure is not restricted to parameter uncertainty. A much deeper form of uncertainty refers to the model's correct specification – which equations should be included in the model, which variables should

enter each equation, etc. This is clearly the worst form of uncertainty for policymaking, as the optimal rule derived from a specific model may perform very poorly if that model differs significantly from the “true” model. It is also a much harder problem to analyze, as specification possibilities are virtually infinite.

Some authors have tackled this issue by applying robust control techniques to the problem of finding policy rules that are reasonably robust to model misspecification (Sargent (1999), Tetlow and von zur Muehlen (1999)). However, we do not pursue this approach here. Instead, we follow Levin et al. (1999a,b) who restrict their analysis to a few macroeconomic models and then investigate the performance of policy rules derived from each model under each one of the alternative models. This approach provides an informal check on the robustness of the various rules considered by identifying those rules that perform relatively well over a range of different models. Although the results are obviously dependent on the arbitrary selection of models to be analyzed, they may provide useful indication as to which rules are most likely to perform badly in the case of model misspecification. The evidence in Levin et al. (1999a,b) suggests that simple policy rules are generally more robust than more complicated rules, which is a very intuitive result: as we adjust a rule to the specific dynamics of a given model, we increase the likelihood of making it inadequate for other models.

This brings us to the second main theme we will be concerned with: should policymakers adopt fully optimal monetary policy rules or simple rules in practice? The answer is closely related to how confident we are that the macroeconomic model at hand provides a reasonable approximation to economic reality. It is clear that within a specific model a simple rule could never outperform the optimal rule in terms of expected welfare loss; therefore, if we consider the model to be “good”, we should in principle opt for the optimal rule. In practice, however, the adoption of the optimal rule does not guarantee the best performance. As discussed above, there may be considerable uncertainty regarding the correct model specification, which means that the choice between optimal and simple rules should take into account not only each rule’s performance in a given model but also their robustness across other possible model specifications. Besides, even within a specific model it is possible for the optimal rule to deliver larger losses than a simple rule, if we happen to be in a particularly unfavorable situation drawn from the model’s probability distribution. After all, the optimal rule only ensures that *expected* losses will be minimized.

In order to shed some light on these issues we will investigate the following questions: How much do we gain within a specific model by adopting the optimal rule instead of a simple one? Does the optimal rule perform well “under risk”, i.e. under unfavorable conditions? How robust are simple and optimal rules over a range of possible model specifications?

Some evidence on these questions suggests that the performance of efficient simple rules is very similar to the performance of more complex optimal rules (Rudebusch and Svensson (1998), Peersman and Smets (1998), Drew and Hunt (2000)). This is an interesting result but it may simply be due to specific model structures that generate simple rules that are “similar” to the fully optimal rules. If, for example, the coefficients on contemporaneous output gap and inflation in the optimal rule account for a large part of the policy reaction we should expect an optimized Taylor rule to be “similar” to the optimal rule and therefore to have similar stabilization properties. We therefore find that

a more interesting exercise might be to compare the performance of optimal rules and *ad hoc* simple rules such as the original Taylor rule, as we do in Section 4.

3. Alternative model specifications for Brazil

According to McCallum (1999), the standard framework for monetary analysis in the recent literature has relied on macroeconomic models with three basic components: (i) an IS-type relation (or set of relations) that specifies the effects of monetary policy on aggregate demand and output; (ii) a price adjustment equation (or set of equations) that specifies the effects of the output gap and price expectations on inflation; and (iii) a monetary policy rule that specifies the policymakers' setting of a short-term instrument (usually the interest rate) in response to the state of the economy as given by expected, current and lagged values of the system's variables.

We follow this standard framework and set up three alternative model specifications, labeled Model 1 to 3. We assume from the outset that all three models are equally likely descriptions of the world. Our basic model is centered on the following set of equations:

$$u_t = \alpha_1^u + \alpha_2^u u_{t-1} + \alpha_3^u u_{t-2} + \alpha_4^u (r_{t-1} - E_{t-1} \pi_t) + \varepsilon_t^u \quad (1)$$

$$\pi_t = \alpha_1^\pi + \alpha_2^\pi \pi_{t-1} + \alpha_3^\pi u_{t-4} + \alpha_4^\pi d_{t-1} + \varepsilon_t^\pi \quad (2)$$

where u_t is the output gap; r_t is the nominal interest rate; π_t is monthly inflation in the consumer price index; $E_{t-1} \pi_t$ are inflation expectations based on information available at time t ; d_t is monthly nominal exchange rate depreciation.⁴ Equation (1) is a reduced form IS relation whereby real interest rates affect aggregate demand and thus the output gap. Equation (2) is a price adjustment equation in which current inflation depends on previous inflation, on exchange rate depreciation and on the output gap. The lag structure was chosen in the estimation process, as described below.

We assume for simplicity that inflation expectations are backward looking and given by the observed average rate in the previous quarter, i.e.

$$E_{t-1} \pi_t = (\pi_{t-1} + \pi_{t-2} + \pi_{t-3})/3 \quad (3)$$

The nominal exchange rate is determined by an uncovered interest parity condition; assuming that changes in exchange rate expectations, foreign interest rates and the risk premium all follow random walks, we may express exchange rate depreciation as:⁵

$$d_t = -\Delta r_t + \varepsilon_t^d \quad (4)$$

where ε_t^d captures innovations to the risk premium, foreign interest rates and expectations.

Equations (1) to (4) complete Model 1. The output gap variable used in the model is measured as:⁶

⁴ See the Appendix 1 for a detailed description of the data.

⁵ See for example Bogdanski et al. (2000).

$$u_t = y_t - y_t^* \quad (5)$$

where y and y^* are seasonally adjusted actual and potential GDP, respectively. We assume a deterministic trend for potential GDP, which is given by

$$y_t^* = y_{t-1}^* + \delta \quad (6)$$

From the actual GDP series and equations (5) and (6) we may calculate the output gap and use it in equations (1) and (2) above to estimate and solve Model 1.

Next we consider two alternative specifications. In Model 2 we keep the basic structure given by equations (1)-(4) intact and simply change the potential GDP definition (and consequently the output gap u_t). We now assume that potential GDP has a stochastic trend and is given by:

$$y_t^* = \phi y_{t-1}^* + (1 - \phi) y_{t-1} + \delta \quad (7)$$

so that the output gap is calculated from (5), (7) and the actual GDP series.

Comparison of results from models 1 and 2 may therefore help us infer the implications of alternative definitions of potential GDP and the output gap. Equations (6) and (7) may be interpreted as polar cases in the class of potential GDP models that is so popular in the literature on monetary policy rules.

In Model 3, on the other hand, we keep the hypothesis of a deterministic potential product and change the specification of aggregate demand. We replace equation (1) with a more disaggregated specification that includes equations for the first differences of consumption (c_t), investment (i_t) and net exports of goods and non-factor services (x_t):⁷

$$\Delta c_t = \alpha_1^c + \alpha_2^c \Delta c_{t-1} + \alpha_3^c (r_{t-1} - E_{t-1} \pi_t) + \alpha_4^c D_1 + \alpha_5^c D_2 + \varepsilon_t^c \quad (8)$$

$$\Delta i_t = \alpha_1^i + \alpha_2^i \Delta i_{t-1} + \alpha_3^i \Delta i_{t-2} + \alpha_4^i (r_{t-1} - E_{t-1} \pi_t) + \alpha_5^i + \varepsilon_t^i \quad (9)$$

$$\Delta x_t = \alpha_1^x \Delta x_{t-1} + \alpha_2^x (d_{t-1} - \pi_{t-1}) + \alpha_3^x D_1 + \alpha_4^x D_2 + \alpha_5^x D_3 + \alpha_6^x D_4 + \varepsilon_t^x \quad (10)$$

GDP is given by

$$y_t = c_t + i_t + x_t \quad (11)$$

and we may find an expression for the output gap by using (5) together with (11) and (6):

⁶ Note that variables are not in logarithms, so that (5) is an unusual definition for the output gap. Similarly, equations (6) and (7) are unusual specifications and provide only local approximations to the behavior of potential GDP. The reason for adopting this particular specification is that it makes our life much easier when comparing results from Model 3. But we should point out that our estimation results are relatively unchanged whether we use variables in levels or in logs.

⁷ D1, D2, D3 and D4 denote dummy variables for 1996.1, 1999.3, 1999.1 and 1999.11, respectively. The first dummy seems necessary to take into account possible measurement errors in the calculation of monthly consumption and exports series (see Appendix 1). The other dummies refer to periods of exchange rate instability following the transition to a floating exchange rate system in 1999.

$$u_t = u_{t-1} + \Delta c_t + \Delta i_t + \Delta x_t - \delta \quad (12)$$

Equations (8), (9), (10) and (12) therefore replace equation (1) as determinants of the output gap. The other equations in the model are basically unchanged, with the exception of equation (2), to which we now add two dummy variables D5 and D6:⁸

$$\pi_t = \alpha_1^\pi + \alpha_2^\pi \pi_{t-1} + \alpha_3^\pi u_{t-4} + \alpha_4^\pi d_{t-1} + \alpha_5^\pi D_5 + \alpha_6^\pi D_6 + \varepsilon_t^\pi \quad (2')$$

Model 3 thus includes equations (2'), (3), (4), (8), (9), (10) and (12). By comparing the results from this model with results from Model 1 we may infer the effects on policy rules of alternative specifications of aggregate demand.

We should note that, unlike models 1 and 2, Model 3 is able to capture a direct effect of real exchange rate devaluation (proxied by $d_{t-1} - \pi_{t-1}$) on aggregate demand.

3.1 Estimation results

Models 1 and 2 are estimated using monthly data from 1995.1 to 2000.12, while model 3 uses data from 1995.11 to 2000.12. The use of such short datasets is warranted by Brazil's macroeconomic environment, which was very unstable up to mid-1994, with exceptionally high inflation rates and frequent regime changes, and relatively stable afterwards, as a result of the 1994 Real Plan. This is a clear structural break and we therefore believe that the estimation of economic relations involving nominal variables should only begin in 1995, i.e. when inflation rates finally declined to reasonably low levels. Of course, the limited amount of degrees of freedom in the estimation should make us particularly cautious in interpreting our results. Model 3 uses an even smaller sample due to the fact that some of the relevant series are available only from 1995.8.

The lag structure and variables in each equation were selected according to the following criteria: (i) key coefficients should have the expected signs and be significant at least at 10% significance level; (ii) there should be no residual autocorrelation; (iii) we should not reject parameter constancy; (iv) models satisfying the first three criteria should be ranked according to information criteria. After conducting a specification search based on these criteria, we arrived at the final specification reported above.⁹

Table 1 presents the estimated coefficients. We estimated by SURE equations (1) and (2) for Models 1 and 2, and equations (2'), (8), (9) and (10) for Model 3.

⁸ D5 is a dummy variable with value 1 in 1996.3, 1996.8, 1997.2, 1997.8 and 1998.8 and zero otherwise; it refers to periods of unusually large reductions in the inflation rate within the fixed exchange rate regime subsample 1995.11-1998.12. D6 is a dummy variable with value 1 in 1999.7, 1999.10 and 2000.7 and zero otherwise; it refers to periods of unusually large increases in the inflation rate within the floating exchange rate regime subsample 1999.1-2000.12.

⁹ We used a Lagrange-multiplier test to test for residual autocorrelation and Chow 1-step ahead and break point tests to test for parameter constancy.

Table 1 – Estimation results

Model	Equation	Coefficients					
		1	2	3	4	5	6
M1	1	0.686 (1.3)	0.415 (3.9)	0.397 (3.8)	-0.548 (1.6)	-	-
	2	0.167 (2.2)	0.682 (8.5)	0.032 (2.0)	0.018 (1.7)	-	-
M2	1	0.954 (1.9)	0.265 (2.5)	0.261 (2.6)	-0.945 (2.9)	-	-
	2	0.213 (2.8)	0.679 (8.7)	0.048 (2.4)	0.019 (1.8)	-	-
M3	2	0.141 (2.3)	0.690 (9.2)	0.022 (1.5)	0.014 (2.0)	0.475 (3.5)	1.096 (6.6)
	8	0.703 (1.9)	-0.154 (1.9)	-0.390 (1.6)	8.646 (7.1)	2.838 (2.4)	-
	9	0.867 (3.5)	-0.821 (7.8)	-0.474 (4.5)	-0.526 (3.2)	-	-
	10	-0.280 (3.1)	0.060 (1.5)	-8.030 (6.3)	-1.500 (0.9)	1.630 (1.4)	-2.700 (2.2)

Estimation period - M1/M2: 1995.1-2000.12; M3: 1995.11-2000.12

Estimation method – SURE (t-statistics in parentheses)

Coefficients in the potential GDP equations were selected so as to maximize 6-step-ahead predictive power. Using the sample period from 1991.1 to 2000.12 we estimated the following coefficients: $\delta = 0.3, \phi = 0.92$.

Estimation results are not particularly good, as we note some rather low t-statistics and the need for several dummy variables in Model 3. However, our basic selection criteria are satisfied: all key coefficients have the expected signs and we do not reject either the absence of residual autocorrelation or parameter constancy.¹⁰ Appendix 2 presents graphs with Chow tests for each model.

It is interesting to compare the estimated coefficients for the real interest rate in equation (1) and for the output gap in equation (2) under models 1 and 2. As expected, both coefficients are smaller in Model 1, in which the output gap is derived from a deterministic potential product. For obvious reasons, these results will have important implications for the calculation of the optimal policy rule in the next section.

4. Calculation of the optimal monetary policy rule

We assume that the policymakers' problem is to minimize a quadratic loss function subject to the restrictions given by a linear macroeconomic model:

$$\text{Min}_z G = E_0 \sum_{t=1}^T \rho^t (\mathbf{y}_t - \mathbf{y}_t^*) \mathbf{R} (\mathbf{y}_t - \mathbf{y}_t^*) \quad (13)$$

$$\text{s.t. } \mathbf{y}_t = \mathbf{A} \mathbf{y}_{t-1} + \mathbf{B} \mathbf{z}_{t-1} + \mathbf{a}_t + \mathbf{e}_t, \quad \mathbf{e}_t \sim (0, \mathbf{S})$$

where \mathbf{y} are target variables with target level \mathbf{y}^* , \mathbf{z} is a vector of control variables, \mathbf{a} is a vector of aggregate effects from exogenous variables, \mathbf{e} is a vector of stochastic shocks and \mathbf{A} and \mathbf{B} are non-stochastic parameter matrices, and \mathbf{R} is the weight matrix that determines the relative importance of each endogenous variable's squared deviations from target. \mathbf{R} is usually taken to be diagonal.¹¹ Let ρ be a discount factor.

It is possible to show that the solution to this stochastic linear-quadratic problem is equal to the solution obtained in the non-stochastic case ($\mathbf{e}_t=0$) and is a linear deterministic function of the target variables (Chow, 1975):

$$\mathbf{z}_t = \mathbf{F}_t \mathbf{y}_t + \mathbf{f}_t \quad (14)$$

¹⁰ According to the 1-step Chow tests we might reject parameter stability for a couple of periods, but we believe this may be due to the presence of outliers.

¹¹ Note that this is a quite general problem, as difference equations of any order may be reparameterized as first-order difference equations by including additional endogenous variables.

This is a remarkable result known as the *certainty equivalence principle*: the solution to an optimal control problem in a stochastic framework may be obtained analytically and is identical to the one that would be obtained in a similar model without the stochastic term. Function (14) is called the optimal reaction function (ORF).¹²

It is clear that given the model parameters $C = (A, B)$ we may calculate the parameters in the reaction function, $F|C$. However, model parameters are usually estimated, so that we may only know their probability distribution $C \sim N(\hat{C}, W)$. In this case, the policymakers' problem is to

$$\text{Min}_z G = E_0 \sum_{t=1}^T \rho^t (y_t - y_t^*)' R (y_t - y_t^*) \quad (15)$$

$$\text{s.t. } y_t = A y_{t-1} + B z_{t-1} + a_t + e_t, \quad e_t \sim (0, S), \quad (A, B) = C \sim N(\hat{C}, W)$$

In this new problem the presence of multiplicative uncertainty implies that certainty equivalence no longer holds, but we may still calculate a linear optimal reaction function, as shown by Chow (1975).

Following Chow (1975), we now show how to calculate the optimal reaction function for problems (13) and (15) in turn.¹³

The solution to problem (13) is obtained by using the difference equations (19-20):

$$H_{t-1} = R + \rho A' R A - \rho A' H_t B (B' H_t B)^{-1} B' H_t A \quad (19)$$

$$h_{t-1} = R y^* - (A + B F_t)' (H_t a_t - h_t) \quad (20)$$

These equations are solved iteratively from the last to the first period, starting with the initial conditions $H_T = R$, $h_T = R y^*$. This procedure gives us values for H_t and h_t in each period, which are then used in the calculation of the optimal reaction function:

$$z_t = -D_t^{-1} B' H_t A y_t - D_t^{-1} B' (H_t a_t - h_t) = F_t y_t + f_t \quad (21)$$

$$\text{where } D_t = (B' H_t B)$$

As the econometric model may include time-dependent exogenous variables (a_t), we use the indicated nonstationary solution. Nonetheless, the reported optimal reaction function refers to the initial period.

For problem (15) with unknown parameters the solution is found by using the same terminal condition ($H_T = R$, $h_T = R y^*$) and the approximated equations (22)-(23)

$$\begin{aligned} H_{t-1} &= R + \rho E[A' R A - \rho A' H_t B (B' H_t B)^{-1} B' H_t A] \\ &\cong R + \rho \sum_w [A_w' R A_w - \rho A_w' H_t B_w (B_w' H_t B_w)^{-1} B_w' H_t A_w] / N \quad (22) \end{aligned}$$

$$h_{t-1} \cong R y_t^* - E(A_w + B_w F_t)' (H_t a_t^w - h_t) \cong R y_t^* - \sum_w (A_w + B_w F_t)' (H_t a_t^w - h_t) / N \quad (23)$$

¹² The linear-quadratic framework is crucial for the derivation of certainty-equivalence. It is sometimes criticized as being "unrealistic" and "arbitrary" but, as pointed out by Blinder (1998, p.10), for small changes in macroeconomic variables "any model of an economy is approximately linear and any convex objective function is approximately quadratic". Besides, some studies show that, under certain conditions, the hypothesis of a quadratic loss function is not necessary for the validity of the certainty-equivalence principle [Chadha and Schellekens (1999)].

¹³ We only provide a sketch of the procedures. For further details, see Chow (1975).

The optimal reaction function is:

$$z_t = -[\tilde{D}_t^{-1} E[B'H_t A y_t] - \tilde{D}_t^{-1} E[B'(H_t a_t - h_t)]] = F_t y_t + f_t \quad (24)$$

where

$$\tilde{D}_t = E[B'H_t B] \cong \sum_n B_w' H_t B_w / N$$

$$F_t \cong -\tilde{D}_t^{-1} \sum_n B_w' H_t A_w y_t / N$$

$$f_t \cong -\tilde{D}_t^{-1} \sum_n B_w' (H_t a_t^* - h_t) / N$$

The functions were approximated by simulation using $N=1000$ replications of model parameters $(A_w, B_w) \sim N(\hat{C}, W)$ using the estimated distribution of each macroeconomic model in Section 3. Once again, we use the nonstationary solution and the optimal reaction function refers to the initial period.

The above procedures allow us to estimate the optimal reaction function for each one of the models presented in Section 3. But we do not know which empirical model provides the best approximation to the "true" model and are therefore uncertain as to which policy rule should be regarded as the best overall rule.

One way to approach this question is to investigate each rule's robustness by calculating for each model n the best reaction function F_n and then estimating the loss $G_m | F_n$ under each model m , given each reaction function F_n . For each pair (m, n) and model realization w we have one trajectory $\{y_{t, mn}^w\}$. Expected loss¹⁴ is then estimated by replicating each model 1000 times, and taking the expected value of the loss function.

$$y_{t, mn}^w = (A_w^m + B_w^m F_t^n) y_{t-1, mn}^w + f_t^n + a_t^w \text{ where } (A_w^m, B_w^m) \sim N(\hat{C}^m, W^m) \quad (25)$$

$$k_{mn}^w = \sum_t \rho^t (y_{t, mn}^w - y_t^*) R(y_{t, mn}^w - y_t^*), E(k_{mn}) \cong \sum_w k_{mn}^w / N \quad (26)$$

In the following exercises we assume that policymakers target a 0.487% monthly inflation rate (corresponding to 6% per year) and a zero monthly output gap, with equal weights. Results would obviously be different if we adopted some other criterion, such as to target average inflation over one year. In all cases we assume the discount factor to be 1.

4.1 Results

Table 2 presents the optimal reaction function coefficients for each model estimated in Section 3, both under the hypothesis of purely additive uncertainty, i.e. known parameters $[F|E(C)]$, and under the hypothesis of additive and multiplicative uncertainty, i.e. unknown parameters $[F|C]$.

¹⁴ The loss function $G = E_0 \sum_t \rho^t (y_t - y_t^*) R(y_t - y_t^*) = K + E_0 V$ can be decomposed in one controlled component K and one non-controlled V , where $K = \sum_t \rho^t (E y_t - y_t^*) R(E y_t - y_t^*)$ and $V = \sum_t \rho^t (y_t - E y_t) R(y_t - E y_t)$.

Table 2 - Optimal reaction functions

Model	RF	Δc	Δi	Δi_{-1}	Δx	π	π_{-1}	π_{-2}	u	u_{-1}	r_{-1}
M1	F E(C)	-	-	-	-	0.359	0.330	0.330	0.760	0.724	0.000
	F C	-	-	-	-	0.352	0.330	0.329	0.568	0.540	0.000
M2	F E(C)	-	-	-	-	0.350	0.330	0.330	0.283	0.278	0.000
	F C	-	-	-	-	0.346	0.330	0.329	0.250	0.243	0.000
M3	F E(C)	0.828	0.180	-0.480	0.700	0.256	0.311	0.311	0.979	0.000	0.058
	F C	0.750	0.160	-0.439	0.637	0.262	0.313	0.313	0.887	0.000	0.053

From this table we may draw some very interesting conclusions. First, comparison between response coefficients in F|E(C) and in F|C within each model shows that the presence of multiplicative uncertainty should make policymakers react less aggressively to the economy's state variables, as suggested by Brainard's "conservatism principle". However, this effect seems to be relatively small in most cases, which is consistent with the findings in Peersman and Smets (1998), Estrella and Mishkin (1999) and Rudebusch (2000), *inter alia*.

Second, comparison among coefficients across models shows remarkable similarities in the optimal response to inflation but quite different responses to the output gap. For example, for each unit increase in the current output gap the interest rate should be raised about 0.57-0.76 according to Model 1 and about 0.25-0.28 according to Model 2. In part, this may be explained by the different potential GDP definitions used in each model. In Model 1, the use of a deterministic potential output implies: (i) an output gap with larger variance; (ii) smaller effects of monetary policy on the output gap; (iii) smaller effects of the output gap on inflation (see Table 1). As a consequence, optimal policy must respond more aggressively to the output gap in order to keep both the output gap and inflation under control.

Third, it is interesting to analyze the coefficient on the lagged nominal interest rate, which enters the optimal reaction function through equation (4) and may be interpreted as a measure of interest-rate smoothing - the higher the coefficient, the more sluggish interest rate movements would tend to be. According to our findings, this coefficient is very small in all cases, especially in Models 1 and 2 (when they are nearly zero); we therefore conclude that according to our empirical models optimal policy should incorporate very little interest rate smoothing.

The following results refer to the application of the optimal rules derived under the hypothesis of both additive and multiplicative uncertainty, i.e. [F|C].

Table 3 presents each model's largest characteristic roots before applying the optimal policy rule (the "free" case) and after applying it (the "controlled" case). We note that the propagation of shocks, measured by the system's characteristic roots, is larger in the free case than in the controlled case.

Table 3 - Characteristic roots modulus (>0.5) and cycles

Model	Free			Controlled
	1	2	3	1
M1	0.93	0.57(16.8)		0.70
M2	0.90	0.65	0.65(10.2)	0.70
M3	1	0.69(2.8)	0.68	0.70

* Cycles in parentheses (in months)

The welfare loss obtained under each model m given the optimal reaction function from each model n is a random variable $\{k_{mn}^w\}$ whose distribution measures loss uncertainty. We summarise this distribution by presenting the expected value and the 95% percentile, which we call “loss at risk”. To make interpretation easier, each measure was normalized by the corresponding model’s optimal rule, so that the expected loss in M1 given M1’s optimal rule (RF|M1) is 1; the expected loss in M1 given M2’s optimal rule (RF|M2) is 36% higher; and so on. We also calculate losses given that monetary policy follows the classic simple Taylor rule (TR).

Table 4 - Loss $K(m,n)$:normalized by the model’s own ORF

Model	$E(k_{mn}^w)$				Loss at risk			
	TR	RF M1	RF M2	RF M3	TR	RF M1	RF M2	RF M3
M1	1.16	1.00	1.36		3.35	9.42	4.40	
M2	1.04	2.65	1.00		2.69	2159	2.41	
M3	1.27	7.09	2.33	1.00	3.39	113	6.88	5.11

Given any specific model, we note that the adoption of the optimal rule calculated from some other model always leads to very poor results in terms of expected loss. The additional loss caused by using the “wrong” optimal policy ranges from 36% in the best case (use of RF|M2 in M1) to 600% in the worst case (use of RF|M1 in M3).

On the other hand, the performance of the simple Taylor rule across models 1-3 is quite robust. In every model, the Taylor rule is ranked second, losing only to the model’s own optimal rule in terms of expected loss. In other words, given any specific model it is always better to adopt the Taylor rule than to adopt the “wrong” optimal rule calculated from some other model. Even more important, the performance of this simple rule is reasonably close to each model’s own optimal rule; the additional loss from using the Taylor rule varies from only 4% in Model 2 to 27% in Model 3.

The Taylor rule’s robustness is even more remarkable when we consider “loss at risk”, i.e. under particularly unfavourable realizations from the model’s probability distribution. In such circumstances, the Taylor rule’s performance deteriorates much less than the optimal rules. Under models 1 and 3, the Taylor rule performs even better than those models’ own optimal rules. This is a very interesting result: even within a specific model, the adoption of the optimal rule does not guarantee the best performance in practice. The reason for this should be clear: the optimal rule is calculated so as to minimize *expected* losses, but the probability distribution for losses may be such that in practice other rules may provide better performance, as Table 4 shows.

Table 5 gives a more detailed account of the results from our exercise. It reports the variances of the output gap, inflation and interest rates calculated in each model under each possible policy rule. We also report variances of consumption, investment and net exports for Model 3.

Table 5 – Variance of each $V(y)$:normalized by the model's own ORF

		Expected value				Loss at risk			
		TR	RF M1	RF M2	RF M3	TR	RF M1	RF M2	RF M3
Model 1	$V(u)$	1.18	1.00	1.38	-	3.56	10.35	4.70	-
	$V(\pi)$	0.90	1.00	1.02	-	3.15	4.08	3.96	-
	$V(r)$	0.87	1.00	0.81	-	1.47	2.31	1.38	-
Model 2	$V(u)$	1.06	2.79	1.00	-	2.84	2356.90	2.52	-
	$V(\pi)$	0.92	1.33	1.00	-	2.67	55.86	2.98	-
	$V(r)$	1.18	1.70	1.00	-	1.75	461.81	1.49	-
Model 3	$V(u)$	1.29	7.31	2.37	1.00	3.46	118.43	7.16	5.29
	$V(\pi)$	0.97	2.59	1.50	1.00	4.19	12.19	6.03	4.88
	$V(r)$	0.89	1.61	0.86	1.00	1.51	14.49	1.40	2.00
	$V(\Delta c)$	0.82	2.81	0.71	1.00	3.00	99.45	2.16	7.11
	$V(\Delta i)$	0.81	1.77	0.78	1.00	2.07	43.66	1.84	3.45
	$V(\Delta x)$	1.00	1.14	1.02	1.00	3.42	4.67	3.46	3.78

5. Conclusion

Based on three versions of a small macroeconomic model for Brazil, this paper has provided empirical evidence on the relation between uncertainty and monetary policy rules and on the robustness of optimal and simple rules across different model specifications. Our main findings were as follows:

- (1) The presence of multiplicative uncertainty should make policymakers react less aggressively to the economy's state variables, as suggested by Brainard's "conservatism principle", although this effect seems to be relatively small.
- (2) Optimal policy should respond more aggressively to the output gap when potential output is deterministic than when it is stochastic.
- (3) The simple Taylor rule is relatively robust across model specifications, whereas the optimal rules derived from each model perform very poorly under alternative models.
- (4) Even within a specific model, the Taylor rule may perform better than the optimal rule under particularly unfavourable realizations from the policymaker's loss distribution function.

Needless to say, these conclusions are dependent on the specific model structures used, so that we should be careful in interpreting them. One particular feature of our models, which may seem inappropriate in the case of Brazil and therefore recommends extra caution in accepting the above conclusions, refers to the absence of links between monetary and fiscal policy.

As changes in interest rates affect the public debt service, the existence of a large outstanding debt places an additional constraint on monetary policymaking, which probably should be taken into consideration by the authorities when setting the interest rate. As a useful extension to our results, it would be interesting to build a similar model that accounts for such effects.

We should also remark that in this text losses were calculated using expected trajectories and optimal reaction functions were time dependent due to the time dependent exogenous variables. If these variables were included in the model we could get a stationary solution for the optimal reaction function. It would be interesting to evaluate the effects of stationary optimal reaction functions and of simulated shocks on the loss distribution.

References

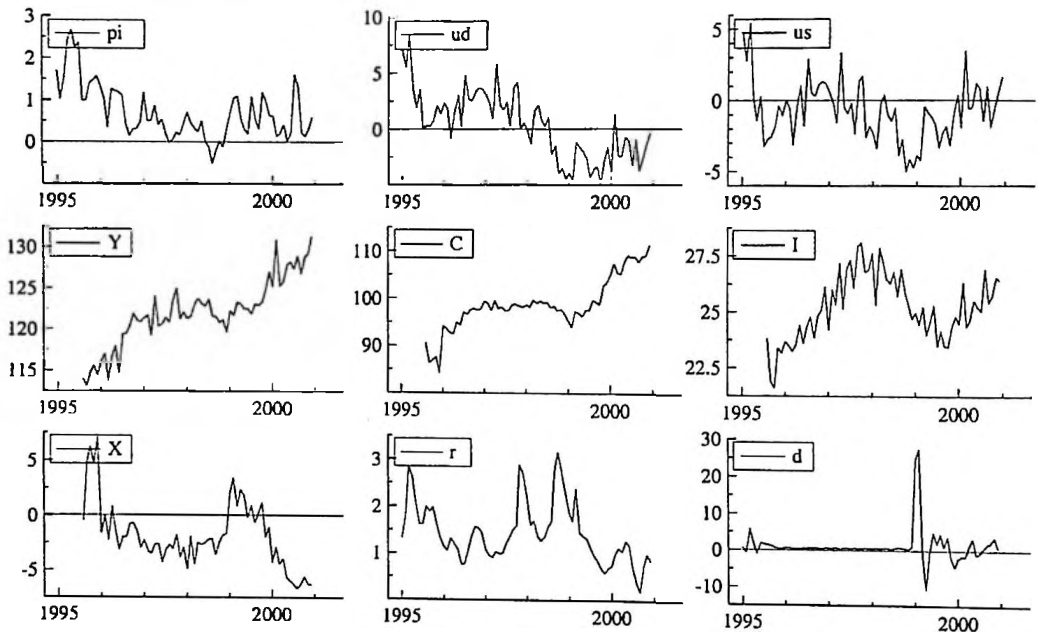
- Andrade, J. and Divino, J.A. (2000) Optimal rules of monetary policy for Brazil, manuscript.
- Ball, L. (1999) Efficient rules for monetary policy, *International Finance*, 2: 63-83.
- Bernanke, B.S., Laubach, T., Mishkin, F.S. and Posen, A.S. (1999) *Inflation Targeting: Lessons from the International Experience*, Princeton University Press.
- Blinder, A. (1998) *Central Banking in theory and practice*, Cambridge: MIT Press.
- Bogdanski, J., Tombini, A., Werlang, S. (2000) Implementing inflation targeting in Brazil, Working Paper Series 1, Central Bank of Brazil.
- Brainard, W. (1967) Uncertainty and the effectiveness of policy, *American Economic Review*, Vol.57, 411-425.
- Chadha, J.S. and Schellekens, P. (1999) Monetary Policy Loss Functions: Two Cheers for the Quadratic, DAE Working Papers, Department of Applied Economics, University of Cambridge.
- Chow, G. (1975) *Analysis and Control of Dynamic Economic Systems*, John Wiley and Sons, New York.
- Drew, A. and Hunt, B. (2000) Efficient simple policy rules and the implications of potential output uncertainty, *Journal of Economics and Business*, 52: 143-160.
- Estrella, A. and Mishkin, F. (1999) Rethinking the role of the NAIRU in monetary policy: Implications of model formulation and uncertainty, in *Monetary Policy Rules* (J.B. Taylor, Ed.). Chicago: Chicago University Press.
- Hall, S., Salmon, C., Yates, T. and Batini, N. (1999) Uncertainty and simple monetary policy rules: An illustration for the United Kingdom; Working Paper Series no. 56, Bank of England.
- Levin, A., Wieland, V. and Williams, J.C. (1999a) Robustness of simple monetary policy rules under model uncertainty, in *Monetary Policy Rules* (J.B. Taylor, Ed.). Chicago: Chicago University Press.
- Levin, A., Wieland, V. and Williams, J.C. (1999b) The performance of forecast-based monetary policy rules under model uncertainty, manuscript, Board of Governors of the Federal Reserve System.
- Martin, B. and Salmon, C. (1999) Should uncertain monetary policy-makers do less?, in *Monetary Policy Under Uncertainty* (B. Hunt and A. Orr, Ed.). Wellington, New Zealand: Reserve Bank of New Zealand.
- McCallum, B. (1999) Recent developments in the analysis of monetary policy rules, *Federal Reserve Bank of St. Louis Review*, 81(6).
- McCallum, B. (2000) The present and future of monetary policy rules, NBER Working Paper 7916.
- Orphanides, A. (1998) Monetary policy evaluation with noisy information, FEDS Working Paper 1998-50, Federal Reserve Board.
- Peersman, G. and Smets, F. (1999) The Taylor rule: A useful policy benchmark for the Euro Area?, *International Finance* 2, 85-116.

- Rudebusch, G. (2000) Is the Fed too timid? Monetary policy in an uncertain world, manuscript, Federal Reserve Bank of San Francisco, forthcoming in the *Review of Economics and Statistics*.
- Rudebusch, G. and Svensson, L. (1998) Policy rules for inflation targeting; Seminar Paper No. 637, Institute for International Economic Studies, Stockholm University.
- Sack, B. (1998) Does the Fed act gradually? A VAR analysis, FEDS Working Paper 1998-17, Federal Reserve Board.
- Sargent, T. (1999) Discussion of "Policy rules for open economies" by Laurence Ball, in *Monetary Policy Rules* (J.B. Taylor, Ed.). Chicago: Chicago University Press.
- Shuetrim, G. and Thompson, C. (1999) The implications of uncertainty for monetary policy, Reserve Bank of Australia, Research Discussion Paper 1999-10.
- Simon, H. (1956) Dynamic Programming under uncertainty with a quadratic criterion function, *Econometrica*, XXIV, 74-81.
- Smets, F. (1998) Output gap uncertainty: Does it matter for the Taylor rule? , in *Monetary Policy Under Uncertainty* (B. Hunt and A. Orr, Ed.). Wellington, New Zealand: Reserve Bank of New Zealand.
- Svensson, L. (1996) Inflation forecast targeting: Implementing and monitoring inflation targets; Working Paper Series no. 56, Bank of England.
- Svensson, L. (1997) Inflation targeting: some extensions; *Scandinavian Journal of Economics* 101, 337-361.
- Svensson, L. and Woodford, C. (2000) Indicator variables for optimal policy, NBER Working Paper 7953.
- Swanson, E. (2000) On signal extraction and non-certainty equivalence in optimal monetary policy rules, manuscript, Federal Reserve Board.
- Taylor, J.B. (1993) Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214.
- Tetlow, R.J. and von zur Muehlen, P. (1999) Can model structure uncertainty imply policy attenuation?, manuscript, Federal Reserve Board.
- Theil, H. (1957) A note on certainty-equivalence in dynamic planning, *Econometrica*, XXV, 346-349.
- Walsh, C.E. (1998) *Monetary Theory and Policy*, Cambridge: MIT Press.
- Woodford, M. (1999) Commentary: How should monetary policy be conducted in an era of price stability?, Federal Reserve Bank of Kansas City.

Appendix 1 : Data description and graphs

Variable	Name	Source/Definition
π	Inflation	Monthly percent change in the broad consumer price index (IPCA) from IBGE
R	Nominal interest rate	Monthly average SELIC overnight interest rate from Brazil's Central Bank
D	Nominal exchange rate depreciation	Percent change in the monthly average rate from Brazil's Central Bank
Y	GDP	Seasonally adjusted monthly chained index series at 1990 Reais; raw data are from IBGE
y^*	Potential GDP	Calculated from equations (6) or (7) in the text
U	Output gap	Calculated from equation (5) in the text
I	Investment	Seasonally adjusted monthly index series from IPEA at 1990 Reais
C	Consumption	Seasonally adjusted monthly index series at 1990 Reais calculated by the authors as follows: first, we calculated nominal monthly GDP based on IBGE's monthly chained index series and on FGV's general price index (IGP-DI), using a factor to make the series consistent with the annual National Accounts figures; second, we calculated nominal investment based on IPEA's monthly index series and on price indices for construction (INCC from IBGE) and for machinery and equipment (IPA-máquinas e equipamentos from FGV), also using a correcting factor to make the series consistent with the National Accounts; third, we calculated net exports of goods and non-factor services based on Balance of Payments and exchange rate data from the Central Bank, again making the series consistent with the National Accounts; fourth, we calculated nominal consumption as a residual from $C = Y - I - (X-M)$; finally, we calculated real consumption at 1990 Reais by using the IPCA as the consumption deflator and making the series consistent with the National Accounts.
X	Net exports	Net exports of goods and non-factor services, calculated as a residual from $x = y - c - i$

Graphs

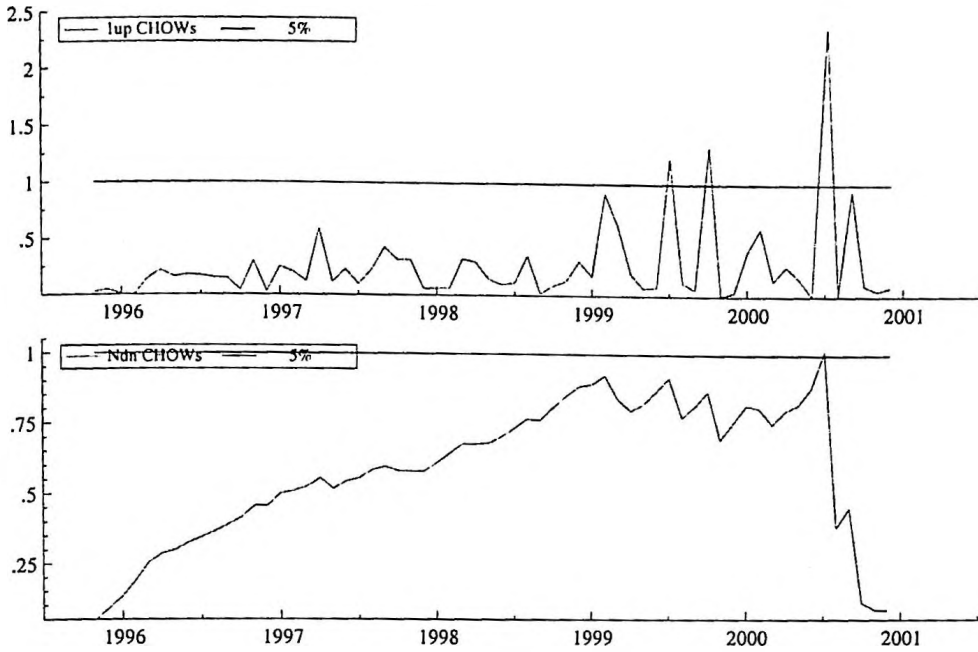


us: u with deterministic potential output

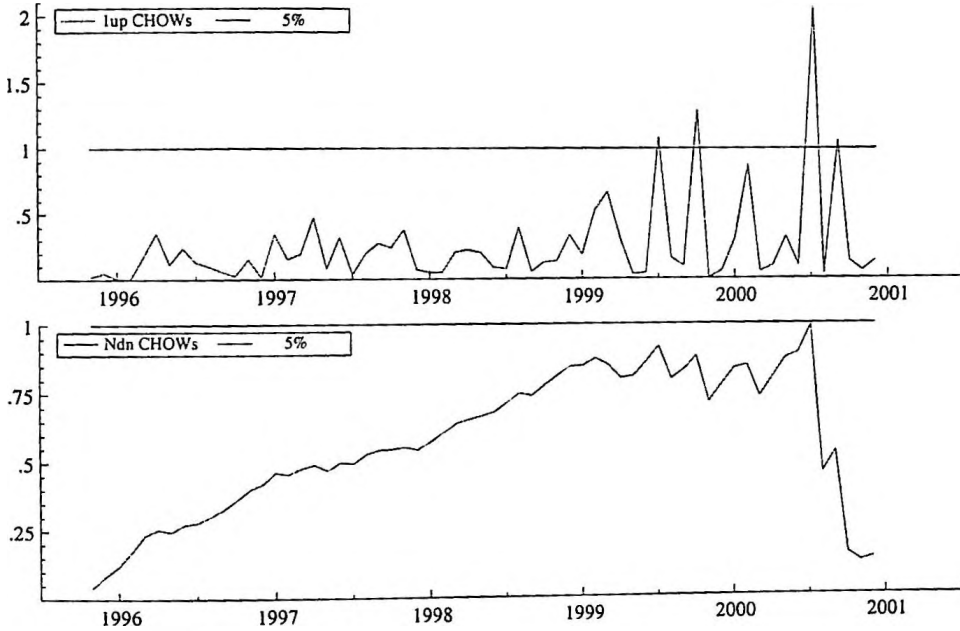
ud: u with stochastic potential output

Appendix 2 : Parameter constancy tests for models 1 to 3

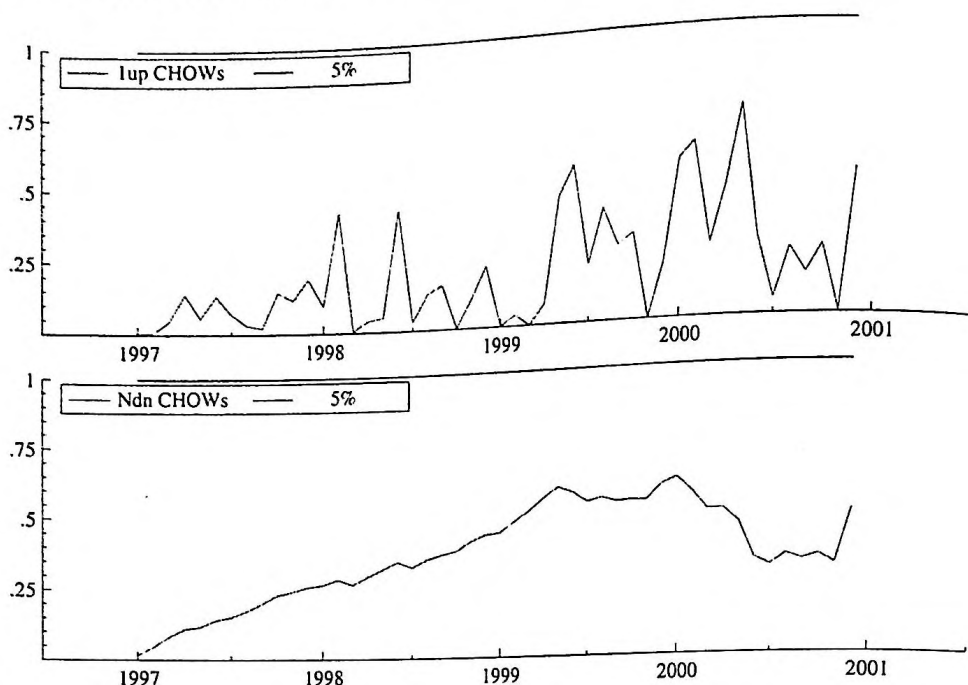
Model 1: 1-step ahead (1up) and break-point (Ndn) Chow tests



Model 2: 1-step ahead (1up) and break-point (Ndn) Chow tests



Model 3: 1-step ahead (1up) and break-point (Ndn) Chow tests



Appendix 3: Loss under each model's optimal reaction function

Table A.1 - Loss given E(Y) in models 1 to 3 under each model's optimal reaction function (RF)†

	Mean				Value under risk*			
	TR	RF M1	RF M2	RF M3	TR	RF M1	RF M2	RF M3
V(u)+V(π)								
M1	0.64	0.55	0.75	-	1.84	5.18	2.42	-
M2	0.97	2.46	0.93	-	2.50	2008	2.24	-
M3	1.36	7.61	2.51	1.07	3.64	121	7.38	5.49

† We also report results for the Taylor rule (TR) * Value for the 95th percentile of the loss distribution function

Table A.2 – Variance of E(Y) in Models 1 to 3 under each model's optimal reaction function (RF)†

		Mean				Value under risk*			
		TR	RF M1	RF M2	RF M3	TR	RF M1	RF M2	RF M3
Model 1	V(u)+V(π)	0.64	0.55	0.75	-	1.84	5.18	2.42	-
	V(u)	0.58	0.50	0.69	-	1.76	5.13	2.33	-
	V(π)	0.04	0.05	0.05	-	0.15	0.20	0.19	-
	V(r)	3.11	3.56	2.87	-	5.22	8.21	4.93	-
Model 2	V(u)+V(π)	0.97	2.46	0.93	-	2.50	2008.84	2.24	-
	V(u)	0.90	2.38	0.85	-	2.42	2008.08	2.15	-
	V(π)	0.06	0.09	0.07	-	0.18	3.69	0.20	-
	V(r)	3.61	5.19	3.05	-	5.33	1409.90	4.55	-
Model 3	V(u)+V(π)	1.365	7.614	2.506	1.074	3.638	121.97	7.385	5.489
	V(u)	1.323	7.525	2.441	1.029	3.562	121.86	7.365	5.442
	V(π)	0.031	0.083	0.048	0.032	0.134	0.390	0.193	0.156
	V(r)	3.039	5.483	2.922	3.404	5.154	49.312	4.780	6.796
	V(Δc)	0.155	0.533	0.134	0.190	0.570	18.895	0.410	1.351
	V(Δi)	0.253	0.553	0.243	0.313	0.649	13.667	0.575	1.079
	V(Δx)	0.127	0.145	0.130	0.127	0.434	0.593	0.440	0.480

† We also report results for the Taylor rule (TR) * Value for the 95th percentile of the loss distribution function

